Int. J. Adv. Appl. Math. and Mech. 10(3) (2023) 21 – 37 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Spinors, Poles, Space-Time Undulations, Torsion and Contour Integrals

Research Article

Prof. Dr. Francisco Bulnes[*]

IINAMEI Director,Investigación internacional Avanzada en Matemáticas e Ingeniería, Chalco, México

Received 17 November 2022; accepted (in revised version) 04 January 2023

Abstract: In the space-time the field sources are identified as fields ϕ_{AB} such that their integrals are Cauchy type integrals as the Conway integrals to any loop generated in the local causal structure of the space-time around of these fields. These integrals are solutions of the spinor equation associated to the corresponding twistor field equation. A theorem is enunciated on the evidence of field torsion as field invariant and geometrical invariant in poles of Cauchy type integrals in spinor-twistor frame. Sources are evidence at least locally of torsion existence. Then exists curvature here. Some conjectures and technical lemmas are obtained too.

MSC: 53C28 • 32L25 • 32S05 • 32S30

Keywords: Cauchy integrals • Conway integrals • Field Sources • Integral Transforms • Momentum-Space Functions • Penrose Transforms • Spinor Equation • Poles • Orbits • Torsion • Twistor Functions • Twistor spaces

1. Introduction

The Universe is a unfinished source of energy, because the Universe is itself the universal source of energy. In the interstellar process of the formation of the sidereal objects are observed different manifestations of the interactions of particles that give shape to the themselves sidereal objects and the existence to all field that is present in the space-time and give their causality and holonomy of this. Then if we consider the sources of these fields from a microscopic level using their spinor frame, we can characterize this sources as monopoles or multi-poles which could be used as sources in the space-time where the torsion can be obtained by the values of their integrals. However, beyond of these considerations we need establish the existence of a field observable related to the space agitation and the values of the field sources related with this space agitation (waves). A field observable that is studied in the electromagnetic fields with the interaction of matter of the space-time is the torsion, which in last studies, have been established strong relations with the CPT-violating, quantum gravity production and the twistor frame used in the study of the ruled surfaces to minimal surfaces in string theory and their moduli stacks. In this string theory is evident that each undulation (or string) can be viewed as a twistor in a duality defined in an integrable system [1].

After, studies related with strictly geometry of the physical stacks suggest that the sources are the poles that can conform a field carpet of the space-time always that the correspondence between twistor space and space-time stays to the equivalences determined through an integral operators cohomology between both spaces [2]. These are precisely the ideas of the twistor geometry of Penrose [3].

In the realistic space-time the existence of mono-poles is due to the torsion of the space where the curvature due to the form $F_{\mathscr{A}} \in SU(2)$, with connection $D_{\mathscr{A}} = d + \mathscr{A}$, is related with a Higgs field ϕ, through the field equation $\bigstar D_{\mathscr{A}}\phi =$

[*] *E-mail address(es):* francisco.bulnes@tesch.edu.mx

Nomenclature

M^I	Real affine Minkowski space when I, is the line defined by $\phi_\pm$, on $\mathbb{M}^\pm$, in $\phi_\lambda(x)$
$H^1_{\mathscr{L}}(U'',\mathscr{O}(-2))$	Cohomological space of first integrals of Conway type.
ϕ	Solutions or integrals of the equation (10).
ω^A,ω_A	Covariant and contra-variant spinors.
$Z_\alpha=(\omega^A,\pi_{A'})$	Twistor field.
$Z_\alpha=\omega_A-ix_{AA'}\pi^{A'}$	Associate spinor field to twistor field, called Robinson field whose flagpole is the Robinson congruence associated with the twistor.
π^A,π_A	Covariant and contra-variant twistors (or twistors-spinors).
$\phi_{A_1'\cdots A_{2h}'}(x)$	Spin field of spinors $A_1',\cdots,A_n^1$, with helicity h.
$f(Z)$	Homogeneous twistor function.
$\omega^{\alpha\alpha'}$	four vectors components of spinorial momentum.
$\phi_{A'B'}(x)=\frac{1}{2\pi i}\oint_\Gamma \pi_{A'}\pi_{B'}\mathrm{F}_h(Z)\pi_{B'}d\pi^{B'}$	4-dimensional case we have the Penrose transform to celestial sphere $\mathbb{CP}^1$.

$F_{\mathscr{A}}$, [4]. Then is natural consider the construction of monopoles from a twistor space $\mathbb{T}$, where these monopoles are viewed as duals of these twistors. From a point of view of the vector bundle, these poles will represent the zeros of the corresponding polynomials of the corresponding homogeneous lines bundle.

We consider E, as the associate rank 2 vector bundle to P, then we define the rank 2 vector bundle $\tilde{E}$, on the twistor space $\mathbb{T}$, as

$$\tilde{E}=\{s\in\Gamma(\gamma_p,E)\mid(i_U D_A-i\phi)s=0\} \tag{1}$$

,[1]

These monopoles have a magnetic nature and its spin bundle is the space:

$$\Theta^{A'}=\{\theta^{AA'}\in SL(2,\mathbb{C})\mid, d(\theta^{AA'})=(1/2)T^{AA'}\}, \tag{2}$$

However, how is possible to calculate around of sources or holes (in each case) the cohomological functionals of such luck that the integrals $\int\omega$, are contour integrals whose value is determined by the residue theorem and determines evidence of field torsion?

How to evidence the curvature of a Universe modeled as a complex Riemannian manifold with singularities interpreted these as sources through the value of its integrals? Can be generated loops from a causal structure of the space-time around of the fields "sources" that evidence said curvature?

The answer is yes, if we consider the fields “sources” inside the corresponding light cones having locally the field characterized as poles. Then loops are closed curves around of pole that represent co-cycles of spinor-waves arising from an electromagnetic field and identified in a Cauchy type integral. The values collection of these integrals (of said sources) are added to other integrals on space-time geodesics and determine the curvature of the space-time.

However, to talk of co-cycles we have that to identify a dual space where the spectrum of curvature can be identified as perturbations in the space-time from the energy states in a Hamiltonian manifold whose spectra are Higgs bosons [5] or any another particle type that represents a deformation in microscopic context of the space-time [6] as singularity.

Then we can to obtain measures in the singularities (that are poles in Cauchy integrals) as the value of cohomological functionals given by the residue value when certain class of Cauchy integrals are evaluated in these points. But, in twistor framework these poles as singularities of the complex Riemannian manifold, are represented as the dual elements called spinors, which can reveal torsion and curvature. Then twistor-spinor sources implies space-time undulations, which implies torsion and its contour integrals curvature values.

[1] *A special case we obtain when i = ϕ, then the homogeneous equation takes the form:*

$$(i_U D_A+1)s=0,$$

or equivalently along to curve $\Gamma(\gamma_p,E)\cong C^\infty(\gamma_p)$, (we consider a lines bundle with a flat connection):

$$\left(\frac{d}{dt}+1\right)s=0,$$

where solving said equation we have the solutions

2. Field sources as spinor fields in the space-time: arising the torsion

We consider a conductor in $\mathbb{M}$ defined for a spiral line, which determines induced current lines by a magnetic field on which is extended this spiral, for example, could be the part of a galaxy arm (see the figure 1B). We consider the conformal structure of the space-time where the spacelike infinity includes the spiral line mentioned (see the figure 1 A). Then considering different steps in timelike infinity and the increasing in spacelike we go conforming the object to its causal structure given by the set of light cones of the causal structure of the space-time (see the figure 1 B).

