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A new approach for local similarity solutions of an unsteady hydromagnetic free convective heat transfer flow along a permeable flat surface

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ABSTRACT

In this study a new class of similarity transformations is used to obtain local similarity solutions for an unsteady hydromagnetic free convective heat transfer flow along a permeable vertical plate. The resulting local similarity equations are solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta integration scheme. Steady solutions are compared with previously published results which show excellent agreement. The results show that this new similarity approach has the advantage that one can separately obtain the steady and unsteady solutions. The results also show that unsteadiness is found to reduce the hydrodynamic as well as thermal boundary layer growths.

Keywords: Unsteady flow, Similarity solution, Convection, Suction, hydromagnetic flow.

MSC 2010 codes: 76M55, 65L06.

1 Introduction

The phenomenon of free convection has many important technological applications such as in cooling of a nuclear reactor and providing heat sinks turbine blades, etc. Soundalgekar (1977) studied the free convection flow past an impulsively started infinite vertical impermeable wall, when it is cooled or heated by the free convection currents. Because of the importance of suction for the boundary layer control in the field of aerodynamics and space science, Kafousias et al. (1979) studied the effects of free convection currents on the flow field of an incompressible viscous fluid past an impulsively started infinite vertical porous limiting surface when the fluid is subjected to suction with uniformly velocity. Raptis et al. (1981) studied the free convection effects on the flow past an accelerated vertical porous plate with time-dependent suction-injection and heat flux. Free convection flow and heat transfer through a porous vertical flat plate immersed in a porous medium with variable suction has been investigated by Sattar et al. (2000). In recent years the subject of Magnetohydrodynamics (MHD in short) has attracted many authors in view not only of its own interest but also of the applications to geophysics and engineering. When the fluid is a

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conductor of electricity the free convection-heat transfer flow can be influenced by an imposed magnetic field. MHD phenomenon results from the natural effect of a magnetic field and a conducting fluid flowing across it. Thus, an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid's motion. Examination of flow models will reveal the influence of magnetic fields on the velocity profile, temperature profile and the local heat transfer rate. In light of these various applications, hydromagnetic free convection flow in the Stokes problem for a porous vertical limiting surface with constant suction has been analyzed by Nanousis et al. (1980). Singh (1982) studied MHD free convection flow in the Stokes problem for a porous vertical plate. Sattar (1992) obtained a similarity solution of an unsteady one-dimensional hydromagnetic heat transfer flow past a suddenly accelerated porous plate with variable suction or injection. In this study, a time dependent similarity parameter was introduced on the basis of the proposal put forward by Hasimoto (1956). Introducing this similarity parameter, the governing boundary layer equations were reduced to non-linear ordinary differential equations, which are similar in time. Later many papers have been published in this line, some of which that are worth mentioning are due to Sattar et al. (2000), Sattar and Hossain (1992), Sattar (1994), Alam et al. (2005) and Rahman and Sattar (2007). Following Sattar and his co-workers, very recently the same idea of a time dependent similarity parameter has been taken into account by Chamkha et al. (2011).

Compared to the above steady two-dimensional and unsteady one-dimensional problems, similarity solutions of unsteady two-dimensional problems are rare where as non-similar solutions of 2D boundary layer problems are readily available in literature. However, Wang (1990) introduced a similarity transformation to solve the hydrodynamic unsteady two-dimensional boundary layer flows past a thin liquid film on a stretching sheet. This similarity transformation reduces the unsteady Navier-Stokes equations to a non-linear ordinary differential equations governed by a nondimensional unsteady parameter. He obtained the asymptotic as well as numerical solutions to the transformed similarity equation for several values of the unsteadiness parameter. Following the works of Wang (1990), the momentum and heat transfer in a liquid film on an unsteady stretching surface was studied by Andersson et al. (2000). Unlike the works of Wang (1990) and Andersson et al. (2000), the aim of the present work is to extend the idea of a time-dependent similarity parameter in order to obtain a local similarity solution of the unsteady two-dimensional hydromagnetic boundary layer equations along a permeable vertical flat plate which may allow someone to obtain both steady and unsteady solutions separately.

