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Peristaltic transport of a couple stress fluid permeated with suspended particles

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A B S T R A C T

The peristaltic motion of a couple stress fluid permeated with suspended particles through a two dimensional flexible channel under long wave length approximation is studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity in fluid phase and particle phase. The expression for pressure rise and friction force has been computed numerically using mathematics software mathematica. The graphical results have been presented to discuss the physical behavior of various physical parameters on pressure rise and friction force. It is observe that pressure rise decreases with increase in couple stress parameter and Dust concentration parameter. The friction force has an opposite behavior compared with pressure rise.

Keywords: Peristaltic motion, Couple stress fluid, suspended particles, pressure rise and Friction force.

MSC 2010 codes: 74F10, 92C10

1 Introduction

Peristalsis is a natural mechanism of fluid transport for many physiological fluids. This is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. This mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferentes of the male reproductive organ, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct.

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Mechanical devices like finger pumps, roller pumps use peristalsis to pump blood, slurries and corrosive fluids.The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment. Several studies [Fung and Yih (1968); Shapiro and Jaffrin *et al.*(1969); Raju and Devanathan(1972); Radhakrishnamacharya (1978); Elshehawey and Mekheimer (1994); Elshehawey and Sobh (2001)] have been made on peristalsis with reference to mechanical and physiological situations.

The theory of couple stress was first developed by [(Stokes 1966)] and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A few examples of such fluids consisting of rigid, randomly oriented particles (red cells), suspended in a viscous medium, such as blood, lubricants containing small amount of polymer additive, electro-rheological fluids and synthetic fluids. Several authors [(Valanis and Sun 1969; Chaturani 1978; Chaturani and Rathod 1981; Srivastava 1986; Mekheimer 2002; Sobh 2008; Raghunatha Rao and Prasada Rao 2012)] have studied couple stresses in peristaltic flow.

The study of two-phase flows finds applications in many branches of Engineering, Environmental, Physical Sciences, etc. A few examples of such flows in diverse fields are the flow of dissolved micro molecules of fiber suspensions in paper making, flow of blood through arteries, propulsion and combustion in rockets, dispersion and fall out of pollutants in air, erosion of material due to continuous impingement of suspended particles in air etc. [(Frederick 1949)] studied two phase fluid-solid flow. In order to develop a mathematical theory of blood flow in arteries, [(Alihasan Nayfeh 1966)] considered blood as binary system of plasma (liquid phase) and blood cells a (solid phase). [(Saffman 1962)] dusty fluid serves as a better model to describe blood as a binary system. Solid-particle motion in two-dimensional peristaltic flows has been discussed by [(Hung and Brown 1976)]. [(Nag 1980)] studied the two-dimensional flow of unbounded dusty fluid induced by the sinusoidal transverse motion of an infinite wall. Dust velocity shear driven rotational waves and associated vertices in a non-uniform dusty plasma has been investigated by [(Shukla et al. 2004), (Rashmi 2007)] studied unsteady flow of a dusty fluid between two oscillating plates under varying constant pressure gradient, [(Ravi kumar et al.2010)] discussed the peristaltic flow of a dusty couple stress fluid in a flexible channel.

The present research aimed is to investigate the interaction of peristalsis for the motion of a Couple stress fluid permeated with suspended particles in a two dimensional flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity in fluid phase and particle phase. The effects of various physical parameters on pressure rise and friction force have been computed numerically.

2 Formulation of the problem

We consider a peristaltic flow of a couple stress-dusty fluids through two-dimensional channel bounded by flexible walls. The geometry of the flexible walls are represented by

$$y = \eta\,(X,t) = d + a\ \operatorname{Sin}\frac{2\pi}{\lambda}\,(X - c\,t) \tag{1}$$

Where '*d*' is the mean half width of the channel, '*a*' is the amplitude of the peristaltic wave, '*c*' is the wave velocity, 'λ' is the wave length and *t* is the time.

The equations governing the two-dimensional flow of a couple stress fluid permeated with suspended particles in fluid phase and particle phase are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\nabla^2 u-\eta^*\nabla^4 u-K\,N(u^p-u) \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\,\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\nabla^2 v-\eta^*\nabla^4 v-K\,N(v^p-v) \tag{4}$$

$$\frac{\partial N}{\partial x}+N\left(\frac{\partial u^p}{\partial x}+\frac{\partial v^p}{\partial y}\right)=0 \tag{5}$$

$$\frac{\partial u^p}{\partial t}+u^p\,\frac{\partial u^p}{\partial x}+v^p\,\frac{\partial u^p}{\partial y}=\frac{k}{m}(u-u^p) \tag{6}$$

$$\frac{\partial v^p}{\partial t}+u^p\,\frac{\partial v^p}{\partial x}+v^p\,\frac{\partial v^p}{\partial y}=\frac{k}{m}(v-v^p) \tag{7}$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, \quad \nabla^4=\nabla^2\nabla^2$

u, u^p, *v* and v^p denote the velocities of the fluid and dust particles respectively, '*p*' is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of viscosity, and η^* is the coefficient of couple stress, *m* is the mass of the dust particle, $K=6\pi\mu r, r$ being the particle radius, is the Stoke's drag coefficient, *N* is the number density of the particles, *k* is the stokes resistance coefficient. The particles are assumed to be uniform in size and uniformly distributed in the fluid so that *N* remains a constant.

