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Magneto elastic SH-type waves in a two layered infinite plate

Research Article

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Abstract: In this paper a study has been made of the propagation of magnetoelastic surface(SH) waves in a two layered infinite plate, under a bias magnetic field. The frequency equation for different cases of the real wave velocity have been found and numerically solved. Dispersion curves for group and phase velocity have been drawn.

MSC: 74Bxx • 74Jxx

Keywords: Magnetoelasticity • SH waves • Perfect conductivity

1. Introduction

Problems of wave propagation in perfect magnetoelastic conductors have been studied, among others, by Knopoff [1], Dunkin and Eringen [2], Yu and Tang [3], Paria [4]. Kalishki and Rogula [5] have discussed Rayleigh waves in a semi-infinite perfect electrical conductor for a single component magnetic field. The propagation of Rayleigh-type magnetoelastic waves in a semi-infinite perfect electrical conductor has also been studied by Viktorov [6] under a two component magnetic field.

In a review article by Maugin [7] a detailed study of wave motion, particularly harmonic waves in a hyperelastic non-linear dielectrics have been made.

A study of SH waves in solids as a perturbation of the boundary conditions for SH bulk waves were also made by Maugin [8]. Lee and Its [9] have considered in detail the effect of a magnetic field on the propagation of Rayleigh waves, particularly under high magnetic permeability. Chakraborty and Chattopadhyay [10] have considered the problem of magnetoelastic SH waves in a perfectly conducting half-space with an upper layer and established the fact that surface waves will exist due to the presence of a magnetic field even in those cases where ordinary Love waves do not exist. Recently Acharya and Roy [11] studied the propagation of surface waves in fibre-reinforced electrically conducting elastic solids. The presence of magnetic field was seen to modulate the Rayleigh wave velocity although no discernible effect was seen on Love waves.

The problem of wave propagation in plates is analogous in many respects to wave propagation in layered spaces. In the present problem, we have considered a two-layered thick plate, both layers being perfectly conducting magnetoelastic solids, and we have shown that SH-type waves can propagate either with a velocity intermediate between the respective magnetoelastic S-wave velocities in the two layers , or with a velocity greater than both of these S-wave velocities. It is also shown that the effect of the magnetic field is to alter the group velocity if the field is unidirectional with the direction of wave propagation. Numerical results have been presented to demonstrate the effect of the field in the phase and group velocities.

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2. Formulation of the problem

Let us consider a two-layered medium of infinite extent, the two layers being of thickness y_1 and y_2 respectively. The domain under consideration is given by $(-\infty < x < \infty, -y_2 \le y \le y_1, -\infty < z < \infty)$, where the interface of two layers is taken as y = 0, the positive direction of y-axis as pointing away from the free surface y = y_1 of the upper layer.

We want to study the propagation of two dimensional waves in this medium propagating along x-direction under a bias magnetic field (H_1, H_2, 0). Both the mediums are taken to be perfectly conducting linear elastic solids. Let medium $(-\infty < x < \infty, 0 \le y \le y_1, -\infty < z < \infty)$ be M_1, and the medium $(-\infty < x < \infty, -y_2 \le y < 0, -\infty < z < \infty)$ be M_2 and G_1 , G_2 respectively denote the rigidity moduli of these two.

The basic equations for magneto elastic disturbances are given by

(I). Maxwell's equations of the electromagnetic field are

$$\nabla \cdot \mathbf{D} = 0, \quad \nabla \cdot \mathbf{B} = 0;$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \tag{1}$$

(where the displacement current is neglected) **D** the electric displacement, **B** the magnetic induction, **H** the magnetic field intensity, **E** electric field intensity and **J** current density vectors respectively.

(II). The equations of motion are

$$\tau_{ij,j} + [\mathbf{J} \times \mathbf{B}]_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \ (i, j = 1, 2, 3) \tag{2}$$

where $\mathbf{J} \times \mathbf{B}$ is the Lorenz force due to the electromagnetic field and τ_{ij} being the usual elastic stress tensor in the medium .

(III). The constitutive equations are

$$\mathbf{B} = \mu \mathbf{H}, \quad \mathbf{D} = \epsilon \mathbf{E} \tag{3}$$

with Ohm's law as

$$\mathbf{J} = \sigma(\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}) \tag{4}$$

where σ is the electric conductivity, μ is the magnetic permeability, $\dot{\mathbf{u}}$ the particle velocity and ϵ the permittivity of the material.