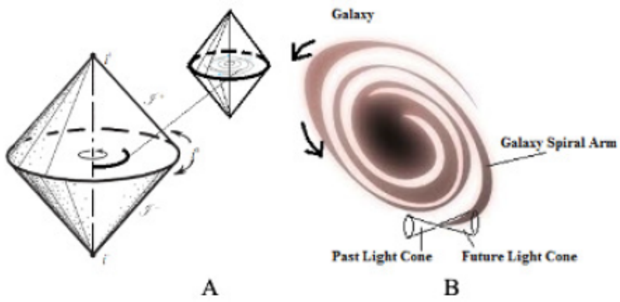

Fig. 1. A). Conformal structure of the space-time given by the Infinity. B). Model of the line source on the spiral arm of galaxy.

Then in the space-time infinity we have the twistor space as the points set[2]:

$$\mathbb{T} = \{Z^{\alpha} \mid Z^{\alpha} = (\omega^{A}, \phi_{A'})\}, \tag{3}$$

for all coordinates systems A, and A'. We define the twistor infinity tensor $I_{\alpha\beta'}$[3] as the obtained directly of whole space-time whose structure obeys a Minkowski space $\mathbb{M}$. Then the surface Σ, which is a 3-dimensional surface is resulted of the twistor fields Z^{α}, and Z^{β}, that is to say:

$$\Sigma = \Sigma(Z^{\alpha}, Z^{\beta}), \tag{4}$$

which has a metric defined when $\alpha = \beta$, and $Z^{\beta} = \bar{Z}^{\bar{\beta}}$, (its complex conjugate). Then in the infinity of the space-time, we have the sequence of mappings:

$$\mathbb{T} \xrightarrow{I^{\alpha\beta}} \mathbb{T}(S) \xrightarrow{I_{\Sigma}^{\alpha\beta}} \mathbb{T}(\Sigma), \tag{5}$$

whose correspondence rule is

$$Z^{\alpha} \mapsto I^{\alpha\beta} S_{\beta\beta'} Z^{\beta'} \mapsto I^{\alpha\beta} \Sigma_{\beta\beta'} Z^{\beta'}, \tag{6}$$

We consider the symmetric part of the fields Z^{α}, and Z^{β}, given by the spinors ω^{AB}, which satisfy the valence-2 twistor equation:

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[2] *$\omega^{A} : \mathbb{T}^{*} \to \mathbb{T}$, with rule of correspondence on points of the space-time $\pi_{A'} \mapsto i x^{AA'} \pi_{A'}$. Also its dual $\pi_{A'} : \mathbb{T} \to \mathbb{T}^{*}$, with correspondence rule of points of the space-time $\omega^{A} \mapsto -i x^{AA'} \omega^{A}$. Likewise, the corresponding twistor spaces in this case are:*

$$\mathbb{T} = \{Z^{\alpha} = (\omega^{A}, \pi_{A'}) \mid \omega^{A} = i x^{AA'} \pi_{A'}\},$$

and

$$\mathbb{T}^{*} = \{W_{\alpha} = (\pi_{A}, \omega^{A'}) \mid \omega^{A'} = -i x^{AA'} \pi_{A}\},$$

[3] *$I_{\alpha\beta} : \mathbb{T}^{*} \to \mathbb{T}$, with the correspondence rule $W_{\alpha} \mapsto Z^{\alpha} I^{\alpha\beta} W_{\beta}$.*

$$\nabla^{A}_{A'}\omega^{AB} = -i \in^{A(B} k^{C)}_{A}, \tag{7}$$

which has solution in a 10-dimensional space. We need bound to the space region of our study to spinor-waves in a 4-dimensional space, that is to say, on a component of (6). The solution space of (7) is spanned by spinor fields ω^{AB}, of the form[4]

$$\omega^{AB} = \omega_1^{(A}\omega_2^{B)} = \omega^{A}\omega^{B}, \tag{8}$$

where each ω_i^A, is a valence-1 twistor satisfying the equation:

$$\nabla^{A}_{A'}\omega^{B} = -i \in^{AB} \pi_{A'}, \tag{9}$$

We need in all time, for our measurements that the conservation condition given for the equation:

$$\nabla^{\alpha} T^{\alpha\beta} = 0, \tag{10}$$

exist, that is to say, we suppose that the energy-matter is always present in the space and is constant, at least in the space region where is bounded the 3-dimensional surface Σ [5] . Likewise, when exist a supermassive body that perturbs the space-time, the energy-matter of its tensor can be carried to (see the figure 2A and 2B):

$$A_{\alpha\beta} = \frac{1}{16\pi G}\int_{S} R_{AB}\omega^{A}_{\alpha}\omega^{B}_{\beta}, \tag{11}$$

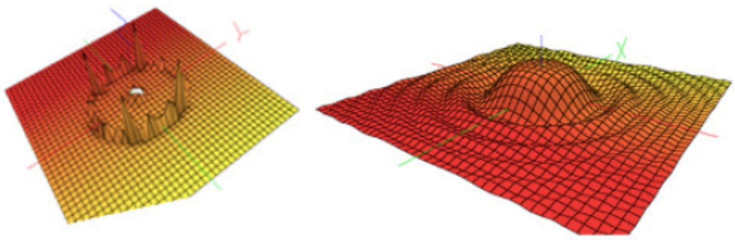

Fig. 2. A). Singularity and around waves for existence of torsion. B). Kinematic twistor tensor due to the energy-matter tensor perturbation of the supermassive body, which is determined on sphere $\mathscr{S}$. Then we can check and describe the screw effect [7] of twistor-spinors that generate the undulations in the two directions to this 2-dimensional model. These undulations reveal torsion in the space-time (see theorem [7]).

This shows that the space-time is undulated under presence of a supermassive body interpreted as source or singularity in terms of Riemannian geometry.
The equation (10) is the permanent energy-matter production.
Finally, we can establish the following commutative diagram of twistor spaces mappings on the gauging and detection mechanism of the torsion:

$$\begin{array}{ccccc} \mathbb{T}(\Sigma) & \xleftarrow{I^{\alpha\beta}\Sigma_{\beta\beta'}} & \Sigma & \xrightarrow{A_{\alpha\beta}} & (\mathbb{T}(S)\odot\mathbb{T}(S))^{*} \\ I^{\alpha\beta}_{\Sigma}\uparrow & & \uparrow T^{\alpha\beta} & & \uparrow A_{\alpha\beta}Z^{\alpha}Z^{\beta}, \\ \mathbb{T}(S) & \xleftarrow{I^{\alpha\beta}} & S & \xrightarrow{\omega^{AB}} & \mathbb{T}(S)\odot\mathbb{T}(S) \end{array} \tag{12}$$

where $\odot$, is a symmetric tensor product.