2 Governing equations of the flow and similarity analysis

We consider an unsteady two-dimensional MHD free convective laminar boundary layer flow of a viscous incompressible fluid along a vertical porous moving plate. A uniform magnetic field B_0 is considered to acting perpendicular to the plate. The x-axis is considered along the plate in the upward direction and y-axis is normal to it. Fluid suction/injection is imposed at the plate surface. The flow configuration and coordinate system are shown in figure 1.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u,v are the velocity components along x,y co-ordinates respectively, υ is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, ρ is the density of the fluid, σ is the magnetic permeability, B_0 is the uniform magnetic field acting normal to the plate , T is the temperature of the fluid within the boundary layer, T_∞ is the free stream temperature, c_p is the specific heat of the fluid at constant pressure and k is the thermal conductivity.

The corresponding boundary conditions to the above model are as follows:

$$\left.\begin{aligned} &u=U(x,t)=\frac{\upsilon x}{\delta^2}, v=\pm v_w(x,t), T=T_w(x,t)=T_\infty+(T_0-T_\infty)\frac{\upsilon x}{U_0\delta^2} \quad \text{at} \quad y=0 \\ &u=0, \quad T=T_\infty \quad \text{as} \quad y\to\infty \end{aligned}\right\} \tag{4}$$

where U_0 is the mean fluid velocity, T_0 is the mean fluid temperature and δ is a time dependent similarity parameter which is taken to be (see Sattar and Hossain 1992; Sattar 1994) as

$$\delta=\delta(t) \tag{5}$$

In order to obtain similarity solution of the problem considered, we now introduce the following transformations which are new in the literature

$$\left.\begin{aligned} &\eta=\frac{y}{\delta}, \psi=\frac{\upsilon x}{\delta}f(\eta), \\ &T=T_\infty+(T_0-T_\infty)\frac{\upsilon x}{U_0\delta^2}\theta(\eta). \end{aligned}\right\} \tag{6}$$

Now introducing (5)-(6) in equations (1)-(4) we obtain the following non-linear ordinary differential equations

$$f'''+\frac{\delta}{\upsilon}\frac{d\delta}{dt}(2f'+\eta f'')-(f')^2+ff''+Gr\theta-Mf'=0, \tag{7}$$

$$\theta''+\frac{\delta}{\upsilon}\frac{d\delta}{dt}\Pr(2\theta+\eta\theta')+\Pr(f\theta'-f'\theta)=0, \tag{8}$$

with transformed boundary conditions:

$$\left.\begin{aligned} f = f_w,\ f' = 1, \theta = 1 \quad \text{at} \quad \eta = 0, \\ f' = 0, \quad \theta = 0 \quad \text{as} \quad \eta \to \infty, \end{aligned}\right\} \tag{9}$$

where $f_w = -\frac{\delta}{\upsilon} v_w(x,t)$ is the dimensionless suction parameter, $M = \frac{\sigma B_0^2 \delta^2}{\rho\, \upsilon}$ is the magnetic field parameter, $Gr = \frac{g\beta(T_0 - T_\infty)\delta^2}{U_0 \upsilon}$ is the local Grashof number and $\Pr = \frac{\upsilon \rho c_P}{k}$ is the Prandtl number.

Now in order to make the equations (7)-(8) locally similar, let

$$\frac{\delta}{\upsilon}\frac{d\delta}{dt} = K \tag{10}$$

where K is taken to be a constant and thus can be treated as a dimensionless measure of the unsteadiness.

Hence the equations (7)-(8) becomes

$$f''' + K\left(2 f' + \eta f''\right) - (f')^2 + ff'' + Gr\theta - Mf' = 0, \tag{11}$$

$$\theta'' + K \Pr\left(2\theta + \eta\theta'\right) + \Pr(f\theta' - f'\theta) = 0, \tag{12}$$

Now integrating (10), we obtain

$$\delta = \sqrt{2K\upsilon t} \tag{13}$$

Choosing $K = 2$ in (13) , we get

$$\delta = 2\sqrt{\upsilon t} \tag{14}$$

The length scale $2\sqrt{\upsilon t}$ for the ordinate similar to one seen in (14) was initially used by Stokes' (1856) for an unsteady parallel flow but $\delta(t)$ form of the length was initially developed by Sattar and Hossain (1992) in case of a solution of an unsteady one-dimensional boundary layer problem. The characteristics length scale $\delta(t)$ defined particularly in (14) physically related to the boundary layer thickness which can be viewed in Schlichting (1968).

