International Journal of Advances in Applied Mathematics and Mechanics
Volume 1, Issue 2 : (2013) pp. 146-156
Available online at www.ijaamm.com
ISSN: 2347-2529

Unsteady free convection MHD flow between two heated vertical plates – one adiabatic

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Received 15 October 2013; Accepted (in revised version) 27 November 2013

ABSTRACT

The problem of unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates is formulated here keeping one plate adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by the Integral Transform Technique. The obtained results of velocity and temperature distributions are shown graphically and are discussed on the basis of it. The effects of Hartmann number, Prandtl number and the decay factor, and effects of adiabatic plate on the velocity and temperature fields are discussed.

Keywords: MHD flow, Unsteady Flow, Laplace Transform, Adiabatic Plate, Heat Transfer..

MSC 2010 codes: 76W05, 80A20.

1 Introduction

Sharma and Kumar (1998) investigated the unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous. Borkakati and Chakrabarty (2000) unsteady free convection MHD flow between two heated vertical plates. Ray *et al*. (2001) studied the problem of "on some unsteady MHD flows of a second order fluid over a plate". The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field is discussed by Sreekant *et al*. (2001), Gourla and Katoch (1991) studied an unsteady free convection MHD flow between two heated vertical plates. But, they did not discuss about the thermodynamic case on the boundary condition on which the plate is adiabatic. Here our aim is to analyze the unsteady free convection magnetohydrodynamic flow of an incompressible and electrically

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conducting fluid past between two heated vertical plates in presence of the transverse magnetic field where the temperature of one of the plates changes while the other plate is adiabatic.

2 The reduced differential transform method (RDTM)

Let us consider an unsteady free convection MHD flow of a viscous incompressible electrically conducting fluid between two heated vertical parallel plates. Let x-axis be taken along the vertically upward direction through the central line of the channel and the y-axis is perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field B_0 is applied in the plane of y-axis and perpendicular to the both x-axis and y-axis. u' is in the direction of velocity of fluid, along the x-axis and v' is the velocity along the y-axis. Consequently u' is a function of y' and t', but v' is independent of y'. The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible.

In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $t > 0$, the temperature of the plate at $y = h$ changes according to the temperature function:
$T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where T_w and T_0 are the temperature at the plates $y = h$ and at $y = -h$ respectively, and $n'(\geq 0)$ is a real number, denoting the decay factor.
Hence the flow field is seen to be governed by the following equations-

Equation of continuity: $$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

Equation of motion: $$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) - \frac{\sigma B_0^2}{\rho} u' \tag{2.2}$$

Equation of energy: $$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \qquad (2.2)$$

Where ρ = density of the fluid,
B_0 = uniform magnetic field applied transversely to the plate,
σ = electrical conductivity of the fluid,
υ = co-efficient of kinematics viscosity,
K = thermal conductivity of the fluid,
C_p = specific heat at constant pressure,
β = co-efficient of thermal expansion,
g = acceleration due to gravity,
T' = temperature of the fluid.
[-h, h] = space between the plates,
n' or n =decay factor,
T_0 = initial temperature of the plates and liquid,

T_w = wall temperature,
$\mu =$ dynamic viscosity of the fluid.

The initial and boundary conditions for the problem are:

$$t' = 0 : u' = 0, T' = T_0 \qquad \forall \;\; y' \in [-h, h]$$

$$t' > 0 : u' = 0, T' = T_0 + (T_w - T_0)\left(1 - e^{-n't'}\right) \quad \text{for} \;\; y' = +h$$

$$: u' = 0, \frac{\partial T'}{\partial y'} = 0 \qquad \text{for} \quad y' = -h \tag{2.4}$$

We now introduce the following non-dimensional quantities:

$$u = \frac{\upsilon u'}{\beta g h^2 (T_w - T_0)}, \quad y = \frac{y'}{h}, \quad T = \frac{T' - T_0}{T_w - T_{0,}},$$

$$t = \frac{\upsilon t'}{h^2}, \Pr = \frac{\mu C_p}{K}, \quad n = \frac{h^2 n'}{\upsilon}, \quad M = B_0 h \sqrt{\frac{\sigma}{\mu}} \tag{2.5}$$

Where Pr is the Prandtl number and M is the Hartmann number.

