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Fitting Zambia's Currency Market Returns with the Poisson Compound Model with Normal Inverse Gaussian Jumps

Research Article

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Abstract: The first stage in creating a trustworthy forecast for the returns on the forex market is identifying a model that can be applied and fully describes the return behavior. In this work, the market data is fitted using the compound Poisson model with the Normal Inverse Gaussian jumps (NIG), Poisson model with the Gaussian jumps (Norm), and the geometric Brownian motion (gBm). The models' AIC and BIC scores were utilized to validate them. The data from Zambian forex market corresponds well with a model compound Poisson with Normal Inverse Gaussian jumps (NIG), which performs better than the model compound Poisson with Gaussian jumps (Norm) and geometric Brownian motion (gBm) across all currencies.

MSC: 62E17 • 62D05

Keywords: Forex • Compound Poisson model with Normal Inverse Gaussian jumps (NIG) • Compound Poisson model with Gaussian jumps (Norm) • Geometric Brownian motion (gBm)

1. Introduction

A number of scientific disciplines rely heavily on Levy processes, including physics (turbulence, laser cooling, quantum field theory), engineering (networks, queues, dams), economics (continuous time-series models), actuarial science (calculating insurance and reinsurance risk), and of course mathematical finance [1].

Both the Compound Poisson measure and the Brownian movement model belong to a certain family of stochastic cycles that are influenced by the Levy process. These models have applications in fields ranging from material science and physics to finance and security. They make it possible to express unexpected moves by hops, which is similar to the Poisson cycle and required for particular applications. Over the past ten years, levy measures are believed to have grown significantly because they have been examined from two angles: academic, where they offer stochastic

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models of business sectors related to money, and financial, where they draw from an abundance of amazing alumni communications. This has persisted in stimulating additional study in theoretical and applied domains [2].

Recently, levy measures have attracted a lot of attention in the field of numerical accounting, mostly because of their wide range of applications. Consequently, they are currently regarded as a fundamental component in the presentation of foreign currency. Toll cycle models are a good place to start when creating and assessing quantitative processes because they contain the most crucial elements of market returns and act as "first-request approximations" for more accurate models. It should be mentioned that the broadest Levy measure is produced when a geometric Brownian and a compound Poisson measure are combined [3].

The good news is that all of the market return's behavior can be explained by these models. More recently, many models based on Levy have been put up to explain more stylized aspects of market returns. Retail currency traders utilize forex analysis to decide which currency pairings to buy and sell. Gaining an understanding of the macroeconomic principles governing currency value could make currency traders more successful. In order to do this, traders can fix exchange rates, buy and sell currencies in advance on futures swaps, or sell them in futures swaps markets. Both "Forex trading" and "How to trade the ForeX market" enable users to profit from the growth or decline of various currencies [4].

For millennia, financial markets have been significant, a popular topic of conversation around water coolers, and a field of study. Regulators and investors have always worked to improve our understanding of these markets and how they interact. A robust macroeconomy depends on well -functioning financial markets, as demonstrated by the Global Financial Crisis of 2008 to 2009, which resulted from a collapse in financial intermediation and a negative feedback loop with the economy. Following the European debt crisis and the subsequent Covid pandemic, these markets have gained even greater significance for scholars, investors, and decision-makers [5].

Dean Teneng used three models in [6] to forecast foreign currency rates. Teneng used the conventional reverse Gaussian (NIG) and Variance Gamma (VG) Toll measurements to demonstrate that the conjecture of unknown trade day-by-day shutdown costs outperforms the guileless Random Walk model. This paper did not employ both the NIG and Norm models. Additionally, no effort was made to fit all three models for every cash and evaluate which one works best.

The goal of this work [7] was to trade shutting costs by fitting the Typical Inverse Gaussian (NIG) circulation with the open programming bundle R and choosing the least complex models by applying the K¨a¨arik and Umbleja approach. With a Kolmogorov-Smirnov test with p worth of 0.062 and 0.08 for each, the daily closing costs of NZD/USD, QAR/CHF, QAR/EUR, CHF/JPY, SAR/CHF, AUD/JPY, GBP/JPY, SAR/EUR, TND/CHF, and TND/EUR tend to be spectacular fits, while EGP/EUR and EUR/GBP are acceptable fits. As a result, the daily shutdown costs of the following currencies have been consistently represented using ordinary opposite Gaussian circulation: CHF/JPY, GBP/JPY, QAR/EUR, SAR/EUR, TND/CHF, EGP/EUR, EUR/GBP, and TND/EUR (12/04/2008-07/08/2012). Their future costs have also been estimated using the NIG L´evy measure. He did not make use of NIG, gBm and Norm.

Reddy and Clinton examine the Geometric Brownian development model as a way to simulate stock value in [8]. They also offer three ways to assess the model's validity. Thomson One provided daily stock valuation information from January 1, 2013, until December 31, 2014. This paper's estimation is highly inconsistent due to its year-based constraints, rendering it largely untrustworthy.

How many radioactive particles a model generates is shown by the study [9]. The quantity of $\alpha-$ particles delivered up to time t can be represented by a Poisson cycle, $N(t) : t \geq 0$, with force α, given a vast array of radioactive centers that transmit $\alpha-$particles on schedule and for times remarkably not the half presence of the radioactive substance (this can range from a fraction of a second to billions of years). Poisson cycles can illustrate the challenge of coupon collection, Poisson associations can show client purchasing attentiveness, and Poisson cycles can show shot up heaval.

