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A Study on The Extensions and Developments of Generalized $\mathfrak{B}K-$Recurrent Finsler Space

Research Article

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Abstract: This paper builds upon existing work on generalized $\mathfrak{B}K-$recurrent Finsler space. We define a new type of Finsler space that considers extension for the above mentioned space. In other words, we introduce a new class of Finsler spaces call "second generalized $\mathfrak{B}K-$recurrent Finsler space,". Within these spaces, we prove that the tensor $R^i_h y_k$ is symmetric and exhibits the remarkable property of aligning perfectly with the $h(v)-$torsion tensor H^i_{hk}. Furthermore, we demonstrate that certain tensors exhibit generalized recurrent behavior under specific conditions. Also, we infer that $K-$Ricci tensor K_{jk} and $R-$Ricci tensor R_{jk} both are equal in the main space.

MSC: 53B40 • 53C60

Keywords: Second generalized $\mathfrak{B}K-$recurrent Finsler space • Symmetry property • Quadrature fundamental function

1. Introduction and Preliminaries

The generalized recurrent spaces for different curvature tensors in Berwald sense discussed by [2, 4, 13–15, 19–22]. Various theorems for types of curvature tensors was proved by many authors ([5] - [9]). The generalized $\mathfrak{B}K-$recurrent Finsler space was introduced by Qasem and Baleedi [22]. Recently, Al-Qashbari and Saleh [10] discussed the generalized recurrent Finsler spaces for K^i_{jkh} of higher order with Berwald's curvature tensor.

Goswami [16] studied certain types of special Finsler spaces. Emamian and Tayebi [12] defined the generalized Douglas-Weyl in Finsler metrics. Further, Atashafrouz and Najafi [11] studied $D-$recurrent Finsler metrics. The several properties of various curvature tensors were investigated by [17, 24, 26].

Let us consider a generalized $\mathfrak{B}K-$recurrent Finsler space satisfying the relations [22]

$$\mathfrak{B}_m H^i_{kh} = \lambda_m H^i_{kh} + \mu_m\left(\delta^i_h y_k - \delta^i_k y_h\right). \tag{1}$$

$$\mathfrak{B}_m H^i_h = \lambda_m H^i_h + \mu_m\left(\delta^i_h F^2 - y^i y_h\right). \tag{2}$$

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$$\mathfrak{B}_m H_k = \lambda_m H_k + (n-1)\mu_m y_k. \tag{3}$$

$$\mathfrak{B}_m H = \lambda_m H + \mu_m F^2. \tag{4}$$

$$\mathfrak{B}_m K_{jk} = \lambda_m K_{jk} + (n-1)\mu_m g_{jk}. \tag{5}$$

$$\mathfrak{B}_m K_j = \lambda_m K_j + (n-1)\mu_m y_j. \tag{6}$$

$$\mathfrak{B}_m K = \lambda_m K + n(n-1)\mu_m + (\mathfrak{B}_m g^{jk})K_{jk}. \tag{7}$$

The metric tensor g_{ij} and Kronecker delta δ^i_h are satisfying the relations:

$$\begin{cases} a)\ g_{ij}y^i y^j = F^2, \quad b)\ g_{ij}y^j = y_i, \quad c)\ \delta^i_l = \dot\partial_l y^i, \\ d) g_{ij}g^{ik} = \delta^k_j = \begin{cases} 1 \text{ if } j = k, \\ 0 \text{ if } j \neq k, \end{cases} \quad e) g_{hk} = \dot\partial_h y_k, \\ f)\ y_i y^i = F^2, \quad g)\ \delta^i_h g_{ik} = g_{hk}, \quad h)\ \delta^i_k y^k = y^i, \\ i)\ \delta^i_i = n, \quad j)\ g^{jh}y_h = y^j, \quad k)\ \delta^i_k y_i = y_k. \end{cases} \tag{8}$$

The Berwald covariant derivatives of the vectors y^i and y_i are vanishing, i.e.

