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Group theoretic analysis of Rayleigh problem for a non-Newtonian MHD Sisko fluid past semi infinite plate

Research Article

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Abstract: Present investigation is made of MHD of Rayleigh problem past semi-infinite plate impassively set into a motion, moves with constant velocity in non-Newtonian Sisko-fluid of infinite extend. The group theoretic technique is applied to transform present highly non-linear partial differential equation of boundary value type into non-linear ordinary differential equation with appropriate boundary conditions. And then obtained boundary value problem transform into initial value problem using new systematic deductive group method and it has been solved numerically using MAPLE. The results are plotted in graph form.

MSC: 76A05 • 76M55 • 54H15

Keywords: Rayleigh problem • MHD • Sisko fluid • Non-Newtonian

1. Introduction

Recently, there has been considerable interest in the flow behavior of fluids which do not obey NewtonâĂŹs law of viscosity, that is, which is based on the assumption of linear relationship between the stress and the rate of the strain. This is because most of the fluids used in industry do not obey this linear relationship such fluids are characterized as non-Newtonian fluids. A number of empirical models for the relation between the stress and the rate of the strain for various non-Newtonian fluids have been proposed in literature [1–4]. Each model is usually adequate for describing fluids properties in a certain restricted range. The flows of such a non Newtonian fluids present exiting challenge to mathematician, physicists, engineers and numerical analyst. Moreover, the steady of flow of electrically conducting non Newtonian fluids as attracted many scientist and engineers because when external magnetic field is applied to such flow geometric then

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it could influence the flow in a significant way. Specifically, the interest in MHD of non Newtonian fluid flow stems from defect that liquid metals that occurs in nature and industry are electrically conducting. These fluids with finite or infinite electrical conducting have attracted both mathematician and engineering experts.

The differential equations governing the motion of non Newtonian fluids are of higher order boundary layer equation and hence, they are more complicated then second order Navier-Stokes equation. The exact solution of flow equation containing such fluid models is indeed tough. This is because, in addition to the inherent difficulties in the solution of such equation which are normally higher order non linear partial differential equations and moreover, contains non linear stress-strain relationship, that cause extra difficulties.

In the analysis of boundary layer problems, the class of solution known as similarity solutions plays an important role because it is only class of exact solution for the boundary layer equation. For Newtonian case, it is well known that similarity solution exists for the class of bodies known as the Rayleigh problems, which includes many practical geometry. Non-linear relation between the shearing stress and rate of strain, however, placed further restriction on the class of problems which can be solved by similarity transformation. Rossow [5] was probably first to study magnetic Rayleigh problem, where a semi finite plate is given an impulsive motion and there after moves a constant velocity in a Newtonian fluid of infinite extend. He has studied both the case: when transfers magnetic field is fixed to the plate and fixed to the fluids. In 1970, Sapunkov [6] studied non Newtonian flow of electrically conducting fluids. He obtained approximate solution to the problem solved in his paper but only in the special case of very strong or very weak magnetic fields. The solution was obtained only for a power law fluid for $n = 2$.

A theoretical solution for the laminar flow of non-Newtonian Sisko fluid [7]between two parallel circular disks was discussed [8]. They have checked the model experimentally to fit accurately the viscosity data of greases over a very wide range of shear rates. Further they have shown that their model equation reduce to power law equation by equating to zero one of the constant of the model and thus it represent behavior of a large number of non-Newtonian fluids. Molati et al.(2009), have studied unsteady unidirectional flow of incompressible Sisko fluid pass suddenly accelerated plat in the presence of time dependent applied transverse magnetic field, they have used lie symmetry method to drive similarity equations. Darji and Timol (2011) have derived similarity equations for magnetohydrodynamic boundary layer flow of non-Newtonian Sisko fluid past infinite porous rigid surface. They have used deductive group theoretic method to derive similarity equation, which is highly non-linear ODE, further they have solved their similarity equation with respective boundary conditions by MATLAB ODE solver.

