International Journal of Advances in Applied Mathematics and Mechanics
Volume 1, Issue 2 : (2013) pp. 103-115
Available online at www.ijaamm.com
ISSN: 2347-2529

Supply-and-Demand model involving fuzzy parameters

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Received 04 October 2013; Accepted (in revised version) 12 November 2013.

ABSTRACT

In this article, we investigate supply-and-demand model a basic application in economics. In the simplest form the supply and demand can be considered as function of price only keeping other parameters constant. In this form the functions are represented by straight lines whose intersection is the equilibrium state. When supply and demand are represented as function of parameters other than price the curves are shifted straight lines. Thus, in a way to incorporate the changes in supply and demand due to the determinants other than price, we can consider the supply-and-demand curves being represented by the parameters involving imprecision. This imprecision in the supply-and-demand realistic models can be represented using fuzzy parameters. To obtain equilibrium state for such systems we have to solve fuzzy linear system involving negative fuzzy numbers. We show that such systems are efficiently solved using techniques by (Pandit 2012), (Pandit 2013).

Keywords: Fuzzy Systems, Negative Fuzzy numbers, α-cut, Supply-and-Demand, Economics.

MSC 2010 codes: 03E72, 03E75

1 Introduction

The interaction of consumers and suppliers determines the quantity of a goods or service sold in a market and the price at which it is sold. To describe it supply and demand model is used. This model can be used if we know three things: buyer's behaviour, seller's behaviour, and how buyers and sellers actions affect price and quantity. It is well known fact that change in price of goods over the fixed time is the vital factor in determining the change in supply or demand, incorporating the change in other factors influence on the change in supply or demand leads us to solve the problem in fuzzy setup.

In the initial sections we describe the supply-and-demand problem as in price theory. In the following section, we justify the need to consider the supply-demand problem in fuzzy

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domain and give the solution for it. Fuzzy treatment to supply-demand model due to (Nasseri *et al.* 2011) gives rise to a system with crisp coefficients and fuzzy right hand side. They have solved such a system by using QR-decomposition method. The fuzzy model for supply-demand market proposed by us is fully fuzzy with negative fuzzy numbers. We have solved the model using the technique by (Pandit 2013) and substantiated with the illustrative examples. One of the illustrations shows that the technique for solving fuzzy systems by us is more general and it can provide strong fuzzy solutions to the problems where the QR-decomposition method fails.

2 Supply-and-Demand model

The supply-and-demand model relies on a high degree of competition, meaning that there are enough buyers and sellers in the market for bidding to take place. Buyers bid against each other and thereby raise the price, while sellers bid against each other and thereby lower the price. The equilibrium is a point at which all the bidding has been done; nobody has an incentive to offer higher prices or accept lower prices.

Perfect competition exists when there are so many buyers and sellers that no single buyer or seller can unilaterally affect the price on the market. Imperfect competition exists when a single buyer or seller has the power to influence the price on the market.

The supply-and-demand model applies most accurately when there is perfect competition. This is an abstraction, because no market is actually perfectly competitive, but the supply-and-demand framework still provides a good approximation for what is happening much of the time.

2.1 Demand

The dictionary meaning of word demand is almost any kind of wish or desire or need. But to an economist, demand refers to both willingness and ability to pay. Quantity demanded Q_d is the total amount of goods that buyers would choose to purchase under given conditions. The given conditions include:

- price of the goods
- income and wealth
- prices of substitutes and complements
- population preferences (tastes)
- expectations of future prices

Thus the demand function shows the correspondence between the quantity demanded, price, and other factors that influence purchases. We can consider $Q_d = D(p, p_s, p_c, Y, \ldots)$ where, Q_d is the quantity demanded of a particular goods in a given time period, p price per unit of the goods, p_s is the price per unit of a substitute goods (a goods that might be consumed instead of this goods), p_c is the price per unit of a complementary goods (a goods that might be consumed jointly with this goods, such as cream with coffee), and Y is consumers income etc.

