Int. J. Adv. Appl. Math. and Mech. 10(3) (2023) 78 – 88 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

On the Adomian Decomposition Method for the Analytical Modelling of Steady State Condensation Film in an Inclined Rotating Disk

Research Article

Liberty Ebiwareme[a, *], Kubugha Wilcox Bunonyo [b], Nicholas Nkasiovu Tasie[c]

[a] *Department of Mathematics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt. Nigeria*

[b] *Department of Mathematics and Statistics, Federal University Otuoke, Bayelsa, Nigeria*

[c] *Department of Physics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt, Nigeria*

Received 05 January 2023; accepted (in revised version) 27 February 2023

Abstract: In this paper, our objective is to examine the steady three-dimensional flow of a condensation film in an inclined disk with rotation in the presence of magnetic field. Appropriate similarity transformation technique is employed to transform the basic governing partial differential equations into a system of nonlinear ordinary differential equations. The transformed equations are solved numerically using Adomian decomposition method (ADM). Using MATLAB Bvp4c solver as the benchmark against the analytical solution obtained using Adomian decomposition method. The behaviour of the analytical solution for the different profiles using different methods are displayed in tables and explicitly discussed. Effects of pertinent parameters on the velocity and temperature profile is presented in tables. The comparison of our results with established results in literature showed excellent agreement.

MSC: 76D05 • 34G20

Keywords: Inclined Disk • nanofluid flow • rotating system • condensation film • Adomian Decomposition Method (ADM) • Similarity transformation

1. Introduction

Condensation refers to the phenomenon of removing a condensate liquid from a cooled and saturated vapour. This phenomenon has useful practical applications in many engineering disciplines especially in mechanical and chemical engineering. Many researchers have conducted studies to explore this problem under different conditions. Pioneering studies on this subject was first caried out by Von Karman and Wang [1] - [2] about a rotating disk in an infinite fluid. Sparrow and Gregg [3] have examined the problem of condensate removal using centrifugal forces in a cooled rotating disk. In this study, the governing Navier-Stokes equations were converted to a system of ordinary differential equations using similarity transformation technique, where solutions for several film thickness was obtained via numerical integration. Beckett et al [4] also investigated the same problem but incorporated the novelty of drag force. Charry and Sarma [5] solved the same problem with the addition of the suction term. This problem bears similarities with the problem of chemical deposition when a fluid film is deposited in a cooled rotating disk [6].

Many real-world problems of practical importance that arise in mathematical physics, engineering, technological, fluid dynamics and biological sciences especially in fluid motion and heat transfer analysis are best modelled using

* Corresponding author.

E-mail address(es): liberty.ebiwareme@ust.edu.ng (Liberty Ebiwareme), wilcoxbk@fuotuoke.edu.ng (Kubugha Wilcox Bunonyo), nkasiovu.tasie@ust.edu.ng (Nicholas Nkasiovu Tasie).

Nomenclature

$f(\eta)$	Normalized radial velocity profile
$g(\eta)$	Gravitational acceleration profile
$k(\eta)$	Normalized velocity profile
$s(\eta)$	Lateral flow profile
$\theta(\eta)$	Normalized temperature profile
$\overline{g}$	Gravity acceleration
h	Thickness of the fluid film
W	Spraying velocity
P_0	Ambient Pressure on the film surface
Pr	Prandtl number
T	Temperature
T_w	Disk temperature
T_0	Film temperature
u	velocity component in the x direction
v	Velocity component in the y direction
w	Velocity component in the z direction

Greek symbols

Ω	Angular velocity
β	Angle between disk and horizontal axis
ρ	Density
ν	Kinematic viscosity
α	Thermal diffusivity of the fluid
δ	Constant Normalized thickness
η	Nondimensional variable

governing equations that are strongly or weakly nonlinear with imposed boundary or initial conditions [7]. Due to the inherent nonlinearities, they are seldom solvable by elementary conventional mathematical methods for closed-form solutions [8]. Nevertheless, the advancement and availability of high-speed computers, algebraic computational software's and programming techniques, researchers have adopted several numerical and exact solution techniques to routinely solve them [9]. However, most of the exact analytical methods have inherent drawbacks in the form of restrictive assumptions and linearization [10]. Similarly, their numerical counterparts have equally proven costly and time consuming as it gives complete solutions at discrete points which gives discontinuities points of a curve. Also, employing numerical methods for most of these nonlinear problems give rise to multiple or inappropriate solutions that gives divergence solution except the convergence and stability criterion for problem is checked [11]. Unsteady MHD free convection heat and mass transfer flow past a semi-infinite porous vertical plate in a rotating system with hall current, heat source and suction has been studied using explicit finite difference scheme by [12]. [13] used Crank-Nicolson finite difference method to analyse the problem of second order chemical on MHD flow past a vertical porous plate with Soret and Dufour. The Crank-Nicolson scheme has been utilised to examine the numerical simulation changes in Soret-Dufour members, radiation, chemical reaction, and viscous dissipation on unsteady MHD flow past an inclined porous plate embedded in porous medium with heat generation or absorption [14].