According to the study [10], a numerical model for the Mathematical Brownian evolution might be applied to forecast the longer-term cost of the stock. Prior to assessing stock value, the 95% assumption level and the stock expected value plan must be determined. By using the mathematical Brownian development model, the value of return is first calculated, then the value of float and volatility is assessed, an estimate of stock value is obtained, the gauge MAPE is computed, the stock expected cost is registered, and the 95% egotism level is computed. The yield investigation shows that the mathematical Brownian development model is the highest.

In order to analyze shifts in power Option Adjusted Spreads (OAS), or the spreads between a spot Treasury wind and an enrolled OAS record of all bonds in a certain rating group, the research [21] provides a framework that swaps the Levy model. An OAS record is created using the OAS of each constituent security and is weighted by market capitalization. OAS keeps an eye on the level of credit risk in chosen bonds. There are specific reasons why these are intriguing. One such argument is the notion that corporate security yields reflect the cost of borrowing for private enterprises. Wide spreads are a sign of increased capital expenditures and, hence, reduced opening upside experience. This [30] and [31] inspired the authors to publish with this journal. another paper that motivated the authors to publish with the journal is [32].

This paper makes the following contributions: (i)Using a compound Poisson model with Gaussian jumps to fit the returns of the Zambian forex market. (ii) Fitting the returns on the Zambian forex market with the Geometric Brownian motion. (iii) Using a compound Poisson model with normal inverse Gaussian jumps to fit the returns of the

Zambian forex market. (iv) Finally, apply both BIC and AIC function to compare each model's performance.

2. Methodology

2.1. Description of the models used

The inverse of a single continuous normal distribution is the inverse Gaussian distribution model, which is widely used in complex applications. This type of distribution was first inferred by Schrödinger in 1915 during his early investigation of the Brownian movement, as this work [12] discusses. This distribution was proposed by Étienne Halphen [13]. The inverse Gaussian name for Tweedie was adopted in 1945 [14]. The models of inverse normal distribution show an exponentially composite model with a single pattern and a large tail to the right. This distribution [15] is one of the ones utilized in the generalized linear model tracking procedure. This distribution is used to track data with positive deviation in addition to its many diverse applications in finance, survival analysis, economics and business, medicine, and even labor dispute settlement [16].

First studied in 1956, the normal inverse Gaussian distribution model attempted to follow Brownian motion in physics. M.C.K. Tweedie initially used it as an inverse Gaussian since there is an inverse connection that expresses the time required to cover the unit's distance and the distance covered at the time the unit was approved [17]. The normal inverse Gaussian distribution is a variance-mean mixture of a normal distribution with the inverse Gaussian serving as the mixing distribution. Brownian motion is subservient to a homogeneous Lévy process, which can be described, by the inverse Gaussian process [18]. [19] proposed the normal inverse Gaussian (NIG) distribution as a possible model for stock prices. An alternative way to depict this process would be a time-changed Brownian motion, where the time change T(t) represents the first passage time of an independent Brownian motion with drift to the level t. As a result, the process of temporal change is inversely Gaussian. At this stage, this suggests the name for a normal inverse Gaussian process for analyzing a Brownian motion.

2.2. Definition: Levy process

A cádlág, adapted, real valued stochastic process $L = (L_t)_{0\leq t\geq T}$ with $L_0 = 0$ a.s. is called a Lévy process if the following conditions are satisfied:

(i) L has independent increments, i.e. $L_t - L_s$ is independent of $\mathscr{F}_s$ for any $0 \leq s < t \geq T$.

(ii) L has stationary increments, i.e. for any $0 \leq s, t \geq T$ the distribution of $L_{t+s} - L_t$ does not depend on t.

(iii) L is stochastically continuous, i.e. for every $0 \leq t \geq T$ and $\epsilon > 0 : \lim_{s\to t} P(|L_s - L_t| \geq \epsilon) = 0$

The deterministic linear drift is the most basic Lévy process. The only (non-deterministic) Lévy process with continuous sample routes is Brownian motion. The Poisson and compound Poisson processes are additional instances of Levy processes. Observe that the Lévy process also known as a "jump diffusion" process that results from adding a linear drift, a Brownian motion, and a compound Poisson process is once more. Since there are jump-diffusion processes that are not Lévy processes, we will refer to it as a Lévy jump-diffusion process. [1]. Suppose H_t is a Levy cycle. At that point, K_t, the interaction, is described as

$$K_t = K_0 e^{H_t}, \ t > 0. \tag{1}$$

A Levy measure that is considered remarkable is equation (1). Usually, resource measurements are displayed using this cycle when autonomous log returns are possible. Undoubtedly, in the improbable scenario where we employ the method's log-returns, K_t is sometimes referred to as a "jump diffusion" process. Since there are jump-diffusion processes that are not Levy processes, we will refer to it as a Levy jump-diffusion process.

$$\log\left(\frac{K_{t+\Delta t}}{K_t}\right) = H_{t+\Delta t} - H_t = \Delta H_t. \tag{2}$$

We call equation (2) an exceptional Levy measure. This cycle is typically used to illustrate resource measurements when autonomous log returns are feasible. Naturally, in the unlikely event that we use the log-returns method [20].

