Int. J. Adv. Appl. Math. and Mech. 11(3) (2024) 18 – 25 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

Abrupt Change-point detection in the statisticals proprieties and location parameter with the modified information criterion

Research Article

Mahamat Ali ISSAKA*

Laboratoire de Modélisation Mathématique, Informatique, Applications et Simulations (L2MIAS), Université de N'Djamena, BP 1027, Tchad

Received 05 September 2023; accepted (in revised version) 01 November 2023

Abstract: The detection of change point in the mean and variance associated with the modified information criterion is aborded in this study. The materials and methods which used in order to estimate the location and shape parameter are the exponential modified Gaussian distribution supposed as a statistical test problem. The application of the proposed model with the powerful is done on the brain electrical activity data.

MSC: 62F03 • 62F05 • 62J15 • 62P10 • 62P30

Keywords: Data Analysis • Statistical data analysis • Mean • Variance • Modified Information Criterion • EEG

© 2024 The Author(s). This is an open access article under the CC BY-NC-ND license (https://creativecommons.org/licenses/by-nc-nd/3.0/).

1. Introduction

Changes occurs frequently in all process. Particulary, the variation of distribution in the statisticals characteristics is a very interesting challenge in stochastic process.
The principal investigation of the change-point issue was gone back to the papers [6, 7] who originally proposed a technique to identify just one change in the distribution. The Application of those change point detection are effective such as in industrial quality management (production monitoring), finance (stock prices), climate studies (global warming) as well as in medicine(online monitoring of intensive-care patients) or geoscience (annual water volume of rivers) to cite a few.
However, we will focus on studying the **Exponentially Modified Gaussian (EMG)** distribution. So, consider a sequence of independent random variables respectively from a normal distribution $N(\mu,\sigma)$ and exponential distribution $\xi(\alpha)$ defined by

$$\forall i = 1,...,n, Y_i \sim N(\mu,\sigma), Z_i \sim \xi(\alpha), X_i = Z_i + Y_i \text{ with } X = (X_1,...,X_n) \sim EMG(\mu,\sigma,\alpha) .$$

In this form, we use modified information criterion (MIC) correlated with test statistics to detect changes in mean and variance parameters and aslso in the location parameters which is α.
So, our testing problem can be defined as follows:

* *E-mail address(es):* mhtali1@gmail.com

$$H_0 : \begin{cases} \underbrace{\mu_1 = \mu_2 = \ldots = \mu_n}_{\mu}, \\ \underbrace{\sigma_1^2 = \sigma_2^2 = \ldots = \ldots = \sigma_n^2}_{\sigma}, \\ \underbrace{\alpha_1 = \alpha_2 = \ldots = \alpha_n}_{\alpha}, \\ \underbrace{MIC(k_1) = MIC(k_2) = \ldots = MIC(k_n)}_{MIC} \end{cases}$$

versus

$$H_1 : \begin{cases} \underbrace{\mu_1 = \mu_2 = \ldots = \mu_k \neq \mu_{k+1} = \ldots = \mu_n}_{\mu}, \\ \underbrace{\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2 \neq \sigma_{k+1}^2 = \ldots = \sigma_n^2}_{\sigma^2}, \\ \underbrace{\alpha_1 = \alpha_2 = \ldots = \alpha_k \neq \alpha_{k+1} = \ldots = \alpha_n}_{\alpha}, \\ \underbrace{MIC(l_1) = MIC(l_2) = \ldots = MIC(l_k) \neq MIC(l_{k+1}) = \ldots = MIC(l_n)}_{MIC} \end{cases}$$

Among the researchers who have worked on this subject, we can cite Wei Ning et al.[2] who proposed a linear regression model with normal distribution errors in detecting change in the regression parameters β_0 and β_1. Particularly, a testing procedure based on a modified information criterion (MIC) for the location of change points has been discussed. Otherwise, Khamis K. Said et al.[3] study the detection procedure based on the modified information criterion (MIC) for change problem with the skew normal distribution model. Recently, Mahamat et al. [4] proposed an exponential modified Gaussian change-point model based on the modified information criterion (MIC) associated with the variance criterion.

The paper is organized as follows. In Section 2 and 3, we discuss on the materials and methodology of this paper which turn about the detection of abnormal derivations with the criterion of the mean and variance combined with the modified information criterion (MIC). The application of the proposed model is done on real data in the section 4 with interpretation of the analysis. We finished this purpose at the conclusion in the last and next section.

