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Comparative Study of Single-Species Models Using the Quantitative Approaches

Research Article

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Abstract: The primary objective of this line of investigation is to develop a distributed Malthusian and Logistic model for the population of a single species in order to create a population. It has also discussed the basic concepts of the mathematical model, population model, Malthusian model, solution of Malthusian model, logistic model, solution of logistic model, difference between Malthusian and logistic model. For the purpose of confirming our primary findings presented in this work, computerized simulations are included. For the purpose of contrasting and comparing the Logistic model with the Malthusian model, the Taylor scheme and the Rk-4 scheme are utilized. It has been shown that the graphical depiction of the Malthusian model and the logistic model for a variety of various values of the parameters is compatible with our findings. Because the graphical representation of the models and our findings are consistent with one another, we were able to arrive at this conclusion.

MSC: 00A71 • 65D17

Keywords: Mathematical model • Population Model • Single Species • Malthusian Model • Logistic Model

1. Introduction

Think of each species as ecosphere. Most biological populations are multi-species; hence there is no single population. Artificial breeding has produced several resources from single populations in human-developed ecosystems. These materials are vital to life and commodities. People must continuously develop a population's resources for economic and production reasons. Maximum usage value with lowest cost consumption is required. Controlling and anticipating one population is essential. In quantitative biology, single-species models are often used for purposes including regulating cellular expansion, preventing ailments, and managing pest populations. In order to gain insightful knowledge from these models, a few study teams have turned to quantitative approaches [1–3]. The findings enable better planning and control of real output. Component changes provide whether it's qualitative or quantitative dynamic modeling. A rigorous differential equation model accurately represents real-world structures. These models cannot be employed in uncertain controlled systems because they lack parameters related to kinetics and mechanistic aspects [4–7] . Descriptive models reflect biological systems with fewer parameters than quantitative models. As with Hill functions, there is mounting indication that sigmoid mechanisms can faithfully describe a wide range of biological regulatory relationships from elementary Michaelis-Menten kinetics through substrate-induced inhibition

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[8] . Assuming linear species formation and degradation rates, Hill-type models are useful for forecasting T cell interface signaling [9] and cardiac -adrenergic networks [10] . Wang et al. [11] found the best harvesting approach for a single species to maximize yield and control production. When population is highest, it can only happen during a brief window. The Non-Independent Dou et al. sought a rational single-species model. Pontryagin's insights on spontaneous system control and optimization yielded optimal harvesting method analytical equations. The single-species demographic model has also garnered interest. In the last few years, single-species population model applicability has increased [12–16]. We could contrast the three models to find out if our conclusions apply elsewhere. This procedure has been tried several times. Chaves et al. [17] suggested changing the hybrid model into a multi-level categorical model before transforming it into a Boolean model to compare them fairly. Large systems may not move to the second level, which introduces state space variables. Wittmann et al. [18] say continuous Hill-type models, like Boolean models, mirror discrete model behavior in T cell receptor signaling networks. More variables to the model, however may not work for large systems. Wittmann et al. [19] found that Boolean and continuous Hill-type models operate equally in a T cell receptor signaling network. Regardless of having to contend that the concentration of components in biological systems is subject to change over the course of a certain period of time, step functions are able to accurately simulate the input and output. sigmoid curves of regulatory relationships [20]. Inevitably use this type of approximation; you will end up with hybrid models, in which manufacturing rates will be represented by logical processes, while decay rates will be linear. These models will be the result of using this approximation. When studying systems, it can be advantageous to use these models, which are utilized relatively frequently in the aforementioned body of research [20–25]. These models are effective when there is an imperfect appreciation for the parameter values. They looked at how time delay affects the system and found that it may be used as a bifurcation parameter to ensure the positive equilibrium and local Hopf bifurcations are asymptotically stable. Investigating how the gap in time influenced this mechanism led to this discovery. Yao et al. [26] investigated the global asymptotic stability of fractional-order complex-valued differential equations with distributed delays. The characteristic equation provided a necessary and adequate stability constraint for a complicated two-dimensional system. This situation and its basis were both established through the Laplace transform. To satisfy the algebraic condition, the fractional exponent, coefficients, and delay must all work together. There is a wealth of information from studying several delayed-observation models of single-species populations [27–33].

