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New exact solutions for the classical Drinfel'd-Sokolov-Wilson equation using the first integral method

Research Article

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Abstract: In this paper, we have established new exact analytical solutions for the classical nonlinear Drinfel'd-Sokolov-Wilson equation (DSWE) by using the first integral method. Many periodic, solitonic, and rational solutions have been found. The results revealed remarkable relations of solitary pattern, periodic solutions or solitons. It is shown that this method is effective and direct one, based on the ring theory of commutative algebra.

MSC: 83C15 • 31B10

Keywords: Exact solutions • Drinfel'd-Sokolov-Wilson equation • First integral method

1. Introduction

It is well known that nonlinear evolution equations are widely used to describe complex phenomena in various fields of science, such as fluid mechanics, plasma physics, optical fibres, mathematical biology, quantum mechanics, chemical engineering, geophysics, meteorology, electromagnetics, ... etc. By the aid of exact solutions, when they exist, the phenomena modeled by these nonlinear evolution equations can be better understood. Therefore, seeking exact solutions of the nonlinear evolution equations are very important and significant in the study of nonlinear physical phenomena. Considerable efforts have been made by many mathematicians and physical scientists to obtain exact solutions of such nonlinear evolution equations and numbers of powerful and efficient methods have been developed by those authors such as the Hirota's bilinear transformation method, the tanh-function method, the extended tanh-method, the Exp-function method, the Adomian decomposition method, the F-expansion method, the Jacobi elliptic function method, Modified Exp-function method, the (G'/G)-expansion method, Weierstrass elliptic function method, homotopy perturbation method, He's polynomial method, and so on [1–6].

On the other hand, the first integral method is a powerful solution method for the computation of exact traveling wave solutions. This method is one of the most important direct and effective algebraic methods for finding exact solutions of nonlinear partial differential equations [7–9]. Different from other traditional methods, the first integral method has many advantages, which is the avoidance of a great deal of complicated and tedious calculations resulting in more exact and explicit traveling solitary solutions with high accuracy [10–12]. This method is reliable, effective, precise and does not require complicated and tedious computations. The main idea of the first integral method is to find the integrals of nonlinear differential equations in polynomial forms. Taking the polynomials with unknown coefficients into account, the method provides exact and explicit solutions. The first integral method is

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widely used by many researchers [13–16] and the references cited therein. Taghizadeh et al. [17–19] proposed the first integral method to solve the modified KdV–KP equation and the Burgers–KP equation. The method was also utilized to construct exact solutions of the nonlinear Schrödinger equation. Taghizadeh and Mirzazadeh [20] used the first integral method to obtain the exact solutions of some complex nonlinear partial differential equations and Konopelchenko-Dubrovsky equation. Moosaei et al. [21] solved the perturbed nonlinear Schrödinger equation with Kerr law nonlinearity by using the first integral method. Recently, it was successfully used for constructing the exact solutions of the Eckhaus equation [22].

In this paper we are going to apply the first integral method on the classical nonlinear Drinfel'd-Sokolov-Wilson equation [23]

$$\left.\begin{array}{l} u_t + 3vv_t = 0 \\ v_t + 2uv_x + u_x v + 2v_{xxx} = 0 \end{array}\right\} \tag{1}$$

The classical nonlinear Drinfel'd–Sokolov–Wilson equation has been studied by several authors [24–26]. Simultaneously, several other authors also study the coupled Drinfel'd–Sokolov–Wilson equation [4, 27, 28]). Here, using the first integral method to obtain the exact solutions of the classical nonlinear Drinfel'd-Sokolov-Wilson equation, and we have obtained new exact solutions of this equation which has not obtained earlier in the literature.

The objective of this article is to apply the first integral method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the classical nonlinear Drinfel'd-Sokolov-Wilson equation. The article is prepared as follows: In Section 2, the first integral method is discussed; In Section 3, we exert this method to the nonlinear evolution equation pointed out above to obtain new exact solutions of this equation, and in Section 4 conclusions are given.