2. Exponentially-Modified Gaussian (EMG) distribution and Estimation

2.1. Model

We introduced the exponentially-modified Gaussian (EMG) distribution, defined as a convolution of an exponential distribution with parameter α and a normal distribution with parameters μ and σ, which are independent of each other. So, $X \sim EMG(\mu, \sigma, \alpha)$ where $X_i = Z_i + Y_i$ as defined in [8]

$$f(x;\mu,\sigma,\alpha) = \frac{1}{2\alpha} \exp\left[\frac{1}{2\alpha}(2\mu + \frac{\sigma^2}{\alpha} - 2x)\right] erfc\left(\frac{\mu + \frac{\sigma^2}{\alpha} - x}{\sqrt{2}\sigma}\right) \tag{1}$$

$$= \frac{1}{\alpha} \exp\left(\frac{\mu - x}{\alpha} + \frac{\sigma^2}{2\alpha^2}\right) \Phi\left(\frac{x-\mu}{\sigma} - \frac{\sigma}{\alpha}\right) \tag{2}$$

$$= \frac{1}{\alpha} \exp\left(\frac{x-\mu}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right) \Phi\left(\frac{x-\mu}{\sigma} - \frac{\sigma}{\alpha}\right) \tag{3}$$

where $erfc(z) = 1 - erf(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty \exp(-t^2)dt$ and $erfc(\frac{z}{\sqrt{2}}) = 2\Phi(-z)$ with $\Phi(.)$ is the Standard Normal distribution function.

2.2. Maximum Likelihood Estimator

The MLEs using the p.d.f. of our model under H_0 are given by $\Theta = (\mu, \sigma, \alpha), \Theta_{1k} = (\mu_1, \sigma_1, \alpha_1), \Theta_{nk} = (\mu_n, \sigma_n, \alpha_n)$ with:

$$L_{H_0}(\Theta) = \prod_{i=1}^{n} f(x_i;\mu,\sigma,\alpha) \tag{4}$$

$$= \prod_{i=1}^{n} \frac{1}{\alpha}\exp\left(\frac{x_i-\mu}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right)\prod_{i=1}^{n}\Phi\left(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha}\right) \tag{5}$$

However, the log-likelihood function under H_0 and H_1 are given respectively by

$$\ln L_{H_0}(\Theta) = -n\ln\alpha + \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right) + \sum_{i=1}^{n}\ln\Phi\left(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha}\right) \tag{6}$$

$$\ln L_{H_1}(\Theta_{1k},\Theta_{nk}) = \left\{-k\ln\alpha_1 + \sum_{i=1}^{k}\left(\frac{x_i-\mu_1}{\alpha_1} - \frac{\sigma_1^2}{2\alpha_1^2}\right) + \sum_{i=1}^{k}\ln\Phi\left(\frac{x_i-\mu_1}{\sigma_1} - \frac{\sigma_1}{\alpha_1}\right)\right\} \tag{7}$$

$$+\left\{-(n-k)\ln\alpha_n + \sum_{i=k+1}^{n}\left(\frac{x_i-\mu_n}{\alpha_n} - \frac{\sigma_n^2}{2\alpha_n^2}\right) + \sum_{i=k+1}^{n}\ln\Phi\left(\frac{x_i-\mu_n}{\sigma_n} - \frac{\sigma_n}{\alpha_n}\right)\right\} \tag{8}$$

The resolution of the the following nonlinear equations give us the maximum likelihood estimators(MLE) under H_0 and H_1 which are :

$$\frac{\partial}{\partial\mu}(\ln L_{H_0}) = -\frac{n}{\alpha} - \frac{1}{\sigma}\sum_{i=1}^{n}\frac{\phi(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha})}{\Phi\left(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha}\right)} = 0 \tag{9}$$

$$\frac{\partial}{\partial\sigma}(\ln L_{H_0}) = -\frac{n\sigma}{\alpha^2} - \sum_{i=1}^{n}\left(\frac{(\frac{x_i-\mu}{\sigma^2} - \frac{1}{\alpha})\phi(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha})}{\Phi\left(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha}\right)}\right) = 0 \tag{10}$$

$$\frac{\partial}{\partial\alpha}(\ln L_{H_0}) = -\frac{n}{\alpha} + \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\alpha^2} - \frac{\sigma^2}{\alpha^3} - \frac{\sigma}{\alpha^2}\frac{\phi(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha})}{\Phi\left(\frac{x_i-\mu}{\sigma} - \frac{\sigma}{\alpha}\right)}\right) = 0 \tag{11}$$