(IV). Elastic Stress-Strain relations

$$\tau_{ij} = \lambda u_{k,k} \delta_{ij} + 2G e_{ij} \tag{5}$$

where e_{ij} are the strain tensor components, τ_{ij} the stress tensor components and

$$2e_{ij} = u_{i,j} + u_{j,i}. \ \theta = e_{ii}, \ (i, j = 1, 2, 3) \tag{6}$$

u_i being particle displacement and λ, G are Lame constants.

(V). The electromagnetic boundary conditions

$$\mathbf{n}.\|\mathbf{B}\| = 0, \quad \mathbf{n} \times \|\mathbf{H}\| = 0, \quad \mathbf{n} \cdot \|\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}\| = 0 \tag{7}$$

where **n** is normal to the interfaces or surface y = constant. $\|.\|$ denotes the jump across the boundary of the vector within.

(VI). Stress continuity conditions across the boundaries y = constant are

$$(\tau_{2j} + \tau^E_{2j})_+ - (\tau_{2j} + \tau^E_{2j})_- = 0, \ (j = 1, 2, 3) \tag{8}$$

where τ^E_{3j} are Maxwell's stress tensor components .

Let the initial constant magnetic field be $\mathbf{H}^{(0)}$ in vacuo, $\mathbf{H}^{(1)}$ in M_1 and $\mathbf{H}^{(2)}$ in M_2, where we take

$$\mathbf{H}^{(p)} = (H_1^{(p)}, H_2^{(p)}, 0), \quad p = 0, 1, 2. \tag{9}$$

There is no initial electric field and the current density **J** is initially zero. The displacement current **D**, being small, is neglected throughout this problem.

The electro magnetic boundary conditions (7) on y = 0 , y = y_1 and y = - y_2 give

$$H_1^{(0)} = H_1^{(1)} = H_1^{(2)} \tag{10}$$

$$\text{and } \mu_0 H_2^{(0)} = \mu_1 H_2^{(1)} = \mu_2 H_2^{(2)} \tag{11}$$

where μ_0, μ_1 and μ_2 are the magnetic permeability of vacuum, M_1 and M_2 respectively. Hence

$$\mathbf{H}^{(1)} = (H_1^{(1)}, H_2^{(1)}, 0) = (H_1, H_2, 0) \tag{12}$$

$$\text{and } \mathbf{H}^{(2)} = (H_1^{(2)}, H_2^{(2)}, 0) = (H_1, r H_2, 0) \tag{13}$$

$$\text{where } H_1^{(1)} = H_1, \ H_2^{(1)} = H_2, \ r = \mu_1/\mu_2. \tag{14}$$

2.1. Propagation of SH wave

We now consider an SH wave, propagating in the x-direction, with displacement given by

$$\mathbf{u}^{(\alpha)} = (0, 0, v^{(\alpha)}(x, y, t)) \tag{15}$$

with $\alpha = 1$ for M_1 , $\alpha = 2$ for M_2 respectively.
The perturbed magnetic field is given by

$$(H_1^{(\alpha)} + h_1^{(\alpha)}, H_2^{(\alpha)} + h_2^{(\alpha)}, h_3^{(\alpha)}) \tag{16}$$

with $\alpha = 1$ for M_1 , $\alpha = 2$ for M_2 respectively.
The media are perfectly conducting, so $\sigma_1, \sigma_2 \to \infty$ and (4) gives the perturbed electric field, which on substituting in (1) gives

$$\frac{\partial \mathbf{h}^{(\alpha)}}{\partial t} = (0, 0, H_1^{(\alpha)} \frac{\partial^2 v^{(\alpha)}}{\partial x \partial t} + H_2^{(\alpha)} \frac{\partial^2 v^{(\alpha)}}{\partial y \partial t}). \tag{17}$$

(neglecting second order terms)
Integrating (17) with respect to time t

$$\mathbf{h}^{(\alpha)} = (0, 0, H_1^{(\alpha)} \frac{\partial v^{(\alpha)}}{\partial x} + H_2^{(\alpha)} \frac{\partial v^{(\alpha)}}{\partial y}) = (0, 0, h_3^{(\alpha)})(s a y). \tag{18}$$

The Lorenz force is

$$\mathbf{J}^{(\alpha)} \times \mathbf{B}^{(\alpha)} = (0, 0, \mu_\alpha (H_2^{(\alpha)} h_{3_y}^{(\alpha)} + H_1^{(\alpha)} h_{3_x}^{(\alpha)})). \tag{19}$$

