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An Efficient Iterative Scheme for Solving the Standard Biological Population Model

Research Article

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Abstract: We apply the semi analytic iterative method to the solution of the standard biological population model. Several test problems are solved and exact solutions are obtained, which demonstrate the efficiency and validity of the method.

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Keywords: Biological population model • Semi-analytic iterative method • Variational iteration method • Homotopy perturbation method • Adomian decomposition method

1. Introduction

The biological population model plays an important role in the biological sciences in the interpretation of the spreading rate of viruses and parasites as well as in the identification of fragile species within an ecosystem. This model, which arises in many important physical phenomena, has been solved using a variety of methods which include the variational iteration method [1], the Adomian decomposition method [2], the homotopy perturbation method [3] and homotopy perturbation transform method [4], to mention but a few.

This paper exploits the well-documented accuracy of the semi analytic iterative method (SAIM), first proposed by Temimi and Ansari [5], to find solutions to the standard biological population model (BPM). This method has been used for solving all kinds of linear and nonlinear ordinary and partial differential equations, as well as higher-order integrodifferential equations [5–7]. More recently it has been applied to solution of the KdV equation [7]. To the best of the author's knowledge, this method has not been applied to the solution of the two-dimensional standard biological population model.

The rest of the paper is organized as follows: Section 2 gives the problem formulation, while Section 3 describes the proposed method of solution. In Section 4 the results of numerical experiments based on a selection of test problems are presented and compared with the exact solution and solutions from previous methods used in the literature. Finally, Section 5 presents some conclusions.

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2. Problem Formulation

Consider the two-dimensional fractional biological population model (FBPM)

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 u^2}{\partial y^2} + f(u),\ t \in [0,\infty),\ 0 < \alpha \le 1, \tag{1}$$

subject to the initial condition

$$u(x,y,0) = h_1(x,y), \tag{2}$$

where $f(u) = hu^a(1-ru^b)$. In equation (1), u denotes the population density (i.e., the number of minimal species per unit volume at position (x,y) and time t), f represents the population supply due to births and deaths of species, $h,\ r,\ a,\ b$ are real numbers, and h_1 is the initial condition. Setting $\alpha = 1$ reduces the FBPM (1) to the standard biological population model (SBPM)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 u^2}{\partial y^2} + hu^a(1-ru^b),\ t \in [0,\infty) \tag{3}$$

which is the focus of this paper. Thus, we apply the SAIM to the solution of the standard nonlinear BPM of the form (3) which may be written in compact form as

$$u_t = u^2_{xx} + u^2_{yy} + hu^a(1-ru^b), \tag{4}$$

subject to the initial condition (2).

3. Description of the Proposed Method

The SAIM was used by Yassein [8] to solve higher order integro-differential equations and by Yassein and Aswhad [7] to solve KdV equations. The method was also used by Kasumo [9, 10] in the solution of the Klein-Gordon, Korteweg-de Vries and Burgers equations. This method uses an iterative approach together with analytical computations to provide a solution of a modified reformulated linear problem. The SAIM was inspired by the homotopy analysis method (HAM) which is a general approximate analytical approach for obtaining convergent series solutions of strongly nonlinear problems [6]. The SAIM offers several advantages over existing methods such as Picard's successive approximations method (SAM) and the ADM in that it is very easy to implement since it avoids the complicated calculation of Adomian polynomials for the nonlinear term in the ADM or Lagrange multipliers in the VIM, thus demanding less computational work [11]. In this paper we propose to use the SAIM to solve the standard biological population model of the form (4), with the initial condition (2), which can be expressed as

$$Lu = Nu, \tag{5}$$

with the condition $B\left(u, \frac{\partial u}{\partial t}\right) = 0$, where L is a linear operator defined by $Lu = u_t$, N is the nonlinear operator given by $Nu = u^2_{xx} + u^2_{yy} + hu^a(1-ru^b)$ and B is the boundary operator. The main requirement of the SAIM is that L should be the linear part of the differential equation, though it is possible to add some linear parts to N in order to simplify the analysis. Assuming that $u_0(x,y,t)$ is an initial guess of the solution $u(x,y,t)$, we take it to be the solution of the equation

$$L[u_0(x,y,t)] = 0 \text{ with } B\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0. \tag{6}$$