The relative boundary conditions are

$$u=0 \quad at \quad y=\pm\eta \tag{8}$$

$$\frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=\pm\eta \tag{9}$$

$$v=0 \quad at \quad y=0 \tag{10}$$

Equation (8) represents no slip on the boundary, (9) indicates the boundary condition related to couple stress fluid and (10) shows velocity at the centre of the channel.

Introducing a wave frame *(x, y)* moving with velocity *c* away from the fixed frame *(X, Y)* by the transformation

$$x = X - c\,t,\ \ y = Y,\ u = U - c,\ v = V,\ \ p = P\,(X,t)$$

Using the following the non-dimensional variables

$$x' = \frac{x}{\lambda},\ y' = \frac{y}{d},\ u' = \frac{u}{c},\ u^{p'} = \frac{u^p}{c},\ v' = \frac{v}{c\,\delta},\ v^{p'} = \frac{v^p}{c\,\delta},\ t' = \frac{c\,t}{\lambda},\ \eta' = \frac{\eta}{d},\ p' = \frac{p\,d^2}{\mu\,c\,\lambda} \tag{11}$$

these equations reduces to (after dropping primes)

$$y = \eta\,(x) = 1 + \varepsilon\ \ \text{Sin}\ 2\,\pi\,x \tag{12}$$

$$R\,\delta\left(\frac{\partial u}{\partial t} + u\ \frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left(\delta^2\,\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - S\left(\delta^2\,\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\delta^2\,\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{R\,\alpha}{\tau}(u^p - u) \tag{13}$$

$$R\,\delta^3\left(\frac{\partial v}{\partial t} + u\ \frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\left(\delta^2\,\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - S\,\delta^2\left(\delta^2\,\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\delta^2\,\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{R\,\alpha}{\tau}\delta^2(v^p - v) \tag{14}$$

$$\delta\left(\frac{\partial u^p}{\partial t} + u^p\,\frac{\partial u^p}{\partial x} + v^p\,\frac{\partial u^p}{\partial y}\right) = \frac{1}{\tau}(u - u^p) \tag{15}$$

$$\delta\left(\frac{\partial v^p}{\partial t} + u^p\,\frac{\partial v^p}{\partial x} + v^p\,\frac{\partial v^p}{\partial y}\right) = \frac{1}{\tau}(v - v^p) \tag{16}$$

Where

$\varepsilon = \frac{a}{d}$ and $\delta = \frac{d}{\lambda}$ are geometric parameters

$R = \frac{cd}{\nu}$ is the Reynolds number , $S = \frac{\eta^*}{\mu d^2}$ is the Couple stress parameter

$\alpha = \frac{Nm}{\rho}$ is the Dust concentration parameter, $\tau = \frac{cm}{kd}$ is the Relaxation time

The corresponding dimensionless boundary conditions are

$$u = -1 \qquad at \qquad y = \pm\eta \tag{17}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = \pm\,\eta \tag{18}$$

$$v = 0 \qquad at \qquad y = 0 \tag{19}$$

3 Method of solution

We seek perturbation solution in terms of small parameter δ as follows:

$$u = u_0 + \delta\, u_1 + \delta^2 u_2 + \ldots.. \tag{20}$$

$$u^p = u_0{}^p + \delta\, u_1{}^p + \delta^2 u_2{}^p + \ldots.. \tag{21}$$

$$v = v_0 + \delta\, v_1 + \delta^2 v_2 \ldots\ldots \tag{22}$$

$$v^p = v_0{}^p + \delta\, v_1{}^p + \delta^2 v_2{}^p \ldots\ldots \tag{23}$$

Substituting equations (20) to (23) in equations (13) to (16) and collecting the coefficients of various powers of δ

The zeroth order equations are

$$S\frac{\partial^4 u_0}{\partial y^4} - \frac{\partial^2 u_0}{\partial y^2} - \frac{R\,\alpha}{\tau}(u_0{}^p - u_0) = -\frac{\partial p_0}{\partial x} \tag{24}$$

$$\frac{\partial p_0}{\partial y} = 0 \tag{25}$$

$$\frac{1}{\tau}(u_0 - u_0{}^p) = 0 \tag{26}$$

$$\frac{1}{\tau}(v_0 - v_0{}^p) = 0 \tag{27}$$

The corresponding boundary conditions are

$$u_0 = -1 \qquad at \qquad y = \pm\, \eta \tag{28}$$

$$\frac{\partial^2 u_0}{\partial y^2} = 0 \qquad at \qquad y = \pm\, \eta \tag{29}$$