2.3. Generalities on the Gaussian distribution

2.3.1. Inverse Gaussian Process

The Gaussian distribution that is inverse, the distribution

$$IG(\delta,\gamma) = GIG(-1/2,\delta,\gamma)$$

has the probability density given by equation (3) below:

$$p_{\mathrm{IG}}(x;\delta,\gamma)=\left(\frac{\delta^2}{2\pi}\right)^{\frac{1}{2}}x^{-\frac{3}{2}}\exp\left[-\frac{\delta^2\left(x-\delta\gamma^{-1}\right)^2}{2\left(\delta\gamma^{-1}\right)^2x}\right], \tag{3}$$

for $x>0$, and $\gamma>0, \delta>0$. In a broader sense, the Fourier transform equation (4) suggests that a probability distribution is specified by the function $p_{\mathrm{IG}}(\cdot;\delta,\gamma)$. As such, the class of inverse Gaussian distributions satisfies the following replication property: The expression $\varphi_{\mathrm{IG}}(u;\delta,\gamma)=\int_0^\infty e^{iux}p_{\mathrm{IG}}(x;\delta,\gamma)\,dx$:

$$\varphi_{\mathrm{IG}}(u;\delta,\gamma)=\exp\left[\gamma\delta\left(1-\sqrt{1-2iu/\gamma^2}\right)\right], u\in R. \tag{4}$$

The Lévy process equivalent for $\mathrm{IG}(\delta_1,\gamma)*\mathrm{IG}(\delta_2,\gamma)=IG(\delta_1+\delta_2,\gamma)$ with parameters $IG(\delta,\gamma)$, is referred to as an inverse Gaussian process. With parameters $\delta>0$ and $\gamma>0$, it is obvious that $IG(\delta,\gamma)$ is endlessly divisible. [20].

2.3.2. Normal Inverse Gaussian Process

The NIG conveyance cycle was first presented by Barndoff-Neilson as a log-returns of stock cost model. It falls within the broader heading of disproportionate Levy measures. It became clear after its presentation that the NIG dispersion best fit the log-returns of securities exchange data. Further research have shown that this appropriation shows outbursts when compared to other resource classes. The density function is described below. $NIG(\mu,,\beta,\delta,\alpha)$

$$f_{NIG}(x;\alpha,\beta,\delta,\mu)=\frac{\alpha}{\pi}e^{\delta}\sqrt{\alpha^2-\beta^2+\beta(x-\mu)\frac{K_1\left(\alpha\delta\sqrt{\frac{1+(x-\mu)^2}{\delta^2}}\right)}{\sqrt{\frac{1+(x-\mu)^2}{\delta^2}}}}, \tag{5}$$

where $\alpha>0$ and $\delta>0$. In equation (5), the parameters of the Normal Inverse Gaussian transport are as follows: The tail steepness is denoted by α, the skewness by β, the scale by δ, and the area by μ. Of the collection of generalized heightened distributions, the NIG dispersion is the only one that can be closed under convolution [21].

An inverse Gaussian process with $\mathscr{L}\{K_1\}=IG\left(\delta,\sqrt{\alpha^2-\beta^2}\right)$ given by $K=(K_t)_{t\in R_+}$ where $\gamma=\sqrt{\alpha^2-\beta^2}$ and $\delta,\alpha\in(0,\infty), \beta\in R$ fulfill this equation. Define $X_t=t\mu+K_t\beta+W_{tk}$ to define $X=(X_t)_{t\in R_+}$. $W=(W_t)_{t\in R_+}$ is a one-dimensional wiener process that is independent of K, and $\mu\in R$. The inverse Gaussian process K, which is subordinate to a one-dimensional drifting Wiener process, can be represented as $\beta K+W_k$. $Y_k=\beta k+W_k$ As thus, the Lévy processes are X and $Y_k=\beta k+W_k$. For the process X, let $NIGP(\alpha,\beta,\delta,\mu)$ be the normal inverse Gaussian process. Here is how we define the distribution:

$$\mathscr{L}\{X_1\}=\mathscr{L}\{\mu+\beta K_1+W_{k_1}\} \tag{6}$$

Equation (6) is known as a normal inverse Gaussian distribution $NIG(\alpha,\beta,\delta,\mu)$. Through

$$E\left[\exp(iu(\beta k_1+W_{k_1}))\right]=E\left[\exp(iu(\beta+2^{-1}iu)k_1)\right]$$

with the help of equation (4) we can determine the characteristics function given in equation (7)

$$\varphi_{NIG}(u;\alpha,\beta,\delta,\mu)=\exp\left[i\mu u+\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+iu)^2}\right)\right], \tag{7}$$

[20]

2.4. Generalities Standard processes

2.4.1. Geometric Brownian Motion

A Brownian (or Wiener) process is the stochastic process $B=B_t:t>0$ if and only if

(i) When $s<t$, the increment B_t-B_s is independent of the history of the process's actions up to time s. Thus, B_t has independent increments;

(ii) Gaussian increments characterize B_t; thus, the distribution B_t-B_s is $N(0,t-s)$.

(iii) Although B_t is continuous, it is not smooth, which means it cannot be differentiated anywhere.

(iv) $B_0 = 0$.

The Brownian movement is without a doubt the brightest star in the cosmic system of stochastic cycles used to illustrate value fluctuations. A random measure Wt with autonomous fixed additions that circulate according to a Gaussian distribution is called a Brownian movement. The mother of modern stochastic analysis, Brownian movement is the most widely considered stochastic measure [22]. Beginning with the last mentioned, Louis Bachelier offered to display the value K_t of an asset at the Paris Bourse as [23]. This is when the Brownian movement and monetary showing were first merged. $K_0 + \sigma W_t = K_t$. A cycle W_t that satisfies the first, second, and fourth conditions above but has dissemination for $W_t - W_s = N(\mu(t-s),(t-s)\sigma_2)$, where μ is the floating coefficient and σ is the dispersion coefficient, is also referred to as having "Brownian movement." The relationship between Brownian movement and standard Brownian movement is the same as the relationship between $N(0,1)$ and $N(\mu t, \sigma_2 t)$ [24].