The Law of Demand states that when the price of a goods rises, and everything else remains the same, the quantity of the goods demanded will fall. To understand the market behaviour it is interesting to know the effect of the price of the goods on the quantity demanded, holding all other factors constant. Mathematically represented as, $Q_d = a\,p + b$,

2.2 Supply

Supply means a fixed amount, in the world of economists supply is not just the amount of something there, but the willingness and ability of potential sellers to produce and sell it. Quantity supplied Q is the total amount of a goods that sellers would choose to produce and sell under given conditions. The given conditions include:

- price of the goods
- prices of factors of production (labour, capital)
- prices of alternative products the firm could produce
- technology
- productive capacity
- expectations of future prices

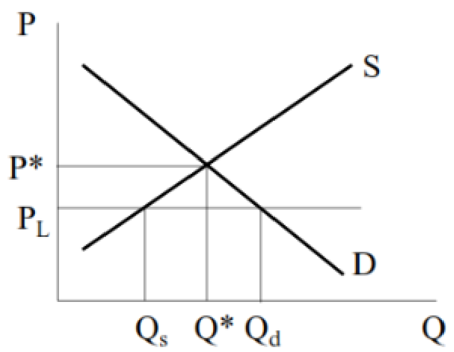

Figure 1: Market Equilibrium

The supply function shows the correspondence between the quantity supplied, price, and other factors that influence the number of units offered for sale. That is, $Q_s = S(p, p_l, p_c, \ldots)$ where, Q_s is the quantity of goods in a given time period, p price per unit of the goods, p_l price of labour involved in the production, p_c price of capital involved in the production, etc. The Law of Supply states that when the price of a goods rises, and everything else remains the same, the quantity of the goods supplied will also rise. Hence, to examine the change in supply function due to change in the price we hold the other involved factors constant. That is, we analyze $Q_s = c\,p + d$.

2.3 Market equilibrium

Putting demand and supply together, we can find equilibrium where the supply and demand curves cross. The equilibrium consists of an equilibrium price P^* and an equilibrium quantity Q^*. The equilibrium must satisfy the market-clearing condition, which is $Q_d = Q_s$, the point where the two lines cross as shown in Figure 1.

3 Supply and demand as fuzzy

As mentioned earlier the supply and demand functions depend on various factors other than price. To measure the effect of price changes on the demand curve all other factors that affect demand are held constant. To have more realistic model for market behaviour points are reinvestigated:

1. The effect on demand curve due to change in multi-factors including price.
2. Price elasticity of demand: The price elasticity of demand (or simply the demand elasticity or elasticity of demand) is the percentage change in the quantity demanded, *Q*, in response to a given percentage change in the price, *p*, at a particular point on the demand curve.

We take both these points one by one. To visualize the effect on demand due to change in multi-factors is difficult. A simpler approach is used by economists to show the effect on demand due to change in a factor other than the price of the goods. It is observed that a change in any factor other than the price of the goods itself causes a shift of the demand curve rather than a movement along the demand curve, as shown in the Figure 2(a).

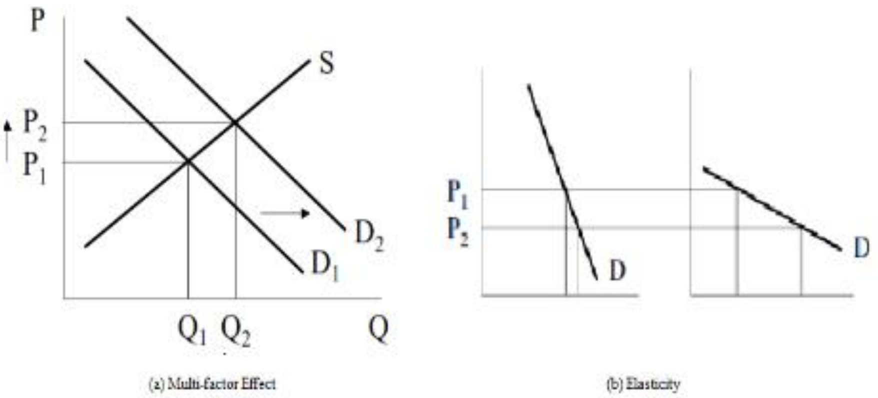

Figure 2: Other effects on Demand Curve

Elasticity refers to the degree of responsiveness of one variable to another. It's not enough to say, for instance, that a rise in price leads to a fall in quantity demanded (the Law of Demand); we want to know how much quantity changes in response to price. A simple way to see the degree of responsiveness is simply to look at the slope. A flatter demand curve represents a greater degree of responsiveness (for a supply or demand curve), as shown in the graphs above, the flatter demand curve produces a larger change in quantity for the same change in price as in Figure 2(b).