Using the quantities (2.5) in the equations (2.2) and (2.3), we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 u + T \tag{2.6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial y^2} \tag{2.7}$$

Under the above non-dimensional quantities, the corresponding boundary conditions reduces to-

$$t = 0: u = 0, T = 0, \qquad \forall \;\; y \in [-1, 1]$$

$$t > 0: u = 0, T = 1 - e^{-nt}, \qquad \text{for } y = +1,$$

$$: u = 0, \frac{\partial T}{\partial y} = 0, \qquad \text{for } y = -1. \tag{2.8}$$

3 Solution of the equations

Now, taking the Laplace Transform of equations (2.6) and (2.7), we obtain

$$\frac{d^2\overline{u}}{dy^2}-\left(M^2+s\right)\overline{u}=-\overline{T} \tag{3.1}$$

$$\frac{d^2\overline{T}}{dy^2}-\Pr s\overline{T}=0 \tag{3.2}$$

Similarly, using Laplace Transform on the boundary conditions (2.8), we get-

$$\overline{u}(\pm 1,s)=0,\quad \overline{T}(1,s)=\frac{n}{s(s+n)},\quad \frac{d\overline{T}(-1,s)}{dy}=0 \tag{3.3}$$

Since the equations (3.1) and (3.2) are 2nd order differential equations in $\overline{u}$ and $\overline{T}$, the solutions of

the equations by use condition (3.3) are obtain as -

$$\overline{u}=\frac{n\sinh\sqrt{M^2+s}(y-1)}{s(s+n)\left(M^2+s-s\Pr\right)\sinh 2\sqrt{M^2+s}.\cosh 2\sqrt{s\Pr}}$$

$$-\frac{n\sinh\sqrt{M^2+s}(y+1)}{s(s+n)\left(M^2+s-s\Pr\right)\sinh 2\sqrt{M^2+s}}+\frac{n\cosh\sqrt{s\Pr}(y+1)}{s(s+n)\left(M^2+s-s\Pr\right)\cosh 2\sqrt{s\Pr}} \tag{3.4}$$

$$\overline{T}=\frac{n}{s(s+n)}\frac{\cosh\sqrt{s\Pr}(y+1)}{\cosh 2\sqrt{s\Pr}} \tag{3.5}$$

Again, using the Inverse Laplace Transform on both sides of the eqns. (3.2) and (3.3), we obtain

$$u=\frac{1}{M^2}\left[1+\frac{\sinh My}{\sinh M}\right]+\frac{e^{-nt}}{n(1-\Pr)-M^2}\left[\frac{\sin\sqrt{n-M^2}(y-1)}{\sin 2\sqrt{n-M^2}\cos 2\sqrt{n\Pr}}+\frac{\sin\sqrt{n-M^2}(y+1)}{\sin 2\sqrt{n-M^2}}\right.$$

$$\left.+\frac{\cos\sqrt{n\Pr}(y+1)}{\cos 2\sqrt{n\Pr}}\right]+\frac{2\sin M\sqrt{\frac{\Pr}{1-\Pr}}(y+1)}{\sin 2M\sqrt{\frac{\Pr}{1-\Pr}}}e^{-\frac{M^2}{1-\Pr}t}$$

$$+\sum \frac{(-1)^k 8k\pi e^{-\left(M^2+\frac{k'2\pi^2}{4}\right)t}}{\left(4M^2+k^2\pi^2\right)^2\left(\frac{1}{n}-\frac{4}{4M^2+k^2\pi^2}\right)\left\{(1-\Pr)-\frac{4M^2}{4M^2+k^2\pi^2}\right\}}\left[\frac{\sin\frac{k\pi}{2}(y-1)}{\cos\sqrt{\frac{\Pr\left(4M^2+k^2\pi^2\right)}{2}}}\right.$$

$$\left.+\sin\frac{k\pi}{2}(y+1)\right]-\sum_{k=0}^{\infty}\frac{(-1)^k 64 e^{-\frac{(2k+1)^2\pi^2}{16\Pr}t}}{(2k+1)^3\pi^3\left\{\frac{1}{n\Pr}-\frac{16}{(2k+1)^2\pi^2}\right\}\left\{\frac{1-\Pr}{\Pr}-\frac{16M^2}{(2k+1)^2\pi^2}\right\}}$$