The model for fitting currency market returns could then be geometric Brownian motion

$$\frac{dK_t}{K_t} = \mu dt + \sigma dB \tag{8}$$

where, in equation (8), $dB\ N(0,dt)$, μ represents the drift, σ represents the volatility, and a starting value $K_0 > 0$. Furthermore, is a unique example of this model because the answers are provided within the structure.

$$K_t = K_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right] \tag{9}$$

Occasionally, the process K_t in equation (9) is referred to as a geometric Brownian motion [25].

2.4.2. Compound Poisson Process

A cycle with just one possible jump size is not very attractive for financial purposes. The compound Poisson measure, where the holding-up events between hops are uncommon in any case, may show a self-evident change in the leap sizes. Alternatively, think of N as a Poisson cycle with boundary λ and an assembly of independent, self-assured pieces $\{Y_i\}_{i\geq 1}$ [26]. The definition of a stochastic process X_t is a compound Poisson process with intensity $\lambda > 0$ and, consequently, the jump size distribution f.

$$X_i = \sum_{i=1}^{N_t} Y_i \tag{10}$$

As a Poisson technique with intensity λ, N_t in this equation (10) is independent of $\{Y_i\}_{i\geq 1}$. With a distribution of $3f$, the jump sizes, Y_i, are identically distributed and independent inside $(i.i.d)$. The following characteristics of a compound Poisson technique are easily determined from its definition:

(i) The sample paths of X are piecewise constant functions

(ii) $\{T_i\}_{i\geq 1}$ is the jump times with the same law as the jump times of the Poisson process N_t: they can take the form of partial sums of free exponential random variables with parameter λ.

(iii) These jump sizes $\{Y_i\}_{i\geq 1}$ are independent and identically distributed $(i.i.d.)$ with law f [23]

Every Lévy cycle has two fundamental structural squares: the Poisson interaction (the leap component) and Brownian motion (the dissemination part). A brief discussion of the Poisson cycle is all totally required, even if the Brownian movement is a natural item for any astute dealer because the Black-Scholes model is present. Let $\{\tau_i\}_{i\geq 1}$ be a set of autonomous remarkable irregular variables with full apportionment and a boundary of λ.

$$N_T = \sum_{n\geq 1} 1_t \geq T_n. \tag{11}$$

It is said that equation (11) is a Poisson process with a boundary of λ. For example, if the holding up events between transports at a bus stop are significantly distributed, the total number of transports exhibited up to time t is a Poisson interaction. Piecewise steady bearings (right continuous with as much as possible, or RCLL) or bounces of size 1 are characteristics of a Poisson measure. The leaps occur sporadically T_i, and the intervals (the holding up events) between them are dramatically appropriated. Every time a date is greater than zero, N_t has the Poisson transport with boundary $t\lambda$; that is, it is a whole number valued and

$$P[Nt = n] = \frac{e^{t\lambda}(\lambda t)^n}{n!}, \quad n = 0,\ 1,\ 2,\ \ldots \tag{12}$$

The compound Poisson process with $E[N_t] = \lambda t$ is represented by equation (12), where the intensity function is denoted by the parameter λ. The addition $Nt\ Ns$ for each $t > s$ has a comparative law as $Nt\ s$ and is independent of the association's true background up to time s. This is the fundamental attribute of the Poisson collaboration that gives the Brownian development the freedom from augmentations and stationary nature. The term "toll interaction," which honors the renowned French mathematician Paul Levy, refers to the cycle with independent and fixed additions. [26].

3. Data Analysis and Discussion

3.1. Description of the data in terms of returns

The return for the GBP, USD, EUR, and Rand against the Zambian Kwacha is shown in the current figure. According to Fig. 1, the value of the pound sterling (GBP) fluctuates over time and has a tendency to bunch, with a discernible enormous (small) peak or decline in return. With high return periods from 2016 to 2017 and also from 2021 upto date and high bunch periods in between, the euro (EUR) exhibits diminishing clustering and is not constant and from 2016 to 2020 there is no clustering. Low time-series behavior of the US dollar (USD) is comparable to other currencies' behaviors and exhibits diminishing clustering and is not constant. From 2015 to 2016 and from 2018 to 2023, there is no cluster and no trend. While it tends to cluster with time, the South African Rand (Rand), which is the regional currency, fluctuates over time and has a tendency to bunch, with a discernible enormous (small) peak or decline in return. With high return periods in 2014 and high bunch periods in between.