2. The first integral method

Consider the nonlinear partial differential equation in the from

$$F\left(u, u_x, u_t, u_{xx}, u_{tt,} u_{xt}, \ldots\right) = 0 \tag{2}$$

where $u = u(x,t)$ is a solution of the nonlinear partial differential Eq. (2). We use the transformation

$$u(x,t) = f(\xi) \tag{3}$$

where $\xi = x + \lambda t$. This enables us to use the following changes

$$\frac{\partial}{\partial \mathrm{t}}(\ldots) = \lambda \frac{\partial}{\partial \xi}(\ldots), \frac{\partial}{\partial \mathrm{x}} = \frac{\partial}{\partial \xi}(\ldots), \frac{\partial^2}{\partial \mathrm{x}^2}(\ldots) = \frac{\partial^2}{\partial \xi^2}(\ldots) \tag{4}$$

Using Eq. (4) to transfer the nonlinear partial differential equation (2) to a nonlinear ordinary differential equation

$$G\left[\mathrm{f}(\xi), \frac{\partial \mathrm{f}(\xi)}{\partial \xi}, \frac{\partial^2 \mathrm{f}(\xi)}{\partial \xi^2}, \ldots\right] = 0 \tag{5}$$

Next, we introduce a new independent variable

$$\mathrm{X}(\xi) = \mathrm{f}(\xi), \quad \mathrm{Y} = \frac{\partial \mathrm{f}(\xi)}{\partial \xi} \tag{6}$$

which leads to a system of nonlinear ordinary differential equations

$$\frac{\partial \mathrm{X}(\xi)}{\partial \xi} = \mathrm{Y}(\xi), \quad \frac{\partial \mathrm{Y}(\xi)}{\partial \xi} = \mathrm{F}\left[\mathrm{X}(\xi), \mathrm{Y}(\xi)\right] \tag{7}$$

By the qualitative theory of ordinary differential equations [29], if we can find the integrals to Eqs. (7) under the same conditions, then the general solutions to Eqs. (7) can be solved directly. However, in general, it is really difficult for us to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there logical way for telling us what these first integrals are. We will apply the Division Theorem [30] to obtain one first integral to Eqs. (7) which reduces Eq. (5) to a first order integrable ordinary differential equation. An exact solution to Eq. (2) is then obtained by solving this equation. Now, let us recall the following Division theorem:

Suppose that $P(\omega, z)$ and $Q(\omega, z)$ are polynomials in $C(\omega, z)$, and $P(\omega, z)$ is irreducible in $C(\omega, z)$. If $Q(\omega, z)$ vanishes at all zero points of $P(\omega, z)$, then there exists a polynomial $G(\omega, z)$ in $C(\omega, z)$ such that

$$Q(\omega, Z) = P(\omega, Z)\, G''(\omega, Z)$$

3. Exact solutions for Drinfel'd–Sokolov–Wilson equation

In this section, we study the Drinfel'd–Sokolov–Wilson equation by using the transformation

$$u(x,t)=f(\xi), \quad v(x,t)=g(\xi), \quad \xi=x-\alpha t \tag{8}$$

where Îś is arbitrary constant, and $f(\xi), g(\xi)$ are arbitrary functions.
Substituting from Eq. (8) into Eq. (1), we obtain the following system of ordinary differential equations

$$f'+3gg'=0 \text{ which by integration gives } f=-\frac{3}{2}g^2 \tag{9}$$

$$-\alpha g'+2fg'+f'g+2g'''=0 \tag{10}$$

Substituting from Eq. (9) into Eq. (10), we get

$$-\alpha g'-6g^2g'+2g'''=0 \tag{11}$$

Integrate Eq. (11) with respect to ξ, we have

$$g''=\frac{\alpha}{2}g+g^3+\frac{c}{2} \tag{12}$$

where c is an integration constant. Using Eq. (6), we get

$$X(\xi)=Y(\xi) \tag{13}$$

$$Y(\xi)=\frac{\alpha}{2}X+X^3+\frac{c}{2} \tag{14}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of Eqs. (13) and (14), and

