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On invariance analysis of MHD boundary layer equations for non-Newtonian Williamson fluids

Research Article

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Abstract: Group symmetry invariance analysis for magnetohydrodynamics (MHD) boundary layer equations governing the flow of non-Newtonian Williamson fluids past a permeable surface is made. First time the use of modern group theoretic method so-called deductive group symmetry analysis, based on general group transformation reveals possible similarity transformations of the problem. The important conclusion drawn from the present analysis is that for non-Newtonian fluids, similarity solutions exist only for the flows past 90° wedge. Numerical solutions are produced using MATLAB ode solver for the series of parameters. A comparative effect of imposed magnetic field is studied on the flow and it is observed that as magnetic field strength increase the skin-friction at surface sharply decrease.

MSC: 76M76 • 35Q35

Keywords: Group symmetry • MHD • Boundary layer • Non-Newtonian fluid • Similarity solution • Skin-friction

1. Introduction

The subject of boundary layer flows of non-Newtonian fluids has been a topic of investigation since last many decades, and is important due to their occurrence in several manufacturing and engineering processes. Such processes include the aerodynamic extrusion of plastic sheets, hot rolling, glass-fiber production and many more. In these the flows which are governed by the structure of the boundary layer of a non-Newtonian fluid, occur near the under considered surface. Sakiadis [1] was probably the first to study the boundary layer flow over a continuous solid surface moving with constant velocity. The dynamics of the boundary layer flow over stretching surfaces was originated from the pioneering work of Crane [2]. He examined the steady incompressible boundary layer flow of a Newtonian fluid caused by stretching flat sheet which moves in its own plane with linear velocity due to the application of a uniform stress. Since, then many authors have considered various aspects of this problem and obtained similarity solutions. The case of the boundary layer flow of non-Newtonian power-law fluid has been first considered by Schowalter [3]. He derived the equations governing the self- similar flow of a pseudoplastic fluid. Acrivos et al. [4] provided numerical solution to the same problem for both shear thinning and thickening fluids. Kapur and Srivastava (1963) obtained the similarity solutions for non-Newtonian power-law fluid. Similarity solutions for non-Newtonian power-law fluid are also obtained by Banks (1983). Good lists of work on this problem involving non-Newtonian fluids can be found in the literature. See [3–17]
A literature survey infer that a lot of work has been done on the boundary layer flows over planer stretching sheet power law non-Newtonian fluids in various situations; however, rare studies exist regarding other non-Newtonian fluids whose strain-stress relationship defined by arbitrary functional relation.

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In recent years several industries deal with the non-Newtonian fluid flows with magnetic field. In view of this, some researchers [18–25] have presented works on MHD flow in an electrically conducting power law fluid over a stretching sheet. In many practical situations the material moves in a quiescent fluid with the fluid flow induced by the motion of the solid material. Therefore the resulting flow is determined by the boundary layer mechanisms. At this point it is worth to note that most of the work has been done for non-Newtonian power-law fluids, this is because of its mathematical simplicity. However there are empirical non-Newtonian fluid models based on functional relationship between shear stress and rate of the strain are available [25]. Even though considerable progress has been made in our understanding of the flow phenomena, more works are needed to understand the effects of the various parameters involved in the non-Newtonian models and the formulation of an accurate method of analysis for anybody shapes of engineering significance. Further, from these chart, we noticed that all the similarity solutions presented there in are derived either by adopting or by ad hoc assumption on similarity variables. In context of these work it is necessary to develop the systematically group transformation for similarity solution.
Motivated by these considerations, we represent the deductive group symmetry analysis based on general group of transformation. The analysis is applied to the particular problem of boundary layer theory. We investigate the MHD boundary layer flow of a typical non-Newtonian fluid so-called Williamson fluid, although mathematically more complex model. This is chosen mainly due to two reasons. Firstly, it can be deduce from kinetic theory of liquids rather than the empirical relation as in power-law model. Secondly, it correctly reduces to Newtonian behavior for both low and high shear rate. This reason is somewhat opposite to pseudoplastic system whereas the power-law model has infinite effective viscosity for low shear rate and thus limiting its range of applicability.
Mathematically, the strain-stress model for the Williamson fluid can be written as, [25, 26]

$$\tau_{yx} = \left(\frac{A}{B + \frac{\partial u}{\partial y}} + \mu_{\infty} \right) \frac{\partial u}{\partial y} \tag{1}$$

where A, B and μ_{∞} are rheological parameters, different for different fluids. For $A = 0$ Eq.(1) reduce to those of Newtonian fluid. In the analysis of boundary layer problems, the class of solutions known as similarity solutions traditionally plays an important role because it is the only class of exact solutions for the boundary layer equations. For Newtonian fluids, it is well known that similarity solutions exist for the class of bodies known as the Falkner-Skan problems, which includes much practical geometry. The non-linear relation between the rate of shearing stress and the rate of strain in Eq.(1), however, places further restriction on the class of problems which can be solved by similarity transformations.
In present work first time we systematically search the group of transformations by admitting the deductive group symmetry technique and then the transformed ordinary differential equation is numerically solved by Keller-Box method. Present group symmetry analysis warrants the body shape that mentioned in Hansen and Na (1968), which found that for the boundary layer flows of non-Newtonian fluids, similarity solution exists only for the flow past wedge, as shown in Fig. 1.