$$\begin{cases} a)\ \mathfrak{B}_k y^i = 0, \\ b)\ \mathfrak{B}_k y_i = 0. \end{cases} \tag{9}$$

Cartan's fourth curvature tensor K^i_{jkh} satisfies the relations

$$R^i_{jkh}\, y^j = K^i_{jkh}\, y^j = H^i_{kh}. \tag{10}$$

$$K_{ijkh} = g_{rj}K^r_{ikh}, \tag{11}$$

where K_{ijkh} is associate curvature tensor of K^i_{jkh}. The Ricci tensor R_{jk}, deviation tensor R^i_h and curvature scalar R satisfy the following relations [1]

$$\begin{cases} a)\ R_{jk}g^{jk} = R, \quad b)\ R^r_{jkr} = R_{jk}, \\ c)\ R^i_{jkh}g^{jk} = R^i_h, \quad d)\ R^r_r = R. \end{cases} \tag{12}$$

The $h(v)$–torsion tensor H^i_{kh}, deviation tensor H^i_h, curvature vector H_k, Ricci tensor K_{jk} and curvature scalar H satisfy the following relations [3]

$$\begin{cases} a)\ H^i_{jkh}\, y^j = H^i_{kh}, \quad b)\ H^i_{kh}\, y^k = H^i_h, \quad c)\ R_{jk}y^j = H_k, \\ d)\ K_{jk}\, y^j = H_k, \quad e)\ H^r_{kr} = H_k, \quad f)\ H^r_r = (n-1)H, \\ g)\ K_j y^j = (n-1)H, \quad h)\ K^r_{jkr} = K_{jk}, \quad i)\ K_{jk}y^k = K_j, \\ j)\ K_{jk}g^{jk} = K, \quad k)\ H^i_k y^k = 0, \quad l)\ H^i_{jk}y_i = 0, \\ m)\ H_k y^k = (n-1)H. \end{cases} \tag{13}$$

The $(h)hv$–torsion tensor C_{ijk} satisfies [18, 23]

$$C_{ijk} = \frac{1}{4}\dot\partial_k\dot\partial_i\dot\partial_j\, F^2. \tag{14}$$

$$C_{ijk}y^k = C_{kij}y^k = C_{jki}y^k = 0. \tag{15}$$

A Finsler space whose Berwald connection parameter G^i_{kh} is independent of y^i is called an affinely connected space (Berwald space). Thus, an affinely connected space is characterized by one of the equivalent conditions

$$\begin{cases} a)\ \mathfrak{B}_k g_{ij} = 0 \\ b)\ \mathfrak{B}_k g^{ij} = 0. \end{cases} \tag{16}$$

2. A Second Generalized $\mathfrak{B}K-$Recurrent Finsler Space

In this work, several theorems have been estiblished and proved in second generalized $\mathfrak{B}K-$recurrent Finsler space. Cartan's third curvature tensor R^i_{jkh} in recurrent Finsler space is defined as [25]

$$\mathfrak{B}_m R^i_{jkh} = \lambda_m R^i_{jkh}. \tag{17}$$

Transvecting (17) by y^j and using [(9)a], we get

$$\mathfrak{B}_m (R^i_{jkh} y^j) = \lambda_m (R^i_{jkh} y^j). \tag{18}$$

Using (10) in (18),we get

$$\mathfrak{B}_m (K^i_{jkh} y^j) = \lambda_m (K^i_{jkh} y^j). \tag{19}$$

Which can be written as

$$y^j \mathfrak{B}_m (K^i_{jkh}) = y^j \lambda_m (K^i_{jkh}).$$

Since y^j is non-zero vector, so we can written above equation as

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh}. \tag{20}$$

In Finsler space if exists non-zero covariant vector field $\lambda_m \neq 0$, such that the curvature tensor K^i_{jkh} satisfies the relation (20) then the space is called recurrent Finsler space. We can written the equation (20) as