[4] *Here the spinors product* $\omega_1^{(A}\omega_2^{B)}$*, comes from fields product* $Z_1^{(A}Z_2^{B)}$*, which is a symmetric tensor product, that is to say,*

$$Z_1^{(A}Z_2^{B)} = Z_1 \otimes_{Symm} Z_2 \in \mathbb{T}\otimes_{Symm}\mathbb{T} = \mathbb{T}\odot\mathbb{T}$$

[5] $\Sigma(Z^{\alpha}, Z^{\beta}) = \Sigma.$

3. Torsion energy

The following result obtained in [6, 7], is the fundamental principle that is required to gauge and detect the torsion through of the tensor $A_{\alpha\beta}$,[6] considering the transformation law to pass from a field Z^{α}, to other Z^{β}, to two coordinate system α, and β, to transform the surface Σ, through the transformation law (taking the diagram (12)):

$$\Sigma_{\alpha\beta} = A_{\alpha\beta} I^{\beta\gamma} \Sigma_{\gamma\alpha'}, \tag{13}$$

Then we enunciate the following theorem.

Theorem 3.1.
(F. Bulnes, Y. Stropovsvky, I. Rabinovich). *We consider the embedding*

$$\sigma : \Sigma \to (\mathbb{T}(S) \otimes \mathbb{T}(S))^*, \tag{14}$$

The space $\sigma(\Sigma)$, is smoothly embedded in the twistor space $(\mathbb{T}(S) \otimes \mathbb{T}(S))^*$. Then their curvature energy is the energy given in the interval $M_N \geq A_{\alpha\beta} Z^{\alpha} I^{\beta\gamma} \bar{Z}_{\gamma} \geq 0$.

The before criteria established by the theorem, is an integral geometry criteria considering the values of the integrals of the contours around of singularities (holes or sources) which in the twistor-spinor framework will be of the form [8]:

$$\Psi = \int_{S^1} \frac{\partial}{\partial\omega^A} \frac{\partial}{\partial\omega^B} \frac{\partial}{\partial^C} \frac{\partial}{\partial\omega^D} \in_{AB} \in_{BC} \pi d\pi + \int_{S^1} \pi_A \pi_B \pi_C \pi_D \in_{A'B'} \in_{B'C'} d\pi, \tag{15}$$

which is deduced of the following natural conjecture, in $\mathbb{M}$, on a complex Riemannian manifold:
Conjeture(F.Bulnes)3.1 The curvature in spinor-twistor framework can be perceived with the appearing of the torsion and the anti-self-dual fields.

6. □

The integral (15) where are perceived these distortions with right-handed gravity is a superposition of the corresponding anti-self- dual fields on α and β- planes (see the figure 3).

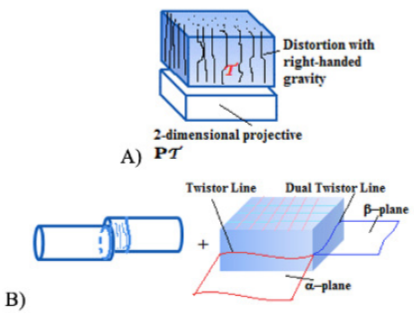

Fig. 3. Distortions with right-handed gravity by spinor $\Psi_{ABCD} + \tilde{\Psi}_{ABCD}(\alpha, \beta)$. Distorted tube more right-handed gravity given by (15).

For other side, the torsion energy is their curvature energy [9]. This can be obtained in the twistor context, considering the duality between twistors and spinors, where one is spectra of another, taking the fact that $A_{\alpha\beta}$, is dual to T^{ab}, (see proposition 1. 1, in [6]) where its spectra is obtained in the twistor space $\mathbb{T}(S) \otimes \mathbb{T}(S))^*$.
Likewise, considering that the curvature energy can be written through the energy densities obtained by twistor fields and its dual (the spinor frame), we can write the dominating energy condition given in the theorem 3. 1, as the integral:

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6 *Kinematic twistor tensor*

$$A_{\alpha\beta}\bar{I}^{\beta}_{\bar{\alpha}}Z^{\alpha}\bar{Z}^{\alpha} = \frac{i}{8\pi G}\int_{S}\bar{\pi}^{A}d\pi^{A'}\wedge\theta_{AA'}$$

$$= \frac{i}{8\pi G}\int_{\mathscr{H}} d\bar{\pi}_{A}\wedge d\pi_{A'}\wedge\theta^{AA'} - \frac{1}{2}G_{ab}\bar{\pi}^{A}\pi^{A'}X^{b}$$

$$\underbrace{\left(= \frac{i}{8\pi G}\int_{\mathscr{H}} d(\theta^{AA'})\right)}_{TorsionEnergy} - \frac{1}{2}G_{ab}\bar{\pi}^{A}\pi^{A'}X^{b}, \tag{16}$$

which is a total Hamiltonian, $\forall X^{b}$, a Hamiltonian vector density (or $\mathscr{H}-$ space element):

$$X^{b} = \frac{1}{6}\epsilon^{b}_{abc}\theta^{c}\wedge\theta^{d}\wedge\theta^{c}, \tag{17}$$

which is a Sparling 3-form mentioned in the footnote 9.
The first integral of (16) represents the torsion energy expressed through spin energy determined from spin bundle (2).

4. Microscopic deformations

Also in deformation theory the anti-self-dual complex space-time has correspondence in duality with the general Ricci-flat space $\mathbb{CP}^{1}$, where circles of the deformed tube have images in a $\pi-$ space. These deformed tubes could be geometrical representations of *2-dimensional superstrings* whose circles of their diameter are points of the infinite line or $\pi-$ space. Then the anti-self-dual complex space-time and the Ricci-flat space are equivalent to the parallelism for $\pi_{A}-$ spinors (locally), that is to say,

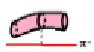
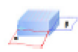
$$[\nabla_{AA'},\nabla_{BB'}]\pi_{C'} = 0, \tag{18}$$

taking place a curvature classification due the products of the summation indices [3, 6, 7]. Likewise, the curvature in Ψ_{ABCD}, represents the non-existence of holomorphic planes[7] in the twistor space to the tube (twistor tube) $\mathbb{C}\mathscr{T}$, then is required the twistor component due to the homogeneous degree -6, $\tilde{\Psi}_{A'B'C'D'}$, which involves a torsion energy (second curvature energy) and the Ricci-flat space condition.

The appearing of curvature needs of torsion as special factor of curvature detection in the deforming of the microscopic space-times in $\mathbb{M}$, is a condition of existence of curvature in these spaces, and in general in geometry[8]. Likewise, in [6] is obtained a particular solution, which could establish curvature in spinor-twistor terms through the second component of curvature given in (12).

Here the problem is to see the cause of second curvature to K_{abcd}, which are the elements mentioned before, then the conjecture 3. 1, is proceeding.