$$Q(X,Y)=\sum_{i=0}^{m}a_i(X)Y^i=0$$

is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$Q[X(\xi),Y(\xi)]=\sum_{i=0}^{m}a_i[X(\xi)]Y^i(\xi)=0 \tag{15}$$

where $a_i(X), \quad i=0,1,\cdots,m$ are polynomials of X and $a_m(X)\neq 0$. Eq. (15) is called the first integral to Eqs. (13) and (14). Due to the Division's Theorem, there exists a polynomial $g(X)+h(X)Y$, in the complex domain $C(X,Y)$ such that

$$\frac{dQ}{d\xi}=\frac{dQ}{dX}\frac{dX}{d\xi}+\frac{dQ}{dY}\frac{dY}{d\xi}=\left[g(X)+h(X)Y\right]\sum_{i=0}^{m}a_i(X)Y^i \tag{16}$$

In this paper, we take two different cases, assuming that $m=1$ and $m=2$ in Eq.(15).

3.1. Case (1)

Suppose that $m=1$, by comparing with the coefficients of $Y^i(i=2,1,0)$ on both sides of (16), we have

$$a_1(X)=h(X)a_1(X) \tag{17}$$

$$a_0(X)=g(X)a_1(X)+h(X)a_0(X) \tag{18}$$

$$a_1(X)\left[\frac{\alpha}{2}X+X^3+\frac{c}{2}\right]=g(X)a_0(X) \tag{19}$$

Since $a_1(X)$, $i=0,1$ are polynomials, then from Eq.(17) we deduce that $a_1(X)$, is constant and $h(X)=0$. For simplicity, we take $a_1(X)=1$, and balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $deg[g(X)]=1$ only. Suppose that $g(X)=AX+B$, then from Eq.(18), we find that $a_0(X)$ can be expressed in the form

$$a_0(X)=\frac{A}{2}X^2+BX+A \tag{20}$$

where A and B are arbitrary constants, and A_1 is an arbitrary integration constant to be determined. Substituting $a_0(X)$ and $g(X)$ into Eq.(19) and setting all coefficients of X powers to be zero, then we obtain system of nonlinear algebraic equations and by solving it, we get

$$A=\pm\sqrt{2}, B=0, \quad A_1=\pm\frac{\alpha}{2\sqrt{2}}, \quad c=0 \tag{21}$$

Using the conditions (21) in Eq.(15), we obtain

$$Y(\xi)=\pm\frac{1}{\sqrt{2}}\left[X^2+\frac{\alpha}{2}\right] \tag{22}$$

Combining Eqs.(13) and (22), we obtain the exact solution to Eqs.(13) and (14), so that the exact solution of the Drinfel'd–Sokolov–Wilson equation can be written as

$$u_{\{1,2\}}(x,t)=-\frac{3\alpha}{4}\tan^2\left[\frac{1}{\sqrt{2}}(x-\alpha t)+\frac{\sqrt{\alpha}}{2}c_1\right] \tag{23}$$

$$v_{\{1,2\}}(x,t)=\pm\sqrt{\frac{\alpha}{2}}\tan\left[\frac{1}{\sqrt{2}}(x-\alpha t)+\frac{\sqrt{\alpha}}{2}c_1\right] \tag{24}$$

where c_1 is an arbitrary integration constant.