$$\times\left[\frac{\sin 2\sqrt{\frac{(2k+1)^2\pi^2-16\Pr M^2}{16\Pr}}(y-1)}{\sin 2\sqrt{\frac{(2k+1)^2\pi^2-16\Pr M^2}{16\Pr}}}-\cos\left\{\frac{(2k+1)\pi}{4}(y+1)\right\}\right] \qquad (3.6)$$

$$T=1-\frac{\cos\sqrt{n\Pr}(y+1)e^{-nt}}{\cos 2\sqrt{n\Pr}}+\sum_{k=0}^{\infty}\frac{(-1)^k 4\cos\frac{(2k+1)\pi(y+1)}{4}e^{-\frac{(2k+1)^2\pi^2}{16\Pr}t}}{(2k+1)\pi\left\{\frac{(2k+1)^2\pi^2}{16n\Pr}-1\right\}} \qquad (3.7)$$

4 Results and discussions

Figure - 1 has been obtained by plotting the temperature distribution T against y at various values of t, considering n = 1, Pr = .025 and M=.25. The graph shows that as t increases the temperature profiles of T also increases. The values of T towards the temperature function plate are much higher than that of adiabatic plate. It is seen that there is no influence of magnetic field on the temperature field. So, there is the effect of temperature on temperature distribution profile only.

Figure-2 shows the temperature profiles for different values of Pr and at fixed values of t (=.1), M (=.25), and n (=1). It is seen that the temperature distribution is less effected due to the fluid having Prandtl number 0.025 whereas the temperature distribution is too much affected due to the fluid whose Prandtl number is .5. So, we can say that the temperature depends on the nature of the fluid.

Figure-3 has been drawn to show the effect of the Decay Factor n on Temperature Profile in the presence of Pr =.025, M = .25, t = .5. It is seen that as n increases the values of T also increases. This increase is uniform. However, more higher values of T at and near the temperature dependent plate.

In figure-4, the velocity profile has been obtained for different values of t, namely; .1, .5, 1. And for n = 1, Pr = .025, M = .25. This shows that for t = .1, there is the rapid increase of values of u uniformly from the adiabatic plate to the temperature function plate. As these values of t increases, the fluctuation of the velocity curves also increases. But the effects between the adiabatic plate and the temperature dependent plate are much higher. Also all three curves intersect at the x-axis at the point (0, 3).

Figure-5, has been obtained by plotting the velocity distribution in the interval [1, -1] for the different values of Magnetic number M (= 0.5, 0.25, 0.025) and for fixed values of t (= .1), n (= 1) and Pr (=.025). It is seen that there is the effect of Magnetic Hartmann number on fluid velocity field.

Figure-6 shows the fluid velocity field u against y for various values of n (=1, 2, 3) and for fixed values of Pr (=.025), M (=.25) and t (=.5). It is seen that as n increases the fluid velocity profiles gets on fluctuating in an increasing manner.

Figure-7 has been found by plotting the values of u in the interval [1, -1] for the different values of Prandtl number, namely; .025, .5, .75. It is observed that for Pr = .025, the values of u are uniformly increasingly. However, for Pr = .75, these values are non – uniform. On the other hand, in the case of values of u great difference can be seen due to temperature differences among the adiabatic plate and non –adiabatic plate.

The velocity profiles have been plotted for different values of Prandtle number Pr and Hartmenn number M and for fixed values of n =1, t = 1 in the figure 8. It is seen that for Pr = .025 and M = .2 the fluid velocity has greater values than for Pr = .5 and M = .6. So, we can say that the less viscous fluids have greater fluid velocity than the high viscous fluid. Also as the values of the Magnetic Hartmann number increases the values of fluid velocity decreases. On the other hand for Pr = .25 and M = .4, we unable to see above cited character. This is perhaps due to the effect of adiabatic and non – adiabatic plate.

