Int. J. Adv. Appl. Math. and Mech. 11(4) (2024) 51 – 70 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Optimal Portfolio Selection of a Constant Proportion Portfolio Insurance when Asset follows Hawkes-Jump-Diffusion

Research Article

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Received 19 June 2024; accepted (in revised version) 08 July 2024

Abstract: We study the firmly of risk asset in the constant proportion portfolio insurance (CPPI) trading strategy in Hawkes-jump-diffusion model where the price of the underlying asset may experience negative jumps. We solve the dynamic of risk asset and cushion by using a mean version stochastic differential Equation under Geometric Brownian Motion. The main goal of portfolio insurance is to protect investors against adverse market movement. We consider the problem of optimal portfolio construction through the dynamics programming and its associate HJB equation of a two-dimensional to solve the supreme of portfolio weights by considering an investors of log, power and exponential utility function. The optimal portfolio model react to each change in jump intensity accordingly. It was observed that, the higher the value of volatility and jump size, the less the expected terminal portfolio. Therefore, the best payoff can be achieved with the increase in number of re-balancing the optimal portfolio weights. And hence reduce the risk of breaching the designed floor.

MSC: 76D10 • 76M20

Keywords: Portfolio Insurance • Optimal Portfolio • Mean Version Stochastic differential equation • Hawkes process

1. Introduction

The idea of optimal allocation in a market between a risk-free bond and a stock was first solved explicitly by [1]. He alternatively solve an investment where the price of the risky asset follows a geometric Brownian motion with Poisson jumps. An investor want to maximized his terminal wealth by allocating money in a risk free and risk asset through the process of solving portfolio optimization problem. Merton problem lead to the problem of solving the Hamilton -Jacobi-Bellman equation from stochastic control theory lead to a non-linear partial differential equation (PDE) for the optimal expected utility as a function of time and current wealth. This was supported by [2], [3] and [4].They solve this PDE, and obtained optimal portfolio in a multi-period discrete time model in similar approach leads to a recursive equation instead of a PDE. Recently investors has paid more attention to corporate with jumps impacts in the market after the recent financial crisis of 2008. [5], observed that this trend might increase due to possible inhomogeneities in the asset prices currently observed in financial markets attention to time-in-homogeneous process could also be paid.

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While the classical way of incorporating jumps in the risky assets dynamics is that of jump-diffusion models, recently the introduction of jumps has been often made with less intuitive instruments: It is expected in real life that some types of events that are observed will naturally cluster in time. Some greater examples to mention, like an earthquake typically increases the geological tension of the region in which it occurs, and aftershocks will likely follow. And also the rival gangs might ignite a spate of criminal retaliations. Or, on a larger scale, the collapse of a wall street investment bank could send shock-waves through the world's financial centre. The mathematical model which is a self-contagion achieved by introducing self-exciting jump in the underlying dynamics was introduced and named after Hawkes [6]. While in Lèvy-driven models the intensity of adverse shocks taken to be constant, self-exciting jump process account for the risk of jump clustering documented in real markets. We study a counting process where risky asset in the market may experience a downward-jumps. After the initial arrival of a jump substantially increased the likelihood of future jumps. The intensity increases by the magnitude of the jump re-scaled by the parameter δ every time it experience an adverse shock and jumps it decays exponentially with common rate k to the level of initial jump. As such, it is a non-Markovian extension of the Poisson process.

Portfolio insurance is a strong tool that created to protect investor against adverse market movement, while still participating in case of upward market opportunities. We study one popular example for portfolio insurance strategy the constant proportion portfolio insurance (CPPI). CPPI is a widely used investment strategy that allows to maintain an exposure to the upside potential of a risky asset while providing a guarantee against the downside risk. This dynamic strategy consists in setting a floor equal to the risky asset as we will describe below. Usually the floor percentage is chosen by the investor depending on the risk tolerance of the client, risk preference client chose lower floor percentage and vice versa. The exposure is equal to the product of cushion which defined as the excess of the portfolio value over the floor and of a predetermined multiple [7]. Likely, multiple depend on the investor's risk tolerance. When the cushion grows exposure will grow and when the portfolio value declines, exposure will decline and approach zero. For this reason asset allocation should solely remain in a risk-free asset when the portfolio value reaches the floor value. However the rise in cushion will eventually put all the portfolio value completely into risky asset. The original CPPI model is formulated in continuous time and assumes instantaneous trading and smooth price changes. However, in practice this assumption is violated, this introduces the notion of gap-risk.

We study the possible acceptable level of risk assets and of that risk-free assets as in the case of [8] and [9] in the constant proportion portfolio strategy under Hawkes jump-diffusion model. We solve the dynamic of risk asset and cushion by using a mean version stochastic differential Equation under Geometric Brownian Motion. The main goal of portfolio insurance is to protect investor against adverse market movement. Therefore the investor choose the floor level depends on her risk preference and always try to maintain it through-out the trading period up to the maturity date, so that the Portfolio value will always lies above it. In this paper we consider the problem of optimal Portfolio construction through the dynamics programming and its associate HJB equation of a two-dimensional to solve the optimal weights of risk free and risk assets, by considering an investors of log, power and exponential utility function.The optimal portfolio model react to each change in jump intensity accordingly, an increases leads to a reduced weights in risk asset allocation and a decrease lead to an increase in risk asset allocation.

[10] use the general theory of calculus of variation to develop Optimal control theory, which is used to derive control strategies . This is based on the work of [11], who use main tools of dynamic programming principle, which is very useful in finding solution to dynamic optimization and the Hamilton-Jacobi equation (HJB). The work was also assisted by [12] from the dynamic programming principle in continuous time.

Merton portfolio optimization problem, consist the two investment options, a risk-less asset with constant interest rate and a risk asset whose price fluctuates from time to time. The problem was explicitly solved by [1] of optimal portfolio allocation in a market with a risk-free bond and a stock as investment alternatives. The price of the risky asset follows a geometric Brownian motion. The investor wants to maximize his terminal wealth under a specific utility function (e.g power utility function).

However, in contradiction the interest rate is not always fixed from time to time in real life, unlike under the classical Merton model. To this, [13] argue that even the money in the bank, the interest rate may fluctuate from time to time. Hence, interest rate fluctuation can be strongly correlated with the price fluctuation of the risk asset.

In related to Merton classical portfolio optimization other study that has been related to classical Merton's portfolio optimization problem [14] considered the cases in which there is no consumption and the the goal is to maximize the long term growth rate of the utility based on the wealth.

The optimal portfolio selection problem with stochastic interest and investment constraints was developed by Detemple and Rindisbacher. Where the main focus problem was to define on finite horizon and the power utility function based on terminal wealth.

We will employ the dynamic programming method approach to solve our Hawkess-Jump-Diffusion optimization problem and the associated Hamilton-Jacobi-Bellman (HJB) integro differential equation.

Dynamic programming is a common technique in solving optimal control problems. Its application requires a Markovian state process and leads to a nonlinear partial differential equation (PDE) of first or second order, known as the Hamilton-Jacobi -Bellaman (HJB) equation, satisfied by the value function.

The process of self-exciting jump have been applied for financial modelling in a number of contexts: [15], in the field for credit modelling. [16]; [17] use Hawkess process for index derivatives modelling, while [18] model the term

structure of interest rates, [19] study optimal portfolio allocation in a multivariate modeling framework allowing for contagion among the risky assets, to fit extremal behaviours of financial time series, [20], [21] estimate conditional Value at Risk. In an insurance context, [22] study the ruin problem in a risk model with Hawkes claims arrivals, while [23] study the ruin probabilities in a jump-diffusion setting, the work of [22] was extended by [24] by using a generalization of the Hawkes process and the Cox process to include both self-excited and externally excited jumps.
Further [25] extend the study using a bi-variate self-exciting process, to quantify also collateral losses. With the availability of high-frequency data, considerable attention has been devoted to the modelling of market price micro structure using Hawkess process. A joint model for transaction times and prices at high frequency was proposed by [26]. Hawkess process based on a generalized Pareto distribution to capture extreme losses and their clustering in order to model intraday Value at Risk was modelled by [21], [27] apply the multivariate Hawkes process to model order books at high frequency. [28] suggest that general Hawkes process on large time scales can be considered asymptotically as Brownian motions.
Constant Proportion Portfolio Insurance was first introduced by [29] for equity instruments and [30] for fixed income instruments, CPPI is a portfolio insurance strategy that allows to maintain an exposure to the upside potential of a risky asset while providing a guarantee against the downside risk. In an idealized setting under continuity assumptions imposed both on the trading frequency and the dynamics of the risky asset, the risk of breanching the floor of the CPPI is zero. However, in reality these assumption are violated.This introduces the notion of gap-risk- the risk that the portfolio value will not meet the guarantee at maturity. Discontinuities in the price of the risky asset, trading frictions and a lack of liquidity all contribute to gap risk. It has been widely documented that price trajectories contain Jumps. The risk of breaching the floor even under continuous-time trading was first introduced by [31], [32]. [5] examine the impact of price jump using historical parameter estimates and find that although there is some gap risk, it is relatively low. [33] compares different re-balancing strategies and ranks their performance for an investor with power utility. She finds that the optimal re-balancing rule should depend both on transaction costs and the changes in the risky asset.
[34] find that in a setting process where the risky asset is driven by a geometric Brownian motion, an individual with Hara utility and susilence level will find the CPPI strategy optimal among different investment strategies with possible guarantee attached. A well contexted study on the problem under discrete-time trading is given in [35] and also [36] investigate the effects of discrete time trading restrictions on the CPPI. They find that the discrete time performance of the CPPI is extremely sensitive to increase in the volatility of the underlying. The problem for discrete time classical CPPI with a fixed-growth floor is studied in [37]. The literature also includes hedging strategies with artificial assets to model jump and price gap risk. [7] apply extreme value theory to allow higher multiplier values when a quantile hedging approach is taken. [38] and [39] use historical data from large indices. Alternative strategies whereby re-balancing is triggered by movements in the risky asset are investigated by [33],[40], [39] and [38].
To end [38] investigates the statistical properties and main issues in the implementation of the CPPI strategy. They show that when considering a realistic level of volatility the CPPI does not perform well in comparison to a riskless investment and gapless portfolio. CPPI returns are highly skewed and in certain cases, fat-tailed.