(neglecting second order terms)
The equation of motion in M_1 is

$$(1 + a_1^2/b_1^2) v_{xx}^{(1)} + (1 + a_3^2/b_1^2) v_{yy}^{(1)} + (2 a_1 a_3/b_1^2) v_{xy}^{(1)} = (1/b_1^2) v_{tt}^{(1)}, \tag{20}$$

$$\text{where} \quad a_1^2 = \mu_1 H_1^2/\rho_1; \;\; a_3^2 = \mu_1 H_2^2/\rho_1; \;\; b_1^2 = G_1/\rho_1 \tag{21}$$

a_1, a_3 are Alfven wave velocities and b_1 is the S-wave velocity in M_1.
The equation of motion for M_2 is

$$(1 + a_2^2/b_2^2) v_{xx}^{(2)} + (1 + r^2 a_4^2/b_2^2) v_{yy}^{(2)} + (2 r a_2 a_4/b_2^2) v_{xy}^{(2)} = (1/b_2^2) v_{tt}^{(2)} \tag{22}$$

$$\text{where} \quad a_2^2 = \mu_2 H_1^2/\rho_2; \;\; a_4^2 = \mu_2 H_2^2/\rho_2; \;\; b_2^2 = G_2/\rho_2; \;\; r = \mu_1/\mu_2 \tag{23}$$

a_2, a_4 are Alfven wave velocities and b_2 is the S-wave velocity in M_2
From (8) stress-free condition on $y = y_1$ and $y = -y_2$ leads to

$$G_1 v_y^{(1)} = 0, \;\; a t \;\; y = y_1 \tag{24}$$

$$G_2 v_y^{(2)} = 0, \;\; a t \;\; y = -y_2 \tag{25}$$

and from stress continuous condition at y =0, we have

$$[G_1 v_y^{(1)}]_{y \to 0^+} = [G_2 v_y^{(2)}]_{y \to 0^-}. \tag{26}$$

The displacement being continuous at y = 0 , we have

$$[v^{(1)}]_{y \to 0^+} = [v^{(2)}]_{y \to 0^-}. \tag{27}$$

2.2. Wave solution

For a SH wave travelling in the x-direction we assume solution of equation (20) of the form $v^{(1)}(x,y,t) = f(y)e^{ik(x-ct)}$.
Substituting $v^{(1)}(x,y,t)$ in(20), then we have the solution in the form

$$v^{(1)}(x,y,t) = (A_1 e^{ikn_1 y} + A_2 e^{ikn_2 y})e^{ik(x-ct)} \tag{28}$$

where A_1 and A_2 are arbitrary constants and

$$n_{1,2} = -L \pm N \tag{29}$$

$$L = a_1 a_3/(a_3^2 + b_1^2), \quad N = b_1\sqrt{b_1^2 + a_1^2 + a_3^2}\sqrt{c^2/\beta_1^2 - 1}/(a_3^2 + b_1^2) \tag{30}$$

$$\text{where} \quad \beta_1^2 = b_1^2(b_1^2 + a_1^2 + a_3^2)/(a_3^2 + b_1^2). \tag{31}$$

We assume solution of equation (22) of the form $v^{(2)}(x,y,t) = g(y)e^{ik(x-ct)}$.
Substituting $v^{(2)}(x,y,t)$ in(22), then we have the solution

$$v^{(2)}(x,y,t) = (B_1 e^{-k(im_1+m_2)y} + B_2 e^{-k(im_1-m_2)y})e^{ik(x-ct)} \tag{32}$$

where B_1 and B_2 are arbitrary constants and

$$m_1 = ra_2 a_4/(r^2 a_4^2 + b_2^2) \tag{33}$$

$$m_2 = b_2\sqrt{b_2^2 + a_2^2 + r^2 a_4^2}\sqrt{1 - c^2/\beta_2^2}\; m_1/ra_2 a_4 \tag{34}$$

$$\text{where} \quad \beta_2^2 = b_2^2 + a_2^2 - r^2 a_2^2 a_4^2/(r^2 a_4^2 + b_2^2). \tag{35}$$

Using boundary conditions (24)-(27), we can eliminate the arbitrary constants A_1, A_2, B_1 and B_2 to obtain the frequency equation

$$[G_1 m_2(N^2 - L^2)\cosh km_2 y_2 \sin Nky_1 - G_2(m_1^2 + m_2^2)N \sinh km_2 y_2 \cos Nky_1]$$

$$+i[G_1 m_1(N^2 - L^2) + G_2(m_1^2 + m_2^2)L]\sinh km_2 y_2 \sin Nky_1 = 0. \tag{36}$$

2.3. Real wave velocity

We look for a SH wave with real wave velocity c in the two following situations (1)when $\beta_1 < c < \beta_2$ and (2)when $\beta_1 < \beta_2 < c$.