$$v_0 = 0 \qquad at \qquad y = 0 \tag{30}$$

Zeroth order in *δ*

On solving the equations (24) to (27) subject to the conditions (28) to (30), we get

$$u_0 = G_1 + G_2 \operatorname{Cos}h\,[\beta y] + G_3\, y^2 \tag{31}$$

$$u_0{}^p = G_1 + (G_4 + G_2)\operatorname{Cos}h\,[\beta y] + G_3\, y^2 \tag{32}$$

$$v_0 = v_0{}^p = -\left(a_1 y + \frac{a_2 \operatorname{Sin}h\,[\beta y]}{\beta} \right) \tag{33}$$

The equations corresponding to the order *δ* are

$$S\frac{\partial^4 u_1}{\partial y^4} - \frac{\partial^2 u_1}{\partial y^2} - \frac{R\,\alpha}{\tau}(u_1{}^p - u_1) = -R\left(\frac{\partial u_0}{\partial t} + u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y} \right) \tag{34}$$

$$\frac{\partial p_1}{\partial y} = 0 \tag{35}$$

$$\frac{\partial u_0{}^p}{\partial t} + u_0{}^p\,\frac{\partial u_0{}^p}{\partial x} + v_0{}^p\,\frac{\partial u_0{}^p}{\partial y} = \frac{1}{\tau}(u_1 - u_1{}^p) \tag{36}$$

$$\frac{\partial v_0{}^p}{\partial t} + u_0{}^p\,\frac{\partial v_0{}^p}{\partial x} + v_0{}^p\,\frac{\partial v_0{}^p}{\partial y} = \frac{1}{\tau}(v_1 - v_1{}^p) \tag{37}$$

The corresponding boundary conditions are

$$u_1 = 0 \qquad at \qquad y = \pm\, \eta \tag{38}$$

$$\frac{\partial^2 u_1}{\partial y^2} = 0 \qquad at \qquad y = \pm\, \eta \tag{39}$$

$$v_1 = 0 \qquad at \qquad y = 0 \tag{40}$$

First order in δ

On solving the equation (34) to (37) subject to the conditions (38) to (40), we obtain

$$u_1 = G_4 + (G_5 + h_3 y^2)\ \mathrm{Cos}h\ [\beta y] + (h_1 y + h_2 y^3)\ \mathrm{Sinh}\ [\beta y] - h_4 y^2 - h_5 y^4 \quad (41)$$

$$u_1^{\,p} = (r_1 + r_3 y^2)\ \mathrm{Cos}h\ [\beta y] + (r_2 y + h_2 y^3)\ \mathrm{Sinh}\ [\beta y] - r_4 \mathrm{Cos}h\ [2\beta y] - h_6 y^2 - h_5 y^4 + r_5 \quad (42)$$

$$v_1 = (n_1 + n_3 y^2)\mathrm{Sin}h\ [\beta y] + (n_2 y + n_4 y^3)\ \mathrm{Cos}h\ [\beta y] + n_5 y^3 + n_6 y^5 - r_5 - a_3 y \quad (43)$$

$$v_1^{\,p} = (n_7 + n_3 y^2)\ \mathrm{Sin}h\ [\beta y] + (n_8 y + n_4 y^3)\ \mathrm{Cos}h\ [\beta y] + r_7\ \mathrm{Sin}h\ [2\beta y] + (n_5 + r_8) y^3 + h_{17} y^5 + h_{20} y - a_3 y \quad (44)$$

The volumetric flow rate in the wave frame is defined by

$$q = \int_0^{\eta} u\ dy = L\,P^2 + M\ P + A_4 \quad (45)$$

The instantaneous flux $Q\ (x,\ t)$ in the laboratory frame is

$$Q = \int_0^{\eta} (u+1)\ dy = q + \eta \quad (46)$$

The average flux $\overline{Q}$ over one period of peristaltic wave is

$$\overline{Q} = \frac{1}{T}\int_0^{T} Q\ dt = q + 1 \quad (47)$$

From equations (36) and (38), the pressure gradient is obtained as

$$\frac{dp}{dx} = \frac{-M \pm \sqrt{M^2 - 4L\,(A_4 - \overline{Q} + 1)}}{2L} \quad (48)$$

Where $L = A_1 + A_2 + A_5$, $M = A_3 + A_6$

The pressure rise over one cycle of the wave can be obtained as

$$\Delta p = \int_0^1 \frac{dp}{dx}\, dx \tag{49}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 \eta^2 \left(-\frac{dp}{dx}\right) dx \tag{50}$$

Where

$B = \frac{R(1+\alpha)}{S}$, $\quad \beta = \frac{1}{\sqrt{S}}$, $\quad G_1 = \frac{P}{\beta^2} - \frac{P\eta^2}{2} - 1$, $\quad G_2 = -\frac{P}{\beta^2}\mathrm{Sec}h[\beta\eta]$, $\quad G_3 = \frac{P}{2}$,