Note that

$$u_0(x,y,t) = u(x,y,0) = h_1(x,y).$$

To generate the next iteration to the solution, we solve the equation

$$L[u_1(x,y,t)] = N[u_0(x,y,t)] \text{ with } B\left(u_1, \frac{\partial u_1}{\partial t}\right) = 0. \tag{7}$$

Continuing in this manner leads to a simple iterative procedure which is effectively the solution of a linear set of problems, i.e.,

$$L[u_{n+1}(x,y,t)] = N[u_n(x,y,t)] \text{ with } B\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}\right) = 0, \tag{8}$$

from which the general iterative formula for solving the standard BPM (4) is

$$u_{n+1}(x,y,t) = u_{n+1}(x,y,0) + L^{-1}\left\{-N[u_n(x,y,t)]\right\}, \tag{9}$$

where $L^{-1} = \int_0^t(\cdot)ds$. Thus the solution to the problem (4) with initial condition (2) is given by

$$u(x,y,t) = \lim_{n\to\infty} u_n(x,y,t).$$

4. Test Examples

In this section we study a selection of test problems illustrating the applicability of the SAIM for solving BPMs. All the computations associated with these examples were performed using a Samsung Series 3 PC with an Intel Celeron CPU 847 at 1.10GHz with 6.0GB internal memory and 64-bit operating system (Windows 8).

All the figures were constructed using MATLAB R2016a. The results are presented in tables and figures accompanying the discussion.

Example 4.1.
Consider the BPM (Zellal and Belghaba [1]):

$$u_t = u_{xx}^2 + u_{yy}^2 + hu^a(1 - ru^b), \tag{10}$$

with initial condition $u(x, y, 0) = \sqrt{x + y + xy}$. For $a = 1,\ r = 0$, this BPM has exact solution $u(x, y, t) = \sqrt{x + y + xy}\mathrm{e}^{ht}$. Thus, our BPM of interest here is

$$u_t = u_{xx}^2 + u_{yy}^2 + hu. \tag{11}$$

To solve (11) using the SAIM, we rewrite it in operator-theoretic form as

$$Lu = Nu,$$

where $Lu = u_t$ and $Nu = u_{xx}^2 + u_{yy}^2 + hu$. The primary problem involves finding the initial approximation by solving the equation

$$L[u_0(x, y, t)] = 0, \text{ with } u_0(x, y, 0) = \sqrt{x + y + xy}. \tag{12}$$

Using the initial condition, the solution of the primary problem is

$$u_0(x, y, t) = u_0(x, y, 0) = \sqrt{x + y + xy}.$$

The general recursive relation for solving (11) is

$$L[u_{n+1}(x, y, t)] = N[u_n(x, y, t)], \text{ with } u_{n+1}(x, y, 0) = \sqrt{x + y + xy}, \tag{13}$$

i.e.,

$$u_{n+1}(x, y, t) = u_{n+1}(x, y, 0) + \int_0^t \left[u_{n_{xx}}^2 + u_{n_{yy}}^2 + hu_n\right] ds. \tag{14}$$

From the recursive relation, we have the approximations

$$\begin{aligned}
u_0(x, y, t) &= \sqrt{x + y + xy},\\
u_1(x, y, t) &= \sqrt{x + y + xy} + \int_0^t \left[u_{0_{xx}}^2 + u_{0_{yy}}^2 + hu_0\right] ds = \sqrt{x + y + xy}(1 + ht),\\
u_2(x, y, t) &= \sqrt{x + y + xy} + \int_0^t \left[u_{1_{xx}}^2 + u_{1_{yy}}^2 + hu_1\right] ds = \sqrt{x + y + xy}\left(1 + ht + \frac{(ht)^2}{2}\right),\\
u_3(x, y, t) &= \sqrt{x + y + xy} + \int_0^t \left[u_{2_{xx}}^2 + u_{2_{yy}}^2 + hu_2\right] ds = \sqrt{x + y + xy}\left(1 + ht + \frac{(ht)^2}{2} + \frac{(ht)^3}{6}\right),
\end{aligned}$$