2. Outline of CPPI Strategy.

Introduced by [29] for equity instruments and [30] for fixed income instruments, the CPPI portfolio insurance strategy guaranteed the minimum amount G to be paid at maturity T to be not less than the desired floor F while providing a guarantee against the downside risk.
The floor F_t which is usually defined as a percentage p of the initial investment

$$F_t = GV_0. \tag{1}$$

CPPI portfolio must be managed to keep the portfolio value V_t, at any time $t \leq T$ above the determined floor value F_t with

$$F_t = Ge^{-r(T-t)} \tag{2}$$

$$dF_t = rF_t dt \tag{3}$$

To ensure this guarantee, First, the CPPI determines the amount to invest in risk asset s_t and allocates the remaining funds to a risk-free asset B_t. Which follows the same dynamics as the floor

$$B_t = V_t - E_t \tag{4}$$

$$dB_t = rB_t dt \tag{5}$$

The floor value F_t is used to calculate the cushion C_t, the difference between portfolio value V_t and the floor F_t.

$$V_t = F_t + C_t \tag{6}$$

$$C_t = V_t - F_t \tag{7}$$

The exposure e_t must be continuously adjusted to a constant multiple m.

$$e_t = m(V_t - F_t) = mC_t \tag{8}$$

3. The underlying lognormal risk asset price process follows a geometric Brownion motion under mean version stochastic differential equation.

We present the dynamics of CPPI strategy when the underlying risky asset follows a Mean Reversion Model

$$dS(t) = kS(t)[\mu - \ln(S(t))]dt + S(t)\sigma dW(t). \tag{9}$$

And the risk-free asset B evolves according to a constant rate of return r

$$dB_t = B_t r dt. \tag{10}$$

The variations of CPPI portfolio value can be written as follows:

$$dV_t = (V_t - e_t)\frac{dB_t}{B_t} + e_t\frac{dS_t}{S_t}. \tag{11}$$

And the variations of the cushion are given by

$$dC_t = (V_t - e_t)\frac{dB_t}{B_t} + \frac{dS_t}{s_t} - dF_t. \tag{12}$$

Knowing that $V_t = C_t + F_t$ and $e_t = mC_t$, the previous equation can be written as follows:

$$dC_t = (C_t + F_t - mC_t)\frac{dB_t}{B_t} + (mCt))\frac{dS_t}{S_t} - \frac{dB_t}{B_t}F_t. \tag{13}$$

And because both the risk-free asset and the floor evolve in the same deterministic way

$$\frac{dB_t}{B_t} = \frac{dF_t}{F_t} = rdt \tag{14}$$

we can rewrite equation (13) as

$$dC_t = (C_t - mC_t)\frac{dB_t}{B_t} + (mC_t)\frac{dS_t}{S_t} \tag{15}$$

which can also be written as

$$dC_t = C_t[r - mr]dt + C_t m(k[\mu - \ln S(t)]dt + C_t\sigma m dW(t)). \tag{16}$$

Finally, the variation of the cushion can be written as follows:

$$\frac{dC_t}{C_t} = (r - mr)dt + m(k[\mu - \ln S(t)]dt + \sigma m dW(t)). \tag{17}$$

By Applying the Ito's lemma

$$d\ln(C_t) = r(1-m)dt + m(k[\mu - \ln(C_t)])dt + \sigma m dW(t) - \frac{1}{2}[r(1-m)dt + m(k[\mu - \ln(C_t)])dt + m\sigma dW(t)]^2 \tag{18}$$

$$= r(1-m)dt + m(k[\mu - \ln(C_t)])dt + \sigma m dW(t) - \frac{1}{2}m^2\sigma^2 dt$$

Let $P = \ln(C_t)$ then

$$d\ln(C_t) = [m(k\mu - r) + r - mkp - \frac{m^2\sigma^2}{2}]dt + m\sigma dW(t) \tag{19}$$

Integrating on both side, so the time t value of the cushion, C_t is given by

$$C_t = C_0 \exp[(m(k\mu - r) + r - mkp - \frac{m^2\sigma^2}{2})t + m\sigma W(t)] \tag{20}$$

Equation (20) shows that the cushion C_t has a log-normal distribution written on expected mean return

$$(m(k\mu - r) + r - mkp - \frac{m^2\sigma^2}{2})$$

and a volatility $m\sigma$.
Since the underlying risky asset S follows a mean reversion model

$$d\ln(S(t)) = k[\mu - P(t)]dt + \sigma dW(t) - \frac{1}{2}k^2\sigma^2 dt$$

$$= k[\mu - P(t) - \frac{k\sigma^2}{2}]dt + \sigma dw(t) \tag{21}$$

and hence

$$S_T = S_0 \exp\left[k[\mu - P(t) - \frac{\sigma^2}{2k}]T + \sigma W_T\right] \tag{22}$$

W_T can be written as follows

$$W_T = \frac{1}{\sigma}\left[log\Big(\frac{S_T}{S_0}\Big) - k(\mu - P(t) - \frac{\sigma^2}{2k})T\right] \tag{23}$$

Substituting this into equation (20) we have

$$C_t = C_0 \exp\left[(m(k\mu - r) + r - mkp - \frac{m^2\sigma^2}{2})t + m\sigma\Big(\frac{1}{\sigma}[log\Big(\frac{S_T}{S_0}\Big) - k(\mu - P(t) - \frac{\sigma^2}{2k})T\right]$$

$$= C_0 \exp\left[(m(k\mu - r) + r - mkP(t) - \frac{m^2\sigma^2}{2})t + \Big(\Big(\frac{S_T}{S_0}\Big)^m - mk(\mu - P(t)) - \frac{\sigma^2}{2k}\Big)T\right]$$

$$= C_0 \exp\left[(mk\mu - mr + r - mkP(t) - \frac{m^2\sigma^2}{2})t + \Big(\Big(\frac{S_T}{S_0}\Big)^m - mk\mu - mkP(t)) + mk\frac{\sigma^2}{2k}\Big)T\right]$$

$$= C_0 \exp\left[(-mr + r - \frac{m^2\sigma^2}{2})t + \Big(\Big(\frac{S_T}{S_0}\Big)^m + mk\frac{\sigma^2}{2k}\Big)T\right]$$

$$= C_0\Big(\frac{S_t}{S_0}\Big)^m \exp\left\{(r - m(r - \frac{\sigma^2}{2}) - \frac{m^2\sigma^2}{2})t\right\} \tag{24}$$

and hence

$$V_t = F_t + (V_0 - F_0)\Big(\frac{S_t}{S_0}\Big)^m \exp\left\{(r - m(r - \frac{\sigma^2}{2}) - \frac{m^2\sigma^2}{2})t\right\}. \tag{25}$$

We obtain equation (25), which is the same result when the risky asset follows a geometric Brownian motion (GBM). The volatility of the risky asset per year is denoted by σ. Here the cushion process will never become negative and thus the value of the portfolio never falls below the floor and the portfolio never gaps.
Still the cushion C_t follow a GBM under mean version stochastic differential equation the expected value is

$$E[V_t - F_t] = C_0 \exp\{((1-m)r + m\mu)t\} \tag{26}$$

$$E[V_t] = F_t + (V_0 - F_0)\exp\{((1-m)r + m\mu)t\} \tag{27}$$

and hence

$$E[V_t] = F_t + (V_0 - F_0)\exp\{(r + m(\mu - r))t\}. \tag{28}$$

and the variance

$$Var[V_t] = (V_0 - F_0)^2 \exp\{((1-m)r + m\mu)t\}(\exp\{m^2\sigma^2 t\} - 1) \tag{29}$$

and therefore

$$Var[V_t] = (V_0 - F_0)^2 \exp\{2(r + m(\mu - r)t\}(\exp\{m^2\sigma^2 t\} - 1). \tag{30}$$