$$\frac{\partial}{\partial\mu_1}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{k}{\alpha_1} - \frac{1}{\sigma}\sum_{i=1}^{k}\frac{\phi(\frac{x_i-\mu_1}{\sigma} - \frac{\sigma}{\alpha_1})}{\Phi\left(\frac{x_i-\mu_1}{\sigma} - \frac{\sigma}{\alpha_1}\right)} = 0 \tag{12}$$

$$\frac{\partial}{\partial\sigma_1}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{k\sigma_1}{\alpha_1^2} - \sum_{i=1}^{k}\left(\frac{(\frac{x_i-\mu}{\sigma_1^2} - \frac{1}{\alpha_1})\phi(\frac{x_i-\mu}{\sigma_1} - \frac{\sigma_1}{\alpha_1})}{\Phi\left(\frac{x_i-\mu}{\sigma_1} - \frac{\sigma_1}{\alpha_1}\right)}\right) = 0 \tag{13}$$

$$\frac{\partial}{\partial\alpha_1}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{k}{\alpha_1} + \sum_{i=1}^{k}\left(\frac{x_i-\mu}{\alpha_1^2} - \frac{\sigma_1^2}{\alpha_1^3} - \frac{\sigma_1}{\alpha_1^2}\frac{\phi(\frac{x_i-\mu}{\sigma_1} - \frac{\sigma_1}{\alpha_1})}{\Phi\left(\frac{x_i-\mu}{\sigma_1} - \frac{\sigma_1}{\alpha_1}\right)}\right) = 0 \tag{14}$$

$$\frac{\partial}{\partial\mu_n}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{(n-k)}{\alpha_n} - \frac{1}{\sigma}\sum_{i=k+1}^{n}\frac{\phi(\frac{x_i-\mu_n}{\sigma} - \frac{\sigma}{\alpha_n})}{\Phi\left(\frac{x_i-\mu_n}{\sigma} - \frac{\sigma}{\alpha_n}\right)} = 0 \tag{15}$$

$$\frac{\partial}{\partial\sigma_n}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{(n-k)\sigma_n}{\alpha_n^2} - \sum_{i=k+1}^{n}\left(\frac{(\frac{x_i-\mu}{\sigma_n^2} - \frac{1}{\alpha_n})\phi(\frac{x_i-\mu}{\sigma_n} - \frac{\sigma_n}{\alpha_n})}{\Phi\left(\frac{x_i-\mu}{\sigma_n} - \frac{\sigma_n}{\alpha_n}\right)}\right) = 0 \tag{16}$$

$$\frac{\partial}{\partial\alpha_n}(\ln L_{H_1}(\Theta_{1k},\Theta_{nk})) = -\frac{(n-k)}{\alpha_n} + \sum_{i=k+1}^{n}\left(\frac{x_i-\mu}{\alpha_n^2} - \frac{\sigma_n^2}{\alpha_n^3} - \frac{\sigma_n}{\alpha_n^2}\frac{\phi(\frac{x_i-\mu}{\sigma_n} - \frac{\sigma_n}{\alpha_n})}{\Phi\left(\frac{x_i-\mu}{\sigma_n} - \frac{\sigma_n}{\alpha_n}\right)}\right) = 0 \tag{17}$$

As $\phi(.)$ and $\Phi(.)$ are nonlinear functions in the resolution, we resolve it by numerical solutions of the MLEs in order to obtain the explicit forms of μ,σ,α under H_0, and $\sigma_1,\alpha_1,\sigma_n,\alpha_n$ under H_1.

3. Modified Information Criterion

In addition to the mean and variance criterions, we used the modified information criterion (MIC) proposed by Chen et al.[1] which is the modification of the approach based on Swartz Information Criterion (SIC) by refining the model complexity as a function of the change location in the context of change point problem. Thus, under the hypothesis H_0 and H_1, we have the MIC respectively defined as follows:

$$MIC(n) = -2\ln L_{H_0}(\hat{\mu},\hat{\sigma},\hat{\alpha}) + 3\ln(n) \tag{18}$$

$$MIC(k) = -2\ln L_{H_1}(\hat{\mu}_1,\hat{\sigma}_1,\hat{\alpha}_1,\hat{\mu}_n,\hat{\sigma}_n,\hat{\alpha}_n) + \left\{6 + \left(\frac{2k}{n} - 1\right)^2\right\}\ln(n) \tag{19}$$