A perfectly elastic demand curve is horizontal, because an infinitely small change in price corresponds to an infinitely large change in quantity. A perfectly inelastic demand curve is vertical, because quantity will never change regardless of the change in price.

Thus, to include the above mentioned points in realistic way for demand curve the parameters may be considered as fuzzy. Similar approach for the supply curve gives us the supply curve with the fuzzy parameters.

The market equilibrium conditions lead us to solving of the following system of linear equations with fuzzy parameters:

$$Q_d = \tilde{a} * p + \tilde{b}$$
$$Q_d = Q_s = \tilde{c} * p + \tilde{d}$$

That is, we have to solve the fuzzy system of linear equation of the form

$$x_1 = \tilde{a} * x_2 + \tilde{b}$$
$$x_1 = \tilde{c} * x_2 + \tilde{d}$$

That is,

$$x_1 - \tilde{a} * x_2 = \tilde{b}$$
$$x_1 - \tilde{c} * x_2 = \tilde{d}$$

The above is the fuzzy system of linear equations of the form

$$\tilde{A}\tilde{x} = \tilde{b} \qquad (1)$$

For obtaining the solution of the system (1) we use the technique by (Pandit 2012), (Pandit 2013), wherein the elements of the coefficient matrix and that of the resource vector can be a positive or negative fuzzy number.

4 Preliminaries

4.1 Definition

Let us denote by R_F the class of fuzzy subsets of the real axis (i.e. u : R →[0, 1]) satisfying the following properties:

(i) $\forall u \in R_F$, u is normal, i.e. $\exists x_0 \in R$ with $u(x_0) = 1$;
(ii) $\forall u \in R_F$, u is convex fuzzy set, That is, $u(tx + (1-t)y) \geq \min\{u(x), u(y)\}, \forall t \in [0,1], x, y \in R_F$
(iii) $u \in R_F$, u is upper semi-continuous on R;
(iv) $\overline{\{x \in R; u(x) > 0\}}$ is compact, where $\overline{A}$ denotes the closure of A.

Then R_F is called the space of fuzzy numbers, as in (Dubois and Prade 1980). Obviously $R \subset R_F$ as R can be regarded as $\{\chi_x : x$ is any usual real number$\}$.

For $0 < \alpha \leq 1$, denote the α-cut as, $[u]^\alpha = \{x \in R : u(x) \geq \alpha\}$ and $[u]^0 = \overline{\{x \in R : u(x) > 0\}}$. Then it is well-known that for each, $\alpha \in [0,1]$, the α-cut, $[u]^\alpha$ is a bounded closed interval

$\left[{}^{\alpha}\underline{u}, {}^{\alpha}\overline{u}\right]$. We will use α in the interval only when specifically required, otherwise it is understood.

4.2 Definition

A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0,\ \forall x \le 0$.

4.3 Definition

A fuzzy number $\tilde{A}$ is said to be non-positive fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0,\ \forall x \ge 0$.

4.4 Definition

A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. $\tilde{A}$ will be positive (negative) and denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$) if each element of $\tilde{A}$ be positive (negative). $\tilde{A}$ will be non positive (non negative) denoted by $\tilde{A} \le 0$ ($\tilde{A} \ge 0$) if each element of $\tilde{A}$ be non positive (non negative).