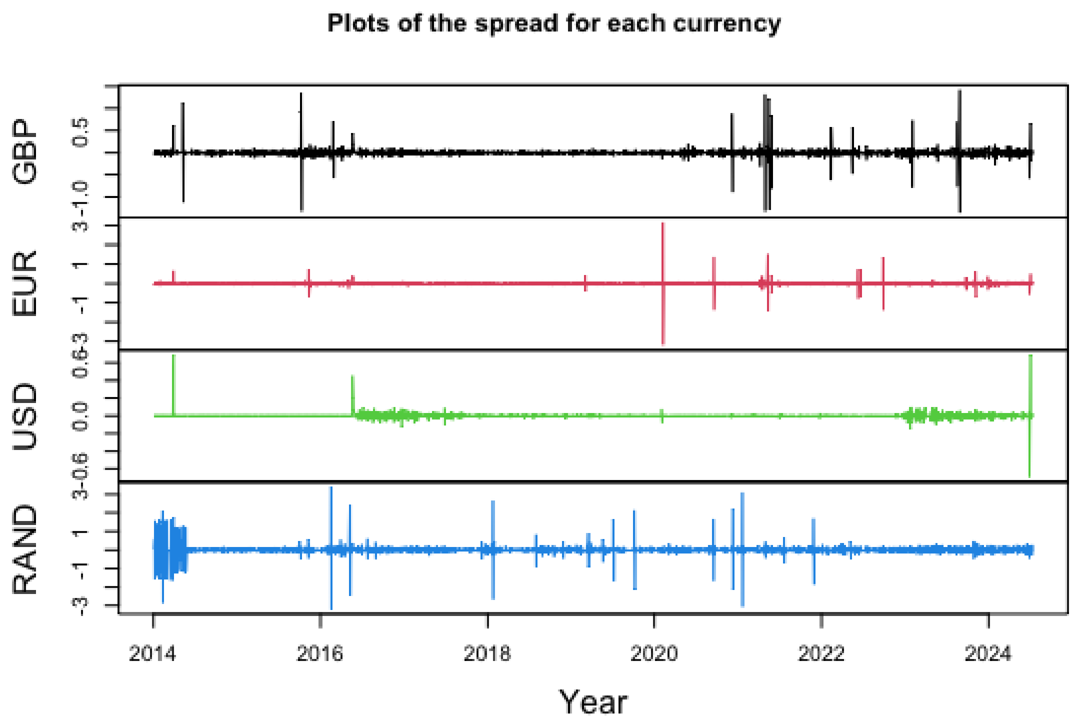

Fig. 1. Shows spread for each currency

Returns are widely used because they make it easier to compare assets with different absolute values and to normalize data. When executing currency models or fitting financial models, an understanding of the distribution and patterns of returns becomes crucial. Models often make assumptions about certain statistical properties of returns, such as normality or stationarity. Through return-based data characterization, you provide the model with the necessary information to forecast or estimate parameters based on these assumptions. In essence, the relationship is about how the chosen data representation, such returns, satisfies the assumptions established about the models used for financial or currency research.

3.2. Results (from fitting the models)

3.2.1. Running the model for international currency EUR, GBP, and USD

The compound Poisson model with Gaussian jumps (Norm) fit, empirical density (continuous line) against the geometric Brownian motion (gBm) fit, and Normal Inverse Gaussian (NIG) model density are shown in the current figures below.

Fig. 2, Fig. 3,Fig. 4 ,and Fig. 5 display the densities of the assumed NIG (blue dashed line), gBm (horizontal dashed green line), and Norm (dashed red line) densities versus the density of the experimental data Y_t (solid line). Equation (10) was utilized to approximate the densities of the postulated gBm, Norm, and NIG by utilizing the Poisson measure of the degenerate mixture. In Fig. 2 it is evident that the data in this dataset are more likely to be of the NIG type than

Gaussian, especially since they are not coming from a geometric Brownian motion. This empirical evidence is further supported by the Akaike information criterion evaluated with the AIC function. In Fig. 3, The empirical density is the highest, followed by the NIG, which is green with a slight difference, the red norm, and the green gBm at the bottom. In Fig. 4, the empirical density is the highest, followed by the Norm which is red and then gBm which is green.

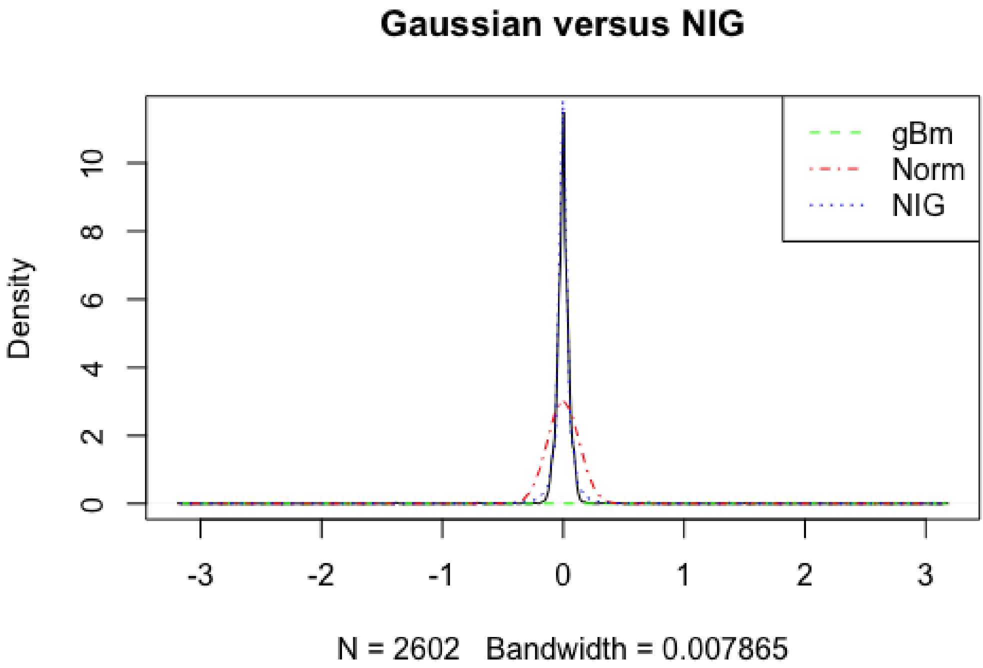

Fig. 2. The graph displays the empirical density as a continuous line, a dashed line representing Norm (red) fit, a dotted line representing horizontal geometric Brownian motion fit (green), and a dotted line representing the predicted NIG Lévy model density (blue).