2.3.1. $\beta_1 < c < \beta_2$

In this case, N and m_2 are both real. So, equating real and imaginary parts of L.H.S. to zero of Eq. (36) to zero, then we have following two equations are satisfied by the wave velocity c

$$[G_1 m_2(N^2 - L^2)\cosh km_2 y_2 \sin Nky_1 - G_2(m_1^2 + m_2^2)N \sinh km_2 y_2 \cos Nky_1] = 0 \tag{37}$$

$$[G_1 m_1(N^2 - L^2) + G_2(m_1^2 + m_2^2)L]\sinh km_2 y_2 \sin Nky_1 = 0. \tag{38}$$

For Eqs. (37) and (38) to be consistent, then the following equations are satisfied

$$\tan Nky_1 = \frac{G_2 N(m_1^2 + m_2^2)}{G_1 m_2(N^2 - L^2)}\tanh km_2 y_2 \tag{39}$$

$$\text{and} \quad G_1 m_1(N^2-L^2)+G_2(m_1^2+m_2^2)L=0. \tag{40}$$

Eq. (40) holds if

$$m_1 = L = 0. \tag{41}$$

Condition (41) valid in either of the following two cases:

Case1: $H_2=0,\ H_1\neq 0$ (Magnetic field in the direction of wave propagation.
Here $a_3=a_4=0, m=L=0$
then $\beta_1^2=(b_1^2+a_1^2),\ \beta_2^2=(b_2^2+a_2^2)$.

$$kN=\frac{k}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{c^2/\beta_1^2-1};\quad m_2=\frac{1}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{1-c^2/\beta_2^2}. \tag{42}$$

The frequency equation is obtained from(39) as

$$\tan[\frac{ky_1}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}]=\frac{G_2 b_1\sqrt{b_2^2+a_2^2}\sqrt{1-\frac{c^2}{\beta_2^2}}}{G_1 b_2\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}}\tanh[\frac{ky_2}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{1-\frac{c^2}{\beta_2^2}}]. \tag{43}$$

The group velocity(c_g) is therefore obtained from following equation

$$\begin{aligned}
&[\frac{ky_1}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}+\frac{ky_1\sqrt{b_1^2+a_1^2}}{b_1\sqrt{\frac{c^2}{\beta_1^2}-1}}(\frac{cc_g-c^2}{\beta_1^2})]\sec^2[\frac{ky_1}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}]\\
&=\frac{G_2 b_1\sqrt{b_2^2+a_2^2}}{G_1 b_2\sqrt{b_1^2+a_1^2}}[-\{\frac{(\frac{cc_g-c^2}{\beta_2^2})}{\sqrt{(\frac{c^2}{\beta_1^2}-1)(1-\frac{c^2}{\beta_2^2})}}+\frac{(\frac{cc_g-c^2}{\beta_1^2})(\sqrt{1-\frac{c^2}{\beta_2^2}})}{\sqrt{(\frac{c^2}{\beta_1^2}-1)^3}}\}\tanh[\frac{ky_2}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{1-\frac{c^2}{\beta_2^2}}]\\
&+\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2}-1}}\{\frac{ky_2}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{1-\frac{c^2}{\beta_2^2}}-\frac{ky_2}{b_2}\sqrt{b_2^2+a_2^2}\frac{(\frac{cc_g-c^2}{\beta_2^2})}{\sqrt{1-\frac{c^2}{\beta_2^2}}}\}\mathrm{sech}^2[\frac{ky_2}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{1-\frac{c^2}{\beta_2^2}}]
\end{aligned} \tag{44}$$

An SH wave propages with a velocity c, $\beta_1<c<\beta_2$ (provided $\beta_1<\beta_2$). If we consider $ky_2\to\infty$, we get the frequency equation for magnetoelastic Love waves as in Chakraborty and Chattopadhyay [10]. These are dispersive in nature, only the velocity range has been modified by the presence of the magnetic field.
Case 2: $H_2\neq 0, H_1=0$(Magnetic field transverse to direction of wave propagation)
Here $a_1=a_2=0, m=L=0$. then $\beta_1^2=b_1^2,\ \beta_2^2=b_2^2$.