and so on, i.e.,

$$u(x, y, t) = \sqrt{x + y + xy}\left(1 + ht + \frac{(ht)^2}{2!} + \frac{(ht)^3}{3!} + \cdots\right) = \sqrt{x + y + xy}\mathrm{e}^{ht},$$

which is the exact solution of (10) with $a = 1,\ r = 0$. This result was also obtained by Zellal and Belghaba [1] using the variational iteration method with He's polynomials (VIMHP). The results are shown in Table 1 (for $-20 \le x, y \le 20$) and Figure 1 (for $0 \le x, y \le 20$).

Table 1. Comparison of approximate and exact solutions for Example 4.1 ($h = -1,\ t = 2$)

(x, y)	$u(x, y, t)$	$u_{\text{SAIM}}(x, y, t)$
$(-20, -20)$	2.567806457	2.567806457
$(-16, -16)$	2.025513049	2.025513049
$(-12, -12)$	1.482523749	1.482523749
$(-8, -8)$	0.937630346	0.937630346
$(-4, -4)$	0.382785986	0.382785986
$(0, 0)$	0	0
$(4, 4)$	0.663004776	0.663004776
$(8, 8)$	1.210475572	1.210475572
$(12, 12)$	1.754145756	1.754145756
$(16, 16)$	2.296715916	2.296715916
$(18, 18)$	2.838816851	2.838816851

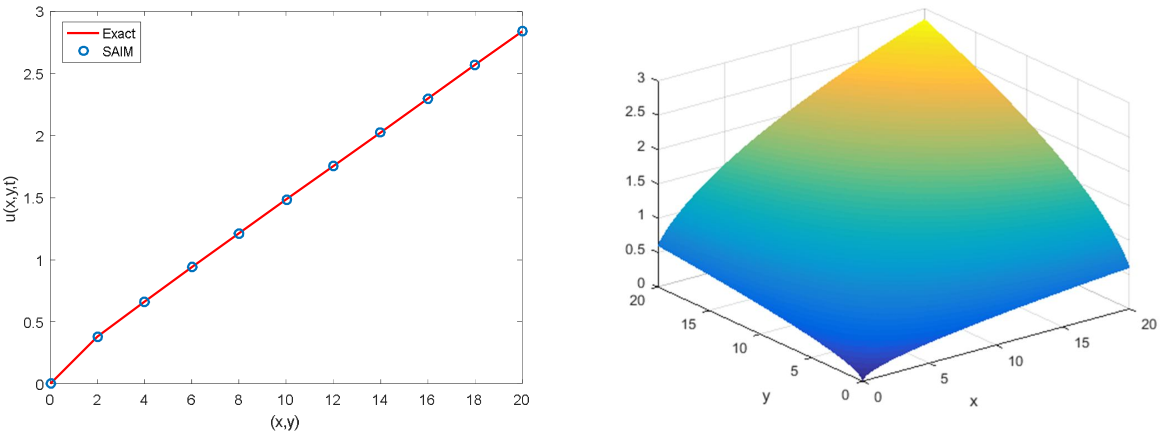


Fig. 1. Comparison of approximate and exact solutions for the BPM in Example 4.1 for $0 \le x, y \le 20$ and a fixed $t = 2$

Example 4.2.
Consider the BPM (4) with $a = b = 1,\ h = -1,\ r = -\frac{8}{9}$ subject to the initial condition $u(x, y, 0) = \mathrm{e}^{\frac{1}{3}(x+y)}$ (Shakeri and Dehghan [2], Roul [3]), i.e.,

$$u_t = u_{xx}^2 + u_{yy}^2 - u\left(1 + \frac{8}{9}u\right). \tag{15}$$

The exact solution of (15) is $u(x, y, t) = \mathrm{e}^{\frac{1}{3}(x+y)-t}$. Rewriting (15) as

$$Lu = Nu,$$

where $Lu = u_t$, $Nu = u_{xx}^2 + u_{yy}^2 - u\left(1 + \frac{8}{9}u\right)$, the general recursive relation is given by

$$L[u_{n+1}(x, y, t)] = N[u_n(x, y, t)], \text{ with } u_{n+1}(x, y, 0) = \mathrm{e}^{\frac{1}{3}(x+y)}. \tag{16}$$