We may represent $n \times m$ fuzzy matrix $\tilde{A}$ with the elements as, where $\tilde{a}_{ij}$ is a trapezoidal fuzzy number $(\tilde{a}_{ij})_{n\times m}$, where, $\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ is defined as

$$\mu_{\tilde{a}_{ij}} = \begin{cases} \dfrac{(x - a_{ij})}{(b_{ij} - a_{ij})} & a_{ij} < x \le b_{ij} \\ 1 & b_{ij} < x \le c_{ij} \\ \dfrac{(d_{ij} - x)}{(d_{ij} - c_{ij})} & c_{ij} < x \le d_{ij} \\ 0 & otherwise \end{cases}$$

In the above representation if $b_{ij} = c_{ij}$ the fuzzy number is known as triangular fuzzy number.

The α-cut of which is given as

$${}^{\alpha}a_{ij} = [\underline{a}_{11}, \overline{a}_{11}] = [(b_{ij} - a_{ij})\alpha + a_{ij}, d_{ij} - (d_{ij} - c_{ij})\alpha]$$

4.5 Operations on fuzzy numbers

For $u, v \in R_F$ and $\lambda \in R^+$, the sum $u + v$ and the product $\lambda \cdot u$ are defined by $[u + v]^{\alpha} = [u]^{\alpha} + [v]^{\alpha}$; $[\lambda \cdot u]^{\alpha} = \lambda [u]^{\alpha}$, $\forall \alpha \in [0,1]$, where $[u]^{\alpha} + [v]^{\alpha}$ means the usual addition of two intervals of R and $\lambda [u]^{\alpha}$ means the usual product between a scalar and a subset of R refer (Dubois and Prade 1980).

4.6 System reduction using level-cuts

Consider the fully fuzzy system in 2-dimension,

$$\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}} \tag{2}$$

i.e.

$$\begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

Using the α-cut of the fuzzy elements, we get,

$$\begin{pmatrix} [\underline{a}_{11}, \bar{a}_{11}] & [\underline{a}_{12}, \bar{a}_{12}] \\ [\underline{a}_{21}, \bar{a}_{21}] & [\underline{a}_{22}, \bar{a}_{22}] \end{pmatrix} \otimes \begin{pmatrix} [\underline{x}_1, \bar{x}_1] \\ [\underline{x}_2, \bar{x}_2] \end{pmatrix} = \begin{pmatrix} [\underline{b}_1, \bar{b}_1] \\ [\underline{b}_2, \bar{b}_2] \end{pmatrix}$$

which becomes,

$$\begin{gathered} [\underline{a}_{11}, \bar{a}_{11}] \otimes [\underline{x}_1, \bar{x}_1] \oplus [\underline{a}_{12} \bar{a}_{12}] \otimes [\underline{x}_2, \bar{x}_2] = [\underline{b}_1, \bar{b}_1] \\ [\underline{a}_{21}, \bar{a}_{21}] \otimes [\underline{x}_1, \bar{x}_1] \oplus [\underline{a}_{22}, \bar{a}_{22}] \otimes [\underline{x}_2, \bar{x}_2] = [\underline{b}_2, \bar{b}_2] \end{gathered}$$

If each a_{ij} is positive the above system can be put into crisp system of linear equations as,

$$\begin{aligned} \underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 &= \underline{b}_1 \\ \underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 &= \underline{b}_2 \\ \bar{a}_{11}\bar{x}_1 + \bar{a}_{12}\bar{x}_2 &= \bar{b}_1 \\ \bar{a}_{21}\bar{x}_1 + \bar{a}_{22}\bar{x}_2 &= \bar{b}_2 \end{aligned}$$

As shown in the modelling the supply-demand system involves even negative fuzzy numbers. To obtain fuzzy solution for such systems after taking α-cut, for the negative parameters interchange $\underline{a}_{ij}$ with $\bar{a}_{ij}$.