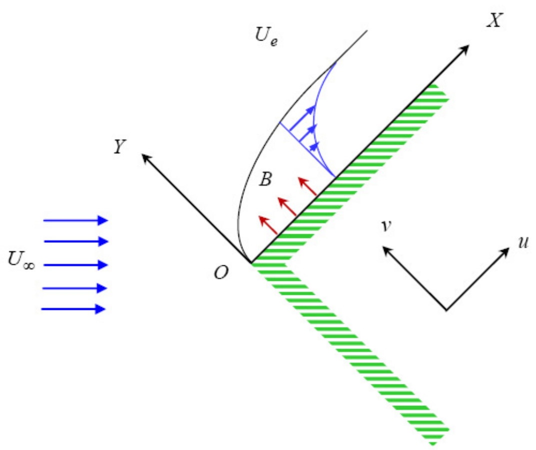

Fig. 1. Schematic of flow past 90° wedge

2. Problem formulation

Let us consider two-dimensional laminar flow of a steady incompressible non-Newtonian Prandtl-Eyring fluid past a permeable surface. The origin of the stationary Cartesian coordinate system is located at the leading edge of the surface. The X-axis is along the surface and Y-axis is taken normal to the surface. The transverse electrically conducting variable magnetic field of the strength $B(x)$ is applied normal to the X-axis. It is assumed that the magnetic Reynolds number Re_m is very small; i.e. $Re_m = \mu_0 \sigma L \ll 1$, where μ_0 is the magnetic permeability, L is the reference length and σ is the electric conductivity. We neglect the induced magnetic field, which is small in comparison with the applied magnetic field. Using boundary layer approximations the appropriate governing equations of continuity and momentum for Williamson fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} + U_e\frac{dU_e}{dx} - \frac{\sigma B^2(x)}{\rho}u \tag{3}$$

where u and v are velocity components along X-axis and Y-axis respectively, ρ is density, $U_e(x)$ is the velocity at the edge of boundary layer, τ_{yx} the extra stress tensor given by (1).Together with boundary conditions,

$$u(x,0) = v(x,0) = 0, \quad u(x,\infty) = U_e(x) \tag{4}$$

The above equations can made dimensionless using the following quantities,

$$\left.\begin{array}{llll} x^* = \dfrac{x}{L}, & y^* = \dfrac{y}{L}\sqrt{\text{Re}}, & u^* = \dfrac{u}{U_\infty}, & v^* = \dfrac{v}{U_\infty}\sqrt{\text{Re}}, \\ U^*_{e^*} = \dfrac{U_e}{U_\infty}, & \tau^*_{y^*x^*} = \dfrac{\tau_{yx}}{\rho U_\infty^2}\sqrt{\text{Re}}, & \text{Re} = \dfrac{U_\infty L}{\nu} & \end{array}\right\} \tag{5}$$

where L is the reference length, $U_\infty(x)$ is the velocity of main stream, ν is the kinematic viscosity, Re is the Reynolds number.
And a non-dimensional stream function $\psi^*(x^*, y^*)$, such that

$$u^* = \frac{\partial \psi^*}{\partial y^*}, \quad v^* = -\frac{\partial \psi^*}{\partial x^*}$$

Substitute the values in (2) to (3) and dropping the asterisks (for simplicity), we get

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial \tau_{yx}}{\partial y} + U_e\frac{dU_e}{dx} - bB^2(x)\frac{\partial \psi}{\partial y} \tag{6}$$

in which $b = \sigma/\rho U_\infty$, together with the boundary conditions

$$\frac{\partial \psi}{\partial y}(x,0) = \frac{\partial \psi}{\partial x}(x,0) = 0, \quad \frac{\partial \psi}{\partial y}(x,\infty) = U_e(x) \tag{7}$$