3.2. Case (2)

Suppose that $m=2$, by comparing with the coefficients of $Y^i(i=2,1,0)$ on both sides of Eq.(16), we have

$$a_2(X)=a_2(X)h(X) \tag{25}$$

$$a_1(X)=a_2(X)g(X)+a_1(X)h(X) \tag{26}$$

$$a_0(X)=-2a_2(X)\left[\frac{\alpha}{2}X+X^3+\frac{c}{2}\right]+g(X)a_1(X)+h(X)a_0(X) \tag{27}$$

$$a_1(X)\left[\frac{\alpha}{2}X+X^3+\frac{c}{2}\right]=g(X)a_0(X) \tag{28}$$

Since $a_i(X), i=0,1,2$ are polynomials, then from (25) we deduce that $a_2(X)$ is a constant and $h(X)=0$. For simplicity, we take $a_2(X)=0$, and balancing the degrees of $g(X), a_1(X)$ and $a_2(X)$, we conclude that $deg[g(X)]=1$ or 0 only.

3.2.1. Set (1)

$deg(g(X))=1$. Suppose that $g(X)=AX+B$, then from Eqs.(26) and (27), we find that $a_1(X)$ and $a_0(X)$ can be written as

$$a_1(X)=\frac{A}{2}X^2+BX+A_1 \tag{29}$$

$$a_0(X)=\frac{1}{4}\left[\frac{A^2}{2}-2\right]X^4+\frac{AB}{2}X^3+\frac{1}{2}\left(AA_1+B^2-\alpha\right)X^2+(BA_1-c)X+A_2 \tag{30}$$

where A and B are arbitrary constants, and A_1, A_2 are arbitrary integration constants.

Substituting $a_0(X), a_1(X)$ and $g(X)$ in Eq.(28) and setting all coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it with aid Maple software, we obtain

$$B=0, \quad c=0 \quad A_2=\frac{\alpha^2}{8}, \quad A=\pm 2\sqrt{2}, \quad A_1=\pm\frac{\alpha}{\sqrt{2}} \tag{31}$$

where α and A_1 are arbitrary constants.
Using the conditions (31) into Eq. (15), we get

$$Y(\xi)=\pm\frac{1}{2}\left[\sqrt{2}X^2+\frac{\alpha}{\sqrt{2}}\right] \tag{32}$$

Combining Eqs.(13) and (32), we obtain the exact solutions of Eqs. (13) and (14), so that the exact solution of the classical Drinfel'd–Sokolov–Wilson equation can be written as

$$u_{\{3,4\}}(x,t)=-\frac{3\alpha}{4}\tan^2\left[\frac{1}{\sqrt{2}}(x-\alpha t)+\frac{\sqrt{\alpha}}{2}c_1\right] \tag{33}$$

$$v_{\{3,4\}}(x,t)=\pm\sqrt{\frac{\alpha}{2}}\tan\left[\frac{1}{\sqrt{2}}(x-\alpha t)+\frac{\sqrt{\alpha}}{2}c_2\right] \tag{34}$$

where c_1 is an arbitrary integration constant. Note that, the solutions given by Eqs.(33) and (34) are similar to the same solutions obtained earlier by Eqs.(23) and (24).

3.2.2. Set (2)

If $deg[g(X)]=0$. Suppose that $g(X)=A$; then from Eqs. (26) and (27), we find that $a_1(X)$ and $a_0(X)$ can be expressed as

$$a_1(X)=AX+B \tag{35}$$

$$a_0(X)=-\frac{1}{2}X^4+\frac{1}{2}\left(A^2-\alpha\right)X^2+(AB-c)X+A_1 \tag{36}$$

where A is an arbitrary constant, B and A_1 are arbitrary integration constants. Substituting $a_0(X), a_1(X)$ and $g(X)$ in Eq.(28) and setting all coefficients of X powers to be zero, then we obtain system of nonlinear algebraic equations and by solving it with aid Maple software, to obtain

$$A=0, \quad B=0, \quad c=c, \quad A_1=A_1 \tag{37}$$

Using the condition (37) into Eq.(15), we get

$$Y^2=\frac{1}{2}X^4+\frac{\alpha}{2}X^2+cX-A_1 \tag{38}$$

where A_1 is an arbitrary integration constant. Combining Eqs.(6), (13), (14) and (38), we get

$$\frac{\partial f(\xi)}{\partial \xi}=\frac{1}{2}f^4(\xi)+\frac{\alpha}{2}f^2(\xi)+cf(\xi)-A_1 \tag{39}$$