That is the system to be solved is:

$$\begin{pmatrix} \underline{a}_{11} & \underline{a}_{12} & 0 & 0 \\ \underline{a}_{21} & \underline{a}_{22} & 0 & 0 \\ 0 & 0 & \bar{a}_{11} & \bar{a}_{12} \\ 0 & 0 & \bar{a}_{21} & \bar{a}_{22} \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} = \begin{pmatrix} \underline{b}_1 \\ \underline{b}_2 \\ \bar{b}_1 \\ \bar{b}_2 \end{pmatrix}$$

with the entries for the negative fuzzy numbers being interchanged. The above system is a crisp system of the form

$$A \times \mathbf{x} = \mathbf{b} \tag{3}$$

where, the coefficient matrix, A is of dimension 4×4, $\mathbf{x}$ and $\mathbf{b}$ are column vectors of dimension 4. The solution of system (3) exists if A invertible. Existence of the solution (Friedman et al. 1998) for the crisp system only gives weak solution for the fuzzy system (Allahviranloo *et al.* 2011). This weak solution is unique fuzzy solution only if it satisfies the conditions given by following theorem, refer (Pandit 2012). Below, we give some theorems and results are taken from (Pandit, 2012).

4.6.1 Theorem 5 (Existence and Uniqueness):

The components of the solution vector x of system (3) can determine the unique fuzzy solution for system (2) if the parameters for the fuzzy system (2) satisfy the following conditions:

(i) $\forall\alpha\in[0,1]$

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\alpha}\underline{b}_{1} \\ {}^{\alpha}\underline{b}_{2} \end{pmatrix} \le \begin{pmatrix} {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\alpha}\overline{b}_{1} \\ {}^{\alpha}\overline{b}_{2} \end{pmatrix}$$

(ii) $\forall\alpha,\beta\in[0,1],\alpha\le\beta$

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\alpha}\underline{b}_{1} \\ {}^{\alpha}\underline{b}_{2} \end{pmatrix} \le \begin{pmatrix} {}^{\beta}\underline{a}_{11} & {}^{\beta}\underline{a}_{12} \\ {}^{\beta}\underline{a}_{21} & {}^{\beta}\underline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\beta}\underline{b}_{1} \\ {}^{\beta}\underline{b}_{2} \end{pmatrix} \le \begin{pmatrix} {}^{\beta}\overline{a}_{11} & {}^{\beta}\overline{a}_{12} \\ {}^{\beta}\overline{a}_{21} & {}^{\beta}\overline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\beta}\overline{b}_{1} \\ {}^{\beta}\overline{b}_{2} \end{pmatrix} \le \begin{pmatrix} {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix}^{-1}\begin{pmatrix} {}^{\alpha}\overline{b}_{1} \\ {}^{\alpha}\overline{b}_{2} \end{pmatrix}$$

The condition guarantees that α-cuts are closed and the support is bounded. Thus, the solution of the crisp system determines ${}^{\alpha}\underline{x}_i$ and ${}^{\alpha}\overline{x}_i$ as x_i and x_{i+2} respectively for i = 1,2. These are used to reconstruct the components of fuzzy $\tilde{\mathbf{x}}$ as shown by the lemma below.

4.6.2 Lemma:

The i^{th} component of the fuzzy solution vector, $\tilde{\mathbf{x}}$ of the fully fuzzy system (2) can be reconstructed form the components ${}^{\alpha}\underline{x}_i$ and ${}^{\alpha}\overline{x}_i$ of the crisp system (3) and is given as

$$\tilde{x}_i = \bigcup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i$$

where, ${}_{\alpha}\tilde{x}_i = \alpha\cdot{}^{\alpha}\tilde{x}_i$ and ${}^{\alpha}\tilde{x}_i = \left[{}^{\alpha}\underline{x}_i, {}^{\alpha}\overline{x}_i\right]$.

Proof: For each particular $y\in R$, let $a=\tilde{x}_i(y)$. Then

$$\left(\bigcup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i\right)(y) = \sup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i(y)$$

$$= \max\left[\sup_{\alpha\in[0,a]} {}_{\alpha}\tilde{x}_i(y), \sup_{\alpha\in(a,1]} {}_{\alpha}\tilde{x}_i(y)\right]$$

For each $\alpha \in (\mathrm{a}, 1]$, we have $\tilde{x}_i(y) = a < \alpha$ and, therefore ${}_\alpha\tilde{x}_i(y) = 0$. On the other hand for each $\alpha \in [0, \mathrm{a}]$, we have $\tilde{x}_i(y) = a \geq \alpha$, therefore ${}_\alpha\tilde{x}_i(y) = \alpha$.