3 Local skin-friction coefficient and Nusselt number

The parameters of engineering interest for the present problem are the local skin-friction coefficient and the local Nusselt number which are given below:
Now the equation for wall shear stress is

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu \upsilon x}{\delta^3} f''(0) \tag{15}$$

Therefore the local skin-friction coefficient is obtained as

$$XCf = \frac{\tau_w}{\rho U^2(x,t)} = f''(0) \text{ whrre } X = \frac{x}{\delta} \tag{16}$$

Again the wall heat flux is

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k(T_0 - T_\infty)\frac{\upsilon x}{U_0 \delta^3}\theta'(0) \tag{17}$$

Therefore the local Nusselt number is

$$Nu = \frac{\delta q_w}{(T_0 - T_\infty)k} = -\frac{1}{\text{Re}}\theta'(0) \tag{18}$$

where $\text{Re} = \dfrac{U_0 \delta^2}{\upsilon x}$ is the local Reynolds number.

4 Numerical solutions

The locally similar and non-linear ordinary differential equations (11)-(12) with boundary conditions (9) have been solved numerically by using sixth order Runge-Kutta method along with Nachtsheim-Swigert (1965) shooting iteration technique [for detailed discussion of this method see also Alam et al. (2006) with *K*, f_w , *Gr*, *Pr* and *M* as prescribed parameter. The computations were done by a program, which uses a symbolic and computational computer language FORTRAN LAHEY. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6}. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ was determined when the value of the unknown boundary conditions at $\eta = 0$ does not change in the successful loop with an error less than 10^{-6}. To assess the accuracy of the present numerical method, we have compared our local skin-friction coefficients with Andersson et al. (1992) in Table-1 and we see that excellent agreement between the results exists.

5 Results and discussion

Numerical calculations have been carried out for different values of *M*, f_w, *Gr* and *Pr* in figures 2-5 and Tables 2-4 for both steady and unsteady cases. Figures 2(a)-(b) represent typical profiles for the velocity and temperature for various values of the magnetic field parameter *M*, respectively for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. The presence of a magnetic field normal to the flow in an electrically conducting fluid introduces a Lorentz force, which acts against the flow. This resistive force tends to slow down the flow and hence the fluid velocity decreases with the increase of the magnetic field parameter as observed in figure 2(a). It is also noticeable that for a fixed value of the magnetic field parameter, the velocity corresponding to the case of steady $(K = 0)$ is lower compared to the case of unsteady $(K = 1.5)$. That is, the unsteadiness parameter accelerates the fluid motion. From figure 2(b) we see that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases.

Figure 3(a) shows the velocity profiles for different values of suction (or injection) parameter f_w for a cooling plate for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. It can be seen that for cooling of the plate the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. For a fixed suction velocity f_w, velocity is found to increase and reaches a maximum value in a region close to the plate, then gradually decreases to zero. On the other hand the fluid velocity increases for injection case. Figure 3(b) indicates the temperature profiles showing the effect of suction (or injection) parameter f_w for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. It can be seen that temperature decreases with the increase of suction while it increases with the increase of injection. Decelerated fluid particles close to the heated wall absorb more heat from the plate and as a consequence the temperature of the fluid within the boundary layer increases. But when these decelerated fluid particles are sucked through the porous plate there is a decrease to the temperature profile. Thus suction (or injection) can be used for controlling the temperature function, which is required in many engineering applications like nuclear reactors, generators etc.

The influence of local Grashof number Gr on the dimensionless velocity and temperature are displayed in figures 4(a)-(b) respectively for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. Figure 4(a) shows the free convection as well as the forced convection effects on the velocity profiles. From this figure we see that velocity increases as Gr increases for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases. This figure also shows that for large $Gr \geq 1$ velocity profiles overshoot near the surface of the plate. From figure 4(b) we observe that the thermal boundary layer thickness decrease with an increasing value of local Grashof number for both steady $(K = 0)$ and unsteady ($(K = 1.5)$ cases.

The effect of Prandtl number (Pr) on the dimensionless velocity and temperature profiles are shown in figures 5(a)-(b) respectively for both steady $(K = 0)$ and unsteady $(K = 1.5)$ cases.