This gives the expected payoff of the CPPI as an increasing function of m and independent of volatility. According to Hull (2005,p282) the expected value and variance of the risky asset is

$$E[S_t] = S_0 \exp\{\mu t\} \tag{31}$$

$$Var[S_t] = S_0^2 \exp\{2\mu t\}(\exp\{\sigma^2 t\} - 1). \tag{32}$$

4. Jump modeling:Hawkes Process.

Let $\{N(t), t \geq 0\}$ be the counting process that we are interest to analyse. Let $\lambda(t|\mathscr{H}_t)$, be the intensity rate at time t, which is not fixed, but depends on some random inputs, including the history of the process $\mathscr{H}_t$. $\mathscr{H}_t$ is the history of the process up to the time t, which digest all that goes into the environment of intensity. The value of the intensity is determined by $\mathscr{H}_t$ in such a way that

$$\mathbb{P}(N(t+h) - N(t)) = 1|\mathscr{H}_t) = \lambda(t|\mathscr{H}_t)h + o(h), \tag{33}$$

and

$$\mathbb{P}(N(t+h) - N(t)) \geq 2|\mathscr{H}_t) = o(h) \tag{34}$$

Note that $\lambda(t|\mathscr{H}_t) = \lambda(t|N(s), s \leq t)$.
Suppose there is a base intensity function $\lambda_0(t) > 0$, and which interrelated with each non-negative random variable event, called a *mark*, whose value is independent of all that has formerly occurred and has distribution given by the cumulative distribution function G. On any occasion an event happens, it is assumed that the present value of the random intensity function rises by the amount of the event's *mark*, with this growth and decline over time at an exponential rate k.
If there have been a total of $N(t)$ events by time t, with

$$0 < T_1 < T_2 < \ldots T_{N(t)} \leq t$$

being the event times and δ_i being the mark of event i, for $i = 1, \ldots, N(t)$, then

$$\lambda(t|\mathscr{H}_t) = \lambda_0(t) + \sum_{i=1}^{N(t)} \delta_i e^{-k(t-T_i)}. \tag{35}$$

We define a self-exciting process $N = \{N(t), t \leq 0\}$ if

$$\lambda(t|\mathscr{H}_t) = \lambda_0(t) + \int_0^t \nu(t-s)dN(s),$$

$$= \lambda_0(t) + \sum_{T_i \leq t} \nu(t - T_i), \tag{36}$$

where $\lambda_0 : \mathbb{R} \to \mathbb{R}_+$ is a deterministic base intensity and function, $\nu : \mathbb{R}_+ \to \mathbb{R}_+$ is called a kernel and it express the positive influence of the past events T_i on the current value of the intensity process.

A Hawkes process is a self-exciting process with exponential kernel. For every jump size distribution with positive mass only on the negative line, we introduce a level effect by including the intensity process also acting to the size of the jumps that have happened in the past. Specifically, we model the $\mathscr{F}_t-$intensity as a function of the magnitude of jumps so that the present level of downside jump events elevate the intensity of future negative jumps,

$$\lambda_t = \bar{\lambda} - \int_{-\infty}^{t} g(t-s)dH_s, \tag{37}$$

For the purpose of this paper we treat the natural case where the influence of a jump in the intensity process is controlled by an exponentially decreasing function. In particular, we regard a kernel function g of the following form

$$g(t-s) = \delta e^{-k(t-s)}, \tag{38}$$

where k and δ are positive constant. We observe that, under the model specification (37), the intensity rises by the magnitude of the jump re-size by the parameter δ every time it undergo an adverse shock and jumps it decays exponentially with regular rate k to the level $\bar{\lambda}$. The parameter k governs the model power to replicate the clustered jump patterns. In fact, by k being accepted positive, the most recent jumps have a higher affect on the intensity path with respect to the jumps, which took place far in the past and whose contribution today is insignificant.
Here we model the intensity λ_t of the counting process by the particular form of Hawkes process that satisfies the Mean Reversion Model

$$d\lambda_t = k\lambda_t[\mu - \ln\lambda_t]dt + \lambda_t\sigma dH_t \tag{39}$$

The solution for λ_t takes the form

$$\frac{d\lambda_t}{\lambda_t} = k(\mu - \ln\lambda_t)dt + \sigma dH_t \tag{40}$$

$$\int_0^t \frac{d\lambda_s}{\lambda_s} = k\int_0^t (\mu - \ln\lambda_s)ds + \int_0^t \sigma dHs \tag{41}$$

if we let $\rho(u) = \mu - \ln\lambda_s$
then

$$\lambda_t = \bar{\lambda}e^{-kt} + k\int_0^t e^{-k(t-u)}\rho(u)du + \int_0^t \sigma e^{ku} dH_u \tag{42}$$

verify by Itȯ formula $e^{kt}\lambda_t$

$$e^{kt}\lambda_t = \bar{\lambda} + k\int_0^t e^{ku}\rho(u)du + \int_0^t \sigma e^{ku} dH_u \tag{43}$$

differentiate on both sides

$$ke^{kt}\lambda_t dt + e^{kt}d\lambda_t = ke^{kt}\rho(t)dt + \sigma e^{kt}dH_t \tag{44}$$

$$k\lambda_t dt + d\lambda_t = k\rho(t)dt + \sigma dH_t \tag{45}$$

$$d\lambda_t = k(\rho(t) - \lambda_t)dt + \sigma dH_t \tag{46}$$

The specification (38) plays a crucial role as it results in λ being mean-reverting and Markovian jointly with the marked Hawkes process. Specifically, under an exponential decaying kernel, jump events in the two separate specifications arrive with intensity λ whose dynamics are given by

$$d\lambda_t = k(\rho(t) - \lambda_t)dt + \sigma dH_t \tag{47}$$

5. Full risky asset dynamics.

In the previous section, we have specified the jump part of our model in a way that enables it to effect enriched intensity specification (37), the sharper the negative shock is, the bigger its impact on the intensity of future jump will be. We now specify the full dynamics for the risky asset involved in an investment strategy by incorporating also a diffusion component which accounts for the 'regular'stochastic behaviour of stock prices S.
On $(\Omega, \mathscr{F}, (\mathscr{F})_{0\le t\le T}, \mathbb{P})$, we assume that the price process for the risky asset under the real-world probability measure $\mathbb{P}$ is modelled by a sermimartingale
$S = (S_t)_{t\in\mathbb{R}_+}$ satisfying

$$\frac{dS_t}{S_{t-}} = k(\mu - \ln S(t) + \gamma_t)dt + \sigma dW_t + dH_t - \mu\lambda_t dt \tag{48}$$

let $\mu k = V$ and $\ln S(t) = \rho$
Then

$$\frac{dS_t}{S_{t-}} = [V - \rho + \gamma_t]dt + \sigma dW_t + dH_t - \mu\lambda_t + dt \tag{49}$$

where σ is a positive real constant,

$$\gamma_t = \epsilon\sigma^2 + \lambda_t(\mu - \grave{\mu}) \tag{50}$$

is the time-t total risk premium, is the instantaneous total equity premium can be written as the sum of two contributions: the premium associated to diffusive risk $\epsilon\sigma^2$ and the premium inferred by altering the distribution of jump,

$\lambda(\mu-\bar{\mu})$, $W=(W_t)_{t\in\mathbb{R}_+}$ is an adapted standard Brownian motion and $H_t=\sum_{j=1}^{H_t}\delta_j^{-k(t-T_j)}$ is the jump size of asset, which is a sequence i.id random variables, which is independent of W and of all the history of jumps and

$$\mu := \mathbb{E}[\sum_{j=1}^{N(t)}\delta_j^{-k(t-T_j)}] \tag{51}$$

is the expected value of the jump. The surplus between the strategy value and the risk exposure is allocated into the non-risky asset. In case the portfolio value breaks the floor B_t at time

$$\tau = inf\{t\in[0,T]|V_t\leq B_t,\} \tag{52}$$

the remaining wealth is invested in the bond and held until maturity to prevent the risk of imperiling the capital further. The risk of violating the floor is referred to as the gap risk.
Therefore, for any $t\leq\tau$, the cushion dynamics for this strategy can be written as in equation (16).

$$dC_t=(C_t-mC_t)\frac{dB_t}{B_t}+(mC_t)\frac{dS_t}{S_t}$$

and, in view of (48) and (49), we find

$$\frac{dC_t}{C_{t-}}=(1-m)\frac{dB_t}{B_t}+m\frac{dS_t}{S_{t-}}$$

$$\frac{dC_t}{C_{t-}}=(1-m)rdt+m((v-\rho+\gamma_t)dt+\sigma dW_t+dH_t-\mu\lambda_t dt) \tag{53}$$

and hence

$$[r+m(-r+v-\rho+\gamma_t)]dt+m\sigma dW_t+m(dH_t-\mu\lambda_t dt) \tag{54}$$

Denoting by $\grave{C}_t$ the forward value of the cushion $\frac{C_t}{B_t}$ and applying the general Itò formula for sermimatingales, we have

$$\frac{d\grave{C}_t}{\bar{C}_{t-}}=m(-r+v-\rho+\gamma_t)dt+m\sigma dWt+m\big(dH_t-\mu\lambda_t dt\big). \tag{55}$$