Hence,

$$\left(\bigcup_{\alpha \in [0,1]} {}_\alpha\tilde{x}_i \right)(y) = \sup_{\alpha \in [0,1]} \alpha = a = {}_\alpha\tilde{x}_i(y)$$

In the following section we give the computational steps to obtain the fuzzy solution taking illustration. The next example shows that the proposed method is more general and it gives fuzzy solution to the problems where QR decomposition method (Nasseri *et al.* 2011) fails.

5 Numerical illustrations

5.1 Example: 1

Step:1 Consider the fully fuzzy supply-demand model as described below:

$$(4, 6, 8)\ \tilde{x}_1 + (-5, -3, -1)\ \tilde{x}_2 = (65, 75, 80)$$

$$(1, 3, 4)\ \tilde{x}_1 + (5, 7, 9)\ \tilde{x}_2 = (40, 50, 60)$$

Step:2 Taking the α-cut of the system we get

$$\begin{aligned} [4+2\alpha, 8-2\alpha]\tilde{x}_1 + [2\alpha-5, -1-2\alpha]\tilde{x}_2 &= [65+10\alpha, 80-5\alpha] \\ [1+\alpha, 4-2\alpha]\tilde{x}_1 + [2\alpha+5, 9-2\alpha]\tilde{x}_2 &= [40+10\alpha, 60-10\alpha] \end{aligned}$$

the corresponding crisp system as follows:

$$\begin{pmatrix} 4+2\alpha & 2\alpha-5 & 0 & 0 \\ 1+\alpha & 5+2\alpha & 0 & 0 \\ 0 & 0 & 8-2\alpha & -1-2\alpha \\ 0 & 0 & 4-2\alpha & 5+2\alpha \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} = \begin{pmatrix} 65+10\alpha \\ 40+10\alpha \\ 80=5\alpha \\ 60-10\alpha \end{pmatrix}$$

Step:3 Since we observe that there are coefficients that negative for all possible values of α we interchange the entries as given in the algorithm, That is a_{12} is interchanged with a_{24}, hence the crisp system to be solved becomes:

$$\begin{pmatrix} 4+2\alpha & -1-2\alpha & 0 & 0 \\ 1+\alpha & 5+2\alpha & 0 & 0 \\ 0 & 0 & 8-2\alpha & 2\alpha-5 \\ 0 & 0 & 4-2\alpha & 5+2\alpha \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} = \begin{pmatrix} 65+10\alpha \\ 40+10\alpha \\ 80=5\alpha \\ 60-10\alpha \end{pmatrix}$$

Step:4 Solving the crisp system we get the components of fuzzy vector as

$$^{\alpha}x_1 = \left[\frac{40\alpha^2 + 270\alpha + 365}{60\alpha^2 + 21\alpha + 21}, \frac{10\alpha^2 - 35\alpha + 700}{4\alpha^2 - 22\alpha + 60}\right]$$

$$^{\alpha}x_2 = \left[\frac{10\alpha^2 + 45\alpha + 95}{60\alpha^2 + 21\alpha + 21}, \frac{70\alpha + 80}{2\alpha^2 - 11\alpha + 30}\right]$$

Step:5 Since the conditions for the existence of the fuzzy solution is satisfied we can construct $\tilde{x}_1$ and $\tilde{x}_2$ using the relation, for j = 1,2 as

$$x_j = \bigcup_{\alpha\in[0,1]} \alpha \cdot {}^{\alpha}x_j$$

The fuzzy solution is as shown in Figure (3).