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + (\dot{\partial}_j y^i g_{hk} - \dot{\partial}_j y^i g_{hk}).$$

Since the metric tensor is symmetric, using [(8)c] and [(8)e] in above equation, we get

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m \left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right). \tag{21}$$

Which is called generalized $\mathfrak{B}K-$recurrent Finsler space $G\mathfrak{B}K - RF_n$ [22]. Using (15) in (21), we get

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m \left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right) + (R^i_h C_{ijk} y^i - R^i_k C_{ijh} y^i).$$

Using (14) in above equation, we get

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m \left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right) + \frac{1}{4}(R^i_h \dot{\partial}_k \dot{\partial}_i \dot{\partial}_j F^2 y^i - R^i_k \dot{\partial}_h \dot{\partial}_i \dot{\partial}_j F^2 y^i).$$

Since $[\dot{\partial}_i y^i = 1]$, using [(8)f] and [(8)e] in above equation, we get

$$\mathfrak{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m \left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right) + \frac{1}{4}\left(R^i_h g_{jk} - R^i_k g_{jh}\right). \tag{22}$$

Equation (22) is extensions and developments of generalized $\mathfrak{B}K-$recurrent Finsler space $G\mathfrak{B}K - RF_n$. Hence, we conclude

Definition 2.1.
A Finsler Space F_n which Cartan's fourth curvature tensor K^i_{jkh} satisfies the condition (22) will be called a second generalized $\mathfrak{B}K-$ recurrent space and the tensor will be called a second generalized $\mathfrak{B}-$recurrent tensor. We shall denoted them briefly by $2nd\ G\mathfrak{B}K - RF_n$ and $2nd\ G\mathfrak{B} - R$, respectively.

Where $\mathfrak{B}_m$ is Berwald's covariant differential operator with respect to x^m, λ_m and μ_m are non-zero covariant vectors field. Transvecting (22) by y^j and using [(8)b], [(9)a] and (10), we get

$$\mathfrak{B}_m H^i_{kh} = \lambda_m H^i_{kh} + \mu_m \left(\delta^i_h y_k - \delta^i_k y_h\right) + \frac{1}{4}\left(R^i_h y_k - R^i_k y_h\right). \tag{23}$$

In view of (1) and (23), we get

$$R^i_h y_k = R^i_k y_h. \tag{24}$$

Thus, we conclude

Theorem 2.1.
In 2nd $G\mathfrak{B}K-RF_n$, the tensor $R^i_h y_k$ is symmetric in h and k.

Transvecting (23) by y^k, using [(13)b], [(8)f,h] and [(9)a], we get

$$\mathfrak{B}_m H^i_h = \lambda_m H^i_h + \mu_m\left(\delta^i_h F^2 - y^i y_h\right) + \frac{1}{4}\left(R^i_h F^2 - R^i_k y_h y^k\right). \tag{25}$$

In view of (2) and (25), we get

$$R^i_h F^2 = R^i_k y_h y^k. \tag{26}$$

Using [(12)c] in right hand side of (26) and using [(8)f] in left hand side of (26), we get

$$R^i_h y_k y^k = R^i_{jhk} g^{jh} y_h y^k.$$

Using [(8)j] in above equation and since $y^k \neq 0$, we get

$$R^i_h y_k = R^i_{jhk} y^j. \tag{27}$$

Using (10) in (27), we get

$$R^i_h y_k = H^i_{hk}. \tag{28}$$

Contracting the indices i and h in (25) and using [(13)f], [(8) f,i] and [(12)d], we get

$$(n-1)\mathfrak{B}_m H = \lambda_m (n-1) H + (n-1)\mu_m F^2 + \frac{1}{4}\left(RF^2 - R^r_k y_r y^k\right). \tag{29}$$