3. Group systematic invariance analysis

The procedure is initiated with the group C_G, a class of transformation of one-parameter $'\varepsilon'$ of the form:

$$C_G: \quad \bar{s} = \mathbb{D}^s(\varepsilon)\bar{s} + \mathbb{T}^s(\varepsilon) \tag{8}$$

where s stands for $x, y, \psi, U_e, \tau_{yx}, B$ whereas $\mathbb{D}'s$ and $\mathbb{T}'s$ are real-valued and are at least differentiable in the real argument ε.
To transform the differential equation (6), transformations of the derivatives of ψ are obtained from C_G via chain-rule operations:

$$\left.\begin{array}{l} \bar{s}_{\bar{i}} = \left(\dfrac{\mathbb{D}^s}{\mathbb{D}^i}\right)s_i \\ \bar{s}_{\bar{i}\bar{j}} = \left(\dfrac{\mathbb{D}^s}{\mathbb{D}^i\mathbb{D}^j}\right)s_{ij} \end{array}\right\}, \quad s = \psi, U_e, \tau_{yx}, B \quad i, j = x, y \tag{9}$$

Eq. (6) is said to be invariantly transformed, for some function $\chi(\varepsilon)$ whenever,

$$\frac{\partial \bar{\psi}}{\partial \bar{y}}\frac{\partial^2 \bar{\psi}}{\partial \bar{y}\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}}\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} - \frac{\partial \bar{\tau}_{\bar{y}\bar{x}}}{\partial \bar{y}} - \bar{U}_{\bar{e}}\frac{d\bar{U}_{\bar{e}}}{d\bar{x}} + b\bar{B}^2(\bar{x})\frac{\partial \bar{\psi}}{\partial \bar{y}}$$
$$= \chi(\varepsilon)\left[\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \tau_{yx}}{\partial y} - U_e\frac{dU_e}{dx} + bB^2(x)\frac{\partial \psi}{\partial y}\right]$$