So, we accept the hypothesis H_0 which means that there is no change point in the criterion of MIC in our derivations if we have

$$MIC(n) \leq \min_{1\leq k\leq n} MIC(k). \tag{20}$$

We reject H_0 if

$$MIC(n) > \min_{1\leq k\leq n} MIC(k) \tag{21}$$

which means that there exists at least one change point in the derivations.
As change point problem, the second step after the detection is the estimation of the change point location $\hat{k}$ who can be defined by

$$MIC(\hat{k}) = \min_{1\leq k\leq n} MIC(k) \tag{22}$$

As our study is based on the detection with the test statistic which perform the acuracy, we define the test statistic K_n by

$$K_n = MIC(n) - \min_{1\leq k\leq n} MIC(k) + 3\ln(n) \tag{23}$$

By substituting the Eq. (18) and (19) into the Eq. (23), we obtain

$$K_n = -2\ln L_{H_0}(\hat{\Theta}) + 3\ln(n) - \min_{1\leq k\leq n}\left[-2\ln L_{H_1}(\hat{\Theta}_{1k},\hat{\Theta}_{nk}) + \left\{6 + \left(\frac{2k}{n} - 1\right)^2\right\}\ln(n)\right] + 3\ln(n) \tag{24}$$

$$= -2\ln L_{H_0}(\hat{\Theta}) + 3\ln(n) - \min_{1\leq k\leq n}\left[-2\ln L_{H_1}(\hat{\Theta}_{1k},\hat{\Theta}_{nk}) + 6\ln(n) + \left(\frac{2k}{n} - 1\right)^2\ln(n)\right] + 3\ln(n) \tag{25}$$

$$= -2\ln L_{H_0}(\hat{\Theta}) - \min_{1\leq k\leq n}\left[-2\ln L_{H_1}(\hat{\Theta}_{1k},\hat{\Theta}_{nk}) + \left(\frac{2k}{n} - 1\right)^2\ln(n)\right] \tag{26}$$

where $\hat{\Theta} = (\hat{\mu},\hat{\sigma},\hat{\alpha})$ and $(\hat{\Theta}_{1k},\hat{\Theta}_{nk}) = (\hat{\mu}_1,\hat{\sigma}_1,\hat{\alpha}_1,\hat{\mu}_n,\hat{\sigma}_n,\hat{\alpha}_n)$.
However, we reject the null hypothesis if K_n is large than the theorical value according to the theorem below.

Theorem 3.1.
Suppose that the Wald conditions $W1 - W7$ and the regularity conditions $R1 - R3$ of [2] are satisfied. Under the null model, as $n \longrightarrow \infty$ then $K_n \rightsquigarrow \chi^2_3$ in distribution, where K_n is defined in equation (26).

Proof: See [2]

4. Application

We applied the proposed model of detection which cumulating statistical characterization μ,σ,α by modified information criterion in the temporal and spatial case on the collected data in this study which are the electrophysiolocial data of patients suffering to epilepsy. We used seven derivations of EEG collected on the frequency of 256Hz and in twenty minutes recorded times.

4.1. Detection in temporal case with MIC

As showed in the Fig. 1, we remark that the detection is so difficult to see but the shape of the curve seems to stand out in two regimes throughout the signal in time. Therefore, we try to analyze the spatial case of the distribution for each derivations in order to localize the abrupt changes.