Defining $Y_t := Y_0-rt+vt-\rho t+\int_0^t(\gamma_s-\mu\lambda_s)ds+\sigma W_t+\sum_{j=1}^{H_t}\delta_j e^{-k(t-T_j)}$, equation (54) can be rewritten as

$$dS(t)=S(t^-)[(-rt+vt-\rho t)dt+\int_0^t(\gamma_s-\mu\lambda_s)ds+\sigma dW_t+\int_{-\infty}^t\delta e^{-k\lambda(t-s)}\tilde{M}(ds,dN_s)] \tag{56}$$

Let $\alpha=-rt+vt-\rho t+\int_0^t(\gamma_s-\mu\lambda_s)$
then we have

$$dX(t)=S(t^-)[\alpha(t)dt+\beta(t)dBt+\int_{\mathbb{R}}\delta e^{-k\lambda(t-s)}\tilde{M}(dt,dN_t)] \tag{57}$$

or

$$\frac{d\grave{C}_t}{\grave{C}_{t-}}=mdY_t \tag{58}$$

and, equivalently,

$$\grave{C}_t=\grave{C}_0\varepsilon(mY)_t, \tag{59}$$

where ε denotes the stochastic Dolèans-Dade exponential.
After time τ, according to the definition of the CPPI strategy, the value of the strategy is entirely invested into a zero-coupon bond to prevent further losses, i.e.

$$V_t=V_\tau e^{r(t-\tau)}, \text{for any} \, t>\tau. \tag{60}$$

It follows that the cushion value, for any time t succeeding τ, can be rewritten as

$$C_t=V_\tau e^{r(t-\tau)}-Ge^{-r(T-t)} \tag{61}$$

and in particular,

$$\grave{C}_t=\frac{V_\tau}{G}e^{r(T-\tau),}-1 \tag{62}$$

where we can see that the forward value of the cushion process remains constant during the post-loss period.
Finally, for any date $t\in[0,T]$, we introduce a new process C^*,defined as the stopped process of $\grave{C}$ by the stopping time τ, i.e.

$$C_t^*=\bar{C}_{t\wedge\tau} \tag{63}$$

where $t\wedge\tau := \min(t,\tau)$, and we find

$$C_t^*=\grave{C}_0\varepsilon(mY)_{t\wedge\tau}. \tag{64}$$

5.1. Model Set-up

We study the risk embedded in the constant proportion portfolio insurance (CPPI) trading strategy in a jump- diffusion model where the process of the underlying asset may experience negative jump over a short-period of time. This negative jump correlation pattern can be modelled by introducing self-exciting jumps in the underlying dynamics based on Hawkes Process.
Like the class of Poisson Process, Hawkes process are defined by their frequency rate, which describes the condition expected number of jumps per unit of time. However, instead of being temporally invariant, the frequency rate of Hawkess is defined as a strictly positive stochastic process whose current value is determined by the number of past events weighted by a kernel function. More precisely, we are interest in a stochastic process
$H = (H_t)_{t\geq 0}$ defined on a filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F})_{0\leq t\leq T}, \mathbb{P})$ which is a semimartingale and admits a decomposition

$$H_t = H_0 + A_t + M_t, t \geq 0. \tag{65}$$

where $A = (A_t) \in \nu$ (a process of bounded variation), $M = (M_t) \in \mathscr{M}_{loc}$ (a local martingale). Further, we have that for each $t \leq 0, A_t$ and M_t are $\mathscr{F}_t-$measurable.
In a filtered probability space $(\Omega, \mathscr{F}, \mathbb{F}, \mathbb{P})$. We consider the market.

$$dS_0(t) = r(t)S_0(t)dt; S_0(0) = 1 \tag{66}$$

$$dS_1(t) = S(t^-)[\alpha(t)dt + \beta(t)dBt + \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}(t,s)\tilde{M}(dt,d\zeta)], \delta > -1, s(0) > 0 \tag{67}$$

where r is a risk free interest rate, δ is the size of the jump, α is the drift parameter, β is the volatility parameter and $\tilde{M}$ is a compensated jump measure. This is a market which has two assets, a risky and non-risky asset. Where the risky asset is driven by a geometric Brownion motion under mean version stochastic differential equation.
Equation (66) can be solved by

$$\int_0^t \frac{dS_0(t)}{S_0} = \int_0^t r(t)ds$$

$$ln|S_0(s)|\Big|_0^t = rs\Big|_0^t$$

$$ln|S_0(t)| - ln|S_0(0)\Big| = rt$$

$$ln\Big|\frac{S_0(t)}{S_0(0)}\Big| = rt$$

$$\frac{S_0(t)}{S_0(0)} = e^{rt}$$

$$S_0(t) = S_0(0)e^{rt}$$

$$= rt$$

The Itô's Lemma for jumps to solve equation (67) is represented by the following: Let $X(t)$ be an Itô-Lèvy process defined as above. Let $f : [0,T] \times \mathbb{R} \in C^2$ function and put

$$Y(t) = f(t, X(t)) \tag{68}$$

Then $Y(t)$ is also an Itô-Lèvy process, with representation

$$dY(t) = \frac{\partial f}{\partial t}(t,S(t))dt + \frac{\partial f}{\partial x}(t,S(t))(\alpha(t)dt + \beta(t)dB(t) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t,S(t))\beta^2(t)dt$$

$$+ \int_{\mathbb{R}} \{f(t, X(t^-) + \delta e^{-k\lambda(t-s)}(t,\zeta)) - f(t, S(t^-))\}\tilde{M}(dt,d\zeta) \tag{69}$$

Solving equation (67) we have

$$S_t = S_0 + \int_0^t \alpha(s,w)ds + \int_0^t \beta(s,w)dB(s) + \int_0^t \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}(s,\zeta,w)\tilde{M}(ds,d\zeta) \tag{70}$$

Trading strategy

We consider strategies in a two-dimensional stochastic process investing in the equity S_t and the risk free asset B_t, $\phi_t = (\phi_t, B_t, \phi_t, S_t)$ such that ϕ_t is $\mathscr{B} \times \mathscr{F}-$measurable and $\mathscr{F}_t-$adapted. Financial we interpret (ϕ_t, B_t) as the number of shares in the risk free asset B_t, while (ϕ_t, S_t) is the number of shares in the risky asset S_t up to time τ.
Up to time τ, the value of the CPPI strategy is defined by

$$V_t(\phi) = \phi_t, B_t + \phi_t, S_t \tag{71}$$

where $(\phi_t, B_t) = \frac{V(\phi)_{t-} - mC_{t-}}{B_t}$ and $(\phi_t, S_t) = \frac{mC_{t-}}{S_{t-}}$
By imposing the self-financing property of the CPPI portfolio implies in particular that

$$V_t = V_0 + \int_0^t \phi_u, BdB_u + \int_0^t \phi_u, SdS_u \tag{72}$$

Alternatively, the investment strategies can be represented by the investment fractions
$\pi_t = (\pi_t^0, \pi_t^1)_{t \in [0,\infty)}$, such that π_t is $\mathscr{B} \times \mathscr{F}-$measurable and $\mathscr{F}_t-$adapted.
Hence the total wealth up to time τ, the value of the CPPI strategy is defined by

$$X^\pi(t) = (\pi_t^0, \pi_t^1).(B_0, S_1) \tag{73}$$

$$X^\pi(t) = \pi_t^0 B_0 + \pi_t^1 S_1 \tag{74}$$

Imposing the self-financing

$$dX^\pi(t) = (\pi_t^0, \pi_t^1).(dB_0, dS_1)$$

$$dX^\pi(t) = \pi_t^0 dB_0 + \pi_t^1 dS_1 \tag{75}$$

or equivalent

$$X^\pi(t) = X^\pi(0) + \int_0^t \pi^0(t) dB_0(t) + \int_0^t \pi^1(t) dS_1(t) \tag{76}$$