We use the iteration

$$u_{n+1}(x, y, t) = u_{n+1}(x, y, 0) + \int_0^t \left[u_{n_{xx}}^2 + u_{n_{yy}}^2 - u_n\left(1 + \frac{8}{9}u_n\right)\right] ds \tag{17}$$

to obtain the successive approximations

$$u_0(x, y, t) = \mathrm{e}^{\frac{1}{3}(x+y)},$$
$$u_1(x, y, t) = \mathrm{e}^{\frac{1}{3}(x+y)} + \int_0^t \left[u_{0_{xx}}^2 + u_{0_{yy}}^2 - u_0\left(1 + \frac{8}{9}u_0\right)\right] ds = \mathrm{e}^{\frac{1}{3}(x+y)}(1 - t),$$
$$u_2(x, y, t) = \mathrm{e}^{\frac{1}{3}(x+y)} + \int_0^t \left[u_{1_{xx}}^2 + u_{1_{yy}}^2 - u_1\left(1 + \frac{8}{9}u_1\right)\right] ds = \mathrm{e}^{\frac{1}{3}(x+y)}\left(1 - t + \frac{t^2}{2}\right),$$
$$u_3(x, y, t) = \mathrm{e}^{\frac{1}{3}(x+y)} + \int_0^t \left[u_{2_{xx}}^2 + u_{2_{yy}}^2 - u_2\left(1 + \frac{8}{9}u_2\right)\right] ds = \mathrm{e}^{\frac{1}{3}(x+y)}\left(1 - t + \frac{t^2}{2} - \frac{t^3}{6}\right),$$

and so on, leading to the solution

$$u(x,y,t)=\mathrm{e}^{\frac{1}{3}(x+y)}\left(1-t+\frac{t^2}{2}-\frac{t^3}{6}+\cdots\right)=\mathrm{e}^{\frac{1}{3}(x+y)}\mathrm{e}^{-t}=\mathrm{e}^{\frac{1}{3}(x+y)-t}.$$

This is the exact solution also obtained using the ADM [2] and the HPM [3]. Table 2 and Figure 2 shows the results for $t=2$ and $-20\le x,y\le 20$.

Table 2. Comparison of approximate and exact solutions from SAIM and HPM for Example 4.2 ($h=-1,\ t=2$)

(x,y)	$u(x,y,t)$	$u_{\mathrm{SAIM}}(x,y,t)$
$(-20,-20)$	0.000000219	0.000000219
$(-16,-16)$	0.000003154	0.000003154
$(-12,-12)$	0.000045399	0.000045399
$(-8,-8)$	0.000653392	0.000653392
$(-4,-4)$	0.009403563	0.009403563
$(0,0)$	0.135335283	0.135335283
$(4,4)$	1.947734041	1.947734041
$(8,8)$	28.03162489	28.03162489
$(12,12)$	403.4287935	403.4287935
$(16,16)$	5806.113346	5806.113346
$(20,20)$	83561.09612	83561.09612