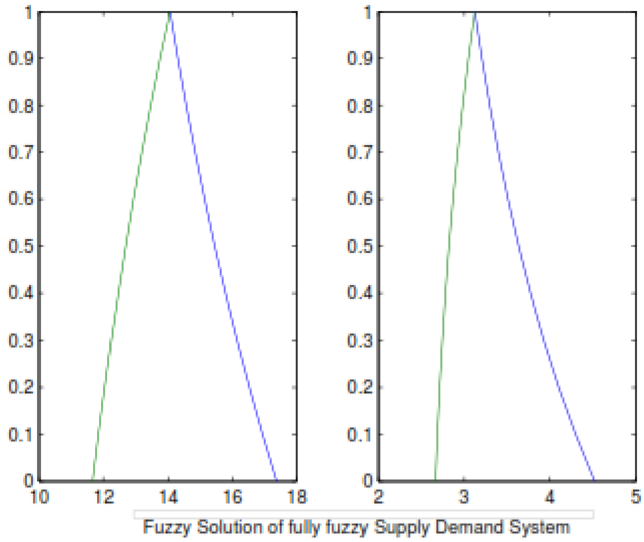

Figure 3: Fuzzy solution components

5.2 Example: 2

Consider the fuzzy supply-demand model with crisp coefficients and fuzzy right hand side as given below:

$$\tilde{x}_1 - \tilde{x}_2 = [3 + 2\alpha, 7 - 2\alpha]$$

$$\tilde{x}_1 + \tilde{x}_2 = [9 + 4\alpha, 17 - 4\alpha] \quad (4)$$

We first try to solve this system by QR decomposition method (Nasseri et al. 2011), for this from the above system (4) we obtain its crisp system as below:

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} = \begin{pmatrix} 3+2\alpha \\ 9+4\alpha \\ 7-2\alpha \\ 17-4\alpha \end{pmatrix}$$

which can be put in the forms

$$\begin{pmatrix} S_1 & S_2 \\ S_2 & S_1 \end{pmatrix} \begin{pmatrix} \underline{x} \\ \overline{x} \end{pmatrix} = \begin{pmatrix} \underline{b} \\ \overline{b} \end{pmatrix}$$

where, $S_1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $S_2 = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$.

To obtain the solution of (4) by apply the QR-decomposition method we should have $S_1 + S_2$ and $S_1 - S_2$ both to be non-singular. Since $S_1 - S_2$ is singular, the solution of (4) cannot be obtained by QR-decomposition method.

Next, we try solving (4) using the method by (Pandit 2013), following which we write
The system as:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} = \begin{pmatrix} 3+2\alpha \\ 9+4\alpha \\ 7-2\alpha \\ 17-4\alpha \end{pmatrix}$$

The obtained system is crisp system of the form $Ax = b$, with non-singular A, hence we can get $\underline{x}_i$ and $\overline{x}_i$ for I = 1,2. This gives the cut for unknown as ${}^{\alpha}x_1 = [3\alpha + 6, 12 - 3\alpha]$ and ${}^{\alpha}x_2 = [\alpha + 3, 5 - \alpha]$. Since the conditions given in the main Theorem are satisfied the fuzzy components can be constructed, which are as shown in Figure 4. Thus the solution technique by (Pandit 2013) is more general.

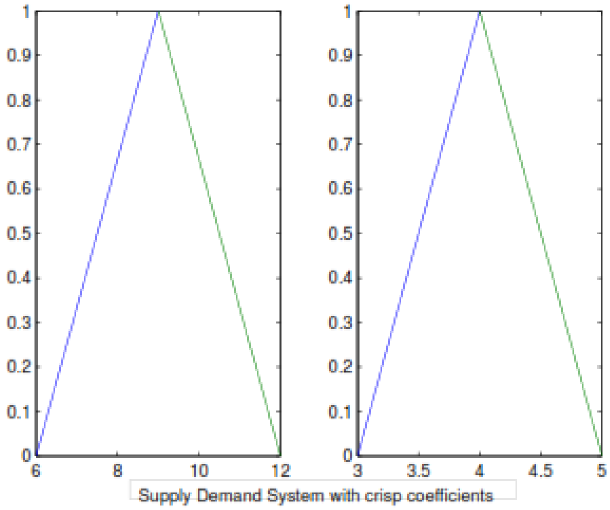

Figure 4: Fuzzy solution components

6 Conclusion

In this paper, we have proposed the supply-demand model as a system involving fuzzy parameters. Such system is solved by the technique by (Pandit 2013). The illustrative examples are given. In Example-2 we took the supply demand model similar to as by (Nasseri *et al.* 2011) and showed that the solution technique by (Pandit 2013) is more general and it can find the fuzzy solutions for the cases where QR-decomposition method cannot be applied.

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