$$kn_1=kN=\frac{kb_1}{\sqrt{(b_1^2+a_3^2)}}\sqrt{(c^2/b_1^2-1)};\quad m_2=\frac{b_2}{\sqrt{(b_2^2+r^2a_4^2)}}\sqrt{(1-c^2/b_2^2)}. \tag{45}$$

The frequency equation is obtained from(39) as

$$\tan[\frac{kb_1y_1}{\sqrt{b_1^2+a_3^2}}\sqrt{\frac{c^2}{b_1^2}-1}]=\frac{G_2\sqrt{1+\frac{a_3^2}{b_1^2}}\sqrt{1-\frac{c^2}{b_2^2}}}{G_1\sqrt{1+\frac{r^2a_4^2}{b_2^2}}\sqrt{\frac{c^2}{b_1^2}-1}}\tanh[\frac{kb_2y_2}{\sqrt{b_2^2+r^2a_4^2}}\sqrt{1-\frac{c^2}{b_2^2}}]. \tag{46}$$

The group velocity(c_g) is therefore obtained from following equation

$$\begin{aligned}
&[\frac{kb_1y_1}{\sqrt{b_1^2+a_3^2}}\sqrt{\frac{c^2}{b_1^2}-1}+\frac{kb_1y_1}{\sqrt{b_1^2+a_3^2}}\frac{(\frac{cc_g-c^2}{b_1^2})}{\sqrt{(\frac{c^2}{b_1^2}-1)}}]\sec^2[\frac{kb_1y_1}{\sqrt{b_1^2+a_3^2}}\sqrt{\frac{c^2}{b_1^2}-1}]\\
&=\frac{G_2\sqrt{1+\frac{a_3^2}{b_1^2}}}{G_1\sqrt{1+\frac{r^2a_4^2}{b_2^2}}}[-\{\frac{(\frac{cc_g-c^2}{b_2^2})}{\sqrt{(\frac{c^2}{b_1^2}-1)(1-\frac{c^2}{b_2^2})}}+\frac{(\frac{cc_g-c^2}{b_1^2})(\sqrt{1-\frac{c^2}{b_2^2}})}{\sqrt{(\frac{c^2}{b_1^2}-1)^3}}\}\tanh[\frac{kb_2y_2}{\sqrt{b_2^2+r^2a_4^2}}\sqrt{1-\frac{c^2}{b_2^2}}]\\
&+\frac{\sqrt{1-\frac{c^2}{b_2^2}}}{\sqrt{\frac{c^2}{b_1^2}-1}}[\frac{kb_2y_2}{\sqrt{b_2^2+r^2a_4^2}}\sqrt{1-\frac{c^2}{b_2^2}}]-\frac{kb_2y_2}{\sqrt{b_2^2+r^2a_4^2}}\frac{(\frac{cc_g-c^2}{b_2^2})}{\sqrt{1-\frac{c^2}{b_2^2}}}\}\mathrm{sech}^2[\frac{kb_2y_2}{\sqrt{b_2^2+r^2a_4^2}}\sqrt{1-\frac{c^2}{b_2^2}}]
\end{aligned} \tag{47}$$

If $k\,y_2 \to \infty$ then the frequency equation reduces to that of ordinary Love waves with the moduli of rigidity, density altered by factors, and the depth of the upper layer is also altered by a factor.
Particular Case: $H_1 = 0,\ H_2 = 0$ (No Magnetic Field)
Here $a_1 = a_2 = a_3 = a_4 = 0$
then $\beta_1^2 = b_1^2,\ \beta_2^2 = b_2^2$.
The frequency equation is obtained from(39) as

$$\tan[k\,y_1\sqrt{\frac{c^2}{b_1^2}-1}] = \frac{G_2}{G_1}\frac{\sqrt{1-\frac{c^2}{b_2^2}}}{\sqrt{\frac{c^2}{b_1^2}-1}}\tanh[k\,y_2\sqrt{1-\frac{c^2}{b_2^2}}] \tag{48}$$

Therefore, if no magnetic field and $y_2 \to \infty$ then we get frequency equation for standard Love waves.