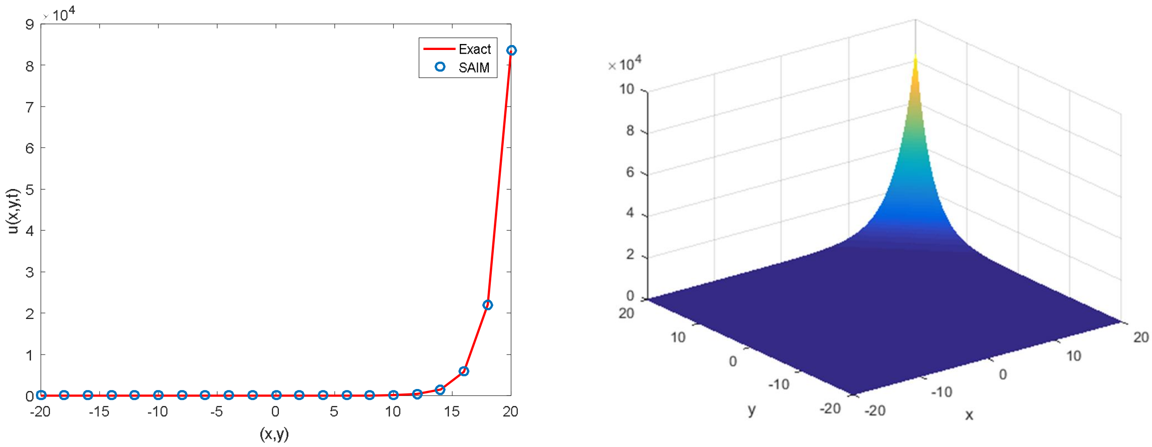


Fig. 2. Comparison of approximate and exact solutions for the BPM in Example 4.2 for $-20\le x,y\le 20$ for a fixed $t=2$

Example 4.3.
Consider the BPM (4) with $a=1,\ r=0$ (Shakeri and Dehghan [2], Roul [3], i.e.,

$$u_t=u_{xx}^2+u_{yy}^2+hu, \tag{18}$$

subject to the initial condition $u(x,y,0)=\sqrt{xy}$ and with exact solution $u(x,y,t)=\sqrt{xy}\mathrm{e}^{ht}$. Here, $Lu=u_t$, $Nu=u_{xx}^2+u_{yy}^2+hu$. Since the primary problem $Lu_0=0$, with $u_0(x,y,0)=\sqrt{xy}$, has a solution $u_0(x,y,t)=\sqrt{xy}$, equation (18) can be solved using the general iterative scheme

$$u_{n+1}(x,y,t)=u_{n+1}(x,y,0)+\int_0^t\left[u_{n_{xx}}^2+u_{n_{yy}}^2+hu_n\right]ds. \tag{19}$$

Thus, the first four approximations are

$$u_0(x,y,t) = \sqrt{xy},$$

$$u_1(x,y,t) = \sqrt{xy} + \int_0^t \left[u_{0_{xx}}^2 + u_{0_{yy}}^2 + hu_0\right] ds = \sqrt{xy}(1+ht),$$

$$u_2(x,y,t) = \sqrt{xy} + \int_0^t \left[u_{1_{xx}}^2 + u_{1_{yy}}^2 + hu_1\right] ds = \sqrt{xy}\left(1+ht+\frac{(ht)^2}{2}\right),$$

$$u_3(x,y,t) = \sqrt{xy} + \int_0^t \left[u_{2_{xx}}^2 + u_{2_{yy}}^2 + hu_2\right] ds = \sqrt{xy}\left(1+ht+\frac{(ht)^2}{2}+\frac{(ht)^3}{6}\right),$$

and so on. Since $u(x,y,t) = \lim_{n\to\infty} u_n(x,y,t)$, the solution to (18) is

$$u(x,y,t) = \sqrt{xy}\left(1+ht+\frac{(ht)^2}{2}+\frac{(ht)^3}{6}+\cdots\right) = \sqrt{xy}\mathrm{e}^{ht},$$

the exact solution also obtained using the HPM [3] and the ADM [2]. Table 3 and Figure 3 compare the results from the SAIM with the exact solution for $h = 0.2,\ t = 2$ and $-20 \le x, y \le 20$.