This equation have the following many new exact solutions [31]

$$f_5(\xi)=\sqrt{-\frac{\alpha}{2}}\tanh\left[\sqrt{-\frac{\alpha}{4}}\xi\right], \quad A_1=-\frac{\alpha^2}{8}, \quad c=0, \quad \alpha<0 \tag{40}$$

$$f_6(\xi)=\sqrt{-\alpha}\sec\left[\sqrt{-\frac{\alpha}{2}}\xi\right], \quad A_1=0, \quad c=0, \quad \alpha<0 \tag{41}$$

$$f_7(\xi)=\sqrt{\frac{\alpha}{2}}\tan\left[\sqrt{\frac{\alpha}{4}}\xi\right], \quad A_1=\frac{\alpha^2}{8}, \quad c=0, \quad \alpha>0 \tag{42}$$

$$f_8(\xi)=sn(\xi), \quad A_1=-1, \quad c=0, \quad \alpha=-2\left(m^2+1\right), \quad m^2=\frac{1}{2} \tag{43}$$

$$f_9(\xi)=cd(\xi),\quad A_1=-1,\quad c=0,\quad \alpha=-2\left(m^2+1\right),\quad m^2=\frac{1}{2} \tag{44}$$

$$f_{10}(\xi)=nd(\xi),\quad A_1=1,\quad c=0,\quad \alpha=2\left(2-m^2\right),\quad m^2=\frac{1}{2} \tag{45}$$

$$f_{11}(\xi)=sc(\xi),\quad A_1=-1,\quad c=0,\quad \alpha=2\left(2-m^2\right),\quad m^2=\frac{1}{2} \tag{46}$$

$$f_{12}(\xi)=sd(\xi),\quad A_1=-1,\quad c=0,\quad \alpha=2\left(2m^2-1\right),\quad m^2\left(m^2-1\right)=\frac{1}{2} \tag{47}$$

$$f_{13}(\xi)=-\frac{\alpha\sec^2\left[\frac{1}{2}\sqrt{-\frac{\alpha}{2}}\xi\right]}{2\sqrt{-\alpha}\tan\left[\frac{1}{2}\sqrt{-\frac{\alpha}{2}}\xi\right]},\quad A_1=0,\quad c=0,\quad \alpha<0 \tag{48}$$

$$f_{14}(\xi)=-\frac{\alpha\sec h^2\left[\frac{1}{2}\sqrt{\frac{\alpha}{2}}\xi\right]}{2\sqrt{\alpha}\tanh\left[\frac{1}{2}\sqrt{\frac{\alpha}{2}}\xi\right]},\quad A_1=0,\quad c=0,\quad \alpha>0 \tag{49}$$

By back substitution we obtain the following new exact solution of the Eqs. (1)

$$u_5(x,t)=\sqrt{-\frac{\alpha}{2}}\tanh\left[\sqrt{-\frac{\alpha}{4}}(x-\alpha t)\right],\quad A_1=-\frac{\alpha^2}{8},\quad c=0,\quad \alpha<0 \tag{50}$$

$$v_5(x,t)=-\frac{2}{3}\sqrt{u_5(x,t)} \tag{51}$$

$$u_6(x,t)=\sqrt{-\alpha}\sec\left[\sqrt{-\frac{\alpha}{2}}(x-\alpha t)\right],\quad A_1=0,\quad c=0,\quad \alpha<0 \tag{52}$$

$$v_6(x,t)=-\frac{2}{3}\sqrt{u_6(x,t)} \tag{53}$$

$$u_7(x,t)=\sqrt{\frac{\alpha}{2}}\tan\left[\sqrt{\frac{\alpha}{4}}(x-\alpha t)\right],\quad A_1=\frac{\alpha^2}{8},\quad c=0,\quad \alpha>0 \tag{54}$$