The mathematical technique used in the present analysis is the general one parameter group transformation. The group methods, are the class of methods, which lead to reduction of the number of independent variables, where first introduced by Birkhoff [9]. He has applied one parameter linear group of transformation. In 1952, Morgan [10] presented a theory, which has led to improvement over earlier similarity method due to [9]. The main drawback of [10] was that it was depended on the use of an assumed group of transformation and hence it yields somewhat restrictive solutions. This drawback was correctly recognized in [11] and he has proposed the deductive methods of analysis using general group of transformation that leads the family of similarity equations. The method has been applied extensively by various authors.

[12–18] to various engineering boundary value problems.

In this work, we present a general procedure for applying a one parameter general group transformation to the Rayleigh problem for a MHD Sisko fluid. The similarity equations derived are highly non-linear ordinary differential equation of boundary value type. Furthermore, via systematic one parameter deductive group theoretic method, this boundary value type problem transform into initial value problem. The initial value problem is solved numerically for various parameters and Effect of the parameters on the velocity has been studied and the results are plotted.

2. Problem formulation

Let an infinite plate occupy $y = 0$ and Sisko fluid the half-space $y > 0$. The applied magnetic field is $H = (0, H(t), 0)$ and hence MaxwellâĂŹs equation $divH = 0$ is satisfied. The motion in the fluid is induced by a plate for $t > 0$. The constitutive equation of a Sisko fluid is

$$\tau = -pI + \nu \tag{1}$$

$$\nu = \left[a + b \left| \sqrt{\frac{1}{2} tr A_1{}^2} \right|^{n-1} \right] A_1 \tag{2}$$

Where, a and b are the material parameters, p is the pressure, I is the identity tensor, $n > 0$ is a characteristics of the non-Newtonian behavior of the fluid and the first Rivlin-Ericksen tensor is given by

$$A_1 = gradV + (gradV)^T \tag{3}$$

In which V is the velocity and grad is the gradient operator. For unidirectional flow, the velocity is

$$V = (u(t, y), 0, 0) \tag{4}$$

Where, u is the velocity in the x-direction and the magnetic body force F under the assumptions considered in the present analysis is the of the form

$$F = -\sigma\mu^2 H^2(t) V \tag{5}$$

In the above expression, σ is the electrical conductivity of the fluid and μ is the magnetic permeability of the medium. The continuity and momentum equations are

$$divV = 0 \tag{6}$$

$$\rho \frac{du}{dt} = div\tau + F \tag{7}$$

In which ρ is the fluid density and $\frac{d}{dt}$ is the material derivative. Note that (6) is identity satisfied through (4), furthermore equations (4), (5), and (7) in the absence of pressure gradient yield the following equation

$$\rho\frac{\partial u}{\partial t}=A\frac{\partial^2 u}{\partial y^2}+B\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]-\sigma\mu^2 H^2(t)\,u \tag{8}$$

The boundary conditions and initial conditions are

$$\left.\begin{array}{l} u(t,0)=U_0 g(t), \quad t>0 \\ u(t,y)\to 0 \quad as \quad y\to\infty, \quad t>0 \\ u(0,y)=0, \quad y>0 \end{array}\right\} \tag{9}$$

Where U_0 is a characteristic velocity and $g(t)$ as yet an unspecified function. It should be pointed out that for $A=0$, equation (8) deduced to the corresponding equation of power law fluid. For $B=0$, it corresponding to Newtonian fluid. The non dimensional form of the governing problem is,

$$\frac{\partial u}{\partial t}=A\frac{\partial^2 u}{\partial y^2}+L\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]-M^2 H^2(t)\,u \tag{10}$$

$$\left.\begin{array}{l} u(t,0)=g(t), \quad t>0 \\ u(t,y)\to 0 \quad as \quad y\to\infty, \quad t>0 \\ u(0,y)=0, \quad y>0 \end{array}\right\} \tag{11}$$

where $u=u^*U_0$, $\quad y=y^*/\rho U_0$, $\quad t=t^*/\rho U_0^{\,2}$, $\quad L=B\left(\rho U_0^{\,2}\right)^{n-1}$, $\quad M^2=\sigma\mu^2/\rho U_0^{\,2}$ and asterisks have been supposed for simplicity.