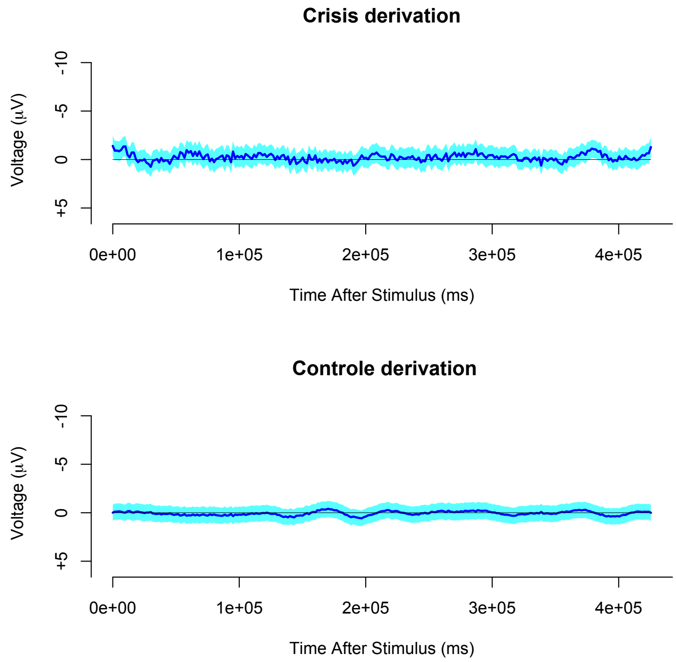

Fig. 1. Temporal evolution of derivation with the parameters μ and σ

4.2. Detection in spatial case with MIC

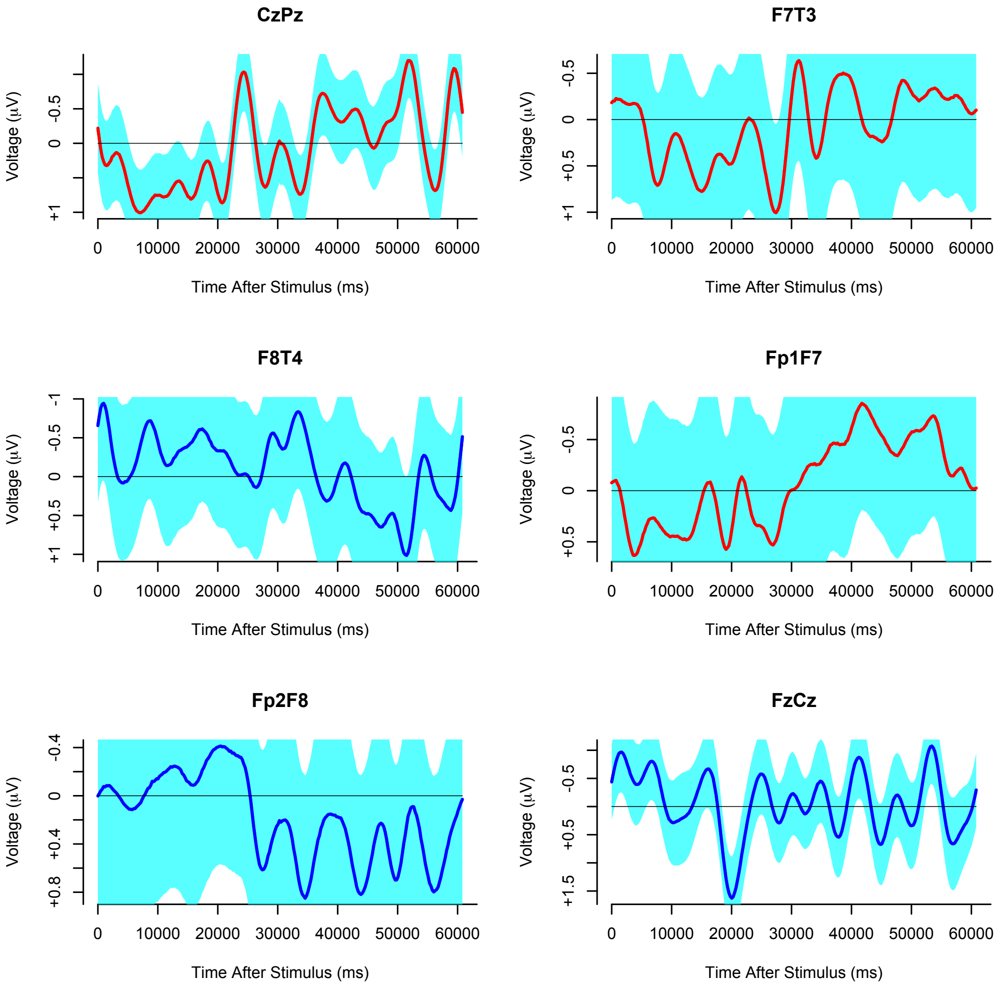

Fig. 2. MIC detection in the spatial case for epileptic patient

In the Fig.2, we displayed the derivation with the mean and variance detection in each derivation of the brain electrical activity. With those derivations named CzFz, F7T3, F8T4, Fp1F7, Fp2F8 and FzCz corresponding respectively to the oxypital, front temporal and fronto-polar, we see that the shape of curve seems stable for the derivations F8T4, Fp2F8 and FzCz as the combination is at zero.

Otherwise, for the F7F3, Fp1F7 and CzPz curves, we observe a typical alpha rhythm at the start of an anomaly in these derivations. We also notice that the departure of the anomaly occurs between the 2nd and the 4th minutes of the recording. This appears to be consistent with the digital signals as well as the interpretation of medical practitioners.