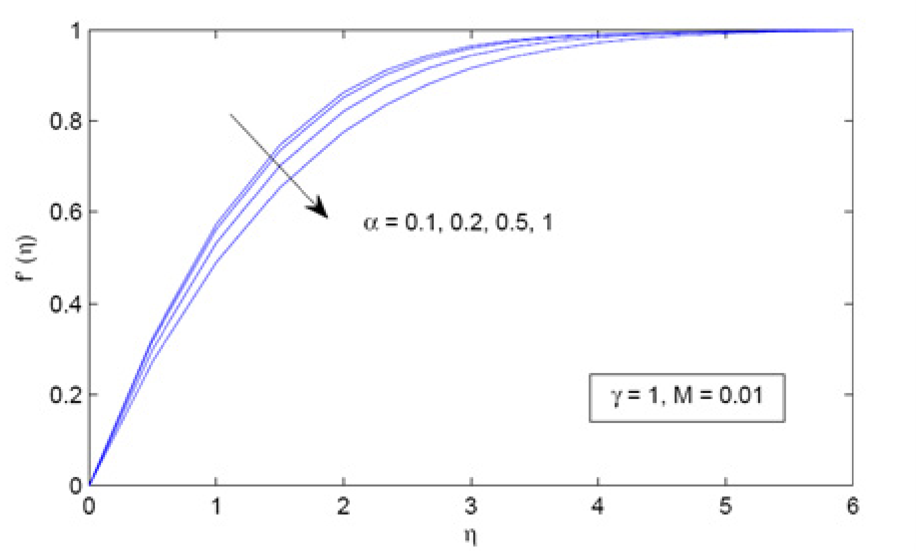

Fig. 2. Effect of α on similarity function related to velocity along x direction

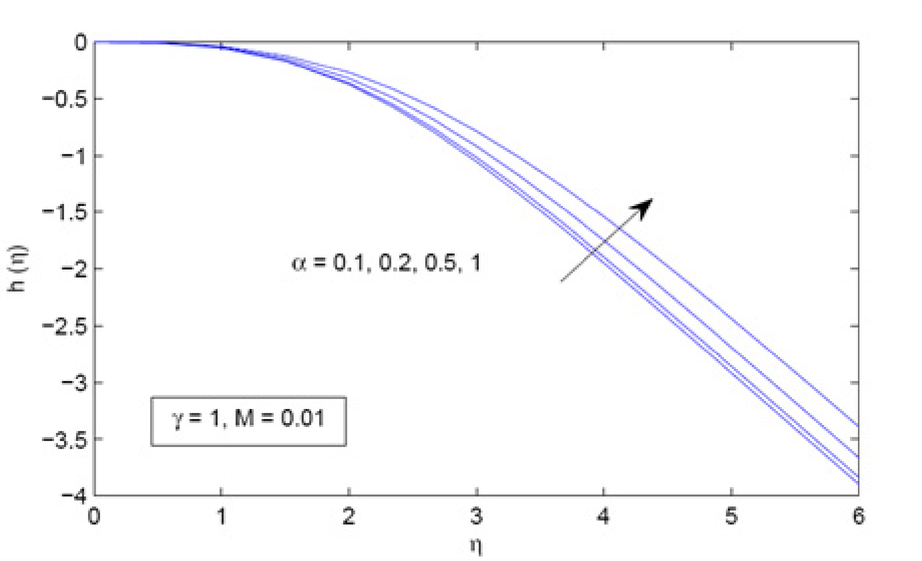

Fig. 3. Effect of α on similarity function related to velocity along y direction

Substituting the values from (8) and (9) in above equation, yields

$$\begin{aligned}&\frac{(\mathbb{D}^{\psi})^2}{\mathbb{D}^{x}(\mathbb{D}^{y})^2}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right]-\frac{\mathbb{D}^{\tau_{yx}}}{\mathbb{D}^{y}}\frac{\partial\tau_{yx}}{\partial y}-\left(\mathbb{D}^{U_e}U_e+\mathbb{T}^{U_e}\right)\frac{\mathbb{D}^{U_e}}{\mathbb{D}^{x}}\frac{dU_e}{dx}\\&+b\left(\mathbb{D}^{B}B+\mathbb{T}^{B}\right)^2\frac{\mathbb{D}^{\psi}}{\mathbb{D}^{y}}\frac{\partial\psi}{\partial y}=\chi(\varepsilon)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial\tau_{yx}}{\partial y}-U_e\frac{dU_e}{dx}+bB^2\frac{\partial\psi}{\partial y}\right]\end{aligned} \tag{10}$$

The invariance of (10) together with boundary conditions (7), implies that

$$\left.\begin{aligned}&\mathbb{T}^{U_e}=\mathbb{T}^{\tau_{yx}}=\mathbb{T}^{y}=\mathbb{T}^{\psi}=\mathbb{T}^{B}=0\\&\frac{(\mathbb{D}^{\psi})^2}{\mathbb{D}^{x}(\mathbb{D}^{y})^2}=\frac{\mathbb{D}^{\tau_{yx}}}{\mathbb{D}^{y}}=\frac{(\mathbb{D}^{U_e})^2}{\mathbb{D}^{x}}=\frac{(\mathbb{D}^{B})^2\mathbb{D}^{\psi}}{\mathbb{D}^{y}}=\chi(\varepsilon)\end{aligned}\right\} \tag{11}$$

These yields,

$$\mathbb{D}^{x}=(\mathbb{D}^{y})^3,\quad \mathbb{D}^{\psi}=(\mathbb{D}^{y})^2,\quad \mathbb{D}^{U_e}=\mathbb{D}^{y},\quad \mathbb{D}^{\tau_{yx}}=1\quad \mathbb{D}^{B}=\frac{1}{\mathbb{D}^{y}} \tag{12}$$