In recent times, semi-analytical methods have caught the attention of researchers. Some of these innovative techniques include Perturbation method (PM), Lyapunov artificial small parameter method, δ expansion method, Homotopy perturbation method (HPM), Variational iteration method (VIM), Differential transformation method (DTM), Homotopy analysis method (HAM), Differential quadrature method (DQM), Abkari-Ganji method (AGM). Although these methods have been extensively applied to solve several problems in the field of Science and Engineering, Nevertheless, their implementation to practical problems have not been without some drawbacks. For instance, the perturbation method is limited in its application, since it is valid only within the range of a small physical parameter and not every problem has a physical parameter present in it. Equally, the Lyapunov small artificial parameter and δ expansion methods give the freedom to choose the artificial parameter to ensure convergence but fall short of not having rules as to where to place the parameter for convergent solution to be obtain [15] - [19]. Variational iteration method requires the calculation of a Lagrange multiplier which complexity is dependent on the overall order of the equation, special Taylor series dependent transformations are employed to transform a given equation to algebraic equation for approximate solutions to be obtained in Differential transform method (DTM) but its computationally time-consuming. Similarly, the Homotopy analysis method of Liao on the otherhand have the disadvantage that it is very difficult to find a good initial guess.

In this study, we employ the novelty of Adomian decomposition method (ADM) to solve the resulting systems of nonlinear ordinary differential equations to obtain approximate analytical solution. This method has been employed to solve many problems in science and Engineering [20] - [31]. Several researchers have solved this problem using different solution techniques. Osman and Arslanturk [32] used variation of parameter method to study the three-dimensional problem of condensation film on an inclined rotating disk. The analytical solution of the steady state condensation film on the inclined rotating disk by a new hybrid method has been investigated using hybrid differential transformation method. Mohammad and Hassan [33] employed optimal homotopy analysis method to obtain the analytical solution for the three-dimensional steady flow of condensation film on inclined rotating disk. Rashidi and Dinarvand [34] and Mohimanian et al. [35] analysed the purely analytic approximate solution for the steady three-dimensional condensation film on an inclined rating disk using homotopy analysis method and differential transform method. Ganji et al. [36] have conducted an analytical investigation of a steady three-dimensional problem of condensation film on inclined rotating disk by Abkari-Ganji method.

The study is organized as follows: Chapter 1 gives the introduction of the condensation film of a rotating inclined disk with magnetic field including pioneering studies on this subject. The formulation of the problem is given in chapter 2. Chapter three presents the fundamentals of the semi-analytical Adomian decomposition method. The analytical procedure of the problem using Adomian decomposition method is given in Chapter four. Discussions of the analytical result obtained using the proposed procedure for the different profiles and the effects of pertinent parameters are displayed in tables and in graphs in Chapter five, while the conclusion of the study is drawn in chapter 6.

2. Mathematical Formulation of Problem

We consider a disk rotating in a plane with angular velocity, Ω. The angle between the horizontal axis and the disk is β. A nanofluid film of thickness h is formed by spraying with the velocity. Assuming the disk radius is large compared to the film thickness so that the end effects can be ignored. Vapour shear effects at the interface of vapour and fluid are usually important. The gravitational acceleration, g acts in the downward direction. The temperature on the disk is T_w and that on the film surface is T_0. Also, the ambient pressure on the film surface is constant at ρ_0. Suppose the pressure is a function of z only and neglecting viscous dissipation. The governing equations of continuity, momentum and energy for steady state are given by [4].

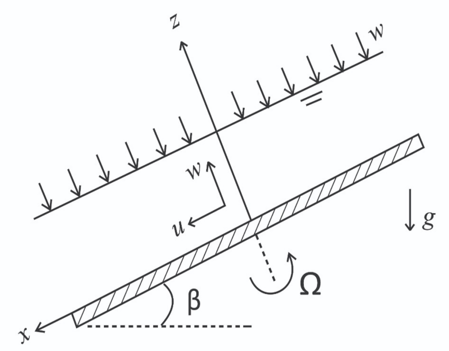

Fig. 1. Configuration of the problem.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\overline{g}\sin\beta \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\upsilon\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\overline{g}\cos\beta-\frac{P_z}{\rho} \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

where (u, v, w) represents the velocity components in the x, y and z directions respectively, T indicates the temperature, ρ is the density, υ is the kinematic viscosity and α is the thermal diffusivity of the fluid. Assuming, zero slip and shear stress on the disk as well as on the film surface, the appropriate boundary conditions are given by

$$u=-\Omega y,\ v=\Omega x, w=0, T=T_w \text{ at } z=0$$

$$\frac{\partial u}{\partial z}=0, \frac{\partial v}{\partial z}=0, w=-W, T=T_0,\ P=P_0 \text{ at } z=h \tag{6}$$