Figure 5(a) shows that for small Prandtl number (Pr) values, the velocity overshoots the free stream velocity and thus there is a larger growth of the boundary layer. But for larger values of Pr, the velocity is found to decrease monotonically and hence there appears a thin boundary layer indicating the decrease of the free convection. From figure 5(b) we also see that the effects of Prandtl number on the thermal boundary layer are similar to those of velocity boundary layer.

From all the above figures a common phenomenon is observed that unsteadiness reduces both the hydrodynamic and thermal boundary layers growths.

Finally, the effects of the above-mentioned parameters on the local skin-friction coefficients and rate of heat transfer are shown in tables II- IV. These effects are in agreement with those seen from the velocity and temperature profiles. The conclusions and discussions regarding the behavior of the parameters on skin-friction and rate of heat transfer coefficients are self evident from Tables 2-4 and hence any further discussions about them seem to be redundant.

6 Concluding remarks

In this paper we have presented a technique which is new in the literature to obtain local similarity solutions for the unsteady hydromagnetic free convective boundary layer equations of a viscous incompressible fluid along a vertical porous flat plate. With the help of this new similarity transformation, the governing boundary-layer equations are reduced to ordinary differential equations, which are then solved numerically by applying Nachtsheim-Swigert shooting iteration method. From this study the major findings are listed below:

1. A new similarity transformation has been introduced to reduce the unsteady hydromagnetic free convective boundary-layer equations into the ordinary differential equations.
2. This new similarity approach has the advantage that one can separately obtain the steady and unsteady solutions.
3. Unsteadiness is found to reduce the hydrodynamic as well as thermal boundary layer growths.
4. Local skin-friction coefficient increases whereas the local Nusselt number decreases as the unsteadiness parameter increases.

7 Figures

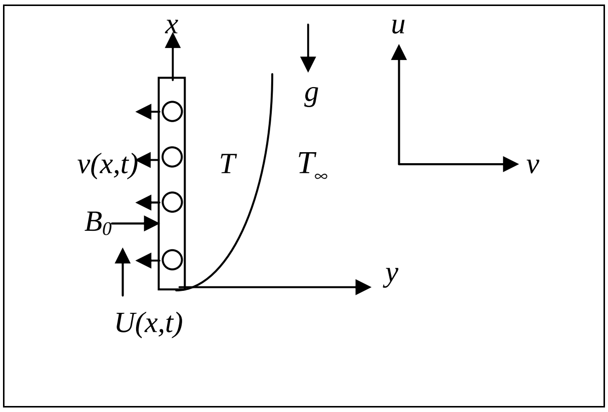


Figure 1: Flow configurations and coordinate system.