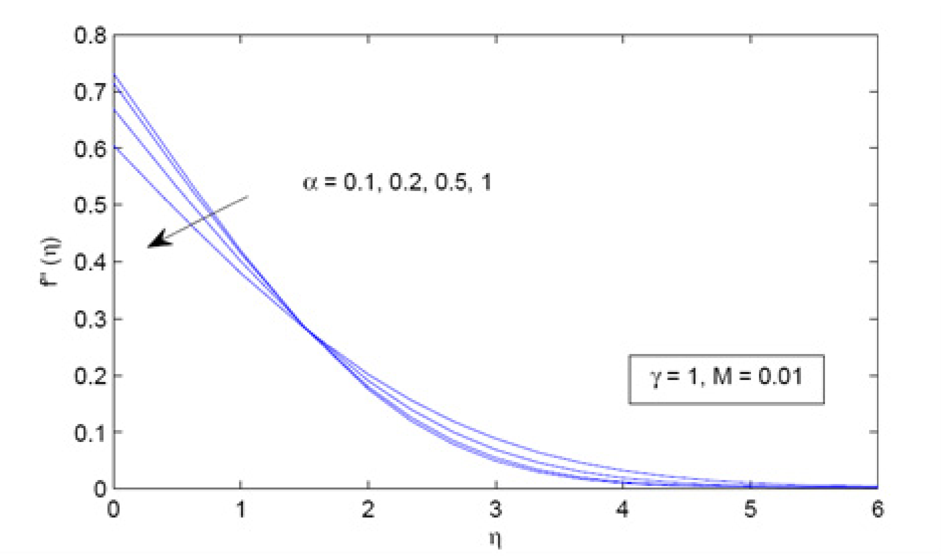

Fig. 4. Effect of α on local skin-friction

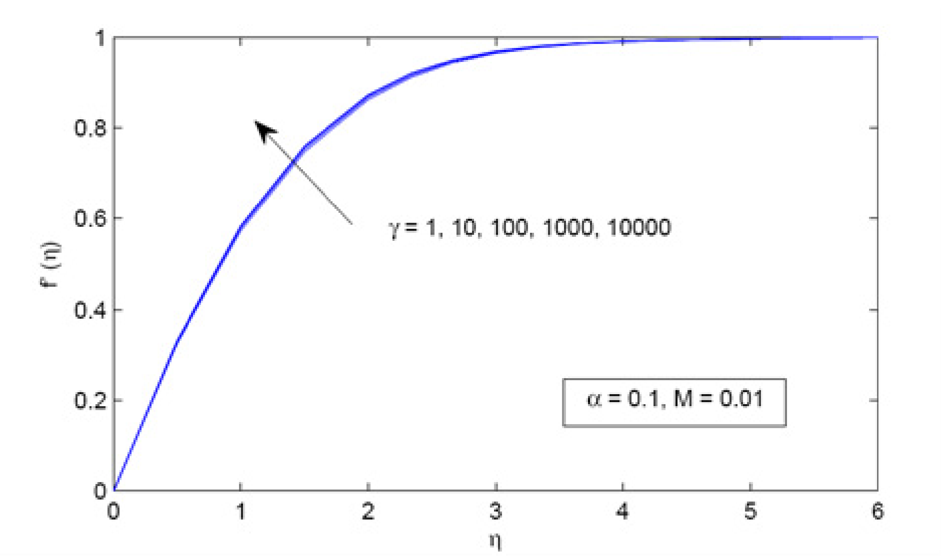

Fig. 5. Effect of γ on similarity function related to velocity along x direction

The one-parameter sub-group G of C_G, which transforms invariantly the governing equations with the auxiliary conditions is

$$G: \begin{cases} H: \quad \{\bar{x} = (\mathbb{D}^y)^3 x + \mathbb{T}^x, \; \bar{y} = \mathbb{D}^y y \\ \bar{\psi} = (\mathbb{D}^y)^2 \psi, \quad \bar{U}_{\bar{e}} = \mathbb{D}^y U_e \\ \bar{\tau}_{\bar{y}\bar{x}} = \tau_{yx}, \quad \bar{B} = \dfrac{1}{\mathbb{D}^y} B \end{cases} \tag{13}$$

3.1. The complete set of absolute invariants

For a one-parameter group, if $\eta = \eta(x, y)$ is the absolute invariant of the independent variables then, the four absolute invariants of for the dependent variables ψ, U_e, B, τ_{yx} are given by

$$F_j(x, y, \psi, U_e, \tau_{yx}, B) = f_j(\eta), \quad j = 1, 2, 3, 4 \tag{14}$$

and can be obtained by following first-order linear partial differential equation: See [27–29]

$$\sum_{i=1}^{6} (\alpha_i s_i + \beta_i) \frac{\partial F}{\partial s_i} = 0, \quad s_i = x, y, \psi, U_e, \tau_{yx}, B \tag{15}$$