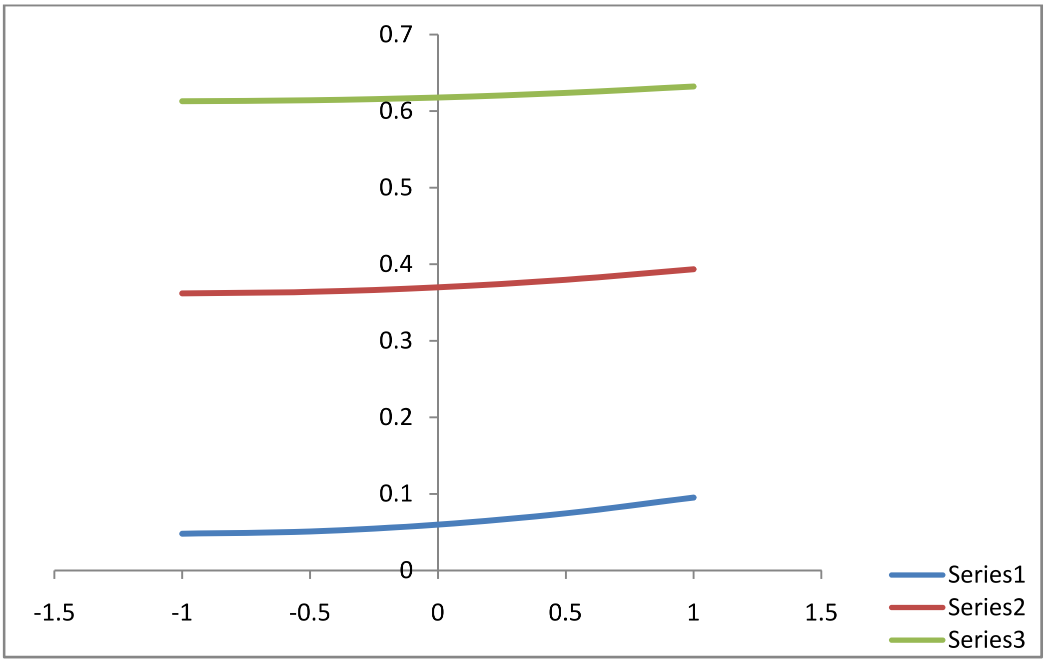

Figure 1: T versus y: for n=1, Pr=.025, M=.25 and SI1 for t=.1, SI2 for t=.5, SI3 for t=1

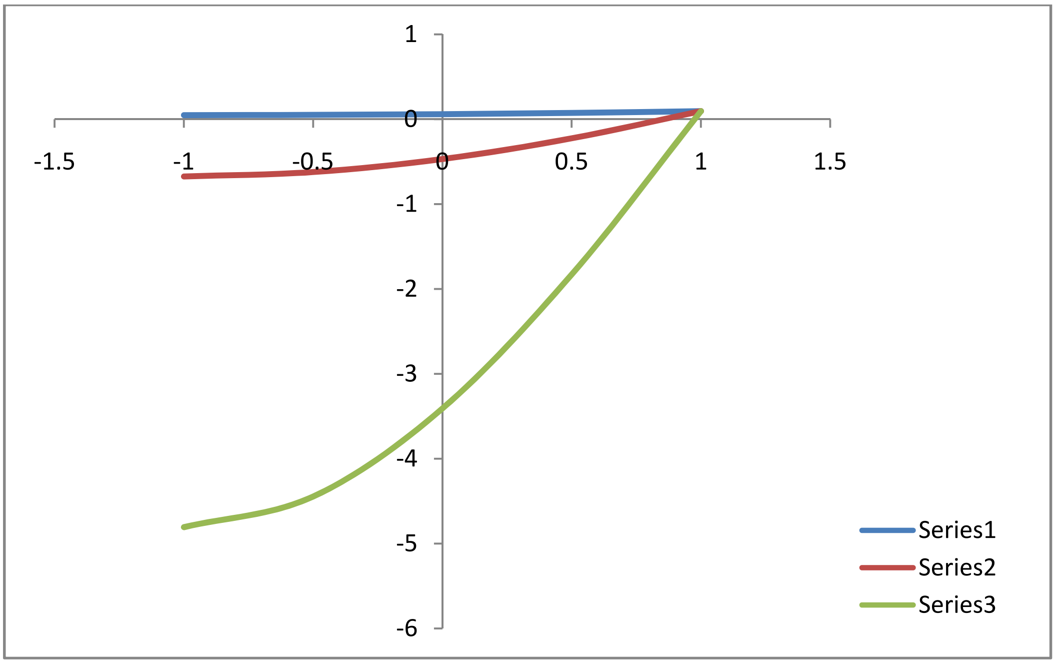

Figure 2: T versus y: for t=.1, M=.25, n=1 and for SI1 for Pr=.025, SI2 for Pr=.25, SI3 for Pr=.5