Since $X^\theta(t) = \pi_t^0 B_0 + \pi_t^1 S_1$ and if we let θ_t to be the fraction of total wealth of the portfolio invested in the risky asset, then we may write

$$\theta_t = \frac{\pi_t^1 S_1}{X^\pi(t)} \tag{77}$$

then

$$1 - \theta_t = \frac{\pi_t^0 B_0}{X^\pi(t)}$$

Using equation (66) above we may write the change in the wealth process by

$$dX^\pi(t) = \pi_t^0 dS_0 + \pi_t^1 dS_1$$

$$= r\pi_t^0 S_0 dt + \pi_t^1 S_1 \left[\alpha dt + \beta dBt + \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta)\right]$$

$$= (1-\theta_t) rX(t)dt + \alpha\pi^1(t) S_1 dt + \beta\pi^1(t) S_1 dBt + \int_{\mathbb{R}} \pi(t) X(t) \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta)$$

$$= (1-\theta_t) rX(t)dt + \alpha\theta_t X(t)dt + \beta\theta_t X(t) dBt + \int_{\mathbb{R}} \pi(t) X(t) \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta)$$

$$= rX(t)dt - \theta_t rX(t)dt + \alpha\theta_t X(t)dt + \beta X(t) dBt + \int_{\mathbb{R}} \pi(t) X(t) \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta)$$

$$= rX(t)dt + (\alpha - r)\theta_t X(t)dt + \beta\theta_t X(t) dB(t) + \int_{\mathbb{R}} \pi(t) X(t) \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta)$$

$$= rX(t)dt + (\alpha - r)\pi(t) X(t)dt + \beta\pi(t) dBt + \int_{\mathbb{R}} \pi(t) X(t) \delta e^{-k\lambda(t-s)}(t,s) \tilde{M}(dt, d\zeta) \tag{78}$$

5.1.1. Admissible

A self-financing portfolio θ is called admissible if there exists $K = K(\theta) < \infty$ such that

$$V^{\theta}(t,w) \geq -K for a.a.(t,w) \in [0,T] \times \Omega$$

(here,and in the following, "almost all t∈ [0,T]" means with respect to Lebesgue measure on $[0,T]$.)
The condition $V^{\theta}(t,w) \geq -K$ is natural from a modeling point of view: There must be a bound on the size of the debt that an agent can have during her portfolio. The condition is also mathematically convenient: It exclude the so-called doubling strategies Øksendal (2003).

5.1.2. Optimal portfolio

In every investment the whole aim is to get a reward in form of returns. We know how our investment performing depend on the amount of reward we receive. When an investor is building a portfolio obviously he/she is trying to select a portfolio that is going to maximize the expected reward, but the question is how do we find an optimal portfolio.
Let

$$\vartheta^* = (\vartheta_t^{*0}, \vartheta_t^{*1}) \tag{79}$$

$$= (\delta^*, \pi^*) \tag{80}$$

such that $\delta^* + \pi^* = 1$, then

$$\delta^* = 1 - \pi^* \tag{81}$$

$$\vartheta^* = (1-\pi^*, \pi^*) \tag{82}$$

Firstly we need to form a reward function, hence our criteria of the portfolio that maximizes our reward function.

5.2. Reward function

Consider the following controlled process on $[0,\tau_G]$

$$dX_t = \alpha(t, X_t, \pi_t)dt + \beta(t, X_t, \pi_t)dBt + \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}(t, X_t, \pi_t)\tilde{M}(dt, d\zeta), X_0 = x_0 \tag{83}$$

with the given performance function

$$J_{x_0}(\vartheta) = \mathbb{E}^{x_0}\Big[\int_0^{\tau_G} f(r, X_r, \pi_r)dr + g(\tau_G, X_{\tau_G})X_{\{\tau_G<\infty\}},\Big] \tag{84}$$

where $\tau_G = inf\{t > 0 : (t, X_t) \notin G\}$ is a stopping time, and the set $G \subset [0,\tau_G] \times \mathbb{R}$ is called the solvency set. The continuous function $f : [0,\tau_G] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is called the "utility function" and the continuous function $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is called the "bequest function". We would like to find an optimal π^* from a set of admissible controls so that this performance function is maximized. To find our optimal control we must use the technique of dynamic programming, as the name suggests we need to consider a family of optimal control problems from which we will select an optimal control. To do this we need to consider different initial times and states along a given trajectory of the controlled diffusion. If we were to consider a diffusion X_t starting at x_0 as in equation (83) then an admissible control π is $(\mathscr{F}_t)_{t\geq 0}$ measurable which means that the control has all information about the system up to time X_t. To be able to use the method of dynamic programming we must vary the initial time and states of the system and choose the best possible control from the set of admissible controls. To do this we consider controlled diffusion of the form

$$dX_t = \alpha(t, X_t, \pi_t)dt + \beta(t, X_t, \pi_t)dBt + \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}(t, X_t, \pi_t)\tilde{M}(dt, d\zeta), X_s = x \tag{85}$$

where $t \in [s, \hat{\tau}_G]$ and the performance function has the form

$$J_{s,x}(\vartheta) = \mathbb{E}^{s,x}\Big[\int_0^{\tau_G} f(r, X_r, \pi_r)dr + g(\hat{\tau}_G, X_{\tau_G})X_{\{\tau_G<\infty\}},\Big] \tag{86}$$

$$\hat{\tau}_G = inf\{t > 0 : (t, X_t) \notin G\}$$

Utility function plays a very important role in our reward function, as we have said the utility is the attitude of an investor toward risk. It is also a satisfaction function. That means we can also choose our portfolio that satisfies our expected utility, in other words the portfolio that maximized our expected satisfaction function. In our performance function (84) if $f=0, \tau_G = T$ and if we choose $g(x) = U(x)$
Then our expected utility is

$$J_{x_0}(\vartheta) = \mathbb{E}[U(X^{\vartheta}(T))] \tag{87}$$

and the performance function is defined by

$$J_{x_0}(\vartheta) = \sup_{\vartheta \in \mathscr{A}} J_{x_0}(\vartheta) \tag{88}$$

$$= \sup_{\vartheta \in \mathscr{A}} \mathbb{E}[U(X^{\vartheta}(T)).] \tag{89}$$

$X^{\vartheta}(T)$ is a value of the wealth at time T the terminal wealth. From equation (76) we can find an optimal portfolio by maximizing the expected utility of the terminal wealth.
In other words supposed $U:(0,\infty) \to [-\infty,\infty)$ is a given utility function, assume to be continuous and concave, we want to find the Portfolio $\pi^*(t) \in A$ and $\Phi(y)$ such that

$$\Phi(y) = \sup_{\vartheta \in \mathscr{A}} J_{x_0}(\vartheta) = J_{x_0}(\vartheta^*) \tag{90}$$

(Hamilton -Jacobi-Bellman (HJB) equation)
(a) suppose we can find a function $\alpha \in \mathscr{C}^2(\mathbb{R})$such that
(i) $A_v\alpha(y) + f(y,v) \le 0$,for all $v \in \mathscr{V}$, where $\mathscr{V}$ is the set of possible control values, and $A_v\alpha(y)$ is given by

$$A_v\alpha(y) = \sum_{i=1}^{k} b_i(y,v)\frac{\partial \alpha}{\partial y_i}(y) + \frac{1}{2}\sum_{ij=1}^{k}(\sigma\sigma^T)_{ij}(y,v)\frac{\partial^2\alpha}{\partial y_i \partial y_j} + \sum_{k=1}^{\ell}\int_{\mathbb{R}}\{\varphi(y+\delta e^{-k\lambda(t-s)}(y,v,\zeta))$$

$$-\alpha(y) - \nabla\alpha(y)\delta e^{-k\lambda(t-s)}(y,v,\zeta)\}v_k(d\zeta). \tag{91}$$

(ii) $\lim_{n\to\infty} \alpha(Y(\tau^n)) = g(Y(\tau\varsigma))1_{\tau\varsigma<\infty}$ for all increasing sequences of stopping times $\tau^{(n)}$ converging to $\tau\varsigma$ a.s.
(iii) "growth condition"

$$\mathbb{E}^y[|\alpha(Y(\tau))| + \int_0^{\tau\varsigma}\{|A\alpha(Y(t))| + |\sigma^T(Y(t))\nabla\alpha(Y(t))|^2$$

$+$

$$\sum_{j=1}^{\ell}\int_{\mathbb{R}}|\alpha(Y(t)) + \delta e^{-k_j\lambda(t_j-s_j)}(Y(t),u(t),\zeta_i) - \alpha(Y(t))|^2 v_j(d\zeta_j)\}dt] < \infty$$