1.1. Mathematical Model

A mathematical model is a symbolic physical system representation. Mathematical modeling represents an issue mathematically. Physics, biology, earth science, chemistry, computer and electrical engineering, and economics, psychology, sociology, and politics use mathematical models.

1.2. Population Model

Population models are mathematical models used to examine population dynamics. The modeling of intricate dynamics and processes. How numbers shift over time or in relation to one another can be explained by studying natural dynamics. Population modeling is used to identify patterns. Age and size are taken into account in ecological population models. Their connections to the outdoors, other creatures, and people could provide light on this. Farmers may increase their yields, prevent biological invasions, and protect the environment with the aid of population models. Parasites, viruses, and other diseases are all explained by population models. Natural population behavior is oversimplified in our models. Varieties of responses to growth are highlighted. The parameters (r and K in the Logistic Model) and behavior patterns of these models have become metaphors for other systems because of their pervasiveness in the field. Animal "carrying capacity" is used by some people.

1.3. Single-Species Models

Ecosystem models reveal broad trends rather than stock variations. Single-species evaluations quantify population collapse risk under varying fishing pressures, even though ecosystem models were not built for this. The UN FSA (1995), EU Council Regulation (EU) 2019/124 (2019), and US Magnuson-Stevens Fishery Conservation and Management Act (MSA) (2014) require "precautionary" fishing to limit stock collapse risk. Ecosystem models with one species are inadequate. Ecosystem models' tropical dynamics and environmental variability explain several mysteries that single-species models cannot. Single-species models focus on population-harvest dynamics; therefore data collecting has addressed these issues. Specificity simplifies design, maintenance, and analysis.

1.4. Methodology

In this study, we compare and contrast the dynamic aspects of the two approaches through connecting them to common numerical technique patterns. The Malthusian and Logistic models are explored, and their solutions and illustrations are compared and contrasted. The next step in the process involves using numerical methods to anticipate solutions. One such method is the Taylor method, while the RK-4 method is the most used numerical approach for predicting future solutions at the moment. Runge-Kutta methods not only provide a more precise estimate than alternative approaches, but they also do away with the necessity for higher-order derivatives. In addition to outperforming competing methods, these techniques also provide superior prior estimation. The Runge-Kutta procedure can be initiated with no further steps or preparations. Without the need to keep track of previous values, more processing time may be devoted to iterative calculations, and greater precision can be achieved by varying the step-length. It is evaluated the model-solving abilities of the Taylor approach and the RK-4 method and make comparisons between the two. The RK-4 approach also appears to provide more exact results than the Taylor approach. By lowering the step-length h, the Runge-Kutta fourth-order technique can increase solution precision. MATLAB code implements Runge-Kutta and Taylor systems.

2. Malthusian Model

The Malthusian growth model, often known as a uncomplicated exponential expansion model, postulates that the quantity of a given asset will rise at a rate proportional to its rate of growth. The model can also been represented as a differential equation:

$$\frac{dN}{dt} = rN \tag{1}$$

With initial condition: $N(0) = N_0$
Where $N_0 = N(0)$ is the initial population size.
r = the population growth rate, sometimes called Malthusian parameter.
t = time.

2.1. Solution of Malthusian Model

The Malthusian model's solution can be found in

$$N(t) = N_0 e^{rt} \tag{2}$$

3. Logistic Model

The model can also been represented as a differential equation:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{k}\right)N \tag{3}$$

Where $k = \frac{r}{a}$. The constant r is called the intrinsic growth rate. That is the growth rate in the absence of any limiting factors.

3.1. Solution of Logistic Model

The Logistic model's solution can be found in

$$N = \frac{kN_0 e^{rt}}{k + N_0\left(e^{rt} - 1\right)} \tag{4}$$

4. Difference between Malthusian and Logistic Model

Table 1. Difference between Malthusian and Logistic Model.

Malthusian Model (MM)	Logistic Model (LM)
A J-shaped typical curve can be seen when the Malthusian model is implemented.	A curve with an S-shaped characteristic can be seen when the logistic model is utilized.
The Malthusian model can be applied to any population that does not have a ceiling on its potential for expansion.	Any population that can be accommodated by a given carrying capacity can be modeled using the logistic approach.
The Malthusian model almost always leads to a dramatic increase in the total number of people.	The population growth rate ends up being relatively stable as a result of using the logistic model. This takes place when the amount of population expansion reaches the maximum allowable capacity of the environment.
The Malthusian model works exceptionally well for populations that are endowed with an infinite supply of resources and room to expand, such as bacterial cultures.	The logistical approach is more realistic and can be used to analyze a variety of populations that currently exist on the globe.
The Malthusian model does not have a maximum population projection.	Carrying capability is the maximum capability of the logistic model, which has other limits as well.
The Malthusian model describes the situation that arises when the rate of growth is proportional to the quantity that is already there.	This is also accurate for the logistic model, but the key difference is that it also incorporates competition as well as assets that are capped at a certain level.