3. Method of solution

The method used in this paper is depends on the application of a one-parameter group transformation. Under this transformation the two independent variables will be reduced by one and the boundary value type partial differential equation (10) which has two independent variables y and t transform into boundary value type ordinary differential equation in only one-independent variable, which is similarity equation. The similarity equation governing the flow is non- linear boundary value problem is further transformed to initial value problem.

First, introduced a one-parameter group transformation of the form

$$G:\left\{\begin{array}{l} \bar{y}=\mathrm{c^y}(a)\mathrm{y}+\mathrm{k^y}(a) \\ \bar{t}=\mathrm{c^t}(a)\mathrm{t}+\mathrm{k^t}(a) \\ \bar{u}=\mathrm{c^u}(a)\mathrm{u}+\mathrm{k^u}(a) \\ \bar{H}=\mathrm{c^H}(a)\mathrm{H}+\mathrm{k^H}(a) \end{array}\right. \tag{12}$$

where $'a'$ is the parameter of the transformation, $.c^s$ and k^s are real valued and at least differentiable in their real argument $'a'$.

3.1. Invariance of differential equation under group of transformation

Equation (10) is said to be invariantly transformed under (12), whenever

$$\frac{\partial \bar{u}}{\partial \bar{t}} - A\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - L\frac{\partial}{\partial \bar{y}}\left[\left|\frac{\partial \bar{u}}{\partial \bar{y}}\right|^{n-1}\frac{\partial \bar{u}}{\partial \bar{y}}\right] + M^2\bar{H}(t)\bar{u} = \lambda(a)\left[\frac{\partial u}{\partial t} = A\frac{\partial^2 u}{\partial y^2} + L\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] - M^2H^2u\right] \quad (13)$$

Under the one parameter group transformation (12), equation (13) becomes

$$\begin{aligned}\left(\frac{\mathrm{c^u}}{\mathrm{c^t}}\right)\frac{\partial u}{\partial t} - A\left(\frac{\mathrm{c^u}}{(\mathrm{c^y})^2}\right)\frac{\partial^2 u}{\partial y^2} - nL\left(\frac{(\mathrm{c^u})^n}{(\mathrm{c^y})^{n+1}}\right)\left(\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial^2 u}{\partial y^2} - \left(\mathrm{c^H}\right)^2\mathrm{c^u}M^2\mathrm{H^2u} + \mathrm{R(a)}\\ = \lambda(a)\left[\frac{\partial u}{\partial t} = A\frac{\partial^2 u}{\partial y^2} + L\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] - M^2H^2u\right]\end{aligned} \quad (14)$$

where,

$$R(a) = M^2\left\{\left(\mathrm{c^H}\right)^2\mathrm{k^uH^2} + 2\mathrm{c^Hk^Hc^uuH} + 2\mathrm{c^Hk^Hk^uH} + \left(\mathrm{k^H}\right)^2\mathrm{c^uu} + \left(\mathrm{k^H}\right)^2\mathrm{k^u}\right\} \quad (15)$$

The invariance of equation (14) will result if

$$R(a) = 0 \quad (16)$$

To vanish $R(a)$, put

$$k^H = k^u = 0 \quad (17)$$

and

$$\frac{\mathrm{c^u}}{\mathrm{c^t}} = \frac{\mathrm{c^u}}{(\mathrm{c^y})^2} = \frac{(\mathrm{c^u})^n}{(\mathrm{c^y})^{n+1}} = \left(\mathrm{c^H}\right)^2\mathrm{c^u} = \lambda(a) \quad (18)$$

The invariance of the auxiliary condition (11) implies that

$$\mathrm{c^u} = 1, \quad k^y = k^t = 0 \quad (19)$$

which yields

$$\mathrm{c^y} = \left(\mathrm{c^t}\right)^{\frac{1}{\mathrm{n+1}}}, \quad \mathrm{c^H} = \frac{1}{\sqrt{\mathrm{c^t}}} \quad (20)$$