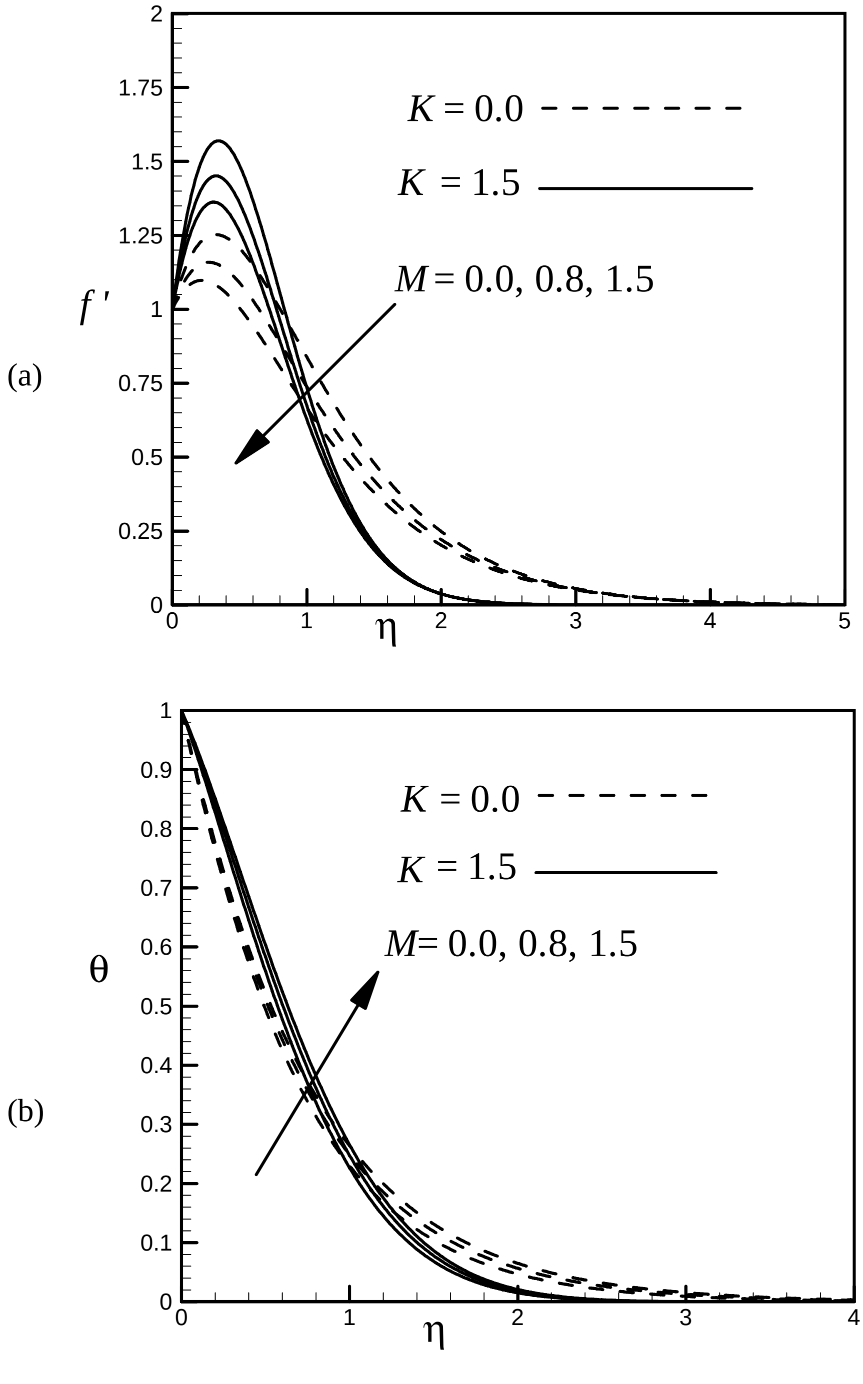
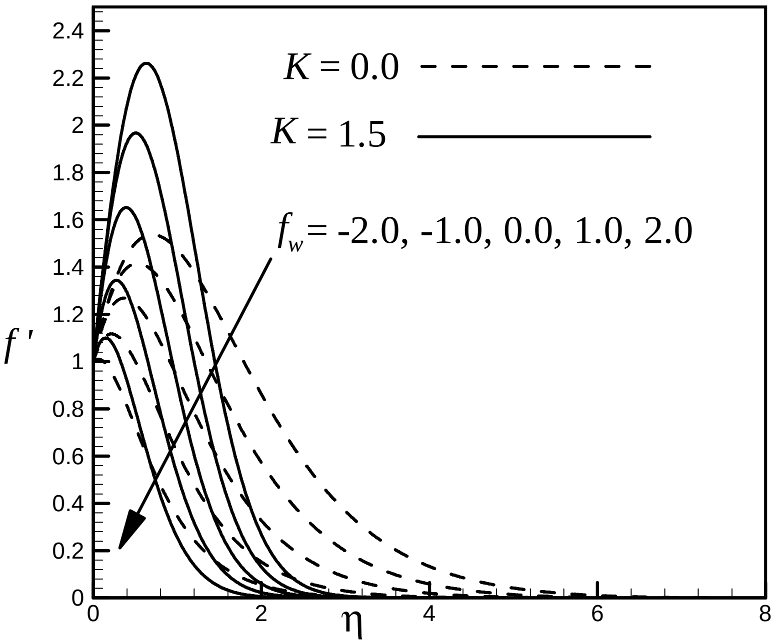


Figure 2: Dimensionless (a) velocity and (b) temperature profiles for different values of *M* and for Pr = 0.70, f_w = 0.50 and *Gr* = 8.0.