5. Comparative study between Malthusian model and Logistic model using Taylor scheme

5.1. Program: When the population growth rate r=1.37, K=1.

Taylor Scheme (Malthusian Model)

```
k=1
r=1.37
dy=@(x,y)r*y;
dy1=@(x,y)r^2*y;
dy2=@(x,y)r^3*y;
y=1;
f=@(x)y*exp(r*x);
x0=0;
xn=2;
h=0.1;
diff=abs(f(x0)-y);
fprintf('x(time)\t\t\ty(tylor)\t\ty(analytical)\t\terror\n')
fprintf('%f \t\t%f \t\t%f\t\t%f\n',x0,y,f(x0),diff);
for x=x0:h:xn-h
    y=y+h*dy(x,y)+(h^2/2)*dy1(x,y)+(h^3/6)*dy2(x,y);
    x=x+h;
diff=abs(f(x)-y);
fprintf('%f\t\t%f\t\t%f\t\t%f\n',x,y,f(x),diff);
end
```

Taylor Scheme (Logistic model)

```
Input:
k=1
r=1.37
x0=0;
xn=2;
y=2;
h=0.1;
dy=@(x,y)r*y-(r*y^2)/k;
dy1=@(x,y)r*dy(x,y)-2*r*y*dy(x,y)/k;
dy2=@(x,y)k*dy1(x,y)-2*r*dy(x,y)*dy(x,y)/k-2*r*y*dy1(x,y)/k;
f=@(x)k*y*exp(r*x)/(k+y*exp(r*x)-y);
diff=abs(f(x0)-y);
fprintf('x(time)\t\t\ty(tylor)\t\ty(analytical)\t\terror\n')
fprintf('%f \t\t%f \t\t%f\t\t%f\n',x0,y,f(x0),diff);
for x=x0:h:xn-h
    y=y+h*dy(x,y)+(h^2/2)*dy1(x,y)+(h^3/6)*dy2(x,y);
    x=x+h;
diff=abs(f(x)-y);
fprintf('%f\t\t%f\t\t%f\t\t%f\n',x,y,f(x),diff);
end
```

Table 2. Comparative study between Malthusian model and Logistic model using Taylor scheme for $r = 1.37$ (MM = Malthusian Model, LM = Logistic Model).

Iteration Number	Taylor scheme of MM	Exact value	Absolute error	Taylor scheme of LM	Exact value	Absolute Error
1	1.146813	1.146828	0.000015	1.770470	1.773003	0.002533
2	1.315180	1.315215	0.000035	1.610346	1.613335	0.002989
3	1.508266	1.508325	0.000060	1.493018	1.495872	0.002853
4	1.729699	1.729790	0.000091	1.404016	1.406573	0.002558
5	1.983641	1.983772	0.000131	1.334739	1.336979	0.002240
6	2.274866	2.275045	0.000180	1.279738	1.281683	0.001945
7	2.608846	2.609086	0.000240	1.235384	1.237070	0.001685
8	2.991858	2.992173	0.000315	1.199168	1.200628	0.001460
9	3.431102	3.431509	0.000406	1.169294	1.170561	0.001266
10	3.934833	3.935351	0.000518	1.144445	1.145546	0.001101
11	4.512518	4.513171	0.000653	1.123631	1.124590	0.000959
12	5.175014	5.175831	0.000817	1.106097	1.106933	0.000836
13	5.934774	5.935789	0.001015	1.091252	1.091983	0.000731
14	6.806076	6.807330	0.001254	1.078633	1.079273	0.000640
15	7.805297	7.806838	0.001541	1.067868	1.068429	0.000561
16	8.951217	8.953101	0.001885	1.058657	1.059150	0.000492
17	10.265372	10.267669	0.002296	1.050757	1.051189	0.000433
18	11.772463	11.775252	0.002789	1.043964	1.044345	0.000381
19	13.500814	13.504190	0.003376	1.038114	1.038449	0.000335
20	15.482910	15.486985	0.004075	1.033067	1.033362	0.000295