Finally, we get the one-parameter group G which transforms invariantly, the differential equations (10) and the boundary conditions (11). The group G is of the form,

$$G: \begin{cases} \bar{y} = (\mathrm{c^t})^{\frac{1}{\mathrm{n+1}}}\mathrm{y} \\ \bar{t} = \mathrm{c^t t} \\ \bar{u} = \mathrm{u} \\ \bar{H} = \frac{1}{\sqrt{\mathrm{c^t}}}\mathrm{H} \end{cases} \quad (21)$$

3.2. Complete set of absolute invariant

Now, to transform the problem in the form of an ordinary differential equation (similarity representation) in a single independent variable (similarity variable) via group method, we have to proceed further in our analysis to obtain a complete set of absolute invariants.

If $\eta = \eta(y,t)$ is the absolute invariants of the independent variables, then

$$g_j(y,t;u,H) = F_j(\eta(y,t)), \quad j = 1,2 \tag{22}$$

which are the two absolute invariants corresponding to u and H.

The application of a basic theorem in group theory (see [11]) states that, A function $g_j(y,t;u,H)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation:

$$\sum_{i=1}^{4}(\alpha_i S_i + \beta_i)\frac{\partial g}{\partial S_i} = 0 \tag{23}$$

Where, $S_i = y,t;u,H$

$$\alpha_i = \left.\frac{\partial c^{s_i}}{\partial a}\right|_{a=a^0}, \quad \beta_i = \left.\frac{\partial k^{s_i}}{\partial a}\right|_{a=a^0} \quad i = 1,2,3,4 \tag{24}$$

where a^0 denotes the value of which yield the identity element of the group.

At first, we seek the absolute invariant of the independent variables. Owing to (23), $\eta(y,t)$ is an absolute invariant if it satisfies the first-order linear partial differential equation:

$$(\alpha_1 y + \beta_1)\frac{\partial \eta}{\partial y} + (\alpha_2 t + \beta_2)\frac{\partial \eta}{\partial t} = 0 \tag{25}$$

Since $\beta_1 = \beta_2 = 0$, The solution of (25) is given by,

$$\eta = y\,t^{-\beta} \quad \text{where} \quad \beta = \frac{\alpha_1}{\alpha_2} \tag{26}$$

The second step is to obtain the absolute invariants of the dependent variables u and H.

By a similar analysis, using (21), (22) and (23), we get

$$u(y,t) = F(\eta) \tag{27}$$

And the second absolute invariant is

$$H(t) = H_0(t) \tag{28}$$

3.3. The reduction to ordinary differential equation:

Substituting from equation (26) to (28) into equation (10), we get

$$-\beta\eta\frac{\partial F}{\partial\eta}-\mathrm{A}t^{-2\beta+1}\frac{\partial^2 F}{\partial\eta^2}-nLt^{(1-\beta(n+1))}\left(\frac{\partial F}{\partial\eta}\right)^{n-1}\frac{\partial^2 F}{\partial\eta^2}+M^2tH_0{}^2F=0 \tag{29}$$

For equations (29), to reduce equation in a single variableÎů, it is necessary that the coefficients should be an arbitrary constants or functions of η only.

$$H_0(t)=\frac{E}{\sqrt{t}} \tag{30}$$

$$\beta=\frac{1}{n+1} \tag{31}$$

Hence, equation (29) will be

$$nL\left(\frac{dF}{d\eta}\right)^{n-1}\frac{d^2F}{d\eta^2}+\gamma A\frac{d^2F}{d\eta^2}+\beta\eta\frac{dF}{d\eta}-\mathrm{NF}=0 \tag{32}$$

$$\text{where}\quad N=E^2M^2\quad\text{and}\quad\gamma=t^{-2\beta+1} \tag{33}$$

Under the similarity variable η, the boundary conditions (11) are

$$F(0)=1,\quad F(\infty)=0 \tag{34}$$

4. Transformation of boundary value problem into initial value problem

In this paper, the completely new and general method is now developed to transform boundary value problem into initial value problem, which is based on the concept of continues transformation groups. The method is simple and its application is straightforward.