, for all $u \in A$ and all stopping times τ
(iv) $\{\alpha^-(Y(\tau))\}_{\tau\le\tau\varsigma}$ is informally integrable for all $u \in A$ and $y \in S$,
where in general, $x^- := max\{-x, 0\}$ for all $x \in \mathbb{R}$
Then

$$\alpha(y) \ge \Phi(y)$$

.
(b) Suppose for all $y \in S$ can find $v = \hat{u}(y)$ such that

$$A_{\hat{u}(y)}\alpha(y) + f(y,\hat{u}(Y)) = 0 \tag{92}$$

and $\hat{u}(y)$ is an admissible feedback control (Markov control), i. e. $\hat{u}(y) means\ \hat{u}(Y(t))$.Then $\hat{u}(y)$ is an optimal control and

$$\alpha(y) = \Phi(y).$$

In two-dimensional Hamilton-Jacobi-Bellman (HJB) equation is given by

$$A\vartheta\alpha(t,x) = b^{(1)}(t,x)\frac{\partial\alpha}{\partial t} + b^{(2)}(t,x)\frac{\partial\alpha}{\partial x} + \frac{1}{2}\sigma_1^2\frac{\partial^2\alpha}{\partial t^2} + \frac{1}{2}\sigma_1\sigma_2^T\frac{\partial^2\alpha}{\partial t\partial x} + \frac{1}{2}\sigma_2\sigma_1^T\frac{\partial^2\alpha}{\partial x\partial t} + \frac{1}{2}\sigma_2^2\frac{\partial^2\alpha}{\partial x^2}$$

$$+\int_{\mathbb{R}}\{\alpha(t+\delta e^{-k\lambda(t-s)}(t,x,\zeta) - \alpha(t) - x\alpha(y)\delta e^{-k\lambda(t-s)}(t,x,\zeta)\}x(d\zeta) \tag{93}$$

6. Calculating the optimal portfolio

From our assumption consider $\pi(t)$ a self-financing portfolio, therefore the corresponding wealth process

$$X^{\pi}(t)$$

satisfies the equation

$$dX(t) = \pi(t)X(t^{-})[(-rt + vt - \rho t)dt + \int_0^t (\gamma_s - \mu\lambda_s)ds + \sigma dW_t + \int_{-\infty}^t \delta e^{-k\lambda(t-s)}\tilde{M}(ds,d\zeta)] \tag{94}$$

Let $\alpha = -rt + vt - \rho t + \int_0^t (\gamma_s - \mu\lambda_s)$
then we have

$$dX(t) = \pi(t)X(t^{-})[\alpha(t)dt + \beta(t)dBt + \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}\tilde{M}(dt,d\zeta)] \tag{95}$$

$$t \in [0,T], X(0) = x_0 > 0.$$

$$dX(t) = \pi(t)X(t)\alpha(t)dt + \pi(t)X(t)\beta(t)dBt + \int_{\mathbb{R}} \pi(t)X(t)\delta e^{-k\lambda(t-s)}\tilde{M}(dt,d\zeta) \tag{96}$$

Case 1 Logarithmic utility function $U(x) = ln(x)$.
Then the expected utility of the terminal wealth is defined by

$$J_{x_0}(\vartheta) = \mathbb{E}[ln(x^{\vartheta}(T))] \tag{97}$$

Now equation (94) does not satisfy the Markov properties, hence we define a new process

$$Y_t = (t, X_t) \tag{98}$$

$$= \begin{bmatrix} t \\ X_t \end{bmatrix} \tag{99}$$

$$dY(t) = \begin{bmatrix} dt \\ dX(t) \end{bmatrix} \tag{100a}$$

$$= \begin{bmatrix} 1 \\ X(t)\pi(t)\alpha(t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ X(t)\pi(t)\beta(t) \end{bmatrix} dB(t) + \begin{bmatrix} 0 \\ X(t)\pi(t) \end{bmatrix} \int_{\mathbb{R}} \delta e^{-k\lambda(t-s)}\tilde{M}(dt,d\zeta) \tag{100b}$$

from the equation (100b)
$b^{(1)} = 1$
$b^{(2)} = X(t)\pi(t)\alpha(t)$
$\sigma_1 = 0$
$\sigma_2 = X(t)\pi(t)\beta(t)$

$$A\vartheta\alpha(t,x) = b^{(1)}(t,x)\frac{\partial\alpha}{\partial t} + b^{(2)}(t,x)\frac{\partial\alpha}{\partial x} + \frac{1}{2}\sigma_1^2\frac{\partial^2\alpha}{\partial t^2} + \frac{1}{2}\sigma_1\sigma_2^T\frac{\partial^2\alpha}{\partial t\partial x} + \frac{1}{2}\sigma_2\sigma_1^T\frac{\partial^2\alpha}{\partial x\partial t} + \frac{1}{2}\sigma_2^2\frac{\partial^2\alpha}{\partial x^2}$$

$$+ \int_{\mathbb{R}} \{\alpha(t + \delta e^{-k\lambda(t-s)}(t,x,\zeta) - \alpha(t) - x\alpha(y)\delta e^{-k\lambda(t-s)}(t,x,\zeta)\}\nu(d\zeta)$$

Substitute $b^{(1)}, b^{(2)}$,σ_1 and σ_2

$$A\vartheta\alpha(t,x) = \frac{\partial\alpha}{\partial t} + x(t)\pi(t)\alpha(t)\frac{\partial\alpha}{\partial x} + \frac{1}{2}(x(t)\pi(t)\beta(t))^2\frac{\partial^2\alpha}{\partial x^2}$$

$$+ \int_{\mathbb{R}} \alpha((t,x) + x\pi\delta e^{-k\lambda(t-s)}(t,N_t)) - \alpha(t,x) - \frac{\partial\alpha}{\partial x}(t,x)x\pi\delta e^{-k\lambda(t-s)}(t,d\zeta)\nu d\zeta$$

We let

$$g(t,x) = lnx, \text{then} \quad \frac{\partial\alpha}{\partial t} = 0, \frac{\partial\alpha}{\partial x} = \frac{1}{x}$$

and

$$\frac{\partial^2 \alpha}{\partial x^2} = -\frac{1}{x^2}$$

By substituting the differentials and the utility function back we have,

$$A\vartheta\alpha(t,x) = x\pi\alpha(t)\frac{1}{x} - \frac{1}{2}x^2\pi^2\beta^2(t)\frac{1}{x^2} + \int_{\mathbb{R}} \alpha((t,x) + x\pi\delta e^{-k\lambda(t-s)}(t,N_t)) - \alpha(t,x) - \frac{1}{x}x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \quad (101)$$

which implies that

$$A\vartheta\alpha(t,x) = \pi\alpha(t) - \frac{1}{2}\pi^2\beta^2(t) + \int_{\mathbb{R}} \alpha((t,x) + x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)) - \alpha(t,x) - \frac{1}{x}x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \quad (102)$$

$$\sup_{\vartheta\in A} J(\vartheta) = J(\vartheta^*) \quad (103)$$

Since in our assumption $f(x) = 0$ then $\alpha(t,x) = lnx$

$$A\vartheta\alpha(t,x) = \pi\alpha(t) - \frac{1}{2}\pi^2\beta^2(t) + \int_{\mathbb{R}} \ln((t,x) + x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)) - \ln(t,x) - \pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \quad (104)$$

$$\frac{A\vartheta\alpha(t,x)}{\partial x} = \pi\alpha(t) - \frac{1}{2}\pi^2\beta^2(t) + \int_{\mathbb{R}} \ln(1 + \pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)) - \pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \quad (105)$$

And so

$$\frac{A\vartheta\alpha(t,x)}{\partial \pi} = \alpha(t) - \pi\beta^2(t) + \int_{\mathbb{R}} \frac{\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)\nu(d\zeta)}{1 + \pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)}(t,\zeta)\nu d\zeta$$

$$\alpha(t) - \pi(t)\beta^2(t) + \int_{\mathbb{R}} \frac{\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)\nu(d\zeta)}{1 + \pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)}(t,\zeta)\nu d\zeta = 0 \quad (106)$$

If $\nu = 0$ then the optimal portfolio is:

$$\pi^* = \frac{\alpha(t)}{\beta^2(t)} \quad (107)$$

Therefore our optimal portfolio is

$$\vartheta^* = (1 - \pi^*, \pi^*) \quad (108)$$

Case 2 Power utility function

$$g(t,x) = \frac{1}{\rho}x^\rho \quad (109)$$

Then the expected utility of the terminal wealth is defined by

$$J_{x_0}(\vartheta) = \mathbb{E}[\frac{1}{\rho}x^\rho(x^\vartheta(T))] \quad (110)$$

$\rho \in (-\infty, 1)$

$$\frac{\partial g}{\partial t} = 0 \quad \frac{\partial g}{\partial x} = x^{\rho-1}, \quad \frac{\partial^2 g}{\partial x^2} = (\rho - 1)x^{\rho-2}$$