2.3.2. $\beta_1 < \beta_2 < c$

In this case, m_2 is purely imaginary and N is real.
Let $m_2 = \mathrm{i}\,\overline{m_2}$ where $\overline{m_2}$ is real and $\overline{m_2} = b_2\sqrt{b_2^2+a_2^2+r^2a_4^2}\sqrt{c^2/\beta_2^2-1}\ m_1/r\,a_2a_4$ Then Eq. (39) can be written as

$$\begin{aligned}&-[G_1m_1(N^2-L^2)+G_2(m_1^2-\overline{m_2}^2)L]\sin k\overline{m_2}y_2\sin N\,k\,y_1\\&\qquad+i[G_1\overline{m_2}(N^2-L^2)\cos k\overline{m_2}y_2\sin N\,k\,y_1-G_2(m_1^2-\overline{m_2}^2)N\sin k\overline{m_2}y_2\cos N\,k\,y_1]=0.\end{aligned} \tag{49}$$

So, equating both side real and imaginary parts of Eq. (49) to zero, then we have following two equations are satisfied by the wave velocity c

$$[G_1m_1(N^2-L^2)-G_2(\overline{m_2}^2-m_1^2)L]\sin k\overline{m_2}y_2\sin N\,k\,y_1=0 \tag{50}$$

$$\text{and } [G_1\overline{m_2}(N^2-L^2)\cos k\overline{m_2}y_2\sin N\,k\,y_1+G_2(\overline{m_2}^2-m_1^2))N\sin k\overline{m_2}y_2\cos N\,k\,y_1]=0. \tag{51}$$

For Eqs. (50) and (51) to be consistent, then the following equations are satisfied

$$\tan N\,k\,y_1 = -\frac{G_2N(\overline{m_2}^2-m_1^2)}{G_1\overline{m_2}(N^2-L^2)}\tan k\overline{m_2}y_2 \tag{52}$$

$$G_1m_1(N^2-L^2)-G_2(\overline{m_2}^2-m_1^2)L=0. \tag{53}$$

Eq. (53) holds if

$$m_1 = L = 0. \tag{54}$$

Condition (54) valid in either of the following two cases:

Case1: $H_2 = 0,\ H_1 \neq 0$ (Magnetic field in the direction of wave propagation.
Here $a_3 = a_4 = 0, m = L = 0$
then $\beta_1^2 = (b_1^2+a_1^2),\quad \beta_2^2 = (b_2^2+a_2^2)$.

$$kN = \frac{k}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{c^2/\beta_1^2-1};\quad \overline{m_2} = \frac{1}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{c^2/\beta_2^2-1}. \tag{55}$$

The frequency equation is obtained from(52) as

$$\tan[\frac{k\,y_1}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}] = -\frac{G_2}{G_1}\frac{\frac{1}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{\frac{c^2}{\beta_2^2}-1}}{\frac{1}{b_1}\sqrt{b_1^2+a_1^2}\sqrt{\frac{c^2}{\beta_1^2}-1}}\tan[\frac{k\,y_2}{b_2}\sqrt{b_2^2+a_2^2}\sqrt{\frac{c^2}{\beta_2^2}-1}\,]. \tag{56}$$

Case 2: $H_2 \neq 0,\ H_1 = 0$ (Magnetic field transverse to direction of wave propagation)
Here $a_1 = a_2 = 0, m = L = 0$
then $\beta_1^2 = b_1^2,\quad \beta_2^2 = b_2^2$.

$$kN = \frac{k\,b_1}{\sqrt{(b_1^2+a_3^2)}}\sqrt{(c^2/b_1^2-1)};\quad \overline{m_2} = \frac{b_2}{\sqrt{(b_2^2+r^2a_4^2)}}\sqrt{(c^2/b_2^2-1)}. \tag{57}$$

The frequency equation is obtained from (52) as

$$\tan[\frac{k\,b_1\,y_1}{\sqrt{b_1^2+a_3^2}}\sqrt{\frac{c^2}{b_1^2}-1}] = -\frac{G_2}{G_1}\frac{\sqrt{1+\frac{a_3^2}{b_1^2}}}{\sqrt{1+\frac{r^2a_4^2}{b_2^2}}}\frac{\sqrt{\frac{c^2}{b_2^2}-1}}{\sqrt{\frac{c^2}{b_1^2}-1}}\tan[\frac{k\,b_2\,y_2}{\sqrt{b_2^2+r^2a_4^2}}\sqrt{\frac{c^2}{b_2^2}-1}]. \tag{58}$$