Multiplying (4) by $(n-1)$ and using it in (29), we get

$$RF^2 = R^r_k y_r y^k$$

Using [(8)f] and [(12)a] in above equation and since $y^k \neq 0$, we get

$$R_{jk} g^{jk} y_k = R^r_k y_r$$

Using [(8)j] in above equation, we get

$$R_{jk} y^j = R^r_k y_r$$

Using [(13)c] in above equation, we get

$$H_k = R^r_k y_r \tag{30}$$

From (28) and (30), we conclude

Theorem 2.2.
In 2nd $G\mathfrak{B}K-RF_n$, the $h(v)-$ torsion tensor H^i_{hk} coincide with the tensor $R^i_h y_k$ and the curvature vector H_k coincide with the tensor $R^r_k y_r$.

Contracting the indices i and h in (23) and using [(13)e], [(8)k,i] and [(12)d], we get

$$\mathfrak{B}_m H_k = \lambda_m H_k + (n-1)\mu_m y_k + \frac{1}{4}\left(R y_k - R^r_k y_r\right). \tag{31}$$

In view of (3) and (31), we get

$$R y_k = R^r_k y_r. \tag{32}$$

Thus, we conclude

Theorem 2.3.
In 2nd $G\mathfrak{B}K-RF_n$, we have the identity (32).

In view of (32) and (30) , we get

$$H_k = R y_k.$$

Using [(13)d] in above equation, we get

$$K_{jk} y^j = R y_k.$$

Using [(12)a] in above equation, we get

$$K_{jk} y^j = R_{jk} g^{jk} y_k.$$

Using [(8)j] in above equation and since $y^j \neq 0$, we get

$$K_{jk} = R_{jk}. \tag{33}$$

Thus, we conclude

Theorem 2.4.
In 2nd $G\mathfrak{B}K-RF_n$, the $K-$Ricci tensor K_{jk} and $R-$Ricci tensor R_{jk} both are equal .

Contracting the indices i and h in (22) and using [(13)h], [(8)g,i] and [(12)d], we get

$$\mathfrak{B}_m K_{jk} = \lambda_m K_{jk} + (n-1)\mu_m g_{jk} + \frac{1}{4}(R g_{jk} - R^r_k g_{jr}). \tag{34}$$

Using (5) in (34), we get

$$R g_{jk} = R^r_k g_{jr}.$$

Using [(12)a] in above equation, we get

$$R_{jk} g^{jk} g_{jk} = R^r_k g_{jr}.$$

Using [(8)d,i] in above equation, we get

$$R_{jk} = \frac{1}{n}(R^r_k g_{jr}). \tag{35}$$

Transvecting (35) by g^{jr} and using [(8)d,i], we get

$$R_{jk} g^{jr} = R^r_k. \tag{36}$$

Transvecting (34) by g^{jk} and using [(13)j], [(8)d,i] and [(16)b], we get

$$\mathfrak{B}_m K = \lambda_m K + n(n-1)\mu_m + \frac{1}{4}(nR - \delta^k_r R^r_k). \tag{37}$$

Using [(16)b] in (7) and using it in (37), we get

$$R = \frac{1}{n}(\delta^k_r R^r_k). \tag{38}$$

From (35), (36) and (38), we conclude

Theorem 2.5.
In 2nd $G\mathfrak{B}K-RF_n$, the $R-$Ricci tensor R_{jk}, deviation tensor R^r_k and curvature scalar R are given by (35), (36) and (38), repectively.

Transvecting (34) by y^k and using[(8)b], [(9)a] and [(13)i],we get

$$\mathfrak{B}_m K_j = \lambda_m K_j + (n-1)\mu_m y_j + \frac{1}{4}(R y_j - R^r_k g_{jr} y^k). \tag{39}$$

In view of (6) and (39), we get

$$R y_j = R^r_k g_{jr} y^k.$$

Transvecting above equation by g^{jr} and using[(8)d,i,j], we get

$$R y^r = n R^r_k y^k.$$

Transvecting above equation by y_r and using[(8)f], we get

$$R F^2 = n R^r_k y_r y^k.$$

Using (30) in above equation, we get

$$R F^2 = n H_k y^k.$$

Using [(13)m] in above equation, we get

$$F^2 = \frac{n(n-1)H}{R}. \tag{40}$$

Thus, we conclude

Theorem 2.6.
In $2nd\ G\mathfrak{B}K - RF_n$, the quadrature fundamental function of Finsler space is given by (40).