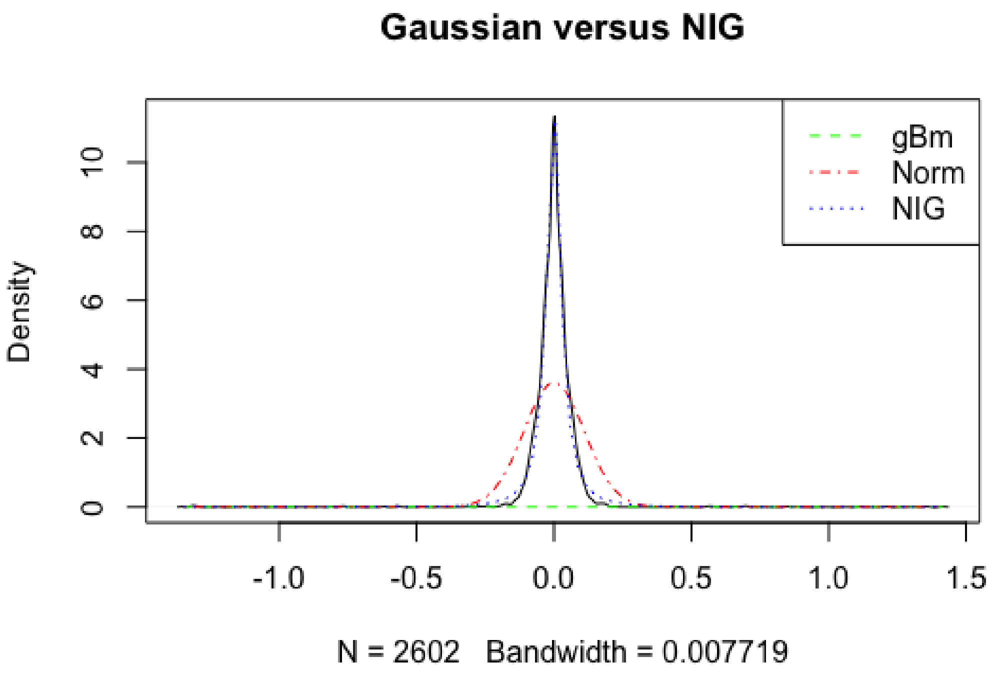

Fig. 3. The graph displays the empirical density as a continuous line, a dashed line representing Norm (red) fit, a dotted line representing horizontal geometric Brownian motion fit (green), and a dotted line representing the predicted NIG Lévy model density (blue).

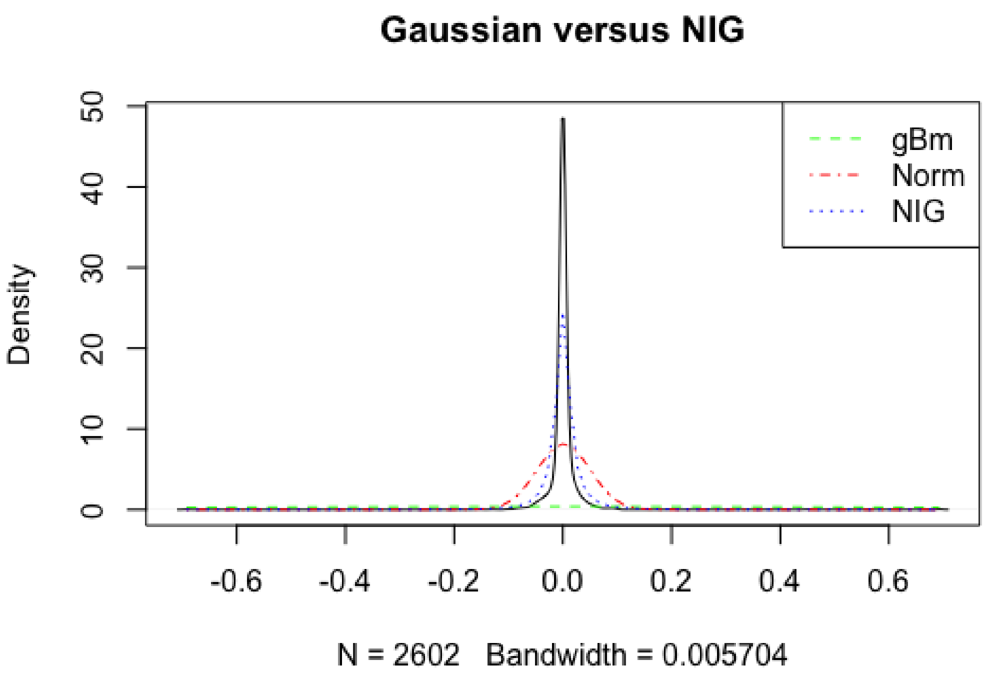

Fig. 4. The graph displays the empirical density as a continuous line, a dashed line representing Norm (red) fit, a dotted line representing horizontal geometric Brownian motion fit (green), and a dotted line representing the predicted NIG Lévy model density (blue).

3.2.2. Running the model for regional currency Rand

The empirical density (continuous line) is plotted against the geometric Brownian motion (gBm) fit, the compound Poisson model with Gaussian jumps (Norm) fit, and the density of the Normal Inverse Gaussian (NIG) model in the current figures.