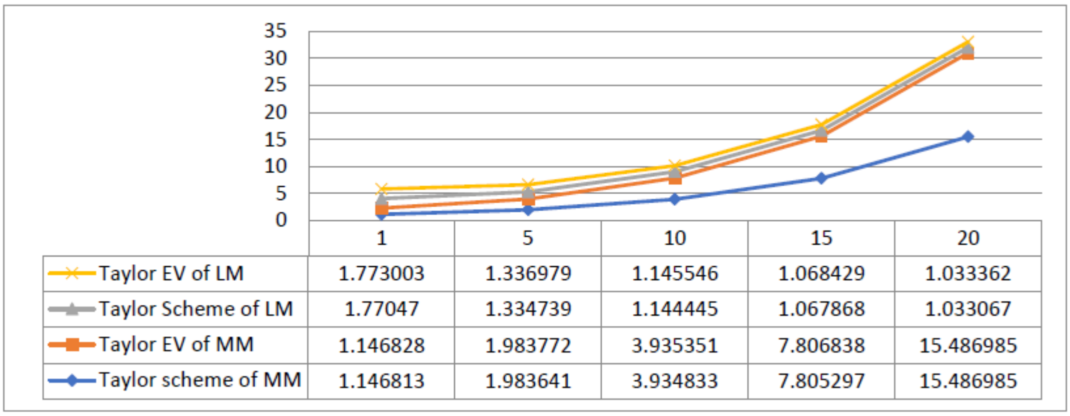

Fig. 1. Solution of comparative study between Malthusian model and logistic model using Taylor scheme by Line graph for $r = 1.37$.

5.2. When the population growth rate $r = 2.18, k = 1$

Table 3. : Comparative study between Malthusian model and Logistic model using Taylor scheme for $r = 2.18$.

Iteration Number	Taylor scheme of MM	Exact value	Absolute error	Taylor scheme of LM	Exact value	Absolute Error
1	1.243489	1.243587	0.000098	1.656070	1.672416	0.016346
2	1.546264	1.546509	0.000245	1.462453	1.477779	0.015326
3	1.922762	1.923218	0.000456	1.338827	1.351316	0.012489
4	2.390933	2.391689	0.000757	1.254393	1.264314	0.009921
5	2.973098	2.974274	0.001176	1.194212	1.202079	0.007868
6	3.697014	3.698769	0.001755	1.150044	1.156310	0.006266
7	4.597195	4.599741	0.002546	1.116941	1.121959	0.005018
8	5.716560	5.720178	0.003619	1.091743	1.095782	0.004039
9	7.108478	7.113540	0.005062	1.072338	1.075602	0.003265
10	8.839312	8.846306	0.006995	1.057258	1.059907	0.002649
11	10.991584	11.001152	0.009568	1.045459	1.047614	0.002155
12	13.667911	13.680890	0.012980	1.036176	1.037934	0.001758
13	16.995893	17.013378	0.017486	1.028843	1.030278	0.001436
14	21.134201	21.157617	0.023417	1.023030	1.024204	0.001174
15	26.280140	26.311339	0.031199	1.018410	1.019371	0.000961
16	32.679057	32.720441	0.041384	1.014730	1.015518	0.000788
17	40.636039	40.690718	0.054679	1.011795	1.012441	0.000646
18	50.530455	50.602450	0.071995	1.009450	1.009980	0.000529
19	62.834050	62.928553	0.094503	1.007575	1.008009	0.000434
20	78.133431	78.257134	0.123703	1.006074	1.006430	0.000356