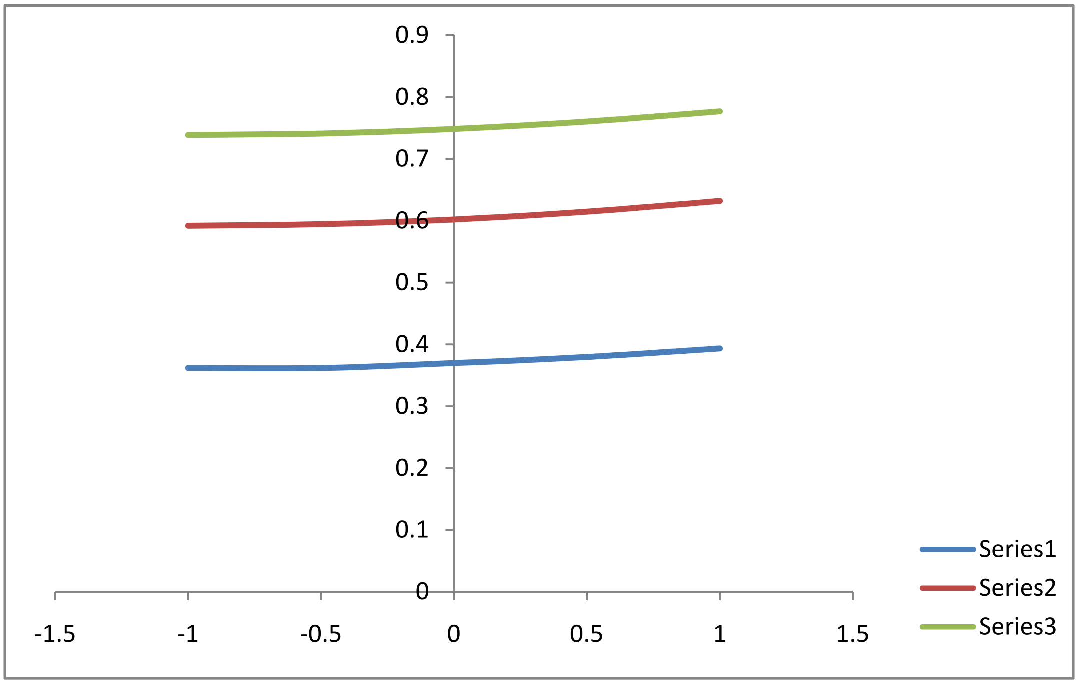

Figure 3: T versus y for M=.25, Pr=.025, t=.5; SI1 for n=1, SI2 for n=2, SI3 for n=3