3. Numerical study and discussion

Real wave velocities have been plotted for different values of elastic and magnetic parameters with the magnetic field range in from 0 to higher values. Two cases have been considered :
(1) When the magnetic field is along the direction of wave propagation.
(2) When the magnetic field is perpendicular to the direction of wave propagation.
In the case when phase velocity lies between β_1 and β_2, we find that an increases in magnetic field intensity in the direction of propagation (which is shown by increase in non dimensional Alfven wave velocity) non dimensional phase and group velocities both increase as shown in Fig. 1(cases i - ix). However not much change in velocity profile is observed if the magnetic field increased in the transverse direction shown in Fig. 5(cases i - vi). Dispersion effect becomes pronounced as the depth of the second stratum (i.e. y_2)increase up to a certain level. However, after that, further increase in $k\,y_2$ does not produce appreciable change in wave velocity as seen in Fig. 3(cases i - iii) and Fig. 6(cases i - ii). The phase velocity is lower where the non magnetic SH-wave velocity ratio $\frac{b_1}{b_2}$ higher, shown in Fig. 4(cases i - iii) and Fig. 7(cases i - ii). All these graphs have been drawn with fixed value of the ratio material density and fixed ratio for the magnetic permeability.
Numerical work has also been done for the case when $\beta_1 < \beta_2 < c$ shown in Figs. 8-11. The observation made earlier regarding increasing the wave velocity with the increase in longitudinal magnetic field also hold in the case too.
Once again no discernible effect of the transverse magnetic field is seen on the wave velocities. The phase velocity decreases if the magnetic field be kept fixed and $\frac{b_1}{b_2}$ gradually increase towards 1.

4. Conclusions

In this paper we have studied the propagation of SH-waves in a medium consisting of two perfectly conducting infinite layers with different physical and magnetic properties. If the depth of one layer increases towards infinity then the frequency equation is the same as that of magnetoelastic Love waves as studied by Chakraborty and Chattopadhyay [10]. It is seen that SH waves exits in the both cases: (1) when phase velocity lies between the magnetoelastic SH-waves velocities i.e. β_1 and β_2 of the two media. (2) when the phase velocity is greater than both of these.
We see that from the frequency equation such wave exist even when $b_1 > b_2$ provided $\beta_1 < \beta_2$ which is possible depending on the magnetic permeability and density of the two layers. For a low ratio of the permeabilities this is possible as shown in Figs. 2.

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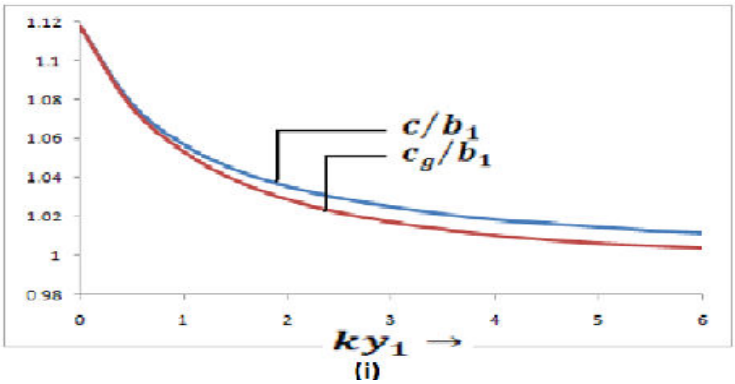
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1.1
1.08
1.06
1.04
1.02
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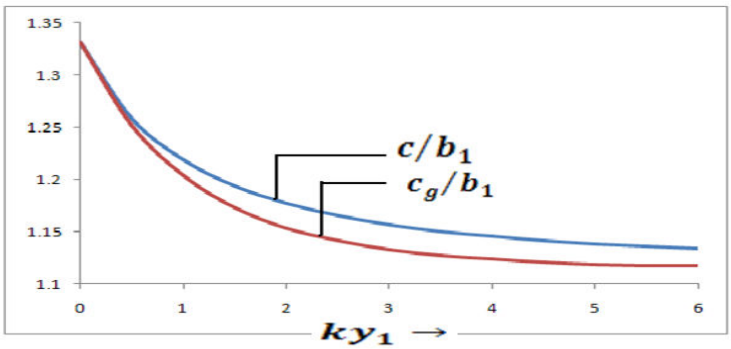
0.98
0
1
2
3
4
5
6
c/b_1
c_g/b_1
$ky_1 \rightarrow$