$G_4 = \frac{P\tau}{R\alpha}\left(1 - S\beta^2\right)\mathrm{Sec}h[\beta\eta]$,

$$G_5 = \left(\frac{L_2}{\beta^2} + \frac{6L_4}{\beta^6} - \frac{2L_5}{\beta^5}\right)\mathrm{Cos}h[\beta\eta] + \left(\frac{L_5}{\beta^4} - \frac{4L_4}{\beta^5}\right)\eta\ \mathrm{Sin}h[\beta\eta] + \frac{L_4}{\beta^4}\eta^2\mathrm{Cos}h[\beta\eta] + \frac{L_3}{12\beta^2}\eta^4 + \left(\frac{L_1}{2\beta^2} + \frac{L_3}{\beta^4}\right)\eta^2 - \left(\frac{L_1}{\beta^4} + \frac{4L_3}{\beta^6} + 1\right),$$

$$G_6 = \left(\frac{L_1}{\beta^4} + \frac{4L_3}{\beta^6}\right)\mathrm{Sec}h[\beta\eta] + \frac{L_3}{\beta^4}\eta^2\mathrm{Sec}h[\beta\eta] + \left(\frac{L_5}{4\beta^4} - \frac{L_4}{4\beta^5} - \frac{L_2}{2\beta^3}\right)\eta\ \mathrm{Tan}h[\beta\eta] + - \frac{L_4}{6\beta^3}\eta^3\ \mathrm{Tan}h[\beta\eta] + \left(\frac{L_4}{4\beta^4} - \frac{L_5}{4\beta^3}\right)\eta^2 + \left(\frac{2L_5}{\beta^5} - \frac{6L_4}{\beta^6} - \frac{L_2}{\beta^4}\right),$$

$L_1 = -B[a_1(G_1 - 1) + G_2 a_2)$, $\quad L_2 = -B[a_2(G_1 - 1) + G_2 a_1)$, $\quad L_3 = {}^{B\,a_1}\!/_{2}$, $\quad L_4 = {}^{B\,a_2}\!/_{2}$,

$L_5 = \frac{a_2}{\beta}(G_2\beta^2 + 1)$,

$a_1 = G_{1X}$, $a_2 = G_{2X}$, $a_3 = G_{5X}$, $a_4 = G_{6X}$, $a_5 = G_1\, G_{1X}$, $a_6 = G_1\, G_{2X}$, $a_7 = G_2\, G_{1X}$,

$a_8 = G_2\, G_{2X}$, $a_9 = G_{1XX}$, $a_{10} = G_{2XX}$, $a_{11} = L_{1X}$, $a_{12} = L_{2X}$, $a_{13} = L_{3X}$, $a_{14} = L_{4X}$, $a_{15} = L_{5X}$,

$a_{16} = {G_{1X}}^2$, $a_{17} = {G_{2X}}^2$,

$h_1 = \frac{L_2}{2\beta^3} + \frac{17L_4}{4\beta^5} - \frac{5L_5}{4\beta^4}$, $\quad h_2 = \frac{L_4}{6\beta^3}$, $\quad h_3 = \frac{L_5}{4\beta^3} - \frac{5L_4}{4\beta^4}$, $\quad h_4 = \frac{L_1}{2\beta^2} + \frac{2L_3}{\beta^4}$, $\quad h_5 = \frac{L_3}{12\beta^2}$,

$h_6 = \frac{L_1}{2\beta^2} + \frac{2L_3}{\beta^4} + 3\,G_3 a_1 \tau$,

$r_1 = G_6 - G_1 a_2 \tau - G_2 a_1 \tau$, $r_2 = h_1 - G_2 \beta a_1 \tau - \frac{P a_2 \tau}{\beta}$, $r_3 = h_3 - G_3 a_2 \tau,$ $r_4 = G_2 a_2 \tau,$

$r_5 = G_5 - G_1 a_1 \tau,$ $r_6 = \tau(G_1 a_9 + a_{16}{}^2),$ $r_7 = \frac{G_2 a_{10} - a_{17}{}^2}{2\beta},$ $r_8 = G_3 a_9$

$n_1 = \frac{a_{12}}{2\beta^5} - \frac{a_4}{\beta} - \frac{3a_{13}}{4\beta^7} - \frac{7a_{15}}{4\beta^6},$ $n_2 = \frac{7a_{15}}{2\beta^5} - \frac{a_{12}}{2\beta^4} - \frac{3a_{14}}{4\beta^6},$ $n_3 = \frac{7a_{14}}{4\beta^5} - \frac{a_{15}}{4\beta^4},$