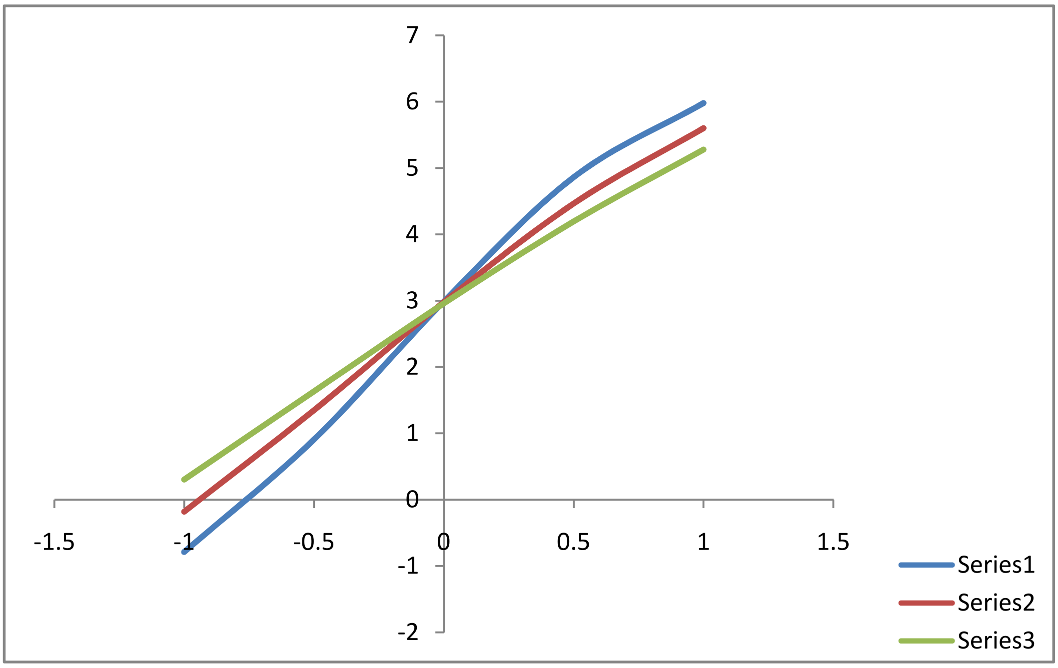

Figure 4: U versus y: n=1, Pr=.025, M=.25 and SI1 for t=.1, SI2 for t=.5, SI3 for t=1

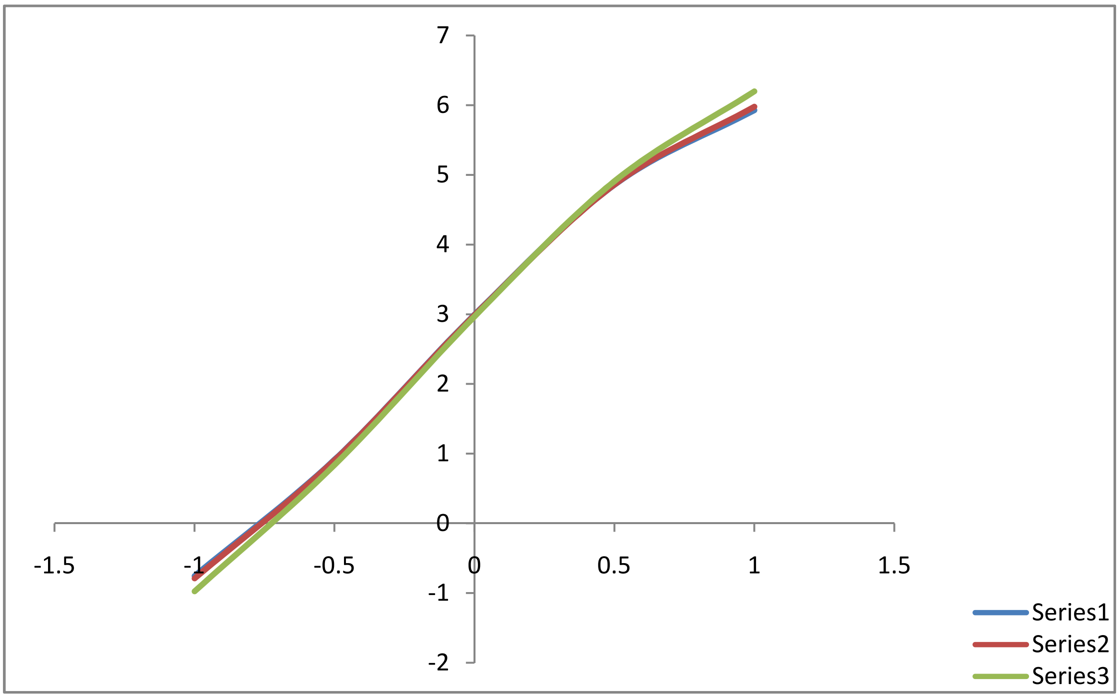

Figure 5: U versus y: for t=.1, n=1, Pr=.025, and SI1 for M=.025, SI2 for M=.25, SI3 for M=.5