Following Wang [7], we use the following transformations of the form

$$\left.\begin{array}{c} u=\Omega y g(\eta)+\Omega x f'(\eta)+\overline{g}\kappa(\eta)\sin\frac{\beta}{\Omega'} \\ \mathrm{v}=\Omega x g(\eta)+\Omega y f'(\eta)+\overline{g}s(\eta)\sin\frac{\beta}{\Omega'} \\ w=-2\sqrt{\Omega\upsilon}f(\eta) \\ T=(T_0-T_w)\Theta(\eta)+T_w \end{array}\right] \tag{7}$$

where η is the similarity variable of the transformation defined in the form

$$\eta=z\sqrt{\frac{\Omega}{\upsilon}} \tag{8}$$

Upon substitution of Eqs. (7) and (8) into Eqs. (1) - (5), the continuity equation is automatically satisfied, and the remaining equations become the form

$$f'''(\eta)-(f'(\eta))^2+(g(\eta))^2+2f(\eta)f''(\eta)=0 \tag{9}$$

$$g''(\eta)-2g(\eta)f'(\eta)+2f(\eta)g'(\eta)=0 \tag{10}$$

$$k''(\eta)-k(\eta)f'(\eta)+s(\eta)g(\eta)+2f(\eta)k'(\eta)+1=0 \tag{11}$$

$$s''(\eta)-k(\eta)g(\eta)-s(\eta)f'(\eta)+2f(\eta)s'(\eta)=0 \tag{12}$$

Suppose the temperature is a function of the distance z alone, Eq. (5) takes the form

$$\Theta''(\eta)+2\mathrm{Pr}f(\eta)\Theta'(\eta)=0 \tag{13}$$

where $Pr=\frac{\upsilon}{\alpha}$ is the Prandtl number of the base fluid. The corresponding boundary conditions for Eqs. (12)- (16) are given by

$$\left.\begin{array}{c} f(0)=0,\ f'(0)=0,\ f''(\delta)=0 \\ g(0)=1,\ g'(\delta)=0 \\ k(0)=0,\ k'(\delta)=0 \\ s(0)=0,\ s'(\delta)=0 \\ \Theta(0)=0,\ \Theta'(\delta)=1 \end{array}\right\} \tag{14}$$

And δ is the normalized thickness defined as

$$\delta=h\sqrt{\frac{\Omega}{\upsilon}} \tag{15}$$

Similarly, the condensation of the spraying velocity is defined as

$$f(\delta)=\frac{W}{2\sqrt{\Omega\upsilon}}=\alpha \tag{16}$$

After the flow field is found, the pressure distribution of the film can be obtained by integrating Eq. (4) as follows.

$$\rho(z)=\rho_0+\rho\left\{\mathrm{v}\left[w_z(z)-w_z(h)\right]-\frac{w^2(z)-w^2(h)}{2-\overline{g}(z)-h\cos\beta}\right\} \tag{17}$$

3. Adomian Decomposition Method (ADM)

To proceed with the analysis, we start by stating the fundamentals of the solution techniques as follows: Let's consider a nonlinear differential equation of the form

$$L(u(x)) + R(u(x)) + N(u(x)) = g(x) \tag{18}$$

Where L is the highest order derivative assumed to be invertible, R is the linear differential operator with order less than that of L, N is a nonlinear term and g is the source term

Rewriting Eq. (17) for $L(u(x))$, we obtain

$$L(u(x)) = g(x) - R(u(x)) - N(u(x)) \tag{19}$$

Taking the inverse operator, L^{-1} on both sides of Eq. (19), we get

$$u(x) = L^{-1}g(x) - L^{-1}R(u(x)) - L^{-1}N(u(x))$$

$$y(x) = \phi - L^{-1}R(u(x)) - L^{-1}N(u(x)) \tag{20}$$

Where ϕ is the term arising from the integration of the source term. It is obtained using the following sequence depending on the order of the given equation.

By the standard Adomian decomposition method, we write the unknown solution as an infinite decomposition series of the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{21}$$

Putting Eq. (18) into Eq. (16), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi - L^{-1}R\left(\sum_{n=0}^{\infty} u_n(x)\right) - L^{-1}N\left(\sum_{n=0}^{\infty} u_n(x)\right) \tag{22}$$

Matching both sides of Eq. (21), we obtain the zeroth order component given by

$$u_0 = \phi$$

Then the recursive relation is given by

$$u_{n+1}(x) = -L^{-1}R(u_n) - L^{-1}N(u_n), n \geq 0 \tag{23}$$

The solution of the problem in Eq. (17) is obtain as limit of the decomposing series

$$u(x) = \lim_{n\to\infty} u_n(x) = u_0(x) + u_1(x) + u_2(x) + \ldots \tag{24}$$

Similarly, the nonlinear term can be determined by an infinite series of the Adomian polynomials given by

$$N(u_0, u_1, u_2, \ldots, u_n) = \sum_{n=0}^{\infty} A_n \tag{25}$$

Then the $A_n^{'s}$ are obtained from the relation

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0}, \quad n = 0,1,2,3 \tag{26}$$