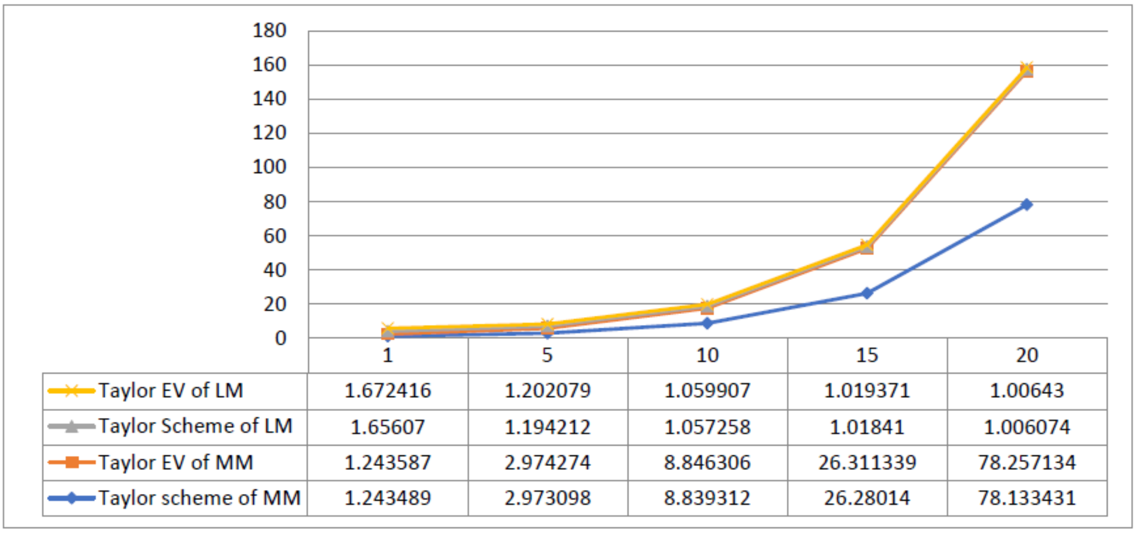

Fig. 2. Comparative study between Malthusian model and Logistic model using Taylor scheme by Line graph for $r = 2.18$.

5.3. When the population growth rate $r = 3.96, K = 1$

Taylor Scheme (Malthusian Model)

Table 4. Comparative study between Malthusian model and Lgistic model using Taylor scheme for $r = 3.96$

Iteration Number	Taylor scheme of MM	Exact value	Absolute error	Taylor scheme of LM	Exact value	Absolute Error
1	1.484758	1.485869	0.001111	1.362934	1.507167	0.144232
2	2.204506	2.207808	0.003302	1.207070	1.292773	0.085703
3	3.273157	3.280514	0.007356	1.126604	1.179823	0.053219
4	4.859 846	4.874415	0.014568	1.080014	1.114301	0.034287
5	7.215695	7.242743	0.027048	1.051520	1.074154	0.022634
6	10.713560	10.761770	0.048210	1.033547	1.048725	0.015178
7	15.907042	15.990583	0.083541	1.021998	1.032278	0.010280
8	23.618105	23.759917	0.141812	1.014490	1.021496	0.007006
9	35.067167	35.304132	0.236964	1.009572	1.014366	0.004794
10	52.066252	52.457326	0.391074	1.006335	1.009623	0.003288
11	77.305777	77.944731	0.638954	1.004198	1.006456	0.002258
12	114.780360	115.815684	1.035325	1.002785	1.004336	0.001551
13	170.421041	172.086972	1.665931	1.001848	1.002914	0.001066
14	253.033980	255.698752	2.664772	1.001227	1.001959	0.000733
15	375.694189	379.934930	4.240741	1.000815	1.001318	0.000503
16	557.814899	564.533654	6.718756	1.000541	1.000886	0.000346
17	828.220053	838.823236	10.603183	1.000359	1.000596	0.000237
18	1229.706230	1246.381709	16.675479	1.000239	1.000401	0.000163
19	1825.815986	1851.960339	26.144354	1.000159	1.000270	0.000112
20	2710.894628	2751.771046	40.876417	1.000105	1.000182	0.000076