Then we have the superposition of two deformations with one component with two interaction planes. Likewise, in both components is considered the spinor fields Ψ_{ABCD}, and $\tilde{\Psi}_{ABCD}$, where the component $\tilde{\Psi}_{ABCD}$, is really the principal contribution of the distortions:

 $\Psi = \Psi_{ABCD}$  $+\tilde{\Psi}_{ABCD}$ 

[7] **Definition 4.1** *A $\beta-$ plane is a holomorphic plane in the twistor space $\mathbb{C}\mathscr{T}$.*
[8] *Of fact, curvature exists if torsion exists. However torsion no necessarily exist if curvature exists.*

But in the component Ψ_{ABCD}, also happens certain distortion understood as twistor waves with image in spinnors, where to the twistor function $f(z)$, the degree +2 has the infinitesimal shunt to wave-spinor $\hat{\omega}^A = \omega^A + \in \eta^{AB}\frac{\partial f}{\partial \omega^B}$, and $\hat{\pi}_A = \pi_A$, with vector field $\eta^{AB}\frac{\partial f}{\partial \omega^B}\frac{\partial}{\partial \omega^A}$, agreeing with the integral:

$$\Psi_{ABCD} = \oint \frac{\partial}{\partial \omega^A} \cdots \frac{\partial}{\partial \omega^D} \pi d\pi, \tag{19}$$

Then $\tilde{\Psi}_{ABCD}$ is incorporated as is signed in (19) using maybe with the differential form in major dimension (for example the Sparling 3-form[9]). But necessarily has that be incorporated in a 3-dimensional space which is inside an *energy states space* which will give an curvature energy state as a spectral density, possibly the image of an integral transform, or in the best case, a censorship condition to the detection and measure of first and second curvature considering the twistor-spinor waves used in the field framework [3, 6].

Likewise, with this spirit of ideas, will be necessary incorporate a 3-form of Sparling type to use the adequate Hamiltonian vector density where their $\mathscr{H}$, space[10] is equal to ASD-space-time whose non-linear graviton twistor space is the space $\mathbb{P}\mathscr{T}$, of twistor lines Z.

5. Poles, contour integrals, and twistor functions

We can create a 2D-numerical model of the screw effect and the re-interpretation in the cosmology objects as black holes or sources as stars or behavior intersidereal magnetic alignment of galaxies, using 2-dimensional complex surfaces considering the Morera's and Cauchy-Goursat's theorems [8] to be evaluated and can be applied in an numerical program. Likewise, for example, on singularities or poles in the space-time, we consider the space-time a complex Riemannian manifold with singularities. This could represent the surface of the real part of a function $g(z) = \frac{z^2}{z-1}$. The moduli of this point is at least 1 and thus lie inside one contour. Likewise, the contour integral can be split into two smaller integrals using the Cauchy-Goursat theorem having finally the contour integral [12] $\oint_C g(z)dz = \oint_C (0 - \frac{1}{z-z_1})dz = 0 - 2\pi i = -2\pi i$, (see the figure 4)).
Likewise, this value is a traditional cohomological functional element of $H^f(\prod - \ell', \Omega^r) = \mathbb{C}$. This element is a contour around the singularity as can be viewed in the figure 4).

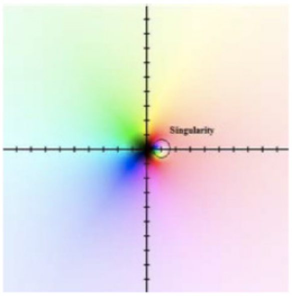


Fig. 4. Pole or singularity of the complex function $g(z) = \frac{z^2}{z-1}$. The surface folds around of this singularity or pole.

We consider the real space-time $\mathbb{M}$, as real component of complexified Minkowski space $\mathbb{CM}$, represented as (complex projective) lines in $\mathbb{PT}$, which will be all points of the complexified compactified Minkowski space $\mathbb{CM}^\#$. Those

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[9] *3-dimensional form* θ, *whose expression in* $\mathbb{M}$, *is:*

$$\theta = \frac{1}{6} \in_{\alpha\beta\gamma\delta} Z^\alpha dZ^\beta \wedge dZ^\gamma \wedge dZ^\delta$$

$$= Z^0 dZ^1 \wedge dZ^2 \wedge dZ^3 - Z^1 dZ^0 \wedge dZ^2 \wedge dZ^3$$

$$+ Z^2 dZ^0 \wedge dZ^1 \wedge dZ^3 - Z^3 dZ^0 \wedge dZ^1 \wedge dZ^2.$$

[10] *For some new developments relating the* $\mathscr{H}$, *space to equations of motion, see [41].*

lines that lie in $\mathbb{PT}$, represent points of the real space-time $\mathbb{M}$ (possibly at infinity), but since these lines are still complex projective lines, they are indeed Riemann spheres [3] (see the figure 5).

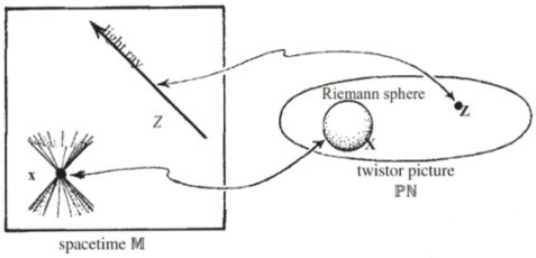

Fig. 5. Twistor correspondence: a ray Z in Minkowski space $\mathbb{M}$ corresponds to a point in $\mathbb{PN}$; a point x of $\mathbb{M}$ corresponds to a Riemann sphere S, in $\mathbb{PN}$.

Let γ, be a curve or geodesic of space-time in M^I, parameterized as $y^a(s)$, with s, the proper time. The geodesic could be a current line produced by the intersidereal magnetic field on which is extended a spiral arm of the galaxy, for example, although here has a context more general. Let $\rho(s)$, be their density source satisfying

$$\Box\phi = 4\pi\rho(s), \tag{20}$$

with

$$\rho_\gamma(s) = \int \rho(s)u(y(s))ds, \tag{21}$$

Then by causality of the space-time and convexity of the light cones corresponding to this causality, (20) has the basic solution ϕ_λ in terms of the distances r_{ret}, and r_{adv}, given for:

$$\phi_\lambda(x) = \lambda\rho(s_{\text{rect}}(x))/r_{\text{rect}} + (1-\lambda)\rho(s_{\text{adv}}(x))/r_{\text{adv}}, \tag{22}$$

If ϕ_λ, is analytically continued in M^4, (complex Minkowski space) assuming that ρ, and γ, are analytics, then ϕ_λ, changes to

$$\hat{\phi}_\lambda(x) = -\lambda\rho(s_{\text{adv}}(x))/r_{\text{adv}} - (1-\lambda)\rho(s_{\text{ret}}(x))/r_{ret}, \tag{23}$$

where $\hat{\phi}_\lambda(x) = \phi_{1-\lambda}(x)$, with density $-\rho(s)$.