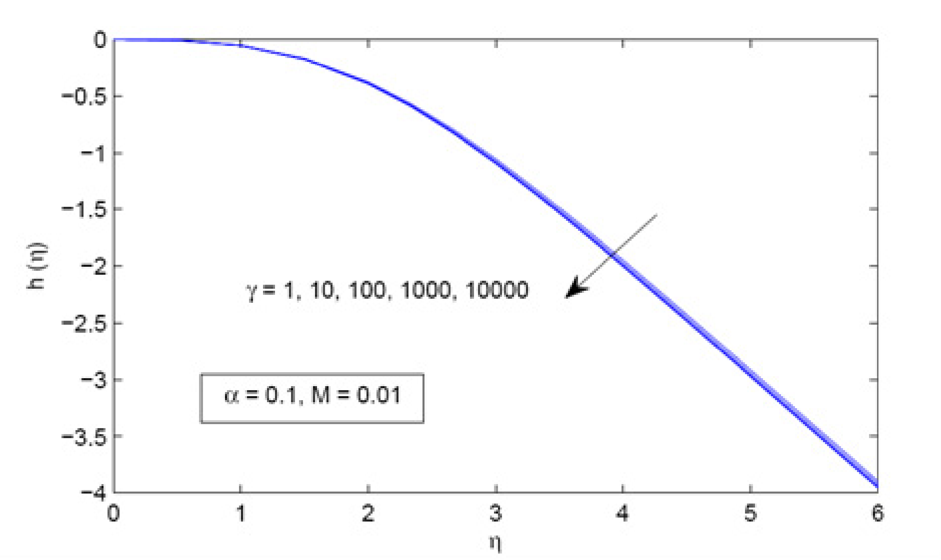

Fig. 6. Effect of γ on similarity function related to velocity along y direction

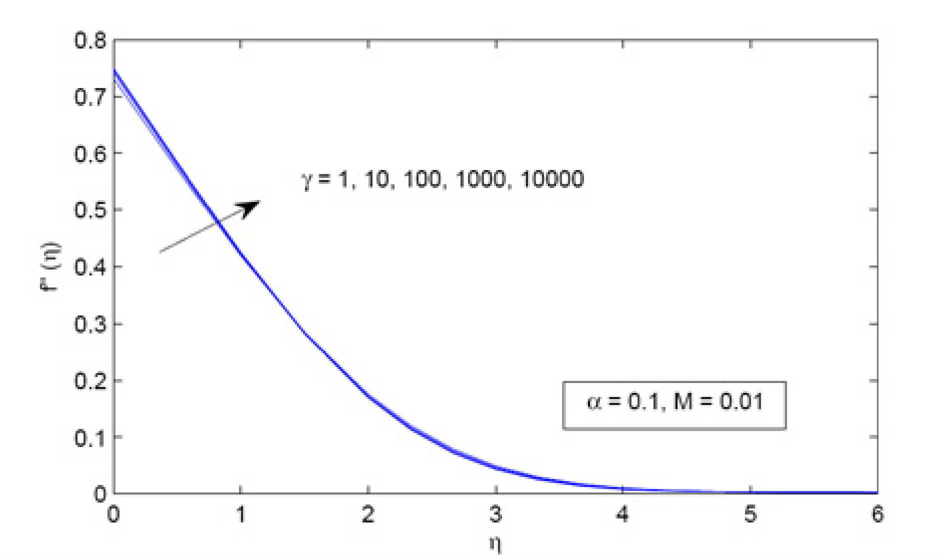

Fig. 7. Effect of γ on local skin-friction

where

$$\alpha_i = \frac{\partial \mathbb{D}^i}{\partial \varepsilon}\Big|_{\varepsilon=\varepsilon^o} \quad \text{and} \quad \beta_i = \frac{\partial \mathbb{T}^i}{\partial \varepsilon}\Big|_{\varepsilon=\varepsilon^o} \quad i = 1, ...6 \tag{16}$$

and $'\varepsilon^{o}'$ denotes the value of parameter ε which yields the identity element of the group G.
The absolute invariant of independent variable owing (15) is $\eta = \eta(x, y)$ if it will satisfies the first order linear partial differential equation

$$(\alpha_1 x + \beta_1)\frac{\partial \eta}{\partial t} + (\alpha_2 y + \beta_2)\frac{\partial \eta}{\partial y} = 0. \tag{17}$$

Using the definition of $\alpha' s$ and $\beta' s$

$$(x + \beta)\frac{\partial \eta}{\partial t} + \frac{y}{3}\frac{\partial \eta}{\partial y} = 0 \tag{18}$$

where $\beta = \beta_1 / 3\alpha_1$

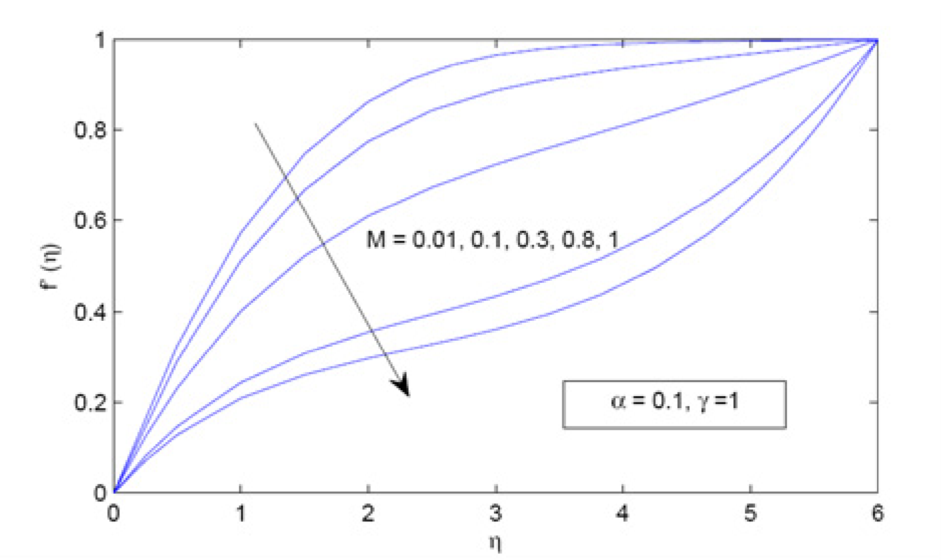

Fig. 8. Effect of magnetic field on similarity function related to velocity along x direction