4.3. Accuracy of the proposed model in the detection

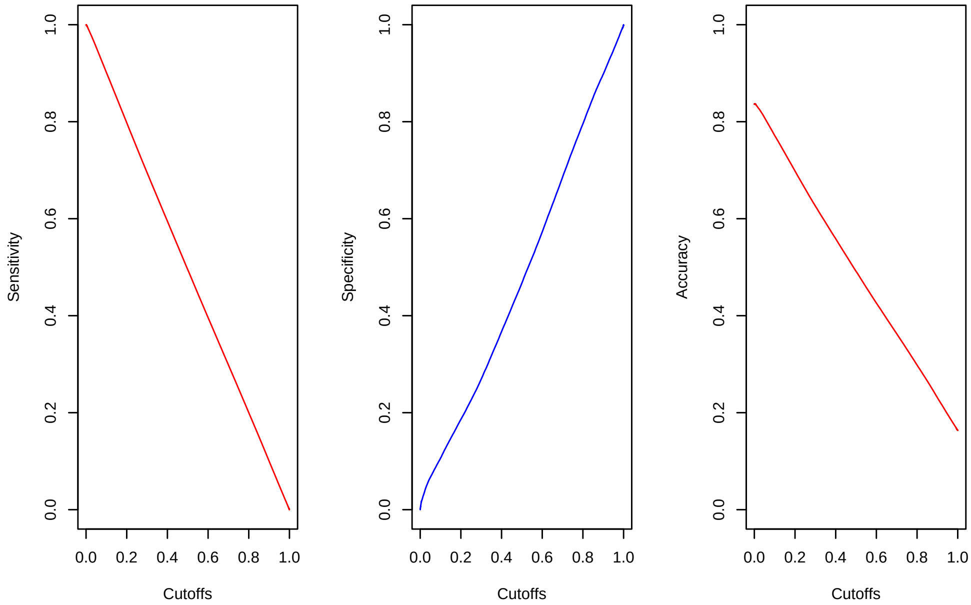

Fig. 3. Accuracy of the proposed model

5. Conclusion

In this paper, we have explored the modified information criterion (MIC) correlated with test statistics to detect changes in mean and variance parameters and also in the location parameters which is α. The innovation is that the detection problem is assimilated in the temporal and spatial case compared with the previous case [3, 4].
The application of this model on the brain electrical activity of epileptic patient and normal shows the shape so typical in crisis as characterization. Even better, the power of the model seems very powerful as the detection of the true positive is very interested as plotted in the Fig. 3 of the previous section.

6. Acknowledgements

The author would like to thank the laboratory Laboratoire Paul Painlévé of the University of Lille for offering a scientific stay of one month.

References

[1] Chen, J., A. K. Gupta, and J. Pan. Information criterion and change point problems for regular models. Sankhya (2006)68:252–282.
[2] Doaa Basalamah, Khamis K. Said, Wei Ning, Yubin Tian, Modified information criterion for linear regression change-point model with its applications. Communications in Statistics - Simulation and Computation (2019.) (15):2740–50. https://doi.org/10.1080/03610918.2018.1554109.
[3] Khamis K. Said, Wei Ning, and Yubin Tian, Modified information criterion for testing changes in skew normal model, Brazilian Journal of Probability and Statistics, (2019) 33 (2):280–300. doi:10.1214/17-BJPS388.
[4] Mahamat Ali Issaka, I.T.Bachar, Variance Change-point Problem in Modified Information Criterion, Far East Journal of Theoretical Statistics,(2020) Vol. 60, No.1-2, pp. 25-39.
[5] Ngunkeng, G., and W. Ning, Information approach for the Change-Point detection in the skew-normal distribution and its applications. Sequential Analysis: Design Methods and Applications 33 (2019) (4):475–90. doi:10.1080/07474946.2014.961845.

[6] Page, E.S., Continuous inspection scheme, Biometrika (1954)(41):100-114.
[7] Page, E.S., A test for a change point in a parameter occuring at an unknown point, Biometrika (1955) (42):523-527.
[8] Scott Haney, Practical Applications and Properties of the Exponentially Modified Gaussian (EMG) Distribution, PhD thesis, Drexel University, 2011.

Submit your manuscript to IJAAMM and benefit from:

► Rigorous peer review
► Immediate publication on acceptance
► Open access: Articles freely available online
► High visibility within the field
► Retaining the copyright to your article

Submit your next manuscript at ► editor.ijaamm@gmail.com