Using Eq. (24), the first five Adomian polynomials are given as

$$\begin{aligned}
A_0 &= N(u_0)\\
A_1 &= u_1 N'(u_0)\\
A_2 &= u_2 N'(u_0) + \frac{1}{2!}u_1^2 N''(u_0)\\
A_3 &= u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!}u_1^3 N'''(u_0)\\
A_4 &= u_4 N'(u_0) + \frac{1}{2}N''(u_0)\left(2u_1u_3 + u_2^2\right) + \frac{1}{2}N'''(u_0)\,u_1^2u_2 + \frac{1}{4!}N^{(iv)}(u_0)\,u_1^4\\
A_5 &= u_5 N'(u_0) + \frac{1}{2}N''(u_0)(2u_1u_4 + 2u_2u_3) + \frac{1}{3!}N'''(u_0)\left(3u_1^2u_3 + 3u_1u_2^2\right) + \frac{4}{4!}N^{(iv)}(u_0)\left(u_1^3u_2\right) + \frac{1}{5!}N^{(v)}(u_0)\,u_1^5\\
A_6 &= u_6 N'(u_0) + \frac{1}{2!}N''(u_0)\left(2u_1u_5 + 2u_1u_4 + u_3^2\right) + \frac{1}{3!}N'''(u_0)\left(3u_1^2u_4 + u_2^3 + 6u_1u_2u_3\right)\\
&\quad + \frac{1}{4!}N^{(iv)}(u_0)\left(4u_1^3u_3 + 6u_1^2u_2^2\right) + \frac{5}{5!}N^{(v)}(u_0)\,u_1^4u_2 + \frac{1}{6!}N^{(vi)}(u_0)\,u_1^6
\end{aligned}$$

4. Analytical Procedure via Adomian Decomposition Method (ADM)

To analytically the dimensionless Eqs. (9) - (13) subject to the boundary condition (14) using ADM, we proceed by writing the equations in operator form as follows

$$L_1 f(\eta) = (f'(\eta))^2 - 2f(\eta) f''(\eta) - g^2(\eta) \tag{27}$$

$$L_2 g(\eta) = 2g(\eta) f''(\eta) - 2f(\eta) g'(\eta) \tag{28}$$

$$L_3 k(\eta) = k(\eta) f'(\eta) - 2f(\eta) k'(\eta) - s(\eta) g(\eta) - 1 \tag{29}$$

$$L_4 s(\eta) = s(\eta) f'(\eta) - 2f(\eta) s'(\eta) + g(\eta) k(\eta) \tag{30}$$

$$L_5 \theta(\eta) = -2Pr f(\eta) \theta'(\eta) \tag{31}$$

where the differential operators are defined as

$$L_1 = \frac{d^3}{d\eta^3}, \; L_2 = L_3 = L_4 = L_5 = \frac{d^2}{d\eta^2}$$

Assuming the inverse of the operators, L_1^{-1}, $L_2^{-1}, L_3^{-1}, L_4^{-1}$ and L_5^{-1} exists and can be integrated from 0 to η as follows

$$L_1^{-1}(.) = \int_0^\eta \int_0^\eta \int_0^\eta (.)\, d\eta d\eta d\eta \tag{32}$$

$$L_2^{-1}(.) = L_3^{-1} = L_4^{-1} = L_5^{-1} = \int_0^\eta \int_0^\eta (.)\, d\eta d\eta \tag{33}$$

Operating L_1^{-1}, $L_2^{-1}, L_3^{-1}, L_4^{-1}$ and L_5^{-1} on Eqs. (27) – (31) and exerting the boundary conditions on it, we have

$$f(\eta) = f(0) + f'(0)\eta + f''(0)\frac{\eta^2}{2} + L_1^{-1} N_1(u) \tag{34}$$

$$g(\eta) = f(0) + f'(0)\eta + L_2^{-1} N_2(u) \tag{35}$$

$$k(\eta) = f(0) + f'(0)\eta + L_3^{-1} N_3(u) \tag{36}$$

$$s(\eta) = f(0) + f'(0)\eta + L_4^{-1} N_4(u) \tag{37}$$

$$\theta(\eta) = \theta(0) + \theta'(0)\eta + L_5^{-1} N_5(u) \tag{38}$$

where

$$\begin{aligned} N_1(u) &= (f'(\eta))^2 - 2f(\eta) f''(\eta) - g^2(\eta) \\ N_2(u) &= 2g(\eta) f''(\eta) - 2f(\eta) g'(\eta) \\ N_3(u) &= k(\eta) f'(\eta) - 2f(\eta) k'(\eta) - s(\eta) g(\eta) - 1 \\ N_4(u) &= s(\eta) f'(\eta) - 2f(\eta) s'(\eta) + g(\eta) k(\eta) \\ N_5(u) &= -2Pr f(\eta) \theta'(\eta) \end{aligned} \tag{39}$$

The standard Adomian decomposition procedures give the following expressions of the form

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta) = f_0(\eta) + L_1^{-1}(N_1 u) \tag{40}$$

$$g(\eta) = \sum_{m=0}^{\infty} g_m(\eta) = g_0(\eta) + L_2^{-1}(N_2 u) \tag{41}$$

$$k(\eta) = \sum_{m=0}^{\infty} k_m(\eta) = k_0(\eta) + L_3^{-1}(N_3 u) \tag{42}$$

$$s(\eta) = \sum_{m=0}^{\infty} s_m(\eta) = s_0(\eta) + L_4^{-1}(N_4 u) \tag{43}$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta) = \theta_0(\eta) + L_5^{-1}(N_5 u) \tag{44}$$