$n_4 = -\frac{a_{14}}{6\beta^4},$ $n_5 = \frac{a_{11}}{6\beta^2} + \frac{2a_{13}}{3\beta^4},$ $n_6 = \frac{a_{13}}{60\beta^2},$ $n_7 = \frac{a_{12}}{2\beta^5} - \frac{3a_{14}}{4\beta^7} - \frac{7a_{15}}{4\beta^6} - \frac{a_4}{\beta} + \frac{\mathrm{G}_1 a_{10} - a_1 a_2}{\beta},$

$n_8 = \frac{7a_{15}}{4\beta^5} - \frac{a_{12}}{2\beta^4} - \frac{3a_{14}}{4\beta^6} + \mathrm{G}_2 a_9 - a_1 a_2,$

$m_1 = \frac{Ba_1}{\beta^6},$ $m_2 = \frac{Ba_2}{\beta^6},$ $m_3 = -\frac{Ba_5}{\beta^4},$ $m_4 = \frac{Ba_6}{\beta^4},$ $m_5 = \frac{Ba_7}{\beta^4},$ $m_6 = \frac{Ba_8}{\beta^4},$

$M_1 = m_3 - m_6 + 3m_2 + m_5,$ $M_2 = \frac{1}{2}\beta^2 m_2 + \beta m_2 + \beta m_5,$ $M_3 = \frac{1}{2}(\beta^2 m_1 + \beta^4 m_3 + 4m_1),$

$M_4 = \beta^2(m_1 + m_5) + m_4,$ $M_5 = \frac{1}{8}(2\beta m_4 + \beta m_5),$ $\mathrm{M}_6 = -\frac{1}{2}\beta m_2(1+\beta),$ $\mathrm{M}_7 = \frac{5}{2}\beta m_2(1+\beta^2),$

$\mathrm{M}_8 = \frac{5}{8}\beta(m_5 + 2m_2),$ $\mathrm{M}_9 = \frac{1}{2}\beta^2(m_6 - m_3),$ $\mathrm{M}_{10} = -\frac{1}{4}\beta(m_4 + 2\beta^3 m_1),$

$\mathrm{M}_{11} = \frac{1}{4}\beta(4\beta^2 m_2 - m_5),$ $\mathrm{M}_{12} = \frac{1}{24}\beta^2 m_1(24 - \beta^2),$ $\mathrm{M}_{13} = \frac{1}{24}\beta^2 m_2(4\beta - 15),$

$\mathrm{M}_{14} = \frac{1}{4}m_2\beta(2\beta + 1),$ $\mathrm{M}_{15} = -\frac{1}{18}\beta^2 m_2 + \frac{17}{8}\beta m_2 + \frac{1}{4}\beta^2 m_5,$ $\mathrm{M}_{16} = -\frac{5}{8}\beta(m_5 - 2m_2),$

$$A_1 = \delta(\eta(M_1 + \frac{\beta^2}{2} m_2 \eta^3)\ \mathrm{Cos}h[\beta\eta] + M_2 \eta^2 \mathrm{Sin}h[\beta\eta] + M_{11}\eta^5 + M_3\eta^3 + M_4\eta),$$

$$A_2 = \frac{\delta\ \mathrm{Sin}h[\beta\eta]}{\beta}\big((M_5 + m_{16}) + m_{15}\ \eta^2)\ \mathrm{Sec}h[\beta\eta] + ((M_7 + m_{17} + m_{20} + m_{21} + M_9)\eta$$
$$+ m_{14}\eta^3)\ \mathrm{Tan}h[\beta\eta] + (m_{11} + m_{14} + M_{10})\eta^2 + (M_6 + m_9 + m_{10} + M_8 + M_{11} + M_{12})\big),$$

$$A_3 = G_1\eta + \frac{1}{\beta}G_2\ \mathrm{Sin}h[\beta\ \eta] + \eta^3,\ A_4 = \frac{\eta^2}{4}\frac{\partial p_1}{\partial x} - \eta$$

$$\mathrm{A}_5 = \delta(\frac{1}{\beta^3}(m_5\,\eta^3\beta^2 + 6m_5\,\eta - 2M_{15}\,\eta\,\beta + (m_8 + M_{14} + M_{16})\eta\,\beta^2)\ \mathrm{Cos}h[\beta\eta] + \frac{M_3}{3}\eta^3$$
$$+ \frac{1}{\beta^4}(2M_{15}\beta - (m_8 - M_{14} + M_{16} + 3m_5\eta^2)\beta^2 + M_{15}\eta^2\beta^3 - 6m_5)\mathrm{Sin}h[\beta\eta] + \frac{m_{10}}{5}\eta^5),$$

$$A_6 = \delta(\frac{1}{\beta}(m_3\eta\beta + m_{19}\eta)\,\mathrm{Cos}h[\beta\eta] + (\frac{1}{\beta}(m_1\,\mathrm{Tan}h[\beta\eta] + m_7\eta\,\mathrm{Sin}h[\beta\eta]\,\mathrm{Tan}h[\beta\eta] + m_6) + \frac{1}{3}(3m_{14}\eta^3 + m_{16}\eta^3)),$$