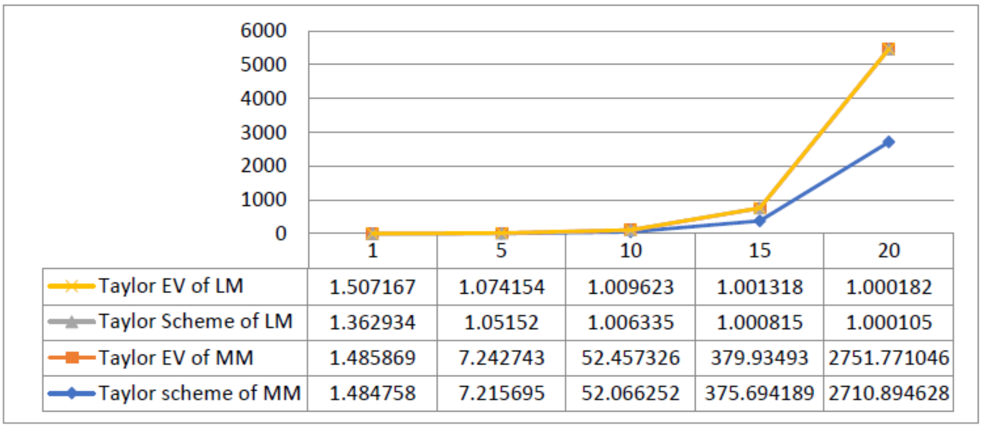

Fig. 3. Comparative study between Malthusian model and Logistic model using Taylor scheme by Line graph for $r = 3.96$.

6. Comparative study between Malthusian model and Logistic model using RK-4 scheme

6.1. Program: When the population growth rate $r = 1.37, k = 1$

Table 5. Comparative study between Malthusian model and Logistic model using Taylor scheme for $r = 1.37$

Iteration Number	Rk-4 scheme of MM	Exact value	Absolute Error	Rk-4 scheme of LM	Exact value	Absolute Error
1	1.146828	1.146828	0.000000	1.773016	1.773003	0.000013
2	1.315214	1.315215	0.000001	1.613351	1.613335	0.000016
3	1.508324	1.508325	0.000002	1.495886	1.495872	0.000015
4	1.729787	1.729790	0.000002	1.406586	1.406573	0.000013
5	1.983768	1.983772	0.000004	1.336990	1.336979	0.000011
6	2.275040	2.275045	0.000005	1.281693	1.281683	0.000010
7	2.609080	2.609086	0.000007	1.237078	1.237070	0.000008
8	2.992165	2.992173	0.000009	1.200635	1.200628	0.000007
9	3.431498	3.431509	0.000011	1.170567	1.170561	0.000006
10	3.935337	3.935351	0.000014	1.145551	1.145546	0.000005
11	4.513153	4.513171	0.000018	1.124594	1.124590	0.000005
12	5.175809	5.175831	0.000022	1.106937	1.106933	0.000004
13	5.935762	5.935789	0.000028	1.091986	1.091983	0.000003
14	6.807296	6.807330	0.000034	1.079276	1.079273	0.000003
15	7.806796	7.806838	0.000042	1.068432	1.068429	0.000003
16	8.953050	8.953101	0.000051	1.059152	1.059150	0.000002
17	10.267606	10.267669	0.000063	1.051191	1.051189	0.000002
18	11.775175	11.775252	0.000076	1.044347	1.044345	0.000002
19	13.504098	13.504190	0.000092	1.038451	1.038449	0.000001
20	15.486874	15.486985	0.000111	1.033364	1.033362	0.000001

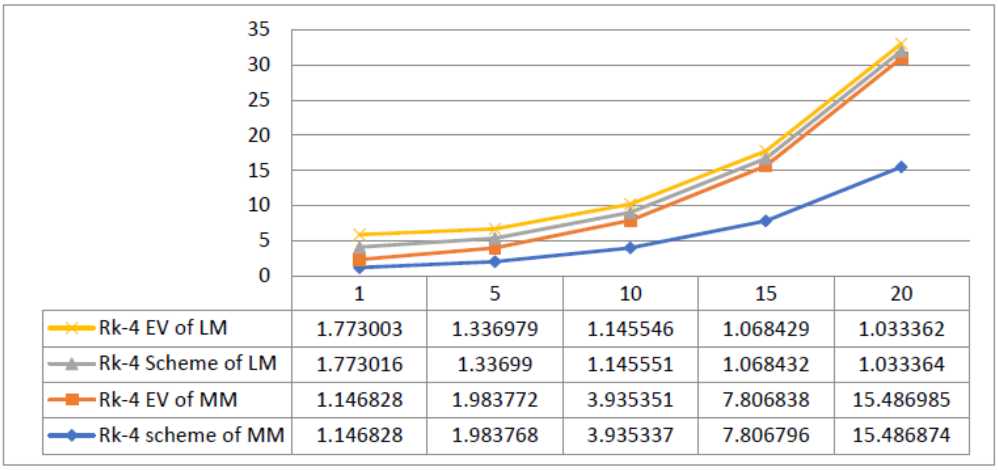

Fig. 4. Comparative study between Malthusian model and Logistic model using Rk-4 scheme by Line graph for $r = 1.37$.