The nonlinear and linear terms of $f(\eta)$, $g(\eta)$, $k(\eta)$, $s(\eta)$ and $\theta(\eta)$ can be expressed as series in the form

$$\sum_{m=0}^{\infty} A_m = (f')^2,\ \sum_{m=0}^{\infty} B_m = ff'',\ \sum_{m=0}^{\infty} C_m = g^2,\ \sum_{m=0}^{\infty} D_m = gf'',\ \sum_{m=0}^{\infty} E_m = fg',\ \sum_{m=0}^{\infty} F_m = kf'$$

$$\sum_{m=0}^{\infty} G_m = fk',\ \sum_{m=0}^{\infty} H_m = sg,\ \sum_{m=0}^{\infty} I_m = sf',\ \sum_{m=0}^{\infty} J_m = fs',\ \sum_{m=0}^{\infty} L_m = gk,\ \sum_{m=0}^{\infty} W_m = f\theta'$$

where

$$\begin{aligned}
A &= (f')^2,\ A_0 = (f_0')^2,\ A_1 = 2f_0'f_1',\ A_2 = 2f_0'f_2' + (f_1')^2 \\
B &= ff'', B_0 = f_0 f_0'',\ B_1 = f_0 f_1'' + f_1 f_0'', B_2 = f_0' f_2 + f_1' f_1 + f_2' f_0 \\
C &= g^2,\ C_0 = g_0^2,\ C_1 = 2g_0 g_1,\ C_2 = 2f_0 f_2 + f_1^2 \\
D &= gf',\ D_1 = g_0 f_1' + g_1 f_0',\ D_2 = g_0 f_2' + g_1 f_1' + g_2 f_0' \\
E &= fg',\ E_0 = f_0 g_0',\ E_1 = f_0 g_1' + f_1 g_0',\ E_2 = f_0 g_2' + f_1 g_1' + f_2 g_0' \\
F &= kf', F_0 = k_0 f_0', F_1 = k_0 f_1' + k_1 f_0',\ F_2 = k_0 f_2' + k_1 f_1' + k_2 f_0' \\
G &= fk', G_0 = f_0 k_0', G_1 = f_0 k_1' + f_1 k_0',\ G_2 = f_0 k_2' + f_1 k_1' + f_2 k_0' \\
H &= sg,\ H_0 = s_0 g_0,\ H_1 = s_0 g_1 + s_1 g_0,\ H_2 = s_0 g_2 + s_1 g_1 + s_2 g_0 \\
I &= kg,\ I_0 = k_0 g_0,\ I_1 = k_0 g_1 + k_1 g_0, I_2 = k_0 g_2 + k_1 g_1 + k_2 g_0 \\
J &= sf', J_0 = s_0 f_0',\ J_1 = s_0 f_1' + s_1 f_0',\ J_2 = s_0 f_2' + s_1 f_1' + s_2 f_0' \\
L &= fs', L_0 = f_0 s_0',\ L_1 = f_0 s_1' + f_1 s_0',\ L_2 = f_0 s_2' + f_1 s_1' + f_2 s_0' \\
W &= f\theta', W_0 = f_0\theta_0',\ W_1 = f_0\theta_1' + f_1\theta_0',\ W_2 = f_0\theta_2' + f_1\theta_1' + f_2\theta_0'
\end{aligned}$$

To determine the components of $f_m(\eta)$, $g_m(\eta), k_m(\eta), s_m(\eta)$, and of $\theta_m(\eta)$, the initial values of $f_0(\eta)$, $g_0(\eta), k_0(\eta), s_0(\eta)$, and of $\theta_0(\eta)$ are defined by applying the boundary conditions

$$f_0(\eta) = \frac{\alpha_1}{2}\eta^2,\ \ g_0(\eta) = 1 + \alpha_2\eta,\ k_0(\eta) = \alpha_3\eta,\ s_0(\eta) = \alpha_4\eta,\ \theta_0(\eta) = \alpha_5\eta \tag{45}$$

$$f''(0) = \alpha_1,\ g'(0) = \alpha_2,\ k'(0) = \alpha_3,\ s'(0) = \alpha_4,\ \theta'(0) = \alpha_5 \tag{46}$$

Similarly, for $m \geq 0$, subsequent approximations can be determined using the expressions for the recursive algorithms

$$f_{n+1}(\eta) = A_{n+1} - 2\,B_{n+1} - C_{n+1}\ ,\ \mathrm{n} \geq 0 \tag{47}$$

$$g_{n+1}(\eta) = 2\,D_{n+1} - 2\,E_{n+1}\ , n \geq 0 \tag{48}$$

$$k_{n+1}(\eta) = F_{n+1} - 2G_{n+1} - H_{n+1} - 1, n \geq 0 \tag{49}$$

$$s_{n+1}(\eta) = I_{n+1} + J_{n+1} - L_{n+1},\quad n \geq 0 \tag{50}$$

$$\theta_{n+1}(\eta) = -2Pr\,W_{n+1}, n \geq 0 \tag{51}$$

Using the symbolic software Mathematica, the constants are obtained using the given second boundary conditions at $\eta = 1$, and the complete solution of the different profiles are obtained and displayed graphically and in tables as follows.