Table 3. Comparison of approximate and exact solutions for Example 4.3 ($t = 2$)

(x,y)	$u(x,y,t)$	$u_{\mathrm{SAIM}}(x,y,t)$
$(-20,-20)$	29.83649395	29.83649395
$(-16,-16)$	23.86919516	23.86919516
$(-12,-12)$	17.90189637	17.90189637
$(-8,-8)$	11.93459758	11.93459758
$(-4,-4)$	5.967298791	5.967298791
$(0,0)$	0	0
$(4,4)$	5.967298791	5.967298791
$(8,8)$	11.93459758	11.93459758
$(12,12)$	17.90189637	17.90189637
$(16,16)$	23.86919516	23.86919516
$(20,20)$	29.83649395	29.83649395

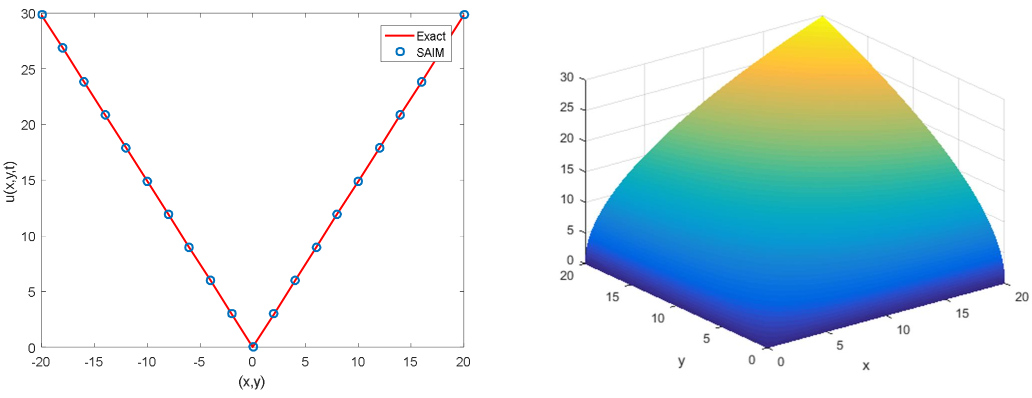


Fig. 3. Comparison of approximate and exact solutions for the BPM in Example 4.3 for $-20 \le x, y \le 20$ and $t = 2$

Example 4.4.
Consider the BPM (4) with $a = b = 1$ (Zellal and Belghaba [1]), i.e.,

$$u_t = u_{xx}^2 + u_{yy}^2 + hu(1-ru), \tag{20}$$

subject to the condition $u(x,y,0) = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)}$. This equation has exact solution $u(x,y,t) = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)+ht}$ and can be rewritten as:

$$Lu = Nu,$$

with $Lu = u_t$, $Nu = u_{xx}^2 + u_{yy}^2 + hu(1-ru)$. The initial problem yields the solution $u_0(x,y,t) = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)}$, so that the first four iterations give the approximations

$$
\begin{aligned}
u_0(x,y,t) &= \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)},\\
u_1(x,y,t) &= \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)} + \int_0^t \left[u_{0_{xx}}^2 + u_{0_{yy}}^2 + hu_0(1-ru_0)\right] ds = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)}(1+ht),\\
u_2(x,y,t) &= \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)} + \int_0^t \left[u_{1_{xx}}^2 + u_{1_{yy}}^2 + hu_1(1-ru_1)\right] ds = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)}\left(1+ht+\frac{(ht)^2}{2}\right),\\
u_3(x,y,t) &= \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)} + \int_0^t \left[u_{2_{xx}}^2 + u_{2_{yy}}^2 + hu_2(1-ru_2)\right] ds = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)}\left(1+ht+\frac{(ht)^2}{2}+\frac{(ht)^3}{6}\right),
\end{aligned}
$$

which approximates the exact solution

$$
u(x,y,t) = \mathrm{e}^{\sqrt{\frac{hr}{8}}(x+y)+ht}.
$$

Table 4. Comparison of approximate and exact solutions for Example 4.4 ($t = 0.2,\ h = -1,\ r = -\frac{8}{9}$)

(x,y)	$u(x,y,t)$	$u_{\mathrm{SAIM}}(x,y,t)$
$(-20,-20)$	0.000001326	0.000001326
$(-16,-16)$	0.000019083	0.000019083
$(-12,-12)$	0.000274654	0.000274654
$(-8,-8)$	0.003952791	0.003952791
$(-4,-4)$	0.056888238	0.056888238
$(0,0)$	0.818730753	0.818730753
$(4,4)$	11.7831043	11.7831043
$(8,8)$	169.5814485	169.5814485
$(12,12)$	2440.601978	2440.601978
$(16,16)$	35124.93888	35124.93888
$(20,20)$	505515.1733	505515.1733

