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The similarity solution of the longitudinal dispersion phenomenon of miscible fluids in porous media

Research Article

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Abstract: The longitudinal dispersion phenomenon of miscible fluids in porous media is discussed by regarding the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of the phenomenon yields a non-linear partial differential equation. This partial differential equation is transformed into ordinary differential equation by using infinitesimal transformations group technique of similarity analysis. An analytical solution of the later is derived in terms of error function.The solution obtained is physically consistent with the results of earlier researchers and which is also more classical than other results obtained by various researchers.This type of phenomenon has been of great concern to hydrologists who have been studying the problem of displacement of fresh water by sea water in coastal areas. The oil industry has also become involved in miscible displacement studies in connection with the possibility of flushing oil by solvents from reservoirs.

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1. Introduction

This paper deals with the discussion of miscible displacement. This type of displacement has been of great concern to hydrologists who have been studying the problem of displacement of fresh water by sea water in coastal areas. The oil industry has also become involved in miscible displacement studies in connection with the possibility of flushing oil by solvents from reservoirs. In spite of the importance of miscible displacement, its study is still in its infancy. A series of theories has been proposed for various types of flow.

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Much of the discussion concerning dispersion originated in references to capillary network models for homogeneous porous media (e.g. [1]) and for heterogeneous porous media (e.g. [2])

In particular, this paper, discusses the phenomenon of longitudinal dispersion which is the process by which miscible fluids in laminar flow mix in the direction of the flow. The phenomenon is discussed by regarding the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of the phenomenon yields a non-linear partial differential equation. This partial differential equation is transformed into an ordinary differential equation by using similarity analysis. An analytical solution of the later is obtained in terms of confluent hypergeometric functions. Many researchers have discussed this phenomenon from different aspects: for example, [3]. Bhathawala [4] has used two parameter singular perturbation method. Raval[5] has obtained the solution of the same in terms of confluent hypergeometric functions. Patel[6] has derived the approximate solution of displacement process in porous media. Rudraiah et al.[7] have provided analytical study of this phenomenon in fluid saturated deformable or non-deformable porous media with or without chemical reaction. Patel and Mehta [8] have considered that the longitudinal dispersion is directly proportional to the concentration and the velocity component in the direction of x-axis is directly proportional to the distance. The solution obtained here is physically consistent with the results of earlier researchers and which is also more classical than other results obtained by various researchers.

2. Mathematical Formulation

Miscible displacement in porous media is a type of double-phase flow in which the two phases are completely soluble in each other. Therefore, capillary forces between the two fluids do not come into effect. At first it must be thought that miscible displacement could be described in a very simple fashion. The mixture under conditions of complete miscibility, could be thought to behave, locally at least, as a single phase fluid which would obey Darcy's law. The change of concentration, in turn, would be caused by diffusion along the flow channels and thus be governed by the bulk coefficient of diffusion of the one fluid in the other. In this fashion, one arrives at a heuristic description which looks, at a first glance, at least very plausible.

The problem is to describe the growth of the mixed region, i.e. to find concentration as a function of time t and position x, as the two miscible fluids flow through porous media. Outside of the mixed zone (*on either side*) the single-fluid equations describe the motion. The problem is more complicated, even in one- dimension with fluids of equal properties, since the mixing takes place both longitudinally and transversely.

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \overline{V}) = 0 \tag{1}$$

where ρ is the density for mixture and $\overline{V}$ is the pore seepage velocity vector. The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial c}{\partial t}+\nabla\cdot(c\overline{V})=\nabla\cdot\left[\rho\overline{D}\nabla\left(\frac{c}{\rho}\right)\right] \tag{2}$$

where c is the concentration of the fluid A into the other host fluid B (i.e. c is the mass of A per unit volume of the mixture) and D is the tensor coefficient of dispersion with nine components D_{ij}.

In a laminar flow through a homogeneous porous medium at constant temperature ? is constant. Then

$$\nabla\overline{V}=0 \tag{3}$$

and the equation (2) becomes,

$$\frac{\partial c}{\partial t}+\overline{V}\cdot\nabla c=\nabla \tag{4}$$

When the seepage velocity $\overline{V}$ is along the x-axis, the non-zero components are $D_{11}=D_L$ (coefficient of longitudinal dispersion) and $D_{22}=D_T$ (coefficient of transverse dispersion) and other D_{ij} are zero. In this case (4) becomes

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D_L\frac{\partial^2 c}{\partial x^2} \tag{5}$$

where u is the component of velocity along the x-axis which is time dependent and $D_L>0$.