In the process of analytic continuing have been elected open orbits B^+, and B^-, such that in $B^+ \cap B^-$, we have $\phi_\lambda(a) = \phi_\lambda(b)$, with $a-b$, the diameter of $B^+ \cap B^-$, (see the figure 6). Then the required description in the solutions given to (20) are the elements of the space of integrals [10]:

$$H^1(B^\pm, \mathcal{O}(V)) = H^1(B^+, \mathcal{O}(V)) \oplus H^1(B^-, \mathcal{O}(V)), \tag{24}$$

But by the description of the Penrose massless fields [10-12], we have that $B^\wedge = B^+ \cup B^-$, (where $B^\wedge$, is some complex neighborhood of M^I) is a double covering $B\backslash B^+ \cup B^-$. This last property permits have a certainty topology to use the complex cohomology of line integrals [10]. Then is wanted to give a description of the massless fields (twistor functions)[11] of the field equation in the space-time with $T^{ab} = 0$, [12-14] that is to say, solutions of the twistor equation through lines bundle Λ, belonging to the complex sheaf $\mathcal{O}(\Lambda)$, of the analytic continuing in B, that is to say, solutions of

[11] *Here the twistor functions represent the massless fields on ruled surfaces in $\mathbb{P}^3(\mathbb{C})$. The idea to explain these fields of this form is to can manipulate in the algebraically way the singularities or sources of these fields through zeros of homogeneous polynomials of lines in M^1.*

$$\nabla^{A'(A}\omega^{B)} = 0, \qquad \omega^B \in O_B(-k), \tag{25}$$

To each ω_α^A, there is a primed spinor $\pi_{\alpha A'}$, such that

$$d\omega_\alpha^A = -\pi_{\alpha A'}\theta^{AA'}, \tag{26}$$

But we want a special line, we want the line $\gamma + L \in \mathscr{O}(\Lambda)$. This helps to understand that the sources of the field are the poles in $s_{\text{rev}}(x)$, with multiplicity $-\lambda$, and $s_{\text{adv}}(x)$, with multiplicity $1-\lambda$. To it, for a resulted in [13], is wanted the description of H^1 -functions whose evaluating is in the cohomology of contours $H^1(\Pi - \coprod, \mathscr{O}(V))$.

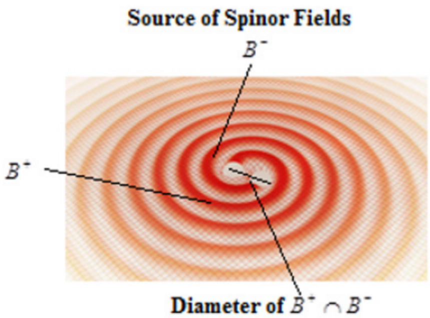

Fig. 6. Spinor fields source in the sidereal element of the space-time.

We consider the spin group of complex Minkowski space given by $SO(2,4)$, whose Lie algebra $\mathfrak{so}(2,4)$, is isomorphic to $SU(2,2)$. Likewise, electromagnetic waves in conformal actions of the group $SU(2,2)$, on a dimensional flat model of the space-time, can be described to two auto-dual Maxwell fields of positive and negative frequency. In this case we can use structures of light cones and the spinor framework to obtain that conformal theories of gauge fields as electromagnetic fields and measure other fields, for example gravity.

We call twistor function $f(Z^\alpha) = \frac{1}{A_\alpha Z^\alpha}$, $\forall Z^\alpha \in \mathbb{T}$. Here A_α, defines a line or ray directed by the field Z^α, in the twistor space.

Lemma 5.1.
Actions of $SU(2,2)$, act on $\mathbb{M}$, to generate twistor functions.

Proof. We consider the Penrose anti-transform of □

$$\phi_{A'B'\dots C'}(x) = \frac{1}{2\pi i}\oint_\Gamma \pi_{A'}\cdots\pi_{C'}[\rho_x f(Z^\alpha)]\pi_{E'}d\pi^{E'}, \tag{27}$$

where ρ_x, denotes restriction to the celestial sphere of space-time point $x^{AA'}$, in $\mathbb{PT}$. Then in the space-time the structure of light cones happen (as is established in the figure 7) where the two cones (future and past light cones) can be characterized, consider a pair of homegeneity -4 twistor functions $\{f_{-4}^{(1,2)}\}$, which will necessarily correspond to electromagnetic solutions $\phi_{A'B'}^{(1,2)}$, in space-time by the integral(28). Combinations of them with a homogeneity -2 function (corresponding to a space-time scalar ϕ) to make a homogeneity -6 function can be generated to give solutions in gravity. □

The before lemma establishes a (linearized) gravity solution $\phi_{A'B'\dots C'}$, in space-time, implying an important space-time relationship between electromagnetic scalar and gravitational fields. This relation of space-time can be described through torsion considering the poles as field sources. Then here can be incorporated the established: the sources of the field are the poles in $s_{\text{rev}}(x)$, with multiplicity $-\lambda$, and $s_{\text{adv}}(x)$, with multiplicity $1-\lambda$, and described these poles by a cohomology of contours. Then we have integrals of Cauchy type to their evaluation. The values corresponding to the dipoles can be obtained by these integrals, which are of Cauchy type.
Likewise using the material of the past section, we recall the Conway integral as those integrals of the form [14]:

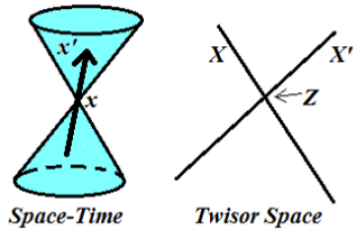

Fig. 7. $\mathbb{M} \cong \mathbb{CP}^3$.

$$\phi(x) = \frac{1}{\pi i} \oint \frac{\rho(s)ds}{(x - y(s))^2}, \tag{28}$$

which yields the solution $\phi_\lambda(x)$, described in (22) as the "contours" (cohomological functionals with integrals $\int \omega$) used surrounds the pole at $s_{\text{ret}}(x)$, with the multiplicity $-\lambda$ and the pole as $s_{\text{adv}}(x)$, with the multiplicity $1-\lambda$. That to the fields represented by (28) in the Higgs fields[12] .

Indeed, we have spin solutions in the spin bundle $\Theta^{A'}$, which are the magnetic monopoles. These solutions are achieved in the space defined in (1).

Then one obtains a twistor function which represents the field $\phi_\lambda(x)$, in the following way: Consider the function ξ^A, of the parameter s, and the twistor $(\omega^A, \pi_{A'})$, defined as:

$$\xi^A(s) = \omega^A - iy^{AA'}(s)\pi_{A'}, \tag{29}$$

Then ω^A, can form a twistor function $f(\omega^A, \pi_{A'})$, by performing the contour integral

$$f(\omega^A, \pi_{A'}) = \frac{1}{\pi i} \oint \frac{\alpha_A \beta^A \rho(s)ds}{\alpha_B \xi^B(s) \beta_C \xi^C(s)}, \tag{30}$$

where $\alpha_A \beta_A$, are arbitrating fixed spinors. f, is homogeneous of degree -2, and one recovers the Conway integral form by use of the s.

One expects any massless field on M^I/γ, to be the same of a source-free field and a field which can be expanded in terms of the multi-pole fields justly described in the cohomological space $H^1_{\mathscr{L}}(U'', \mathscr{O}(-2))$, (special case when $h = 0$) when $\mathscr{L}$, is a part of the ruled surface inside the world line in the causal structure of the space-time.

Theorem 5.1.
The twistor function (30) involves torsion through the poles of its contour integral.