4 Results and discussions

In this analysis we have presented the graphical results of the solutions pressure rise Δp, friction force F. The pressure rise Δp is shown in Figures (1) to (5), for the different values of R, α, ε, δ, S. These figures represent the variation of Δp against $\overline{Q}$ in the fluid phase. It is found that the above all figures Δp gradually decreases in magnitude to attain the maximum at $\overline{Q}$ = 1.0. From figure (1) we find that Reynolds number $R \leq 2$, the profiles are of straight lines and for higher $R \geq 3$ the profiles are parabolic in nature and the Δp experiences depreciation with Reynolds number R. we find that Δp reduces with Dust concentration parameter α in Figure 2. From figures (3-5), the pressure rise Δp against $\overline{Q}$ increases from $\overline{Q}$ = 0 to $\overline{Q}$ = 1.0, the variation of Δp with for different parameters amplitude ratioε, slope parameter δ and couple stress parameter S, shows that Δp experiences an depreciation with ε, δ, and S in the entire flow region.

The Frictional force F is shown in Figures (6) to (10) for the different values of R, α, ε, δ, S. Figures (6-10) represent the variation of Frictional force F against $\overline{Q}$ for a different parametric values. These figures show that frictional force has opposite behavior in comparison with pressure rise [(Sobh 2008)]. Figure (6) represents the Frictional force F with R, it is found that $R \leq 2$, the profiles of F are straight lines and for higher $R \geq 3$ the profiles are almost parabolic in nature. It is found that F experiences an enhancement with increase in R. Figure (7) the Frictional force F enhances with increase in Dust concentration parameter α. From figures (8-10) we notice that the Frictional force F against $\overline{Q}$ decreases from $\overline{Q}$ = 0 to $\overline{Q}$ = 1.0, also we notice that F experiences an enhancement with increase in ε, δ and S in the entire flow region.