These results are in agreement with those obtained by Zellal and Belghaba [1] using VIMHP, by Roul [3] using the HPM and by Shakeri and Dehghan [2] using the ADM, for $h = -1,\ r = -\frac{8}{9}$. Table 4 and Figure 4 show the results. More iterations would result in smaller errors, hence better accuracy of the SAIM.

Example 4.5.
As a final example, we solve the general BPM (4) with $a = b = 1,\ r = 0,\ h \neq 0$ (Mayembo et al. (2019)), i.e.,

$$
u_t = u_{xx}^2 + u_{yy}^2 + hu, \tag{21}
$$

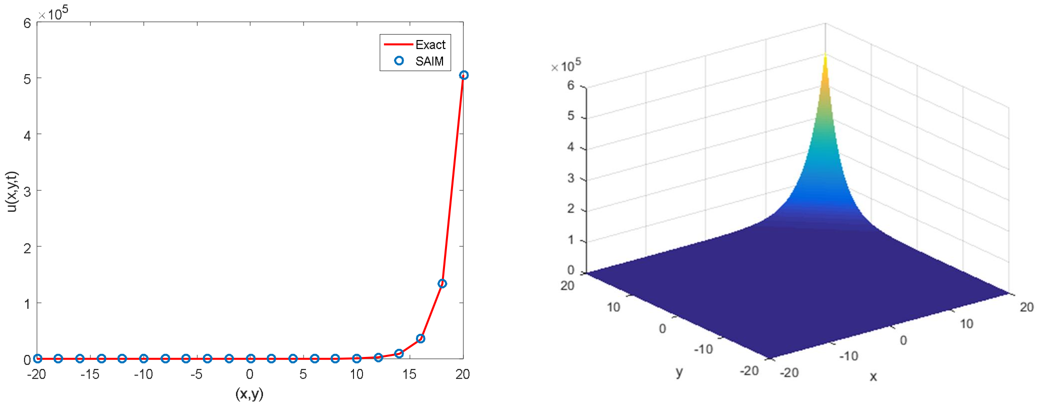
subject to the initial condition $u(x,y,0) = \sqrt{\cos x \cosh y}$. The exact solution is $u(x,y,t) = \sqrt{\cos x \cosh y}\mathrm{e}^{ht}$. We rewrite (21) as

$$
Lu = Nu,
$$

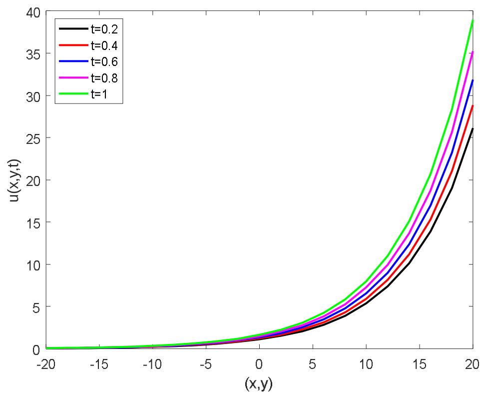
where $Lu = u_t$, $Nu = u_{xx}^2 + u_{yy}^2 + hu$. The first few iterations of the SAIM give the approximations

$$
\begin{aligned}
u_0(x,y,t) &= \sqrt{\cos x \cosh y},\\
u_1(x,y,t) &= \sqrt{\cos x \cosh y} + \int_0^t \left[u_{0_{xx}}^2 + u_{0_{yy}}^2 + hu_0\right] ds = \sqrt{\cos x \cosh y}(1+ht),\\
u_2(x,y,t) &= \sqrt{\cos x \cosh y} + \int_0^t \left[u_{1_{xx}}^2 + u_{1_{yy}}^2 + hu_1\right] ds = \sqrt{\cos x \cosh y}\left(1+ht+\frac{(ht)^2}{2}\right),
\end{aligned}
$$