To introduce the general group theoretic method to transform BVP to IVP, first define a one parameter group of transformation. To apply this method, boundary conditions at initial point must be homogeneous. Otherwise, the method cannot be applied. The difficulty can be overcome by making the change of variable $F=1-f$, equation (32) becomes,

$$nL\left(\frac{df}{d\eta}\right)^{n-1}\frac{d^2f}{d\eta^2}-\gamma A\frac{d^2f}{d\eta^2}-\beta\eta\frac{df}{d\eta}-\mathrm{N}+\mathrm{N}f=0 \tag{35}$$

and boundary conditions (34) become,

$$f(0)=1,\quad f(\infty)=1 \tag{36}$$

Now, define a one parameter group of transformation as follow:

$$\left.\begin{aligned} \bar{\eta} &= H^{\eta}(a)\eta + K^{\eta}(a) \\ \bar{f} &= f^{\eta}(a) f + K^{f}(a) \end{aligned}\right\} \tag{37}$$

where $'a'$ is the parameter of transformation. Now transformation (37) gives

$$\left.\begin{aligned} \eta &= H_1{}^{\eta}(a)\bar{\eta} + H_2{}^{\eta}(a) \\ f &= H_3{}^{f}(a)\bar{f} + H_4{}^{f}(a) \end{aligned}\right\} \tag{38}$$

where $\quad H_1{}^{\eta}(a) = \frac{1}{H^{\eta}(a)}, \quad H_2{}^{\eta}(a) = -\frac{K^{\eta}(a)}{H^{\eta}(a)}, \quad H_3{}^{f}(a) = \frac{1}{H^{f}(a)}, \quad H_4{}^{f}(a) = -\frac{K^{f}(a)}{H^{f}(a)}$

Under transformation (38), boundary condition (36) transform into

$$\left.\begin{aligned} &\bar{\eta} = 0 \quad \Rightarrow \quad \bar{f} = 0 \\ &H_2{}^{\eta}(a) = 0, \quad H_4{}^{f}(a) = 0 \end{aligned}\right\} \tag{39}$$

Under the transformation of (38), equation (35) becomes

$$nL\left(\frac{\left(H_3{}^{f}\right)^{n}}{(H_1{}^{\eta})^{n+1}}\right)\left(\bar{f}'\right)^{n-1}\bar{f}'' - \left(\frac{H_3{}^{f}}{(H_1{}^{\eta})^{2}}\right)\gamma A\bar{f}'' - \left(H_3{}^{f}\right)\beta\bar{\eta}\bar{f}' - \mathrm{N} + \left(H_3{}^{f}\right)\mathrm{N}\bar{f} = 0 \tag{40}$$

Under this transformation equation (40) will remains invariant if

$$\frac{\left(H_3{}^{f}\right)^{n}}{(H_1{}^{\eta})^{n+1}} = \frac{H_3{}^{f}}{(H_1{}^{\eta})^{2}} = \left(H_3{}^{f}\right) \tag{41}$$

Now, equation (40) gives

$$nL\left(\frac{\partial\bar{f}}{\partial\bar{\eta}}\right)^{n-1}\frac{\partial^{2}\bar{f}}{\partial\bar{\eta}^{2}} - \gamma A\frac{\partial^{2}\bar{f}}{\partial\bar{\eta}^{2}} - \beta\bar{\eta}\frac{\partial\bar{f}}{\partial\bar{\eta}} - \mathrm{N} + \mathrm{N}\bar{f} = 0 \tag{42}$$

Now, set missing initial condition is equal to parameter of transformation, that is $f'(0) = a$

Under (38),

$\left(\frac{H_3{}^{f}}{H_1{}^{\eta}}\right)\bar{f}'(0) = a$ which is independent of $'a'$ if $\left(\frac{H_3{}^{f}}{H_1{}^{\eta}}\right) = a$.