The boundary conditions in longitudinal direction are:

$$c(0,t)=c_o(t>0) \tag{6}$$

$$c(L,t)=c_L(t>0) \tag{7}$$

where c_o is the initial concentration of the tracer (one fluid A) and c_L is the concentration of the tracer (of the same fluid) at $x=L$.

Thus (5) together with boundary conditions (6) and (7) represents the boundary value problem for the longitudinal dispersion of miscible fluids flow through a homogeneous porous medium.

3. Similarity Solution

The problem describing the concentration c as a function of time t and position x, as the two miscible fluids flow through a homogeneous porous medium is given by (5).

Since u is the cross-sectional time dependent flow velocity through the porous medium, it is regarded as, for definiteness,

$$u=\frac{1}{\sqrt{t}} \tag{8}$$

From (5), we have

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D_L\frac{\partial^2 c}{\partial x^2} \tag{9}$$

and $c(0,t)=c_0; c(L,t)=c_L$ (where $c_L\neq 1$) and where $D_L=1$

We get,

$$\frac{\partial c}{\partial t}+\frac{1}{\sqrt{t}}\frac{\partial c}{\partial x}=\frac{\partial^2 c}{\partial x^2} \tag{10}$$

Now, we consider all possible groups of infinitesimal transformation that will reduce the equation (10) to an ordinary differential equation. On applying such technique to a given differential equation, it may turn out that for some or all of the groups other than the linear and spiral groups, the boundary condition cannot be transformed although the partial differential equation can be transformed into an ordinary differential equation. For such cases, we are at least assured that the groups of infinitesimal transformations that remain are the groups possible for the given boundary value problems.

We seek a one parameter group of infinitesimal transformation which takes the (x,t,c)-*space* into itself and under which (1) is invariant:

$$G=\begin{cases}\overline{x} &= x+\epsilon X\\ \overline{c} &= c+\epsilon C\\ \overline{t} &= t+\epsilon T\end{cases} \tag{11}$$

where generators X, T and C are functions of x, t and c. Invariance of equation gives,

$$\frac{\partial \overline{c}}{\partial \overline{t}}+\frac{1}{\sqrt{\overline{t}}}\frac{\partial \overline{c}}{\partial \overline{x}}=\frac{\partial^2 \overline{c}}{\partial \overline{x}^2} \tag{12}$$

Applying transformations (11) into the (12) we get the group of infinitesimal transformation explicitly is,

$$\begin{cases}X &= \frac{a_0}{2}x+k\\ T &= a_0t+a_1\\ C &= \left(\frac{a_0}{4}c+a_2\right)x+a_3\end{cases} \tag{13}$$

Thus, the characteristic equations are

$$\frac{dx}{\frac{a_0}{2}x+k}=\frac{dt}{a_0t+a_1}=\frac{du}{\left(\frac{a_0}{4}c+a_2\right)x+a_3} \tag{14}$$

Now, assume that, $a_0=2$ and $k=a_1=a_2=a_3=0$, we have,

$$\frac{dx}{x}=\frac{dt}{2t}=\frac{du}{\frac{cx}{2}} \tag{15}$$

From these, we can have the similarity variable

$$\eta = \frac{x}{\sqrt{t}} \tag{16}$$

and also we have,

$$c = \phi\left(\eta\right) \tag{17}$$

where

$$\eta = \frac{x}{\sqrt{t}}$$

Using (10), (16), (17) to obtain the following ordinary differential equation,

$$\phi''\left(\eta\right) + \left(\frac{\eta}{2} - 1\right)\phi'\left(\eta\right) = 0 \tag{18}$$

This is a second order nonlinear differential equation.

Solution to this equation (18) is given by [9],

$$\phi\left(\eta\right) = c_1 e\sqrt{\pi} erf\left(\frac{\eta}{2} - 1\right) + c_2 \tag{19}$$

$$c = c_1 e\sqrt{\pi} erf\left(\frac{\eta}{2} - 1\right) + c_2 \tag{20}$$

where c_1 and c_2 are constants.

4. Conclusion

Specific problem of the longitudinal dispersion of miscible fluids flow through porous media is discussed under certain assumptions and error function solution is obtained. We have expressed the result in the form which is well suited for meaningful interpretation of the response of the physical phenomenon. Further it is also concluded that the behavior of the concentration in the longitudinal dispersion is oscillatory as well as exponential.

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