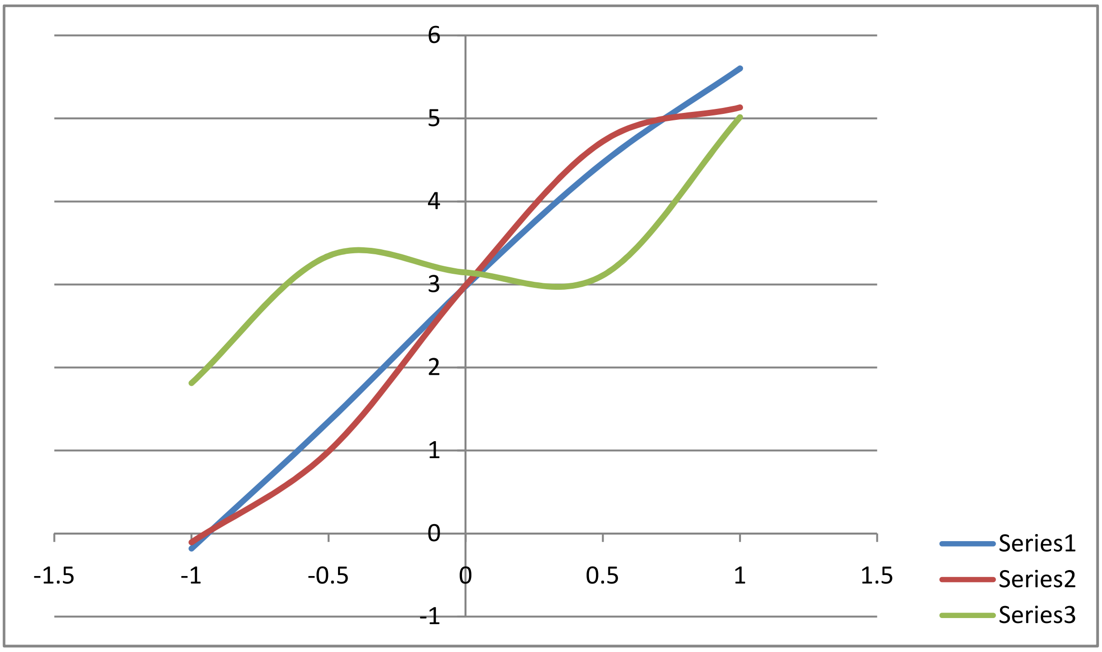

Figure 6: U versus y for M=.25, Pr=.025, t=.5; SI1 for n=1, SI2 for n=2, SI3 for n=3