(a)



(b)

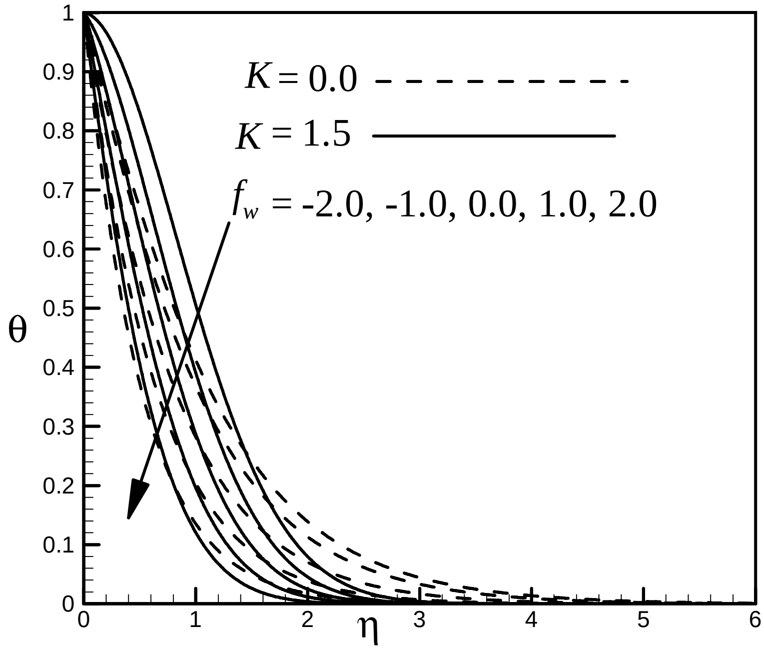
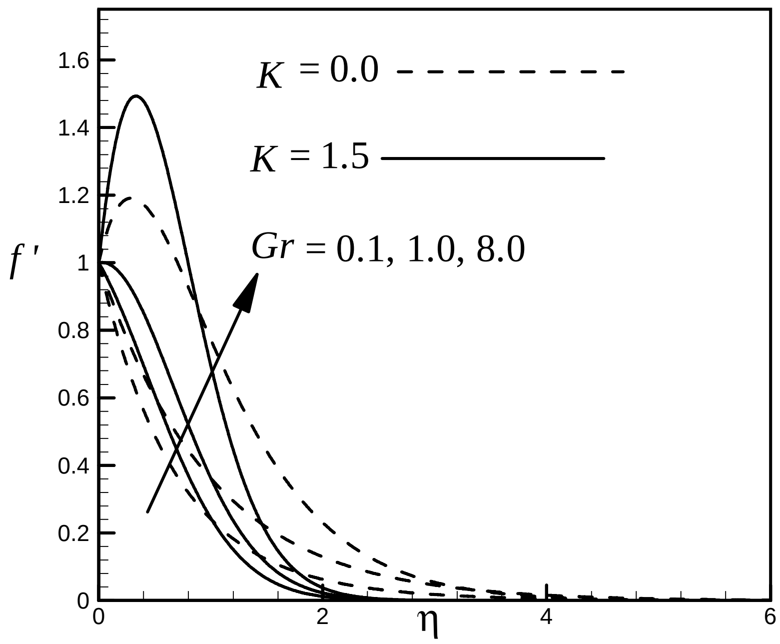


Figure 3: Dimensionless (a) velocity and (b) temperature profiles for different values of f_w and for Pr = 0.70, M = 0.50 and Gr = 8.0.

(a)



(b)