6.2. When the population growth rate $r = 2.18, K = 1$

Table 6. Comparative study between Malthusian model and Logistic model using Taylor scheme for $r = 2.18$ (MM = Malthusian Model, LM = Logistic Model)

Iteration Number	Rk-4 scheme of MM	Exact value	Absolute Error	Rk-4 scheme of LM Exact value	Absolute Error
1	1.243583	1.243587	0.000004	1.672509 1.672416	0.000093
2	1.546498	1.546509	0.000011	1.477869 1.477779	0.000090
3	1.923199	1.923218	0.000020	1.351390 1.351316	0.000074
4	2.391657	2.391689	0.000033	1.264372 1.264314	0.000058
5	2.974223	2.974274	0.000051	1.202125 1.202079	0.000046
6	3.698693	3.698769	0.000076	1.156346 1.156310	0.000036
7	4.599631	4.599741	0.000110	1.121988 1.121959	0.000029
8	5.720022	5.720178	0.000157	1.095805 1.095782	0.000023
9	7.113321	7.113540	0.000219	1.075621 1.075602	0.000018
10	8.846003	8.846306	0.000303	1.059921 1.059907	0.000015
11	11.000738	11.001152	0.000414	1.047626 1.047614	0.000012
12	13.680328	13.680890	0.000562	1.037943 1.037934	0.000010
13	17.012621	17.013378	0.000757	1.030286 1.030278	0.000008
14	21.156603	21.157617	0.001014	1.024211 1.024204	0.000006
15	26.309988	26.311339	0.001351	1.019377 1.019371	0.000005
16	32.718649	32.720441	0.001792	1.015522 1.015518	0.000004
17	40.688350	40.690718	0.002368	1.012444 1.012441	0.000003
18	50.599333	50.602450	0.003118	1.009982 1.009980	0.000003
19	62.924460	62.928553	0.004093	1.008011 1.008009	0.000002
20	78.251777	78.257134	0.005357	1.006432 1.006430	0.000002

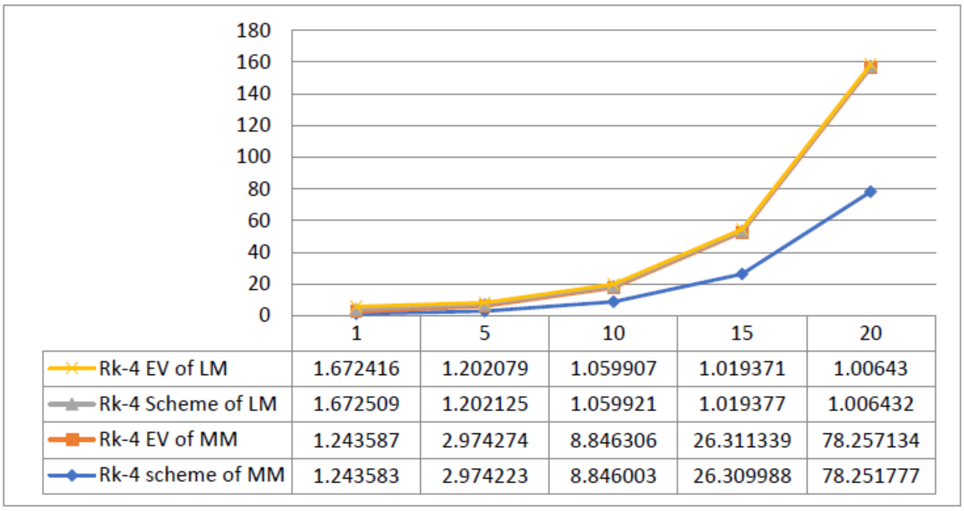

Fig. 5. Comparative study between Malthusian model and Logistic model using Rk-4 scheme by Line graph for $r = 2.18$.