Proof. ω^A, is a 1-spinor derived of twistor equation (29), where the contour integral takes the form □

$$f(\omega^A, \pi_{A'}) = \frac{1}{\pi i} \oint \frac{\alpha_A \beta^A \rho(s)ds}{\alpha_B(\omega^B - iy^{BB'}(s)\pi_{A'})\beta_C(\omega^C - iy^{CC'}(s)\pi_{A'})}, \tag{31}$$

But f, is homogeneous of degree -2, then we have

$$f(\omega^D, \pi_{D'}) = \frac{1}{\pi i} \oint \frac{\rho(s)ds}{(\omega^D - iy^{DD'}(s)\pi_{D'})^2}, \tag{32}$$

[12] *Of fact the denominator of its rational function complies in $\mathbb{R}^3$, the formulation $\lambda(1 - |\phi|^2)^2$, in the Higgs fields.*

which is a Conway integral (28) that express a dipole of field. Of fact this is a Čech co-cycle to an element of the relative cohomology group $H^1_{\mathscr{L}}(U,\mathcal{O}(-2))$, where to a complexified world-line $y^a(s)$, corresponds in twistor space to a piece $\mathscr{L}$, of the ruled surface where the ruling lines correspond to the points of the world-line. Then (32) yields sources $\phi_\lambda(Z^D)$, with multiplicities λ, and $(1-\lambda)$, in each case of the light cones structure (see the figure 7).

Now, we need demonstrate that the integral (32) involves a *torsion indicium* deduced from only microscopic undulation of the space-time given by its twistor to local coordinates systems.
Likewise the poles are the co-cycles of the twistor function considering its integral transform. This does it an integral of Cauchy type, as the Conway integral, or another (Penrose transform, Twistor transform). However, how we can evidence the torsion through the poles?
We will use the torsion result in [9], which establishes formally that:

$$\tau(t_1,t_2) = \pm 2\pi \cot\Psi \mathscr{F}\{K(t_1,t_2)\}\delta(t_1,t_2), \tag{33}$$

that is to say, in a pole spectra there is a delta function [9, 15] that acts as screw effect producing undulations in the space-time. This evidences the torsion. We need transform to the spinor-twistor context the functional relation, having as a momentum-space function[13] $f(\bar{\pi}_\alpha,\pi_{\alpha'})$. If we choice the curvature energy as such function then we have:

$$k(\omega^\alpha,\pi_{\alpha'}) = \int_{\infty}^{-\infty} K(\bar{\pi}_\alpha,\pi_{\alpha'})e^{-\omega^0\bar{\pi}_0-\omega^1\bar{\pi}_1}d\bar{\pi}_0 d\bar{\pi}_1,$$

For it we consider coordinates systems in the momentum-space given by $(\bar{\pi}_\alpha,\pi_{\alpha'})$, and its spectra will be a twistor space whose coordinate system is $(\omega^\alpha,\pi_{\alpha'})$, where this last, involves spinors ω^α, which define the undulations. We have that advertise that in the poles there are delta functions that represents the sources in the screw effect[14] in the space-time [9]. Then for the Lancret criteria, the curvature and torsion to $\alpha(s)$, complies $\frac{\tau(s)}{k(s)} = c$, $c = \cot\psi$. Then

$$k(\omega^\alpha,\pi_\alpha) = c\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\omega^0\bar{\pi}_0-\omega^1\bar{\pi}_1}K(\bar{\pi}_\alpha,\pi_{\alpha'})d\bar{\pi}_0 d\bar{\pi}_1, \tag{34}$$

and how its torsion energy is its curvature energy then

$$\begin{aligned}\tau(\omega^\alpha,\pi_{\alpha'}) &= \frac{c}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2\pi\mathscr{F}\{K(\bar{\pi}_\alpha,\pi_{\alpha'})\}\delta(\omega^\alpha,\pi_{\alpha'})d\bar{\pi}_0 d\bar{\pi}_1\\ &= \frac{2\pi c}{2\pi}2\pi k(\omega^\alpha,\pi_{\alpha'})\delta(\omega^\alpha,\pi_{\alpha'})\\ &\pm 2\pi\cot\psi\, k(\omega^\alpha,\pi_{\alpha'})\delta(\omega^\alpha,\pi_{\alpha'}),\end{aligned} \tag{35}$$

which evidences the existence of torsion in a twistor function. Therefore the theorem is proved. □

[13] *Theorem. The Fourier transform of a momentum-space function $f(\bar{\pi}_\alpha,\pi_{\alpha'})$, is the integral transform:*

$$\begin{aligned}f(\omega^\alpha,\pi_{\alpha'}) &= \frac{1}{(-2\pi i)}\int_{-i\Gamma_\sigma} f(\bar{\pi}_\alpha,\pi_{\alpha'})e^{-\omega^0\bar{\pi}_0-\omega^1\bar{\pi}_1}d\bar{\pi}_0 d\bar{\pi}_1\\ &= \sum_{a,b=0}^{\infty} C_{ab}(\bar{\pi}_\alpha,\pi_{\alpha'})\frac{\mathrm{sgn}\sigma\Gamma(a+1)\Gamma(b+1)}{(-2\pi i)^2(\omega^0)^{a+1}(\omega^1)^{b+1}},\\ &\forall(\omega^0,\omega^1)\in\mathbb{C}^*\times\mathbb{C}^*.\end{aligned}$$

[14] *The appearing of waves in the space-time agitation when a field acts from a microscopic level in the existence of energy in the space produces the screw effect.*

6. Contour sources, iso-rotations and twistor spaces inside space-time with singularities

The iso-rotation of all sidereal object in the material level frame provokes the drag of materials and their composing by the twistor behavior is due to magnetic fields by MHD[15-18] - actions. However, what happens in the massless field where the material actions come from microscopic sources and without mass or matter presence?
These can be detected for appearing of the poles or singularities, which can be considered rips of the space-time due an extra force.
This extra force has as field observable the torsion, and this can be detected by waves and is evidenced formally by spinors. This was demonstrated formally in the theorem 5.1, of this work.
Likewise, from a point of purely cosmological view, the existence of these singular points reveals the existence of sidereal objects where big quantities of flow (using Poincarè arguments) of energy are expulsed or/and attracted along the space-time as twistors satisfying the geometrical Penrose model of a black hole. The torsion as field observable always is present. Since, the spins s and $-s$, corresponds to different field interactions whose images can correspond to the twistor spaces $\mathbb{P}^+$, and $\mathbb{P}^-$.

Here we can incorporate the following cohomology space that re-interprets field theory objects with geometrical objects considering the Universe as a complex Riemannian manifold (topological space) to a field source:

$$H^1(\mathbb{P}\mathcal{T}, O(-2h-2)) \cong \ker(U, \Box_{h(k)}) = \{\phi \in C^2(U) \mid \Box_{h(k)}\phi = 0, \text{in } U \subset \mathbb{M}\}, \tag{36}$$

The field torsion results evident with some work on twistor-spinor framework [13, 19].
A 2-spinor has a phase that is geometrically described by a null flag half-plane, where if the flag plane is rotated about the flagpole through the spinor changes sign then is generated torsion, considering the monopoles that can to appear are a good indicium of the existence torsion. However, how prove the indicium of torsion through only undulations in the space-time?

The twistor behavior is an effects that born from torsion fields (from a microscopic level) which have the helicities that prescribe monopoles that in the causal structure of the space-time (that is to say, with their structure of light cones) these take the form of contours (see the figure 8).