and so on. This is an approximation to the exact solution $u(x,y,t) = \sqrt{\cos x \cosh y}\mathrm{e}^{ht}$ (Mayembo et al. [12]). If the initial condition is changed to $u(x,y,0) = \sqrt{\sin(\theta x)\cosh(\theta y)}$, the exact solution would be $u(x,y,t) = \sqrt{\sin(\theta x)\cosh(\theta y)}\mathrm{e}^{ht}$, as in Zellal and Belghaba [1], with $h = 1$. For the initial condition $u(x,y,0) = \sqrt{\sin x \sinh y}$, the exact solution for $h = 1$ is $u(x,y,t) = \sqrt{\sin x \sinh y}\mathrm{e}^{t}$, as in Roul [3]. The results are shown in Table 5 and Figure 5.



(a)



(b)

Fig. 4. Comparison of approximate and exact solutions for the BPM in Example 4.4 for $h=-1,\ r=-\frac{8}{9}$ and $-20 \le x, y \le 20$ (a) for a fixed $t=0.2$; (b) for different time values

Table 5. Comparison of approximate and exact solutions from SAIM for Example 4.5 ($t=2$)

(x, y)	$u(x, y, t)$	$u_{\text{SAIM}}(x, y, t)$
$(-1, -1)$	6.746859788	6.746859788
$(-0.8, -0.8)$	7.132639817	7.132639817
$(-0.6, -0.6)$	7.308843539	7.308843539
$(-0.4, -0.4)$	7.373276892	7.373276892
$(-0.2, -0.2)$	7.38807083	7.38807083
$(0, 0)$	7.389056099	7.389056099
$(0.2, 0.2)$	7.38807083	7.38807083
$(0.4, 0.4)$	7.373276892	7.373276892
$(0.6, 0.6)$	7.308843539	7.308843539
$(0.8, 0.8)$	7.132639817	7.132639817
$(1, 1)$	6.746859788	6.746859788

5. Conclusion

In this paper, the semi analytic iterative method has been applied to determine exact or closed-form solutions to the standard nonlinear biological population model subject to given initial conditions. The work has further served to demonstrate the ability of this method to produce exact to near-exact solutions to nonlinear biological population models and has confirmed the method's suitability for solving these types of partial differential equations. Future work could explore the use of other methods, such as Haar wavelets, modified simple equation, splines, etc. for solution of

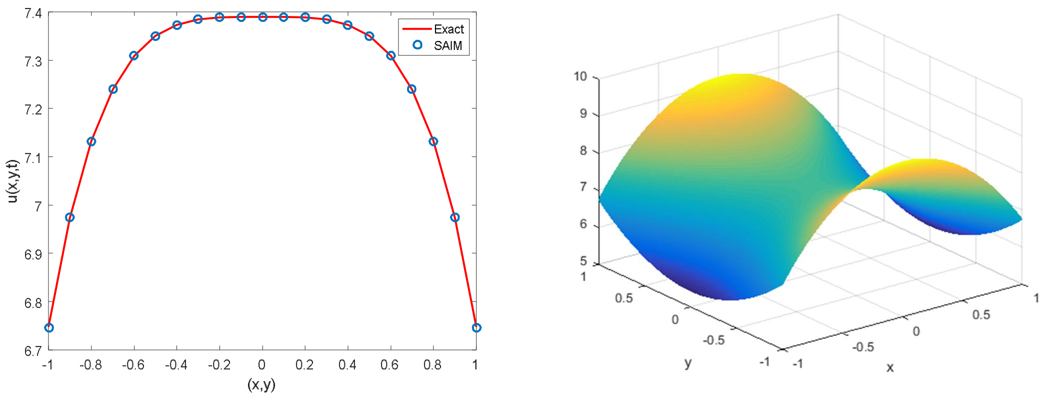


Fig. 5. Comparison of approximate and exact solutions for the BPM in Example 4.5 for $h = 1,\ t = 2$ and $-1 \le x, y \le 1$

biological population models.

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