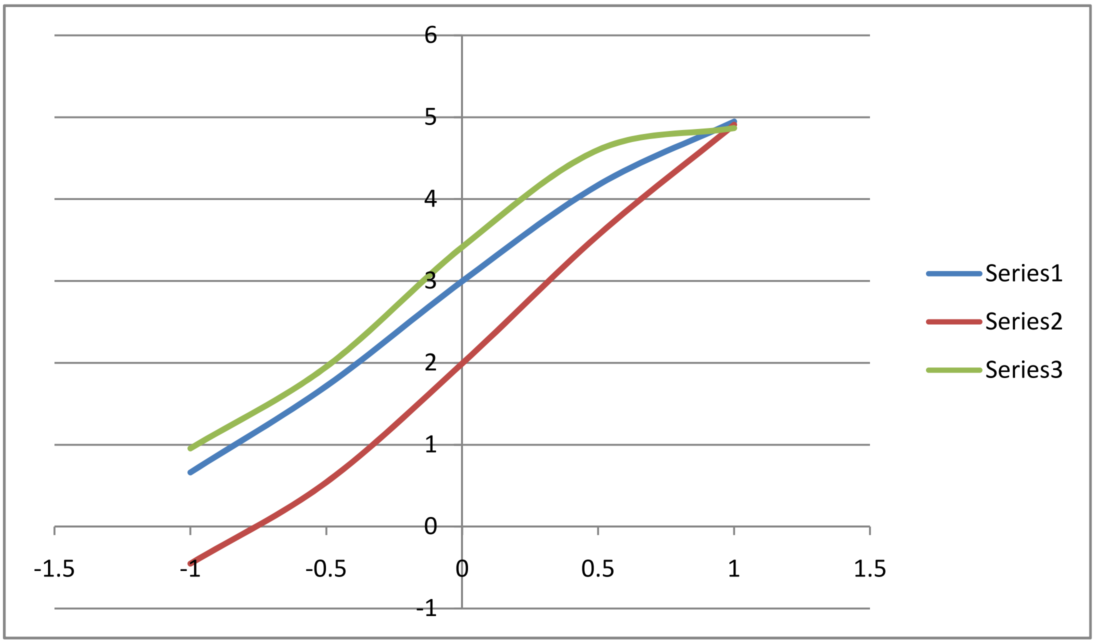

Figure 7: U versus y: n=2, t=1, M=.25; SI1 for Pr=.025, SI2 for Pr=.5, SI3 for Pr=.75

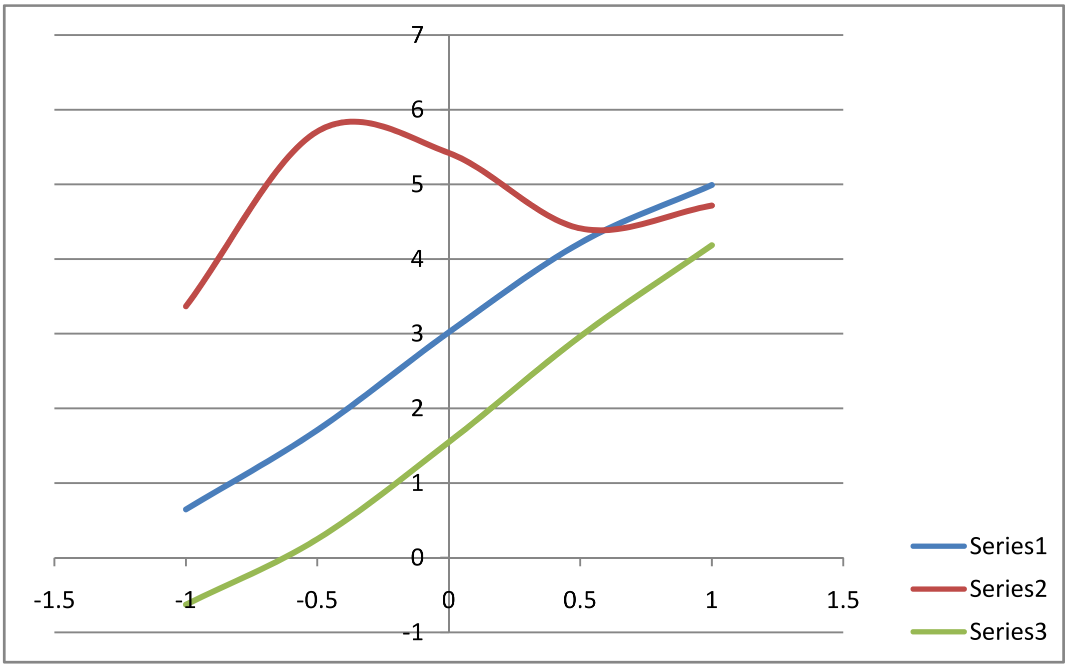

Figure 8: U versus y: n=1, t=1; SI 1forM= .2, Pr=.025, SI2 forM=.4, Pr=.25, SI3 for M=.6, Pr=.5

5 Concluding remarks

As we have studied the paper of Gourla and Katoch (1991) taking one of the two plates as the adiabatic, we able to see clear difference between the graphs obtained by them and us. In case of temperature distribution curves, obtained by them, the values of temperature at and near the walls are higher than at the middle of the channel. But, in our problem, the values of temperature at and near the temperature dependent plate are higher than the adiabatic plate and it is increasing uniformly. Different nature we have obtained for different fluids. Again, if we compare the two velocity profiles obtained by them and us, then also clear difference is visible. In case of them, the values of the velocity are the highest at the middle than that at and near the walls. However, in our case all the obtained values are higher at and near the temperature dependent plate than at and near the adiabatic plate. We also observed that the velocity profiles depend on the Prandl number and Hartmann number.

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