Fig. 5 shows the densities of the estimated NIG (blue dashed line), gBm (horizontal dashed green line), and Norm (dashed red line) densities versus the density of the experimental data Y_t (solid line). Equation (10) was utilized to approximate the densities of the postulated gBm, Norm, and NIG by utilizing the Poisson measure of the degenerate mixture. In Fig. 5 it is evident that the blue color of NIG is intended to balance the density of observational data; red (Norm) follows, and gBm is the final. This empirical evidence is further supported by the Akaike information criterion evaluated with the AIC and the bayessian information criterion BIC function.

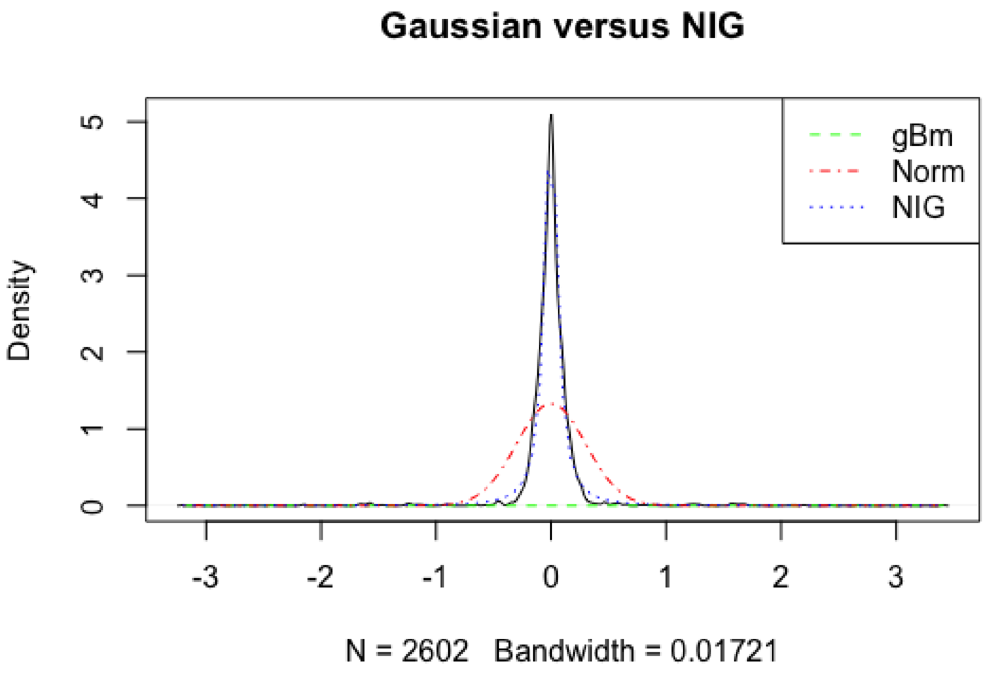

Fig. 5. The graph displays the empirical density as a continuous line, a dashed line representing Norm (red) fit, a dotted line representing horizontal geometric Brownian motion fit (green), and a dotted line representing the predicted NIG Lévy model density (blue).

3.3. Validation of the model

3.3.1. Selection Criteria

In statistics, economics, and many other fields, the model selection criteria is an extremely important topic with a wide range of real-world applications. Many statisticians are now studying this problem theoretically and practically. Over the past 20 years, it has received a lot of attention, particularly in the fields of regression and econometric models. The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are the two most widely used criteria for selecting models.

The two techniques listed below are for selecting the optimal or most effective model using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

(i) Akaike Information Criterion (AIC): Akaike [28] originally suggested the use of AIC as a way to compare several models based on a certain result. The definition of the AIC for a candidate model is as follows::

$$\text{AIC} = -2\ell\left(\hat{\theta}|y\right) + 2H, \tag{13}$$

where H is the total number of estimated model parameters, including the intercept, and $\ell(\hat{\theta}|y)$ is a log-likelihood at the estimated model's maximum point. The choice rule states that a model is better if its AIC value is less.

(ii) Schwarz [29] initially proposed the Bayesian information criterion (BIC), often known as the Schwarz criteria (also SBC, SBIC), which is a criterion for choosing a model from a limited number of options. The definition of the BIC for a candidate model is as follows:

$$\mathrm{BIC} = -2\ell(\hat{\theta}|y) + H\ln(n), \tag{14}$$

where H is the number of estimated model parameters, including the intercept, and $\ell(\hat{\theta}|y)$ is the log-likelihood of the estimated model at its highest point. The sample size is represented by n. The model is better if the BIC value is less, according to the selection rule. These are the steps involved in implementing AIC and BIC:

Step1: Selecting up-and-comer models which can be fitted to the dataset.

Step2: Estimating unknown parameters of models.

Step3: Finding up- sides of AIC and BIC by utilizing the formulas (13) and (14), respectively.

Step4: Basing on the standard of choice, one can choose the most reasonable model [27].