5. Results

Table 1. Comparison Analysis between ADM solution with other methods for velocity profile when $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$.

η	$f(\eta)$						
	MATLAB Bvp4c	HAM [31]	DTM [33]	OHAM [30]	VPM [29]	AGM [32]	ADM
0.00	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.000000
0.10	0.002247	0.002247	0.0022467	0.002277	0.002247	0.002247	0.002247
0.20	0.008328	0.008388	0.008389	0.008387	0.008328	0.008328	0.0083280
0.30	0.017266	0.017266	0.017268	0.017269	0.017266	0.017266	0.0172661
0.40	0.028099	0.028099	0.028098	0.028097	0.028098	0.028099	0.0280991
0.50	0.039873	0.039873	0.039871	0.039872	0.039873	0.039873	0.0398730
0.60	0.051648	0.051648	0.051647	0.051649	0.051647	0.051648	0.0516481
0.70	0.062489	0.062488	0.062489	0.062487	0.062489	0.0624893	0.06248927
0.80	0.714682	0.714682	0.714684	0.714686	0.714682	0.0714682	0.0714682
0.90	0.776586	0.776586	0.776588	0.776589	0.776585	0.0776590	0.0776590
1.00	0.0801362	0.0801361	0.0801362	0.080136	0.0801136	0.0801362	0.08013614

Table 2. Comparison Analysis between ADM solution with other methods for gravitational acceleration profile when $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$g(\eta)$						
	MATLAB Bvp4c	HAM [31]	DTM [33]	OHAM [30]	VPM [29]	AGM [32]	ADM
0.00	1.00000	1.00000	1.000000	1.000000	1.000000	1.00000	1.000000
0.10	0.992286	0.992286	0.992287	0.992286	0.992286	0.992286	0.9922856
0.20	0.985417	0.985417	0.985416	0.985417	0.985418	0.985417	0.9854165
0.30	0.980055	0.980055	0.980056	0.980054	0.980055	0.980055	0.9800545
0.40	0.9766608	0.9766608	0.9766610	0.9766605	0.976660	0.976660	0.976600
0.50	0.9755007	0.9755007	0.9755006	0.9755008	0.975501	0.975501	0.9755010
0.60	0.976654	0.976654	0.9766541	0.9766541	0.976655	0.976654	0.9766535
0.70	0.980007	0.980007	0.980006	0.980007	0.980069	0.980007	0.980007
0.80	0.985263	0.985263	0.9852640	0.9852641	0.985263	0.985263	0.9852630
0.90	0.991941	0.991941	0.9919420	0.9919420	0.991942	0.991941	0.9919409
1.00	0.999383	0.999383	0.999383	0.999383	0.999383	0.999383	0.9993826

Table 3. Comparison Analysis between ADM solution with other methods for temperature profile when $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$k(\eta)$						
	MATLAB Bvp4c	HAM [31]	DTM [33]	OHAM [30]	VPM [29]	AGM [32]	ADM
0.00	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.00000
0.10	0.044418	0.044418	0.044418	0.044418	0.044418	0.044418	0.044418
0.20	0.078877	0.078877	0.078876	0.078878	0.078877	0.078877	0.0788767
0.30	0.103421	0.103421	0.103420	0.103422	0.103421	0.103421	0.1034210
0.40	0.118102	0.118102	0.8118104	0.811810	0.118102	0.118102	0.1181020
0.50	0.122980	0.122980	0.122982	0.122981	0.122980	0.122980	0.122990
0.60	0.474801	0.474801	0.474802	0.474805	0.474801	0.474801	0.474801
0.70	0.588108	0.588108	0.588109	0.588106	0.588107	0.588108	0.588109
0.80	0.710948	0.710948	0.710947	0.710948	0.710948	0.710948	0.710948
0.90	0.843200	0.843200	0.843201	0.843208	0.843200	0.843200	0.843201
1.00	0.984728	0.984728	0.984727	0.984726	0.984727	0.984728	0.9847275

Table 4. Comparison Analysis between ADM solution with other methods for concentration profile when $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$s(\eta)$						
	MATLAB Bvp4c	HAM [31]	DTM [33]	OHAM [30]	VPM [29]	AGM [32]	ADM
0.00	0.00000	0.00000	0.000000	0.000000	0.000000	0.00000	0.00000
0.10	0.003932	0.003932	0.003931	0.003933	0.003932	0.003932	0.0039315
0.20	0.007431	0.007431	0.007435	0.007434	0.007431	0.007431	0.0074310
0.30	0.010163	0.010163	0.010164	0.010165	0.010163	0.010163	0.0101628
0.40	0.011891	0.011891	0.011890	0.011893	0.011891	0.011891	0.0118899
0.50	0.012481	0.012481	0.012482	0.012483	0.012481	0.012481	0.0124810
0.60	0.011894	0.011894	0.011893	0.011892	0.011894	0.011894	0.011894
0.70	0.0101869	0.0101869	0.0101868	0.0101862	0.0101868	0.0101869	0.0101869
0.80	0.0075101	0.0075101	0.0075103	0.0075106	0.0075110	0.0075101	0.0075100
0.90	0.0041066	0.0041066	0.0041065	0.0041065	0.0041065	0.0041066	0.0041067
1.00	0.0003123	0.0003123	0.0003120	0.0003122	0.0003123	0.0003123	0.0003122