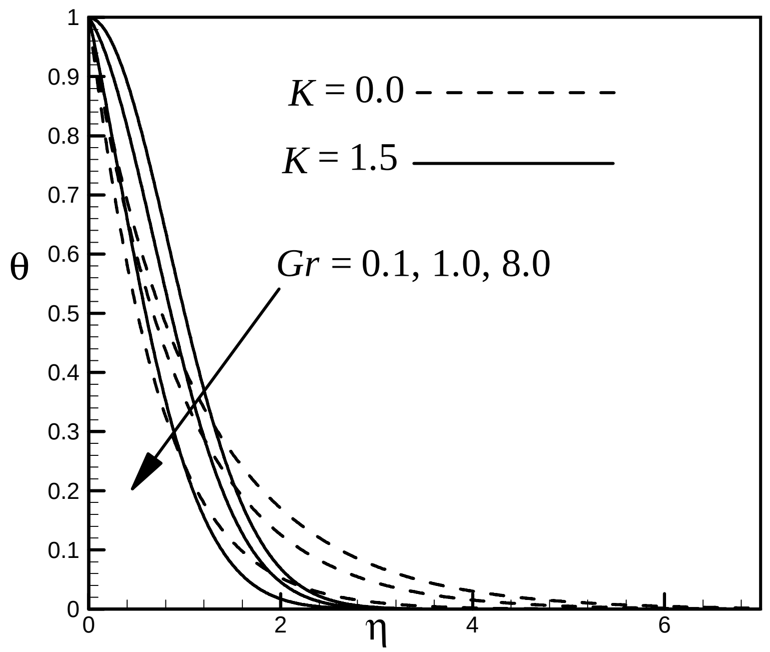


Figure 4: Dimensionless (a) velocity and (b) temperature profiles for different values of *Gr* and for Pr = 0.70, f_w = 0.50 and *M* = 0.50.