3.3.2. Comparison performances of models

Models will be validated in this section based on their AIC and BIC scores. With the lowest Akaike information criterion (AIC) and Bayessian information criterion (BIC) values among the three models, the Compound Poisson Model with Normal Inverse Gaussian Jumps (NIG) performs better than the other Compound Poisson model with gaussian jumps (Norm) and geometric brownian motion model (gBm) in Table 1. This implies that the NIG model offers the optimal trade-off between complexity and model fit. In spite of its simplicity, the gBm model has higher AIC and BIC values, which suggests a worse fit to the data. The Norm model is in the middle, suggesting a decent fit but a little more complexity.
With the lowest Akaike information criterion (AIC) and Bayessian information criterion (BIC) values among the three models, the Compound Poisson Model with Normal Inverse Gaussian Jumps (NIG) performs better than the other Compound Poisson model with gaussian jumps (Norm) and geometric brownian motion model (gBm) in Table 2. This implies that the NIG model offers the optimal trade-off between complexity and model fit. In spite of its simplicity, the gBm model has higher AIC and BIC values, which suggests a worse fit to the data. The Norm model is in the middle, suggesting a decent fit but a little more complexity.
With the lowest Akaike information criterion (AIC) and Bayessian information criterion (BIC) values among the three models, the Compound Poisson Model with Normal Inverse Gaussian Jumps (NIG) performs better than the other Compound Poisson model with gaussian jumps (Norm) and geometric brownian motion model (gBm) in Table 3. This implies that the NIG model offers the optimal trade-off between complexity and model fit. In spite of its simplicity, the gBm model has higher AIC and BIC values, which suggests a worse fit to the data. The Norm model is in the middle, suggesting a decent fit but a little more complexity.
With the lowest Akaike information criterion (AIC) and Bayessian information criterion (BIC) values among the three models, the Compound Poisson Model with Normal Inverse Gaussian Jumps (NIG) performs better than the other Compound Poisson model with gaussian jumps (Norm) and geometric brownian motion model (gBm) in Table 4. This implies that the NIG model offers the optimal trade-off between complexity and model fit. In spite of its simplicity, the gBm model has higher AIC and BIC values, which suggests a worse fit to the data. The Norm model is in the middle, suggesting a decent fit but a little more complexity.

Table 1. Give each model's AIC and BIC scores for EUR

	gBm	Norm	NIG
AIC	12937.56	-3188.607	-7793.571
BIC	12949.29	-3176.88	-7770.116

Table 2. Give each model's AIC and BIC scores for GBP

	gBm	Norm	NIG
AIC	11388.84	-4075.791	-7629.48
BIC	11400.56	-4064.064	-7606.026

Table 3. Give each model's AIC and BIC scores for USD

	gBm	Norm	NIG
AIC	2 exp + 10	-3205.134	-4434.139
BIC	2 exp + 10	-3193.406	-4410.684

Table 4. Give each model's AIC and BIC scores for Rand

	gBm	Norm	NIG
AIC	23047.81	1129.322	-3042.783
BIC	23059.53	1141.05	-3019.329

3.4. Summary of model's performance

The Compound Poisson model with normal inverse gaussian jumps (NIG) is determined to be the most appropriate for our data based on the akaike information criterion (AIC) and bayesian criterion information (BIC) values followed by the compound poisson model with gaussian jumps (Norm) and Geometric brownian motion (gBm). The outcomes after the model is run for the euro, British Pound Sterling, United States Dollar, and South African Rand show how crucial it is to choose a model that takes complexity and model fit into consideration. We have confirmed our findings and established a solid framework for further study by utilizing AIC and BIC.

4. Conclusion and Recommendation

4.1. Conclusion

Assessing the impact of the Levy process on returns from the Zambian foreign exchange market using 2602 data from 2-Jan-24 to 12-Jun-24 was the main objective of the study. The study specifically used the gBm, Norm, and NIG models to try to match the returns from the Zambian FX market. The AIC and BIC metrics were used to assess each model's performance.

After fitting all three models for each currency, the research concluded that the best model for the euro (EUR) is Compound Poisson having Normal Inverse Gaussian jumps (NIG) with $AIC = -7793.571$ and $BIC = -7770.116$. This is in comparison to the models' geometric Brownian motion (gBm) with $AIC = 12937.56$ and $BIC = 12949.29$, and Compound Poisson having Gaussian jumps (Norm) with $AIC = -3188.607$ and $BIC = -3176.88$.

In addition, the study found that the model Compound Poisson having Normal Inverse Gaussian jumps (NIG) with AIC= -7629.48 and BIC= -7606.026 is the best model for Pound sterling (GBP) in terms of the geometric Brownian motion (gBm) with AIC= 11388.84 and BIC= 11400.56 and Compound Poisson having Gaussian jumps (Norm) with AIC= -4075.791 and BIC= -4064.064.

The study also found that the model Compound Poisson having Normal Inverse Gaussian jumps (NIG) with AIC= -4434.139 and BIC= -4410.684 outperformed both the Compound Poisson having Gaussian jumps (Norm) with AIC= -3205.134 and BIC= -3193.406 and geometric Brownian motion (gBm) with AIC= 2exp +10 and BIC= 2exp +10 for the united states dollar (USD).

Furthermore, the study shows that the Compound Poisson model with Normal Inverse Gaussian jumps (NIG) with AIC = -3042.783 and BIC = -3019.329 performs better for South African Rand (Rand) than the Compound Poisson model with Gaussian jumps (Norm) with AIC = 1129.322 and BIC = 1141.05, as well as the geometric Brownian motion model with AIC=23047.81 and BIC=23059.53.

4.2. Recommendation

According to the study's empirical results, the NIG model outperforms the gBm and Norm models for all currencies. The model NIG could be used by the Dares Salaam Stock Exchange (DSE) , Rwanda Stock Exchange (RSE), Lusaka Stock Exchange (LuSE), and Nairobi Securities Exchange (NSE).

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