6.3. When the population growth rate $r = 3.96$, $k = 1$

Table 7. Comparative study between Malthusian model and Logistic model using Taylor scheme for $r = 3.96$

Iteration Number	Rk-4 scheme of MM	Exact value	Absolute Error	Rk-4 scheme of LM	Exact value	Absolute Error
1	1.485782	1.485869	0.000087	1.507700	1.507167	0.000533
2	2.207550	2.207808	0.000258	1.293245	1.292773	0.000472
3	3.279939	3.280514	0.000575	1.180157	1.179823	0.000334
4	4.873275	4.874415	0.001139	1.114531	1.114301	0.000230
5	7.240627	7.242743	0.002116	1.074312	1.074154	0.000158
6	10.757997	10.761770	0.003773	1.048834	1.048725	0.000109
7	15.984044	15.990583	0.006540	1.032354	1.032278	0.000076
8	23.748812	23.759917	0.011105	1.021549	1.021496	0.000053
9	35.285569	35.304132	0.018562	1.014403	1.014366	0.000037
10	52.426681	52.457326	0.030645	1.009649	1.009623	0.000026
11	77.894645	77.944731	0.050087	1.006474	1.006456	0.000018
12	115.734499	115.815684	0.081185	1.004349	1.004336	0.000013
13	171.956293	172.086972	0.130679	1.002923	1.002914	0.000009
14	255.489649	255.698752	0.209103	1.001965	1.001959	0.000006
15	379.602047	379.934930	0.332883	1.001322	1.001318	0.000004
16	564.006075	564.533654	0.527579	1.000889	1.000886	0.000003
17	837.990352	838.823236	0.832884	1.000598	1.000596	0.000002
18	1245.071393	1246.381709	1.310316	1.000403	1.000401	0.000001
19	1849.905277	1851.960339	2.055063	1.000271	1.000270	0.000001
20	2748.556871	2751.771046	3.214174	1.000182	1.000182	0.000001

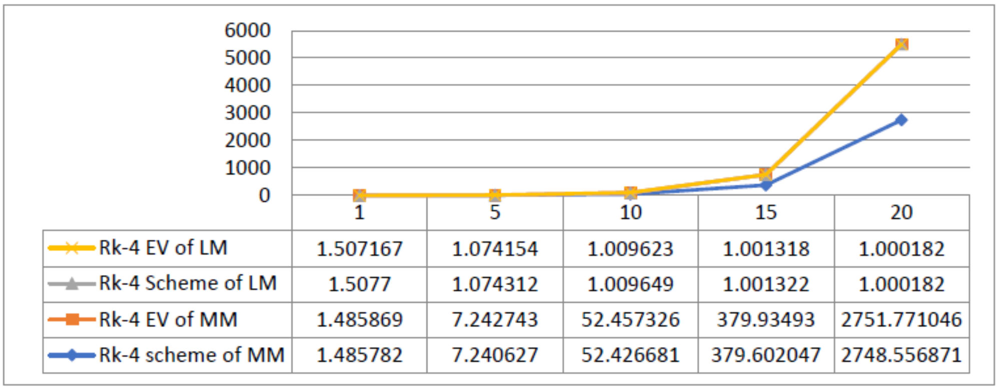

Fig. 6. Comparative study between Malthusian model and Logistic model using Rk-4 scheme by Line graph for $r = 3.96$.

7. Results and Discussions

When compared to other methods, Runge-Kutta methods provide an estimate that is more accurate, and they also eliminate the need for higher-order derivatives. When compared to alternative approaches, these strategies also perform better in terms of prior estimation. There are no special processes or preconditions needed to begin using the Runge-Kutta method. Step-length can be easily modified to obtain desired accuracy, and since intermediate values need not be stored, more computational resources are made available for iterative calculations. The accuracy of the solution provided by the Runge-Kutta fourth-order method can be improved by decreasing the step-length h as needed. The Taylor scheme is employed to contrast the Malthusian and Logistic hypotheses. Logistic modelling clearly provides more reliable results than Malthusian modelling. The Malthusian model and the Logistic model are compared using the Rk-4 method. The Logistic Model is clearly superior to the Malthusian Model in terms of the accuracy of its predictions.

8. Conclusion

For this investigation, ordinary differential equations were determined through mathematical analysis of two models. These two classifications of models are known as the Malthusian model and the Logistic model, respectively. Extensive testing has been done on the proposed numerical solutions for the logistic model and the Malthusian model. Comparative results of the Malthusian model and the logistic model being applied to the Taylor scheme and the RK-4 scheme are shown and discussed. When compared to the Malthusian model, it is clear that the RK-4 approach yields more reliable findings. Finally, the Malthusian and logistic models are graphically shown for different settings. Population increase was analyzed using both models.

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