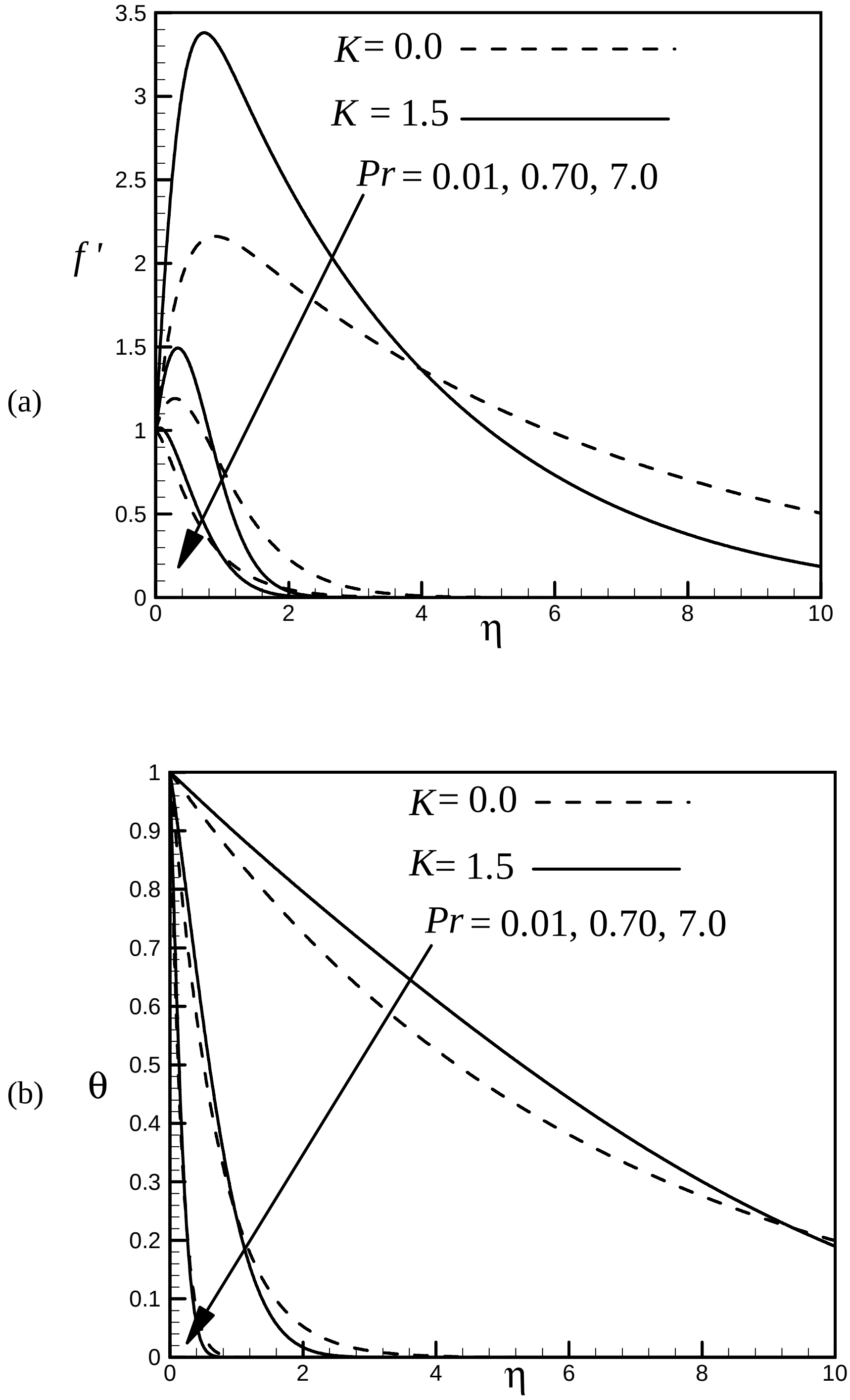


Figure 5: Dimensionless (a) velocity and (b) temperature profiles for different values of Pr and for M = 0.50, f_w = 0.50 and Gr = 8.0.

8 Tables

Table 1: Comparison of $-f''(0)$ with Andersson et al. (1992) for their Newtonian fluid case and for our $\Pr = 0.70$ and $f_w = Gr = K = 0$ case with different M.

M	Andersson et al. (1992)	Present study
0.0	1.000	1.0000092
0.5	1.225	1.2247449
1.0	1.414	1.4142136
1.5	1.581	1.5811388
2.0	1.732	1.7320508

Table 2: Effects of K and Gr on local Skin-friction coefficient and Nusselt number for $f_w = 0.50$, $M = 0.50$ and $\Pr = 0.70$.

1. K	Gr = 0.1		Gr = 1.0		Gr = 8.0	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.0	-1.4449964	0.9499131	-1.0096937	1.0330963	1.5057596	1.2775346
0.2	-1.3320460	0.8257066	-0.8653757	0.9314182	1.7299524	1.2087616
0.5	-1.1605048	0.6409734	-0.6482849	0.7756033	2.0661789	1.1036753
0.8	-0.9870383	0.4583052	-0.4330342	0.6182021	2.4070362	0.9961795
1.0	-0.8697621	0.3294084	-0.2876054	0.5101530	2.6330982	0.9234292
1.5	-0.5746165	0.0120768	0.0670859	0.2443071	3.1921880	0.7386597
2.0	-0.2774149	-0.2970026	0.4298961	-0.0357284	3.7519991	0.5488755

Table 3: Effects of K and M on local Skin-friction coefficient and Nusselt number for $f_w = 0.50$, $Gr = 8.0$ and $\Pr = 0.70$.

2. K	M = 0.0		M = 1.0		M = 2.0	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.0	1.7993595	1.3057303	1.2379360	1.2514699	0.7644272	1.2048581
0.2	2.0345917	1.2394426	1.4516129	1.1801367	0.9592501	1.1283679
0.5	2.3860273	1.1378728	1.7731793	1.0714352	1.2547678	1.0121884
0.8	2.7410220	1.0339002	2.1003233	0.9603126	1.5561035	0.8937310
1.0	2.9642735	0.9638654	2.3178156	0.8852582	1.7573198	0.8140027
1.5	3.5555504	0.7839054	2.8567288	0.6951084	2.2587821	0.6126888
2.0	4.1339544	0.5993202	3.3982315	0.5000207	2.7645001	0.4069908

Table 4: Effects of *K* and *Pr* on local Skin-friction coefficient and Nusselt number for $f_w = 0.50$, $M = 0.50$ and $Gr = 8.0$.

3. *K*	*Pr* = 0.01		*Pr* = 1.0		*Pr* = 7.0	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.0	3.6912027	0.1564937	1.2020774	1.5552405	-0.4068420	5.5262301
0.2	4.1429415	0.1504835	1.4013149	1.4715858	-0.2854902	5.3268542
0.5	4.8853234	0.1408646	1.7041326	1.3440977	-0.1103762	5.0306238
0.8	5.7018593	0.1305071	2.0004225	1.2150295	0.0817096	4.7265887
1.0	6.2844937	0.1231940	2.2015131	1.1275064	0.2098460	4.5228144
1.5	7.8585849	0.1035900	2.6974463	0.9058850	0.4897312	4.0479017
2.0	9.5801407	0.0822694	3.1787761	0.6821694	0.8644746	3.4758708

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