By substituting the above into the equation

$$A\vartheta\alpha(t,x) = \frac{\partial\alpha}{\partial t} + x(t)\pi(t)\alpha(t)\frac{\partial\alpha}{\partial x} + \frac{1}{2}(x(t)\pi(t)\beta(t))^2\frac{\partial^2\alpha}{\partial x^2}$$

$$+\int_R \alpha((t,x)+x\pi\delta e^{-k\lambda(t-s)}(t,\zeta))-\alpha(t,x)-\frac{\partial\alpha}{\partial x}(t,x)x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{111}$$

we have

$$A\vartheta\alpha(t,x)=x(t)\pi(t)\alpha(t)x^{(\rho-1)}+\frac{1}{2}(x(t)\pi(t)\beta(t))^2(\rho-1)x^{(\rho-2)}$$

$$+\int_{\mathbb{R}}\frac{1}{\rho}x^{\rho}((t,x)+x\pi\delta e^{-k\lambda(t-s)}(t,\zeta))-\frac{1}{\rho}x^{\rho}-(x^{\rho-1})(t,x)-x(t)\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{112}$$

$$=x^{\rho}\pi(t)\alpha(t)+\frac{1}{2}(\rho-1)x^{\rho}\pi^2(t)\beta^2(t)+\int_{\mathbb{R}}\frac{1}{\rho}x^{\rho+1}\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)-x^{\rho}\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta$$

$$\frac{A\vartheta\alpha(t,x)}{\partial x}=\rho x^{(\rho-1)}\pi(t)\alpha(t)+\frac{1}{2}\rho(\rho-1)x^{\rho}\pi^2(t)\beta^2(t)+\int_{\mathbb{R}}\rho(\rho-1)x^{\rho-2}\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta) \tag{113}$$

$$-\rho x^{\rho-1}\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{114}$$

$$\frac{A\vartheta\alpha(t,x)}{\partial\pi}=\rho x^{(\rho-1)}\alpha(t)+\rho(\rho-1)x^{\rho}\pi(t)\beta^2(t)+\int_{\mathbb{R}}\rho(\rho-1)x^{(\rho-1)}\delta e^{-k\lambda(t-s)}(t,\zeta)-\rho x^{(\rho-1)}\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{115}$$

The optimal portfolio is

$$\rho(\rho-1)x^{\rho}\pi(t)\beta^2(t)+\int_{\mathbb{R}}\rho(\rho-1)x^{(\rho-1)}\delta e^{-k\lambda(t-s)}(t,\zeta)-\rho x^{(\rho-1)}\delta e^{-k\lambda(t-s)}=-\rho x^{(\rho-1)}\alpha(t) \tag{116}$$

When δ=0

$$\pi^*(t)=-\frac{\rho x^{(\rho-1)}\alpha(t)}{\rho(\rho-1)x^{\rho}\beta^2(t)} \tag{117}$$

Therefore our optimal portfolio is

$$\vartheta^*=(1-\pi^*,\pi^*) \tag{118}$$

Case 3 Exponential utility function Now suppose we are dealing with the case of an exponential utility function to deliver the optimal portfolio. Where our utility function is given by:

$$U(x)=-\exp(-\lambda x) \tag{119}$$

then

$$g(t,x)=-\exp(-\lambda x) \tag{120}$$

Then the expected utility of the terminal wealth is defined by

$$J_{x_0}(\vartheta)=\mathbb{E}[-\exp(-\lambda x)(x^{\vartheta}(T))] \tag{121}$$

and so

$$\frac{\partial\alpha}{\partial t}=0, \tag{122}$$

$$\frac{\partial\alpha}{\partial x}=\lambda e^{-\lambda x} \tag{123}$$

and

$$\frac{\partial^2\alpha}{\partial x^2}=-\lambda^2 e^{-\lambda x} \tag{124}$$

By substituting the above value into the equation

$$A\vartheta\alpha(t,x)=\frac{\partial\alpha}{\partial t}+x(t)\pi(t)\alpha(t)\frac{\partial\alpha}{\partial x}+\frac{1}{2}(x(t)\pi(t)\beta(t))^2\frac{\partial^2\alpha}{\partial x^2}$$

$$+\int_{\mathbb{R}}\alpha((t,x)+x\pi\delta e^{-k\lambda(t-s)}(t,N_t))-\alpha(t,x)-\frac{\partial\alpha}{\partial x}(t,x)x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{125}$$

We have

$$A\vartheta\alpha(t,x)=\lambda e^{-\lambda x}x(t)\pi(t)\alpha(t)-\frac{1}{2}x^2(t)\pi^2(t)\beta^2(t)\lambda^2e^{-\lambda x}+\int_{\mathbb{R}}\alpha((t,x)+x\pi\delta e^{-k\lambda(t-s)}(t,\zeta))-\alpha(t,x)$$

$$-\lambda e^{-\lambda x}x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{126}$$

Here we guess that $\alpha(t,x)=-e^{-\lambda x}$
then

$$A\vartheta\alpha(t,x)=\lambda e^{-\lambda x}x(t)\pi(t)\alpha(t)-\frac{1}{2}x^2(t)\pi^2(t)\beta^2(t)\lambda^2e^{-\lambda x}+\int_{\mathbb{R}}-e^{\lambda x}((t,x)+x\pi\delta e^{-k\lambda(t-s)}(t,\zeta))+e^{-\lambda x}(t,x)$$

$$-\lambda e^{-\lambda x}x\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{127}$$

From here we form

$$\frac{A\vartheta\alpha(t,x)}{\partial x}=-\lambda^2e^{-\lambda x}x(t)\pi(t)\alpha(t)+\lambda e^{-\lambda x}\pi(t)\alpha(t)-x(t)\pi^2(t)\beta^2(t)\lambda^2e^{-\lambda x}+\frac{1}{2}\pi^2x^2\beta^2(t)\lambda^3e^{-\lambda x}$$

$$+\int_{\mathbb{R}}-\lambda e^{\lambda x}((t,x)+\pi\delta e^{-k\lambda(t-s)}(t,N_t))-\lambda e^{-\lambda x}(t,x)+\lambda^2e^{-\lambda x}x\pi-\lambda e^{-\lambda x}\pi\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{128}$$

and

$$\frac{A\vartheta\alpha(t,x)}{\partial\pi}=-\lambda^2e^{-\lambda x}x(t)\alpha(t)+\lambda e^{-\lambda x}\alpha(t)-2x(t)\pi(t)\beta^2(t)\lambda^2e^{-\lambda x}+\pi x^2\beta^2(t)\lambda^3e^{-\lambda x}$$

$$+\int_{\mathbb{R}}-\lambda e^{\lambda x}((t,x)+\delta e^{-k\lambda(t-s)}(t,\zeta))-\lambda e^{-\lambda x}(t,x)+\lambda^2e^{-\lambda x}x\delta e^{-k\lambda(t-s)}-\lambda e^{-\lambda x}\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta \tag{129}$$

So the optimal portfolio is

$$x^2\beta^2(t)\lambda^3-2x(t)\pi(t)\beta^2(t)\lambda^2+\int_{\mathbb{R}}-\lambda e^{\lambda x}((t,x)+\delta e^{-k\lambda(t-s)}(t,\zeta))-\lambda e^{-\lambda x}(t,x)+\lambda^2e^{-\lambda x}x\delta e^{-k\lambda(t-s)}$$

$$-\lambda e^{-\lambda x}\delta e^{-k\lambda(t-s)}(t,\zeta)\nu d\zeta$$

$$=\lambda^2x(t)\alpha(t)-\lambda\alpha(t) \tag{130}$$

when $\delta=0$

$$\pi^*=\frac{\lambda^2x(t)\alpha(t)-\lambda\alpha(t)}{x^2\beta^2(t)\lambda^3-2x(t)\beta^2(t)\lambda^2} \tag{131}$$

Therefore our optimal portfolio is

$$\vartheta^*=(1-\pi^*,\pi^*) \tag{132}$$

7. Analysis and Discussion

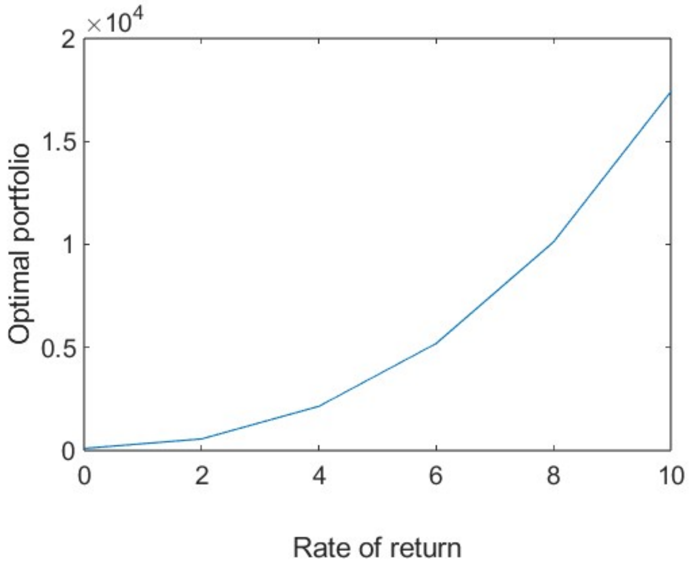

Fig. 1. Optimal portfolio when $S_0 = 100, V_0 = 100, \mu = 0.1$, $T = 10$, $\sigma = 0.02$ and various α