Hence,

$$\bar{f}'(0) = 1 \tag{43}$$

From equations (40) and (41), we get the values of $H_3{}^{f} = a^{\frac{n+1}{2}}$

Now, the parameter $'a'$ can be determine from the boundary condition at infinity, we get $H_3{}^{f} = \frac{1}{\bar{f}(\infty)}$

And finally,

$$a = \left(\frac{1}{\bar{f}(\infty)}\right)^{2/n+1} \tag{44}$$

To determine parameter of transformation $'a'$, we have to find $\bar{f}(\infty)$ For that, we have plotted $\bar{f}$ vs $\bar{\eta}$ graph and find constant value at which graph doesnâĂŹt changed which is $\bar{f}(\infty)$.

Thus equation (42) together with boundary condition (39) and (43) constitute initial value problem.

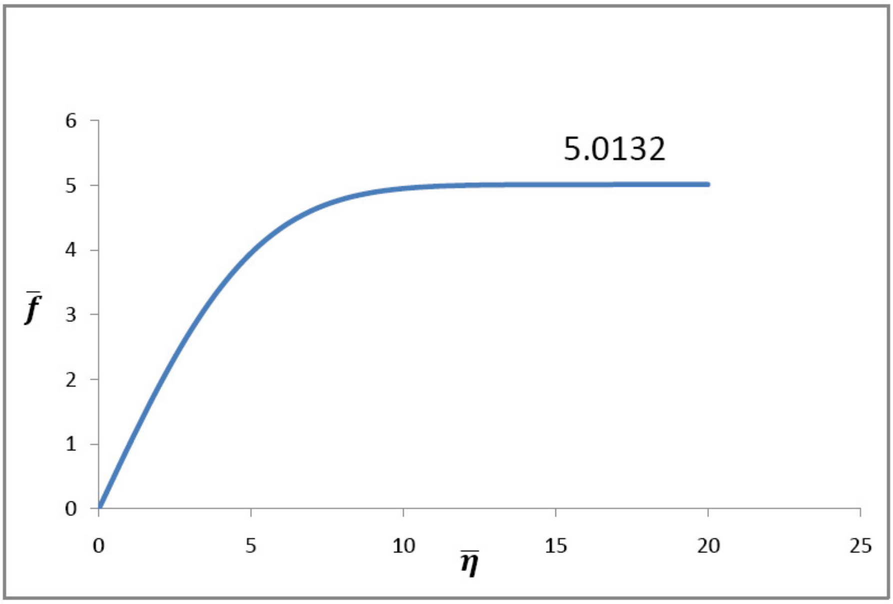

Figure 1. solution of equation (44) for $n = 1,\ N = 0,\ \beta = 0.5,\ L = 2, A = 10$

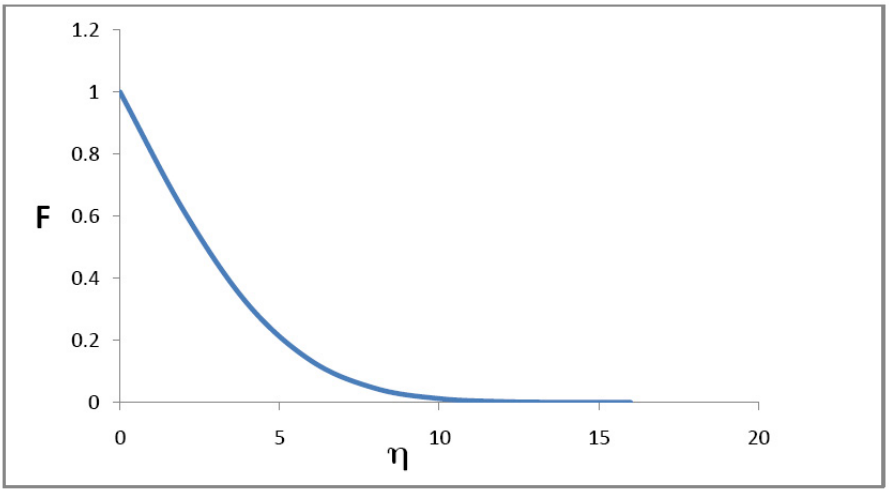

Figure 2. solution of equation (32) for $n = 1,\ N = 0,\ \beta = 0.5,\ L = 2, A = 10$