5 Figures

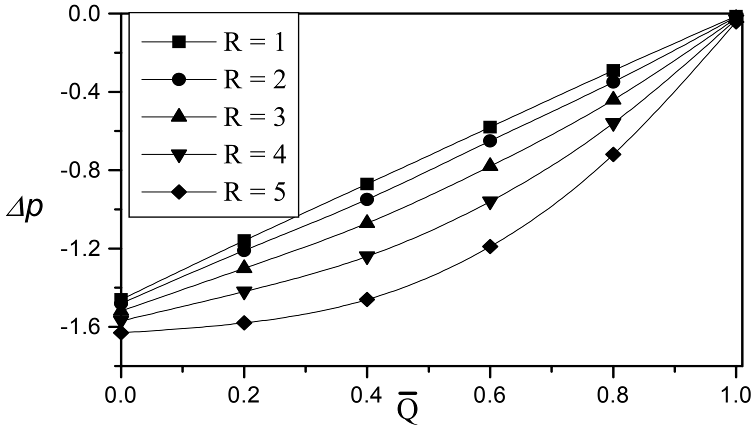

Figure 1: Effect of R on Δp when ε = 0.01, δ = 0.01, α = 1, S = 0.2, τ =1

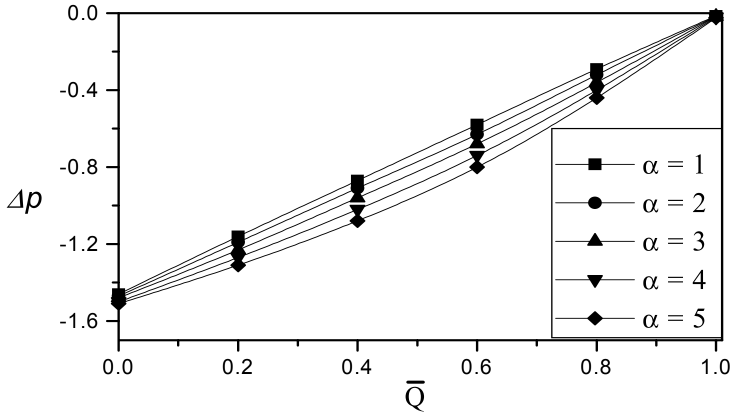

Figure 2 : Effect of α on Δp when ε = 0.01, δ = 0.01, R = 1, S = 0.2, τ =1

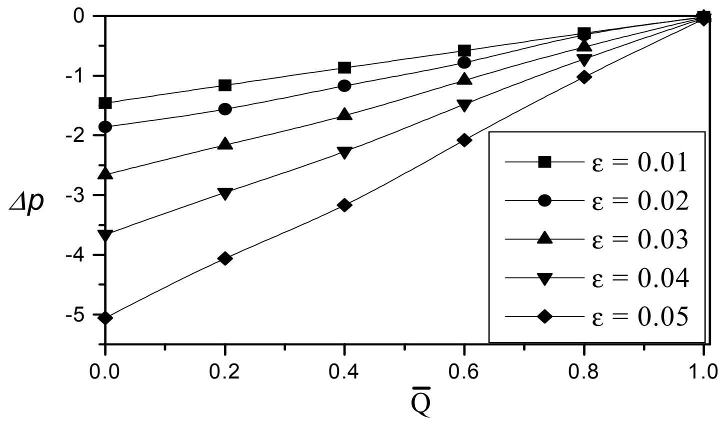

Figure 3: Effect of ε on Δp when δ = 0.01, R = 1, S = 0.2, α = 1, τ =1

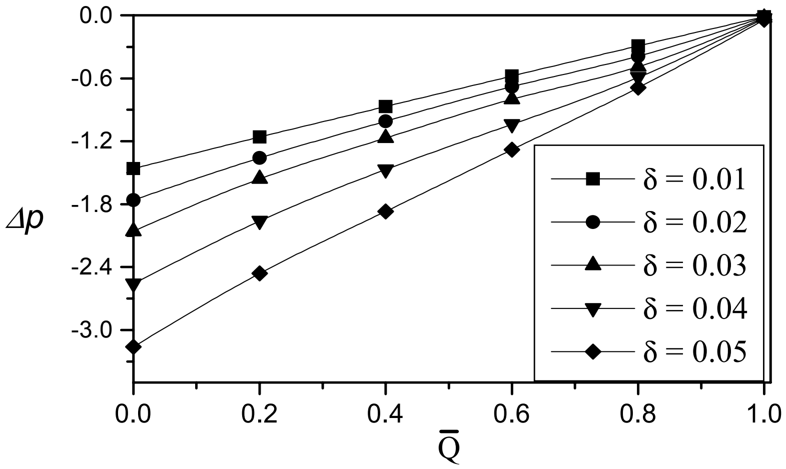

Figure 4: Effect of δ on *Δp* when ε = 0.01, R = 1, S = 0.2, α = 1, τ =1

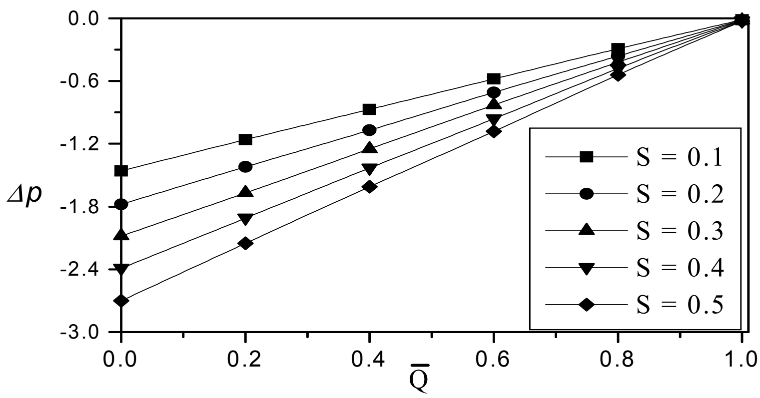

Figure 5: Effect of S on *Δp* when $\varepsilon = 0.01$, $\delta = 0.01$, $\alpha = 1$, $R = 1$, τ =1