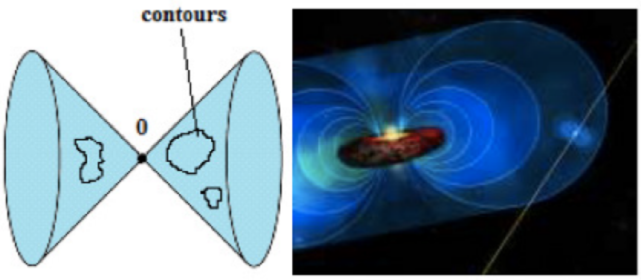

Fig. 8. The contours used to calculate the singular integrals are the sources inside the intersidereal rotations magnetic field. Strong magnetic field around of the singular center of a galaxy, for example. In a side from the center there is a source of electromagnetic energy whose space is characterized as twistor.

Being these contours resulted of sources determined by fields of torsion which are of electromagnetic nature, we establish the following result:

Definition 6.1.
An iso-rotation is a rotation in a plane (around of axis perpendicular to the rotation plane).

Conjecture (F. Bulnes) 6. 1. Any source or hole that displaces and rotates around its axis in the space-time generates torsion.

Proof. To demonstrate this we need to establish rotations in any plane of a source or hole that is displaced on its axis and generate spinors in a 3-dimensional space, which will prove the torsion existence[15]. Of fact there is a correspondence between torsion and spinors established by their tensors. Indeed, we consider the spinorial momentum relations to four-vector components (that is to say, considering a fix axis): □

$$\omega_{\alpha\alpha'} = \pi_\alpha \bar{\rho}_{\alpha'} + \rho_\alpha \bar{\pi}_{\alpha'}, \tag{37}$$

$$\omega_{\alpha\alpha'} = \pi^\alpha \bar{\rho}_{\alpha'} + \rho_\alpha \bar{\pi}_{\alpha'}, \tag{38}$$

Then we have its product:

$$\omega_{\alpha\alpha'}\omega^{\alpha\alpha'} = \pi_\alpha \pi^\alpha \bar{\rho}_{\alpha'}\bar{\rho}_{\alpha'} + \rho_\alpha \bar{\rho}_{\alpha'}\bar{\pi}_{\alpha'}\pi^\alpha +$$

$$\pi_\alpha \bar{\pi}^{\alpha'} \bar{\rho}_{\alpha'}\rho_{\alpha'} + \rho_\alpha \rho_{\alpha'}\bar{\pi}_{\alpha'}\bar{\pi}^{\alpha'}, \tag{39}$$

where using the facts

$$\pi_\alpha \pi^\alpha = \bar{\pi}_{\alpha'}\bar{\pi}^{\alpha'} = 0, \qquad \pi_\alpha \bar{\pi}^{\alpha'} \bar{\rho}_{\alpha'}\rho_\alpha = -\pi_\alpha \bar{\pi}^{\alpha'} \rho_{\alpha'}\rho_\alpha, \tag{40}$$

and the commuting Weyl spinors

$$\rho_\alpha, \bar{\rho}_{\alpha'} = (\rho_\alpha)^*, \qquad \pi_\alpha, \bar{\pi}_{\alpha'} = (\pi_\alpha)^*, \tag{41}$$

Then

$$\omega_{\alpha\alpha'}\omega^{\alpha\alpha'} = -2\rho_\alpha \pi_\alpha \bar{\pi}^{\alpha'} \bar{\rho}_{\alpha'} = -2\pi^\alpha \rho_\alpha \bar{\rho}_{\alpha'} \bar{\pi}^{\alpha'}$$

$$= -2 \mid \pi^\alpha \rho_\alpha \mid^2 = -2S, \tag{42}$$

But by the footnote 16, we have:

$$\omega_{\alpha\alpha'}\omega^{\alpha\alpha'} = -2S = -2(\frac{1}{2})T = -T, \tag{43}$$

which evidences the torsion T.□

Lemma 6.1.
(F. Bulnes). The torsion is always present through a spin connection.

Proof. Let $\theta^{AA'}$ a spin connection on a $\Theta^{A'}$ spin bundle. □

---

[15] *As the field torsion is the product of interaction of two fields, gravitational field and electromagnetic field, we have in general that:*

$$\tau^\gamma_{\alpha\beta} = 2S^\gamma_{\alpha\beta},$$

which due to that $S^\gamma_{\alpha\beta} = \chi^{CC'}_{AA'} \in_{A'B'} + \tilde{\chi}^{CC'}_{A'B'} \in_{AB}$, *we have the torsion*

$$\tau^\gamma_{\alpha\beta} = 2(\chi^{CC'}_{AA'} \in_{A'B'} + \tilde{\chi}^{CC'}_{A'B'} \in_{AB}).$$

This connection is a spin connection of connected Riemann spin manifold (M^n, g), with $n = \dim M \geq 3$. If exists curvature then not necessarily exists torsion. This coincides with the fact that if the spin $s = 0$, then torsion is equal to zero. No being this to the curvature. If exists torsion then exists curvature, where always $s \neq 0$, and its connection on the spin bundle $\Theta^{A'}$, does remains the torsion to any spinor π_A.

Indeed, we consider the spinor of order $s = 2$, with intention to obtain curvature

$$S^{\gamma}_{\alpha\beta} = \chi^{CC'}_{AA'} \in_{A'B'} + \tilde{\chi}^{CC'}_{A'B'} \in_{AB}, \tag{44}$$

(which was mentioned in the section V, of this paper), where $\chi_{A'B'}$ and $\tilde{\chi}_{A'B'}$, are symmetric in the coordinate systems AB, and $A'B'$. Likewise, re-written the spinor equation (44), to torsion, in the twistor-spinor framework[16] [6] we have

$$\omega^{AA'} = \nabla_{AA'}\pi_{B'} = -i \in^{AB} \pi^A, \tag{45}$$

Then a twistor evaluated in a spinor of 2-valence takes the form[17] :

$$\pi^{A'}\omega^{AA'} = \pi^{A'}(\nabla_{AA'}\pi_{B'}) = \pi^{A'}\omega^A\bar{\omega}^{A'}(i\omega^B\pi_{B'})^{-1}$$

$$= \pi^{A'}q^{AA'}, \tag{46}$$

But we consider that in each position vector x^a, is had that:

$$x^a = X^{AA'} + k\pi^{A'}\bar{\pi}^A = X^{AA'} + kp^a, \tag{47}$$

Then $p^{AA'} = k^{-1}(x^a - X^{AA'})$[18]. Then we have in the equation (26) that through twistors we can write $q^{AA'}$,

$$p^a = k^{-1}(x^a - X^{AA'}) = (i\omega^{B'}\pi_{B'})^{-1}(x^a - X^{AA'}), \tag{48}$$

due to that $\tilde{p}^a = q^{AA'}$, in an invariant way, we have:

$$\pi^{A'}(\nabla_{AA'}\pi_{B'}) = \pi^{A'}\left[\frac{1}{i}\frac{\omega^A\bar{\omega}^{A'}}{Z^\beta}\right], \tag{49}$$

Then we consider the following spinors:

$$\omega^A = \xi^A + iy^{AA'}\pi_A, \qquad \bar{\omega}^{A'} = \bar{\xi}^{A'} - iy^{AA'}\bar{\pi}_{A'}, \tag{50}$$

For another side (49) takes the form

$$\pi^{A'}(\nabla_{AA'}\pi_{B'}) = (\omega^A - iy^{AA'}(s)\pi_{A'})\pi_{B'}$$

$$-2\pi^{A'}\pi^{C'}\chi_{A'B'AC'}, \tag{51}$$

which gives

$$\pi^{A'}(\nabla_{AA'}\pi_{B'}) = \xi_A\pi_{B'} - 2\pi^{A'}\pi^{C'}\chi_{A'B'AC'}, \tag{52}$$

---

16 *We use the twistor equation* $\nabla^A_{A'}\pi^B = -i \in^{AB} \pi_{A'}$.