5. Result and Discussion

Case I : Newtonian fluid without magnetic field. That is $n = 1$ and $N = 0$

The equation (42) will reduce to:

$$L\frac{\partial^2 \bar{f}}{\partial \bar{\eta}^2} - \gamma A\frac{\partial^2 \bar{f}}{\partial \bar{\eta}^2} - \beta\bar{\eta}\frac{\partial \bar{f}}{\partial \bar{\eta}} = 0 \tag{45}$$

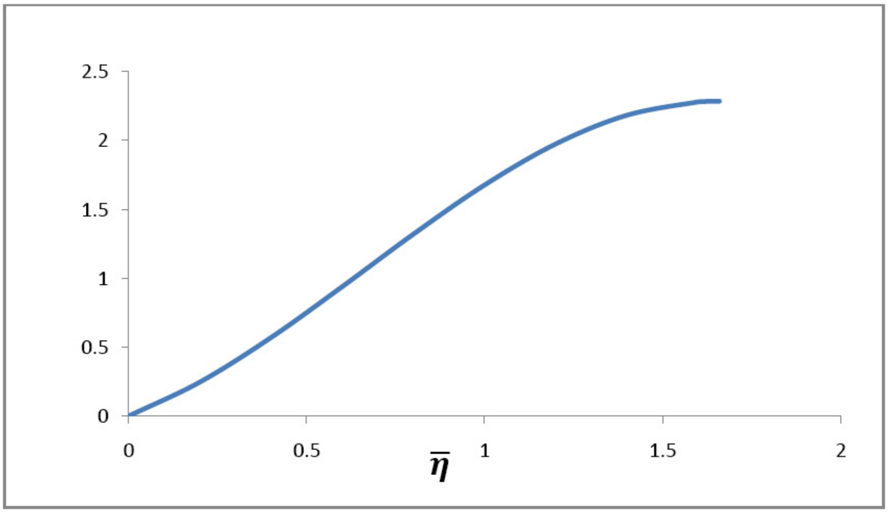

Figure 3. solution of equation (44) for $n = 1$, $N = 5$, $\beta = 0.25$, $L = 7$, $A = 5$

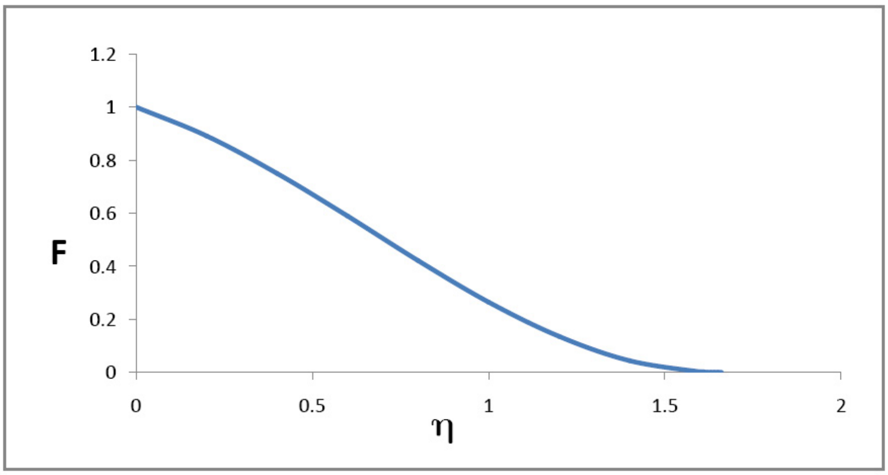

Figure 4. solution of equation (32) for $n = 1$, $N = 5$, $\beta = 0.25$, $L = 7$, $A = 5$

Table 1.