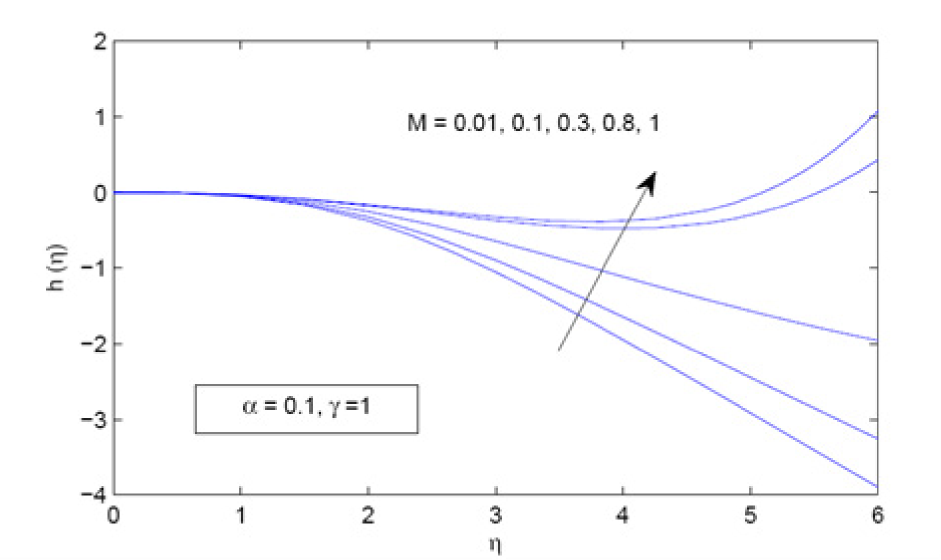

Fig. 9. Effect of magnetic field on similarity function related to velocity along y direction

The characteristic equation of (18) is

$$\frac{dx}{(x+\beta)} = \frac{3dy}{y} = \frac{d\eta}{0} \tag{19}$$

Applying the variable separable method the absolute invariant of independent variables is

$$\eta(x,y) = y(x+\beta)^{-1/3} \tag{20}$$

Similarly the absolute invariants for dependent variables owing (15), one can derive, are:

$$\left.\begin{array}{ll} f_1(\eta) = \psi(x+\beta)^{-2/3}, & f_2(\eta) = U_e(x+\beta)^{-1/3} \\ f_3(\eta) = \tau_{yx}, & f_4(\eta) = B(x+\beta)^{1/3} \end{array}\right\} \tag{21}$$

Since $U_e(x)$ and $B(x)$ are functions of x only, $f_2(\eta)$ and $f_4(\eta)$ must be constants say U_0 and B_0 respectively. The group transformation of absolute invariants is

$$\left.\begin{array}{ll} \psi = (x+\beta)^{2/3} f(\eta), & U_e = U_0(x+\beta)^{1/3} \\ \tau_{yx} = g(\eta), & B = B_0(x+\beta)^{-1/3} \end{array}\right\} \tag{22}$$

where $f(\eta) = f_1(\eta), \quad g(\eta) = f_3(\eta)$

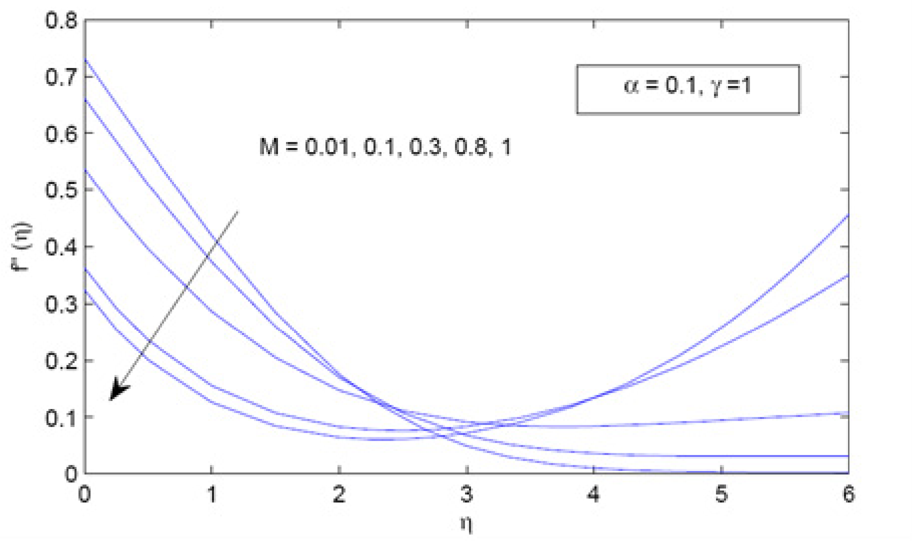

Fig. 10. Effect of magnetic field on local shear-stress

3.2. Reduction to ordinary differential equation

Substituting the values of derivatives from (22) in (6), one can reduce the following differential equation:

$$\left(f'\right)^2 - 2ff'' - 3g' - U_0^2 + 3Mf' = 0 \tag{23}$$

where $M = \sigma B_0^2/\rho U_\infty$ is the magnetic parameter and $g\left(\eta\right)$ is similarity variable related to non-dimensional strain-stress relation (1), whence

$$g'\left(\eta\right) = f''' + \frac{\alpha}{\left\{1+\sqrt{\gamma}f''\right\}^2} f''', \text{where} \alpha = \frac{A}{\mu_\infty B}, \ \gamma = \frac{\rho U_\infty^3}{\mu L B^2}$$

Substituting the value in (23), we get

$$f''' = \frac{\frac{1}{3}\left\{\left(f'\right)^2 - 2ff'' - U_0^2 + 3Mf'\right\}\left\{1+\sqrt{\gamma}f''\right\}^2}{\alpha + \left\{1+\sqrt{\gamma}f''\right\}^2} \tag{24}$$

in which $\alpha's$ and $\beta's$ are dimensionless number and referred as flow parameters and primes denote derivative with respect to similarity variable η.
The boundary conditions (7) transform to

$$f(0) = f'(0) = 0, \quad f'(\infty) = U_0 \tag{25}$$

Further the expression of local skin-friction coefficient C_f is

$$\frac{1}{2}\sqrt{\text{Re}}C_f \equiv \tau_{yx}\Big|_{y=0} = f''(0) + \frac{\alpha}{1+\sqrt{\gamma}f''(0)}$$

4. Numerical solution of problem

The transformed highly non-linear ordinary differential equation (24) subject to the boundary conditions (25) is solved numerically by using Keller-Box method [30, 31]. This method is second order accurate and allows uniform and non-uniform grid size. It is worth mentioning that a uniform grid of $\Delta\eta = 0.01$ was found to be satisfactory for a convergence criterion of 10^{-6} in all most all the cases. The solutions are presented graphically for various flow parameters as show in Figs. 2-10.

5. Results and discussion

In presence of magnetic field M, the effect of flow parameters α and γ on similarity functions $f'(\eta)$related to velocity along x direction, $h(\eta) = -2f(\eta) + \eta f'(\eta)$ related to velocity along y direction and $f''(\eta)$ related to local skin-friction are displayed graphically in the Figs. 2-10.

These figures depict the influence of flow parameters under effect of magnetic field on the velocity component along the wedge of surface, so-called the wedge velocity (Figs. 2, 5, 8), along the normal velocity component to the surface ((Figs. 3, 6, 9) and on the local shear-stress, hence skin friction at the wedge (Figs. 4, 7, 10).

Figs. 2 shows that boundary layer increase as the flow parameter α increase by controlling the flow parameter γ and M. There is increase in normal component of velocity which is negative over whole domain for an increase in α as can be seen in Figs. 3. Figs. 4 depicts that as α increase the local shear-stress $f''(0)$ decrease and hence the local skin-friction C_f increase. A reverse trend can be observed for the parameter γ (Figs. 5-7).

Observe from the definition of dimensionless parameter as rheological parameter A increase that is non-Newtonian behaviour increase the parameter α increase It is therefore natural for the boundary layer to become wider. Also the parameter γ decreases as viscosity μ increase. Hence the boundary layer to become wider for a higher viscosity value. More energy will be dissipated near the boundary which in turn causes the outer velocity to resume higher from the boundary.

Also The rheological parameter B and μ_∞ has an inverse relation with dimensionless parameter. An increase in one of the rheological parameter means a decrease in α and γ. Figs. 8-10 represent the effect of magnetic parameter M on wedge velocity, normal velocity and local skin-friction respectively. It shows that increase in M causes the boundary layers to thicken. It is worth to note that all solutions have derived for non-dimensional quantities and hence these results are applicable for all types of under considered non-Newtonian fluids.

6. Conclusion

Using deductive group symmetry method the similarity transformations are derived first time to covert the system of partial differential equation to system of ordinary differential equation. Reduced system is numerically solved using MATLAB. Effects of rheological parameters on the boundary layers are discussed in detail. It is found that change in all the dimensionless parameters and rheological parameters causes the boundary layers thickness.

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