17 $q^{AA'} = \omega^A\bar{\omega}^{A'}(i\omega^B\pi_{B'})^{-1} = \frac{1}{i}\frac{\omega^A\bar{\omega}^{A'}}{Z^\beta}$.

18 $p^a = \pi^A\bar{\pi}^{A'}$.

which proves that in a contour with field state $\phi_\lambda(Z^E)$, can be given as the spinor products (which appear in spectral image of the momentum-space[19]):

$$\pi^{A'}(\xi^A + iy^{AA'}\pi_A)(\bar{\xi}_{A'} + iy^{AA'}\bar{\pi}^{A'})$$

$$= \pi^{A'}(\xi^A\bar{\xi}_{A'} + \xi^A iy^{AA'}\bar{\pi}^{A'} + \bar{\xi}_{A'} iy^{AA'}\pi_A - y^{AA'}\pi_A y^{AA'}\bar{\pi}^{A'}),$$

Having the condition of that the anti-self-dual complex space-time and the Ricci-flat space are equivalent to the parallelism given through π_A- spinors (locally) that are had with the formalisms (18) with (52), we have the torsion through the integrability condition (18):

$$[\nabla_{C(A'}\nabla^C_{B')} - 2\tilde{\chi}_{A'B'}\nabla_{HH'}]\pi^{C'} = \phi^{C'}_{A'B'E'}\pi^{E'} -$$

$$2\tilde{\Omega}\pi_{(A}\in_{B')} C' + \nabla_{A'B'}\pi^{C'}, \tag{53}$$

where is clear the appearing of torsion in the terms $\phi^{C'}_{A'B'E'}\pi^{E'} - 2\tilde{\Omega}\pi_{(A}\in_{B')} C'$, and the integrability condition to $\alpha-$ surfaces is appeared too, considering $\lambda_A\lambda^A = \pi_{A'}\pi^{A'} = 0$.
Then by the conjecture 3. 1, a total spinor field that detects distortions due to curvature existence in the microscopic level staying described as (15).
Then the spin connection necessarily has torsion component, since the metric connection differs from the Levi-Civita connection by a quantity which is determined by the torsion tensor. We remember that being $\Theta^{A'}$, a spin bundle, we can define a spin connection ∇^s, of the connected Riemannian manifolds $(M^n, g), (n = \dim M \geq 3)$, endowed of a 3-form T, called torsion such that the metric connection ∇^g, is related as:

$$\nabla^s = \nabla^g + 2sT = \nabla^g + d(\theta^a), \tag{54}$$

where by the conjecture 6. 1, $d(\theta^a)$, is a spin.□

7. Conclusions

If we consider the multi-poles as the sources of the fields of different nature of the space-time (of fact their moduli stack is obtained by equivalences in field theory using some *gerbes* of derived categories), we can to use the loops around of these poles as contours of the cohomological functionals. Likewise, these cohomological functionals establish classes of contour integrals that determine and evaluate through the residue theorem and other as the Morera's and Cauchy-Goursat's theorems where from a point of view of the physics are the value of the states of a field. These values can be registered to design and determine spinor waves which, considering the covariant nature of these invariant objects and the local structure of the space-time, carry us to integrability conditions from the curvature of the space-time and therefore its torsion as second curvature.
In this work has been related three fundamental aspects of the space-time, this modeled as a complex Riemannian manifold including singularities as poles of corresponding twistor functions, and complex momentum-space functions; where the poles, spinors and torsion are fundamental geometrical invariants, which has been demonstrated along the paper that these aspects are related in duality, equivalence and realization (of its corresponding representation through twistor framework). These relations were proved in the theorem 5. 1. Likewise, any spin pair s, and $-s$, can generate rotations in a local region $\mathcal{Q}$, of the space-time $\mathbb{M}$, considering the spinors as undulations from electromagnetic field acting on the space. In this paper has been proved that a sufficient condition to the torsion existence is the existence of spinors. In another work [17], and using the construction of monopoles from twistor space, has been demonstrated fundamentally that

$$\{\pm 1\} \to \mathrm{Spin}(\mathcal{Q}) \to SO(\mathcal{Q}),$$

where $\mathrm{Spin}(\mathcal{Q}) \to GL(\mathcal{Q})$, which could demonstrate the necessity, that is to say, to start with spinors to obtain rotations which are effects of torsion.

[19] *Appearing as the spectra*

The Conway integrals can be considered in axisymmetric boundaries and also non-axisymmetric cases where gravity aspects are confined within axisymmetric boundaries, for example locally . The potential and attractions in the classical gravitational fields for the elementary thin disc can be given in closed form in terms of elliptic integrals and elementary functions. However, if we consider the electromagnetic nature of the iso-rotations for fields as sources, we need other formulism based in twistor geometry, where elliptic integrals are analogues in the space $H^1_{\mathscr{L}}(U'', \mathscr{O}(-2))$, that is to say, to purely electromagnetic fields. Then the cohomological analogous are "poles" which can be interpreted as "sources" of field (see the figure 9). Then is had the conjecture 6. 1. Finally, and considering the microscopic deformations of the space-time, we can conclude that the torsion is always present through a spin connection. Important representations of momentum-space functions are obtained through spectra in twistor space, which evidences the torsion as element of the twistor function.
The fields obtained by the contour integrals around of sources represent the fields induced by another field (in this case could be the gravity) in an extended region of the space-time. This is foreseen in theoretical physics and fine electronics experiments to field torsion.
Here the integrals given by (19) are due to the fields of electromagnetic type (Actions of $SU(2,2)$, act on $\mathbb{M}$). If we want involve the gravitational sources we need an integral of the form:

$$\phi_{A_1 \cdots A_{2h}}(x) = \frac{1}{2\pi i} \oint_{\Gamma} \pi_{A'_1} \cdots \pi_{A'_{2h}} F_h(Z) \pi_{B'} d\pi^{B'} \tag{55}$$

where the twistor function $F_h(Z)$, could correspond to a curvature function or related with gravity then the torsion appears.

Fig. 9. The cohomological analogous are poles which can be interpreted as "sources" of electromagnetic radiative energy. The spinor formalism could be used through the responsible electromagnetic energy of the accretion and iso-rotation of a sidereal object as a galaxy, interacting with matter, for example.

Acknowledgements

I am very grateful with Rene Rivera Roldan, Director of Electronics Engineering Program in TESCHA, for your hours support and understanding of the importance of this research to mathematics and field theory research.

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