$\bar{\eta}$	$\bar{f}$	η	f	$F = 1 - f$
0	0	0	0	1
2	1.9197	2	0.3829	0.6171
4	3.4224	4	0.6827	0.3173
6	4.3434	6	0.8665	0.1335
8	4.7851	8	0.9546	0.0454
10	4.9509	10	0.9877	0.0123
12	4.9997	12	0.9974	0.0026
14	5.0109	14	0.9996	0.0004
16	5.0129	16	1	0

Table 2.

$\bar{\eta}$	$\bar{f}$	η	f	$F = 1 - f$
0	0	0	0	1
0.2	0.2464	0.2	0.1079	0.8921
0.4	0.5691	0.4	0.2492	0.7508
0.6	0.9380	0.6	0.4108	0.5892
0.8	1.3187	0.8	0.5775	0.4225
1	1.6751	1	0.7336	0.2664
1.2	1.9727	1.2	0.8640	0.1360
1.4	2.1813	1.4	0.9554	0.0446
1.6	2.2781	1.6	0.9978	0.0022
1.61	2.2797	1.61	0.9985	0.0015
1.62	2.2810	1.62	0.9990	0.0010
1.63	2.2830	1.63	0.9999	0.0001
1.64	2.2830	1.64	0.9999	0.0001
1.65	2.2830	1.65	0.9999	0.0001
1.66	2.2830	1.66	0.9999	0.0001

$$\bar{f}(0) = 0, \quad \bar{f}'(0) = 1 \tag{46}$$

Since the differential equation (45) is non-linear. We could not obtain the exact solution of equation (45). However, we have derived numerical solution of this equation for particular values of the parameters $\gamma = 1,\ L = 2,\ \beta = 0.5,\ A = 10$ by using maple ODE solver and its graphical representation is given in Figs. 1-2.

From the Fig. 1 it is seen that as $\bar{\eta}$ approaches infinity $\bar{f}$ approaches the value of 5.0132, which is taken as $\bar{f}(\infty)$. Substitution into equation (44), we get

$$a = \left(\frac{1}{\bar{f}(\infty)}\right)^{2/n+1} = \left(\frac{1}{5.0132}\right) = 0.1995 \tag{47}$$

$$H_3{}^f = a^{\frac{n+1}{2}} = 0.1995 \quad \text{and} \quad H_1{}^\eta = \left(\frac{H_3{}^f}{a}\right) = 1 \tag{48}$$

From these values, finally we calculate the solution of original equation (32) which satisfies boundary conditions too. These solutions are also included in Table 1 and Fig. 2 .

Case II: Newtonian fluid with magnetic field.

The numerical solution for the special case of Newtonian fluid with magnetic field of equation (42) with boundary conditions (39) and (43) for the values $n = 1,\ N = 5,\ \beta = 0.25,\ L = 7,\ A = 5$ shown in Fig 3 . Figure 3 gives the value of $\bar{f}(\infty) = 2.2830$. From this value, we calculate

$$a = 0.4380, \quad H_3{}^f = 0.4380 \quad \text{and} \quad H_1{}^\eta = 1 \tag{49}$$

Table 2 and Fig. 4 represent the solution of original equation (32) which satisfies boundary conditions.

6. Conclusion

In the present paper, the deductive group theoretic method is applied to derive the proper similarity transformation to transform boundary layer equation governing from the flow of Sisko fluids past semi infinite flat plate. The similarity equation for the present flow problem is highly non-linear second order ordinary differential equation of boundary value type. Finally, using deductive group transformation derived boundary value type ordinary differential equation transform into initial value problem. And numerical solutions of this initial value problem are obtained by ODE solver of MAPLE. The main advantage of present analysis is that any non linear boundary value type partial differential equation satisfying group invariance condition can be transform into boundary value type ordinary differential equation which later on can be transform into initial value problem using linear group transformation. Once initial value problem is achieved it can be quite easy to solve using simple numerical techniques.

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