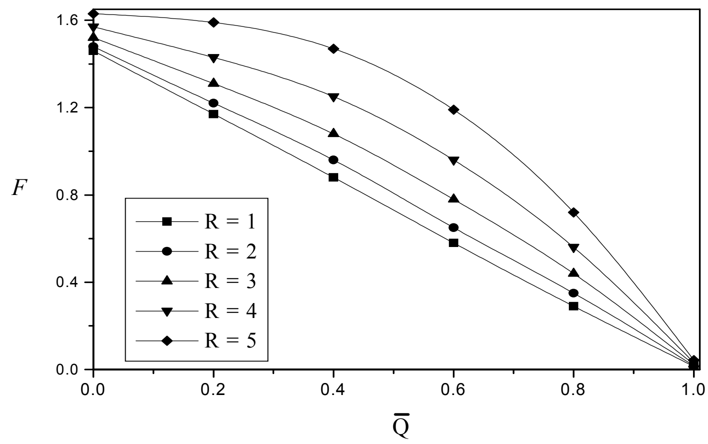

Figure 6: Effect of *R* on *F* when ε = 0.01, δ = 0.01, α = 1, S = 0.2, τ =1

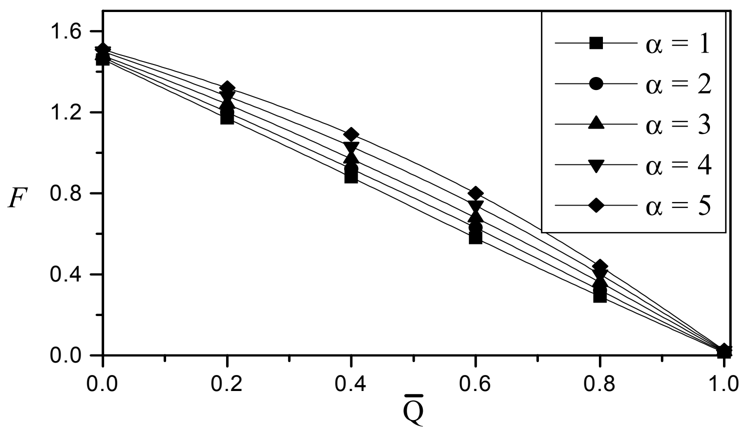

Figure 7: Effect of α on *F* when ε = 0.01, δ = 0.01, R = 1, S = 0.2, τ =1

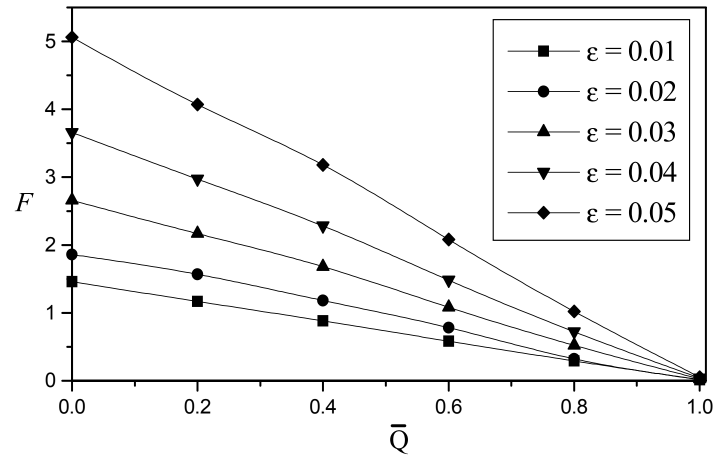

Figure 8: Effect of ε on F when δ = 0.01, R = 1, S = 0.2, α = 1, τ =1

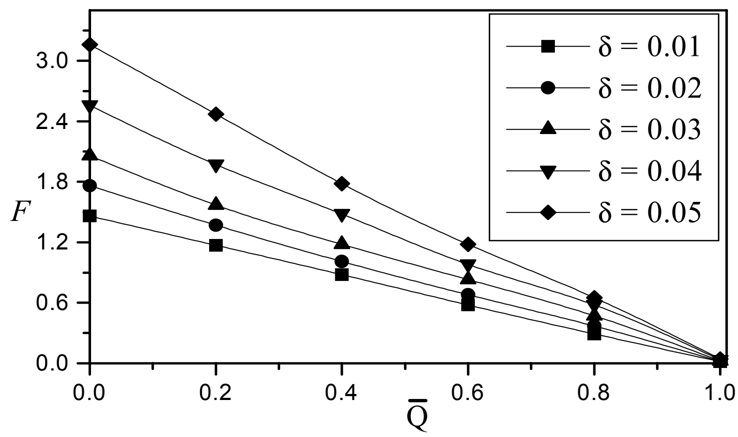

Figure 9: Effect of δ on F when ε = 0.01, R = 1, S = 0.2, α = 1, τ =1

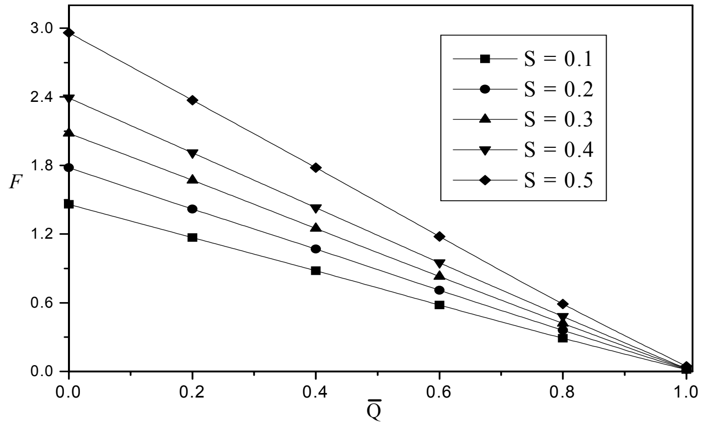

Figure 10: Effect of S on F when ε = 0.01, δ = 0.01, α = 1, R = 1, τ =1

6 Conclusions

In this research paper we studied the peristaltic flow of a couple stress fluid permeated with suspend particles in a flexible channel. The governing equations of motion are solved analytically using long wave length approximation. Furthermore, the effect of various values of parameters on pressure rise and friction force have been computed numerically and explained graphically. We conclude the following observations:

- Pressure rise Δp experiences depreciation with Reynolds number R, amplitude ratio ε, slope parameter δ and couple stress parameter S.
- We also observe that Δp decreases with increase in Dust concentration parameter α.
- F experiences an enhancement with increase in R, ε, δ and S in the entire flow region.
- The Frictional force F enhances with increase in Dust concentration parameter α.
- There is no change in Pressure rise and Frictional force with the Relaxation time.
- We observe that the Friction force F has an opposite behavior compared with Pressure rise.

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