Table 5. Comparison Analysis between ADM solution with other methods for concentration profile when $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$\theta(\eta)$						
	MATLAB Bvp4c	HAM [31]	DTM [33]	OHAM [30]	VPM [29]	AGM [32]	ADM
0.00	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.00000
0.10	0.200554	0.200554	0.200556	0.200555	0.2000554	0.200554	0.200554
0.20	0.401036	0.401036	0.401038	0.401037	0.4010355	0.401036	0.401036
0.30	0.601278	0.601278	0.601277	0.601276	0.6012780	0.601278	0.601278
0.40	0.801032	0.801032	0.801032	0.801032	0.8010320	0.801032	0.8010319
0.50	1.000000	1.000000	1.000000	1.000000	1.0000000	1.000000	1.000000
0.60	1.128880	1.128880	1.128883	1.128887	1.1288801	1.128880	1.1288801
0.70	1.240920	1.240920	1.240921	1.240923	1.2409202	1.240920	1.2409200
0.80	1.338320	1.338320	1.338321	1.338322	1.3383200	1.338320	1.338320
0.90	1.423000	1.423000	1.423002	1.423001	1.4230000	1.423000	1.4230001
1.00	1.4966201	1.4966201	1.4966202	1.4966204	1.4966204	1.496620	1.4966203

6. Conclusion

This present study investigates the steady three-dimensional nanofluid flow of a condensation film in a rotating disk with magnetic field and rotation. The simplified governing equations are solved analytically using semi-analytical Adomian decomposition method (ADM) and the result compared with established published result using Variational of parameter method (VPM), MATLAB Bvp4c solver, Optimal Homotopy analysis method (OHAM), Homotopy analysis method (HAM), Differential transform method (DTM) and excellent agreement is achieved. It is observed that, the solution obtained using Adomian decomposition method (ADM) is convergent with negligible errors which shows that the proposed method is efficient, accurate and dependable.

References

[1] V. T. Karman, Uber laminare und turbulente reibung, Zeitschrift fur Angewandte Mathematik und echanik, 1 (1921) 233–252.
[2] C. Y. Wang, Condensation film on an inclined rotating disk, Applied Mathematical Modelling 31(8) (1925) 1582–1593.
[3] E. M. Sparrow, J. Gregg, A theory of rotating condensation 81 (1959) 113–120.
[4] P. M. Beckett Hudson, P. C. Poots, Laminar film condensation due to a rotating disk, Journal Mathematical Problems in Engineering 15 of Engineering Mathematics 7(1) (1973) 63–73.
[5] S. P. Chary, P. K Sarma, Condensation on a rotating disk with constant axial suction, Journal of Heat Transfer 98(4) (1976) 682–684.
[6] K. F. Jensen, Flow phenomena in chemical vapour deposition of thin films, Annual Revelations in Fluid Mechanics 23 (1991) 197-232.

[7] S. R. Satanu, T. R. Sekhar, H. Jafari, K. Suchandan, Applied Analysis, Computation and Mathematical Modelling in Engineering, Select Proceedings of AACMME, Springer Verlag, New York, 2022

[8] D. D. Ganji, Numerical and Analytical Solutions for solving Nonlinear Equations in Heat Transfer, Advances in Mechatronics and mechanical Engineering, First Edition, IGI Global, USA, 2017

[9] K. Surana, J. N. Reddy, The Finite Element method for Boundary value problems, Mathematics and Computations, Taylor and Francis, CRC Press, London, 2017

[10] S. H. Hashemi Kachapa, D. D. Ganji, Dynamics and Vibration: Progress in Nonlinear Analysis, 202, Springer Dordrecht Heidelberg, New York, 2014

[11] A. I. Ruban, Fluid Dynamics: Boundary Layers Part 3. Oxford University Press, Clarendon Street, Oxford, UK, 2018

[12] L. I. Mahtarin, M. H. Sanjeed, K. P. Sanjit, N. M. Rabindra, MHD free convection heat and mass transfer flow over a vertical porous plate in a rotating system with Hall current, heat source and suction, International Journal of Advances in Applied Mathematics and Mechanics 6(1) (2018) 49-64.

[13] A. K. Shukla, K. D. Yogendra, Second order chemical reaction on MHD flow past a vertical porous plate with Soret and Dufour effects, International Journal of Advances in Applied Mathematics and Mechanics 8(1) (2020) 40-49.