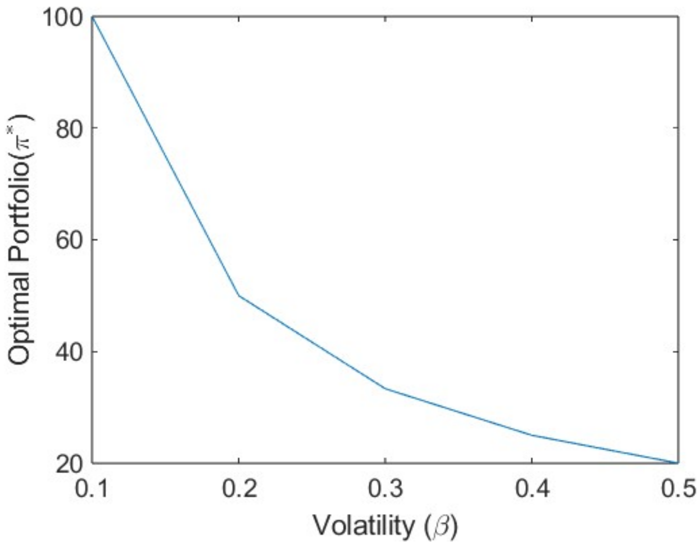

Fig. 2. Optimal portfolio when $S_0 = 100, V_0 = 100, \mu = 0.1$, $T = 10$, and various σ

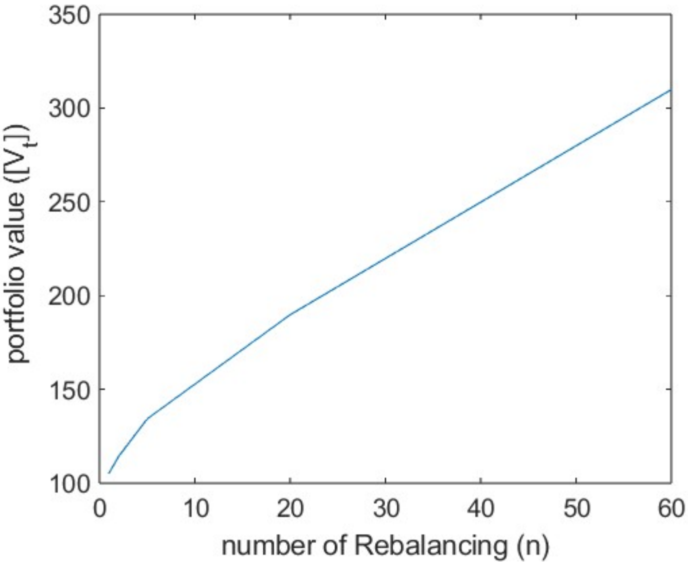

Fig. 3. Optimal portfolio when $S_0 = 100, V_0 = 100, \mu = 0.1, T = 10$, and $\sigma = 0.02$

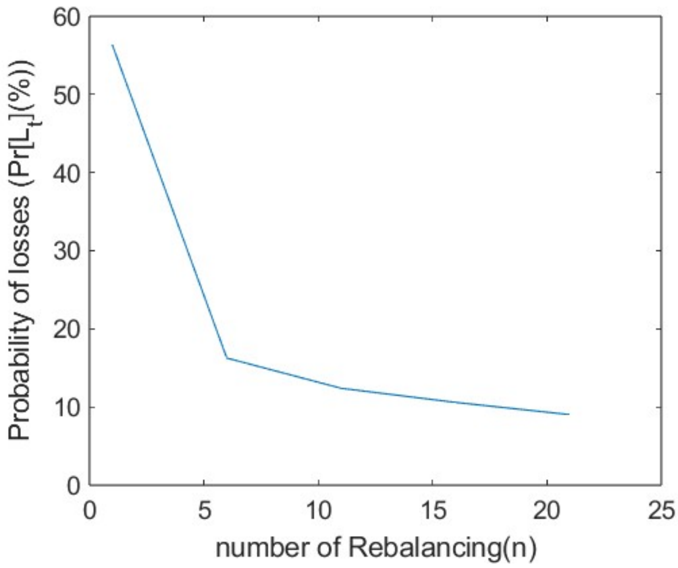

Fig. 4. Optimal portfolio when $S_0 = 100, V_0 = 100, \mu = 0.1, T = 10$ and $\sigma = 0.02$

In the first case, where we have a log investor, of our optimal portfolio in the presence of jumps. We observe that, α the rate of return which can also be regarded as selected portfolio guarantee level in the absence of noise in the market it has a positive impact on the portfolio investment strategy as shown in fig (1), which subsequently leads to a notable impact on the whole portfolio and allows investor to invest more in a risk free asset.
We relate the term $-\pi(t)\beta^2(t)$ with fig (2), as the volatility increase this leads to a decrease in π^* and hence reduce the amount of fraction from risk asset to risk free asset and therefore avoid the portfolio value to fall below the guaranteed amount. For the term

$$\int_{\mathbb{R}} \frac{\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)v(d\zeta)}{1+\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)}(t,\zeta)v d\zeta$$

which can also be expressed as

$$\pi(t)\int_{\mathbb{R}} \frac{\delta e^{-k\lambda(t-s)}(t,\zeta)v(d\zeta)}{1+\pi(t)\delta e^{-k\lambda(t-s)}(t,\zeta)}(t,\zeta)v d\zeta$$

the shaper the δ is, the bigger its impact on the intensity of future jumps will be. And we will expect the value of π^* to decrease which will eventually lead to poor performance in the CPPI strategies.

And in the case of a no negative shock occurs when $\nu = 0$, which is in the continuous case when $\alpha > \beta^2$, it is observed that as α becomes larger and larger than β then the re-balancing will take place to shift more capital from risk asset again to prevents high optimal portfolio losses. This means for the above calculated optimal weights the rate of return α will always have a positive impact on the value of the portfolio. While an increase in β above the required amount will always decrease the value of the portfolio.

In the second case where our investor choose a power utility function an increase in the value of β always leads to a decrease in the value of portfolio for $\rho \in (-\infty, 1)$ and the rate of return α always give a positive impact on the value of portfolio. And in the absence of jumps in the process we are expecting the value of portfolio to always increase with the rate of return as α increases and decreases as β increase. Also an increase in the intensity of jump, thus always leads to a decrease in the value of the portfolio and further a possibility of breaching the floor can be observed.

The third model is exponentially utility. Similarly to the previous observation an increase in volatility above the supreme one always lead to a decrease in the value of the portfolio and the rate of return in the absence of noise in the market always has a positive impact to the value of the portfolio.

Furthermore It was also observed that, the higher the value of volatility and jump size, the less the expected terminal portfolio. Therefore the best payoff can be achieved with the increase in number of re-balancing, the optimal portfolio weights as it is approved in fig (3).

Re-balancing portfolio frequently reduces the portfolio of losses, therefore as we see in fig (4) the probability of losses of the CPPI value to break the designed floor is reduced by frequently increases the number of re-balancing the fractions of portfolio value. So as n increases it reduces the probability of portfolio losses to zero.This results is consistent with what we would expect, as the volatility increase the risky asset involves more uncertainty hence we should reduce the fraction of the wealth invested in the risky asset and increase the fraction in the safe asset. The fraction of wealth in the risky asset is always larger in the no jump case when compared to any of the values in the jumps cases because of the zero impact of jump size δ and the measure ν. It should be surprising that the result does not depend on the sign of π^*. Actually the Jumps positive or not bring out a growth of the risk .

8. Conclusion

We studied the possible acceptable level of risk assets and of that risk-free assets in the constant proportion portfolio strategy under Hawkes jump-diffusion model. We solve the dynamic of risk asset and cushion by using a mean version stochastic differential Equation under Geometric Brownian Motion. The main goal of portfolio insurance is to protect investor against adverse market movement. Therefore the investor choose the floor level depends on her risk preference and always try to maintain it through-out the trading period up to the maturity date, so that the Portfolio value will always lies above it. In this paper we consider the problem of optimal Portfolio construction through the dynamics programming and its associate HJB equation of a two-dimensional by considering an investors of log, power and exponential utility function. Our risky asset is considered in the market where we may experience a downward-jumps. Any accuracy of a jump substantially increased the likelihood of future jumps. The intensity increases by the magnitude of the jump re-scaled by the parameter δ every time it experience an adverse shock and jumps it decays exponentially with common rate k to the level of initial jump. The optimal portfolio model react to each change in jump intensity accordingly, an increases leads to a reduced weights in risk asset allocation and a decrease lead to an increase in risk asset allocation. High volatility and of that high jump size and its intensity leads to a poor performance of CPPI strategy. Focusing more on the analysis of volatility and the impact of jump on the portfolio, we should be able to measure the risk point that possible lead to the payoff that may fall below the designed floor. It was observed that, the higher the value of volatility and jump size, the less the expected terminal portfolio. Therefore the best payoff can be achieved with the increase in number of re-balancing, the optimal portfolio weights. Re-balancing portfolio frequently reduces the portfolio of losses. Expected value of breaching the floor reveals that high volatility does greatly increases the size of losses, indicating less volatility will be preferred. Whatever types of investors we have under the utility function, the investor would rather stake his wealth in a non-jump model rather than in the discontinuous one.

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