Transvecting (23) by y_i and using [(13)l],[(9)b] and [(8)k], we get

$$R^i_h y_i y_k = R^i_k y_i y_h.$$

Using (30) in above equation, we get

$$H_h y_k = H_k y_h. \tag{41}$$

Transvecting (25) by y^h, using [(13)k], [(8)f,h], and[(9)a], we get

$$R^i_h y^h F^2 = R^i_k F^2 y^K.$$

Using (26) in above equation, we get

$$R^i_k y_h y^k y^h = R^i_h y_k y^k y^h.$$

Using (28) and [(8)j,f] in above equation, since $F^2 \neq 0$, we get

$$H^i_{kh} g^{kh} = H^i_{hk} g^{hk}. \tag{42}$$

From (41) and (42), we conclude

Theorem 2.7.
In $2nd\ G\mathfrak{B}K - RF_n$, the tensors $H_h y_k$ and $H^i_{kh} g^{kh}$ are symmetric in h and k.

Now, we have a corollary related to the above theorems. In view of (23), (25), (31), (29), (34), (39) and (37), we have

$$\mathfrak{B}_m H^i_{kh} = \lambda_m H^i_{kh} + \mu_m \left(\delta^i_h y_k - \delta^i_k y_h\right), \tag{43}$$

$$\mathfrak{B}_m H^i_h = \lambda_m H^i_h + \mu_m \left(\delta^i_h F^2 - y^i y_h\right), \tag{44}$$

$$\mathfrak{B}_m H_k = \lambda_m H_k + (n-1)\mu_m y_k, \tag{45}$$

$$\mathfrak{B}_m H = \lambda_m H + \mu_m F^2, \tag{46}$$

$$\mathfrak{B}_m K_{jk} = \lambda_m K_{jk} + (n-1)\mu_m g_{jk}, \tag{47}$$

$$\mathfrak{B}_m K_j = \lambda_m K_j + (n-1)\mu_m y_j \tag{48}$$

and

$$\mathfrak{B}_m K = \lambda_m K + n(n-1)\mu_m \tag{49}$$

if and only if

$$R^i_h y_k - R^i_k y_h = 0, \tag{50}$$

$$R^i_h F^2 - R^i_k y_h y^k = 0, \tag{51}$$

$$R y_k - R^r_k y_r = 0, \tag{52}$$

$$R F^2 - R^r_k y_r y^k = 0, \tag{53}$$

$$R g_{jk} - R^r_k g_{jr} = 0, \tag{54}$$

$$R y_j - R^r_k g_{jr} y^k = 0 \tag{55}$$

and

$$nR - \delta^k_r R^r_k = 0, \tag{56}$$

respectively. Thus, we conclude

Corollary 2.1.
In 2nd $G\mathfrak{B}K-RF_n$, the $h(v)-$torsion tensor H^i_{kh}, deviation tensor H^i_h, curvature vector H_k, curvature scalar H, $K-$Ricci tensor K_{jk}, curvature vector K_j and curvature scalar K behave as generalized recurrent if and only if (50), (51), (52), (53), (54), (55) *and* (56), *respectively hold.*

3. Conclusion

We expanded the generalized $\mathfrak{B}K-$recurrent Finsler space to obtain a new, more generalized Finsler space is called second generalized $\mathfrak{B}K-$recurrent Finsler space. Further, we studied the relations between some tensors and proved them satisfy the symmetry property in $2G\mathfrak{B}K-RF_n$.

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