$$v_7(x,t)=-\frac{2}{3}\sqrt{u_7(x,t)} \tag{55}$$

$$u_8(x,t)=sn(\xi),\quad A_1=-1,\quad c=0,\quad \alpha=-2\left(m^2+1\right),\quad m^2=\frac{1}{2} \tag{56}$$

$$v_8(x,t)=-\frac{2}{3}\sqrt{u_8(x,t)} \tag{57}$$

$$u_9(x,t)=cd(x-\alpha t),\quad A_1=-1,\quad c=0,\quad \alpha=-2\left(m^2+1\right),\quad m^2=\frac{1}{2} \tag{58}$$

$$v_9(x,t)=-\frac{2}{3}\sqrt{u_9(x,t)} \tag{59}$$

$$u_{10}(x,t)=nd(x-\alpha t),\quad A_1=1,\quad c=0,\quad \alpha=2\left(2-m^2\right),\quad m^2=\frac{1}{2} \tag{60}$$

$$v_{10}(x,t)=-\frac{2}{3}\sqrt{u_{10}(x,t)} \tag{61}$$

$$u_{11}(x,t)=sc(x-\alpha t),\quad A_1=-1,\quad c=0,\quad \alpha=2\left(2-m^2\right),\quad m^2=\frac{1}{2} \tag{62}$$

$$v_{11}(x,t)=-\frac{2}{3}\sqrt{u_{11}(x,t)} \tag{63}$$

$$u_{12}(x,t)=sd(x-\alpha t),\quad A_1=-1,\quad c=0,\quad \alpha=2\left(2m^2-1\right),\quad m^2\left(m^2-1\right)=\frac{1}{2} \tag{64}$$

$$v_{12}(x,t)=-\frac{2}{3}\sqrt{u_{12}(x,t)} \tag{65}$$

$$u_{13}(x,t)=-\frac{\alpha\sec^2\left[\frac{1}{2}\sqrt{-\frac{\alpha}{2}}(x-\alpha t)\right]}{2\sqrt{-\alpha}\tan\left[\frac{1}{2}\sqrt{-\frac{\alpha}{2}}(x-\alpha t)\right]},\quad A_1=0,\quad c=0,\quad \alpha<0 \tag{66}$$

$$v_{13}(x,t)=-\frac{2}{3}\sqrt{u_{13}(x,t)} \tag{67}$$

$$u_{14}(x,t)=-\frac{\alpha\,\sec h^2\left[\frac{1}{2}\sqrt{\frac{\alpha}{2}}(x-\alpha t)\right]}{2\sqrt{\alpha}\tanh\left[\frac{1}{2}\sqrt{\frac{\alpha}{2}}(x-\alpha t)\right]},\quad A_1=0,\quad c=0,\quad \alpha>0 \tag{68}$$

$$v_{14}(x,t)=-\frac{2}{3}\sqrt{u_{14}(x,t)} \tag{69}$$

It should be mentioned that, Eqs. (23), (24), (33), (34), and (50)-(69) have been checked with Maple by putting them back into the original equation (1), and found correct.

4. Conclusions

In this paper, the first integral method is used to constract new wide classes of exact analytical traveling wave solutions for the classical nonlinear Drinfel'd-Sokolov-Wilson equation arising in nonlinear physics. The results revealed remarkable relations of solitary pattern, periodic solutions or solitons. The availability of computer systems like Mathematica or Maple facilitates the tedious algebraic calculations. The method which we have proposed in this paper is also a standard, direct and effective method, which allow us to do complicate and tedious calculation, based on the ring theory of commutative algebra. It is worth noting that the proposed method is simple and effective and gives more solutions. The applied method will be used in further work to establish more entirely new solutions for other kinds of nonlinear partial differential equations.

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