[14] A. K. Shukla, K. D. Shubham, Numerical simulation of changes in Soret-Dufour numbers, Radiation, Chemical reaction, and viscous dissipation on Unsteady MHD flow past an inclined porous plate embedded in porous medium with heat generation or absorption, International Journal of Advances in Applied Mathematics and Mechanics 10(2) (2022) 15-26.

[15] I. H. Abdel-Halim Hassan, Comparison differential transformation technique with Adomian decomposition method for linear and nonlinear initial value problems, Chaos Solitons and Fractals 36(1) (2008) 53-65.

[16] M. M. Rashidi, S. A. Mohimanian, S. Abbasbandy, Analytic Approximate Solutions for Heat Transfer of a Micropolar Fluid through a Porous Medium with Radiation, Communications in Nonlinear Science and Numerical Simulations, 2009

[17] A. Arikoglu I. Ozkol, On the MHD and slip flow over a rotating disk with heat transfer, International Journal of Numerical Methods for Heat and Fluid Flow 16(2) (2000) 172-184.

[18] M. Turkyilmazoglu, Heat and mass transfer on the MHD fluid flow due to a porous rotating disk with Hall current and variable properties, J. Heat. Transf. 133(2) (2011) (021701021701-6).

[19] A. Arikoglu, G. Komurgoz, I. Ozkol, Effect of slip on the entropy generation from a single rotating disk, Journal of Fluids Engineering, Transactions of the ASME 130(10) (2008) 1012021-1012029.

[20] G. Adomian, Solving Frontier problems of Physics, The Adomian decomposition method, Kluwer, Dordrecht, 1994

[21] G. Adomian, R. Roch, On the solution of Nonlinear differential equation with convolution product nonlinearities, Journal of Applied Analytical Mathematics 114 (1986) 171-175.

[22] M. A. Al-Jawary, S. G. Al-Razaq, A Semi Analytical Iteration Technique for solving Duffing Equations, International Journal of Pure and Applied Mathematics 18 (2016) 871-885.

[23] E. Farshad, A. Hadi, E. F. Farshad, M. Rohoallah, An Iterative Method for solving Partial Differential Equations and solution of Korteweg-Devries Equations for Showing the Capability of the Iterative Method. World Applied Programming 3 (2013) 320-327.

[24] L. Ebiwareme, F. A. Kormane, E. O. Odok, Simulation of unsteady MHD flow of incompressible fluid between two parallel plates using Laplace-Adomian decomposition method, World Journal of Applied Research and Reviews 14(03) (2022) 136-145.

[25] L. Ebiwareme, Y. A. Da-Wariboko, Modified Adomian decomposition method and Pade approximant for the Numerical approximation of the deterrence model in society, The International Journal of Engineering Science 16(7) Series 1 (2021) 01-12.

[26] L. Ebiwareme, A comparative study between semi-analytical iterative schemes for the reliable treatment of coupled nonlinear partial differential equations, International Journal of Innovation Engineering and Science Research. 6(1) January-February 2021

[27] L. Ebiwareme, Analytical solution of the dynamics of atmospheric C02 using LADM-Pade approximation approach, International Journal of Trend in Scientific Research and Development 6(2) January-February 2022.

[28] L. Ebiwareme, Application of semi-analytical iteration techniques for the numerical solution of linear and nonlinear differential equations, International Journal of Mathematics Trends and Technology 67(2) 146-158, February 2022

[29] L. Ebiwareme, R. E. Akpodee, R. I. Ndu, An application of LADM-Pade approximation for the analytical solution of the SIR infectious Disease Model, International Journal of Innovation Engineering and Science Research 6(2) March-April 2022.

[30] L. Ebiwareme, Analytical study of the Hepatitis E Virus Model (HEV) vis Hybrid semi-analytical Laplace Transformation Adomian decomposition method 5(1) Jan-Feb 2022.

[31] M. Khan, M. Hussain, Application of Laplace Decomposition Method on Semi-infinite Domain, Numerical Algorithm 56(2) (2011) 211-218.

[32] G. Osman, C. Arslanturk, Variation of parameters method for a three-dimensional problem of condensation film ion an inclined rotating disk, Journal of Applied Mathematics and computational Mechanics 18(1) (2018) 15-28.

[33] H. N. Hassan, M. M. Rashidi, Analytical solution of for three-dimensional steady flow of condensation film on inclined rotating disk by Optimal Homotopy Analysis method. Walailak Journal of Science and Technology 10(5) (2012) 479-498.

[34] M. M. Rashidi, S. Dinarvand, Purely analytic approximate solutions for steady three-dimensional problem of condensation film on inclined rotating disk by homotopy analysis method, Nonlinear Analysis 10(4) (2009) 2346-2356.

[35] S. Berkan, S. R. Hoseini, D. D. Ganji, Analytical investigation of steady three-dimensional problem of condensation film on inclined rotating disk by Akbari-Ganji method, Propulsion and power Research (Elsevier), 2016

[36] M. R. Medhi, M. P. Seyyed Amin, Analytic Solution of Steady Three-Dimensional Problem of Condensation Film on Inclined Rotating Disk by Differential Transform Method. Mathematics Problems in Engineering 2010 Article ID: 613230, 2010