(i)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2
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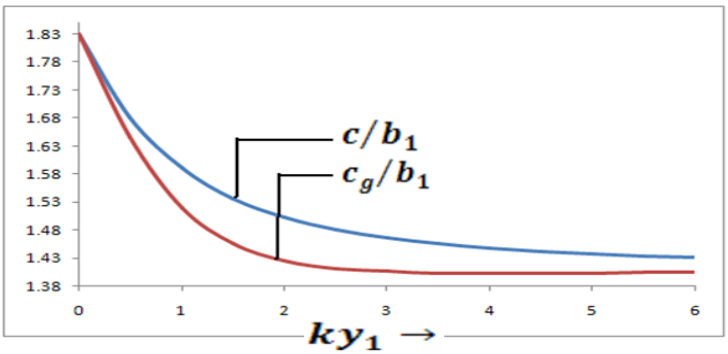
4
5
6
c/b_1
c_g/b_1
$ky_1 \rightarrow$

(ii)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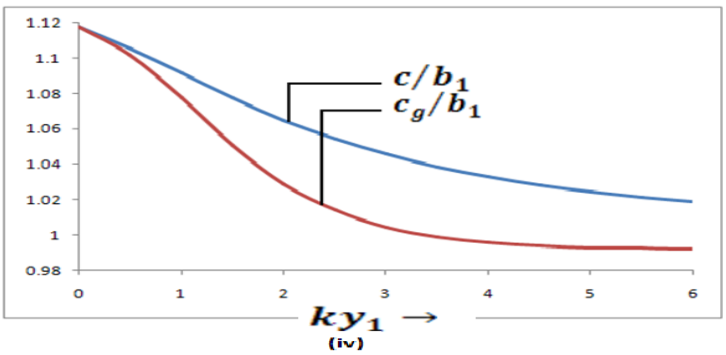
5
6
c/b_1
c_g/b_1
$ky_1 \rightarrow$

(iii)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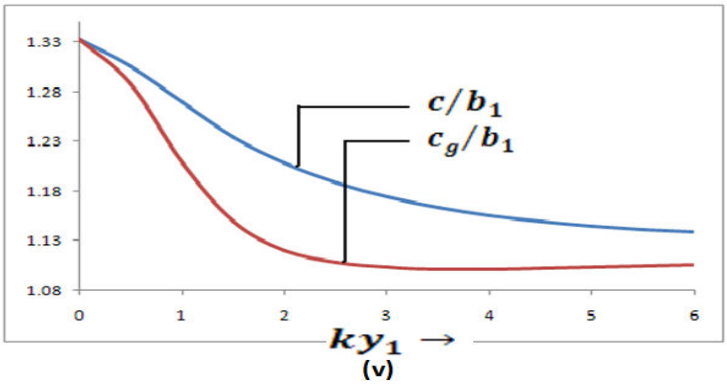
4
5
6
c/b_1
c_g/b_1
$ky_1 \rightarrow$

(iv)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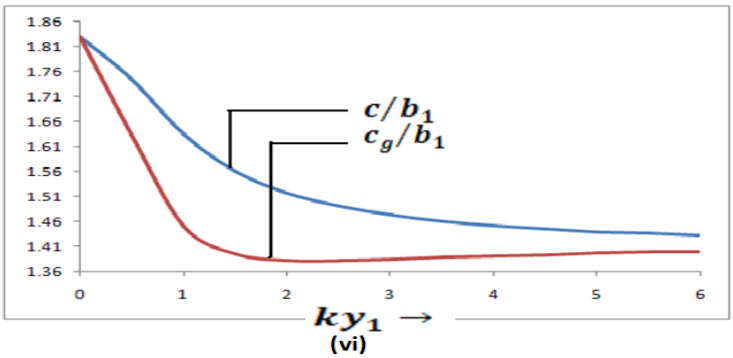
4
5
6
c/b_1
c_g/b_1
ky_1 →

(v)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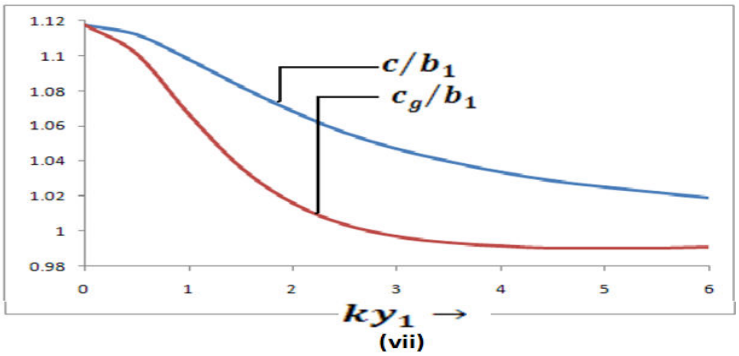
4
5
6
c/b_1
c_g/b_1
ky_1 →

(vi)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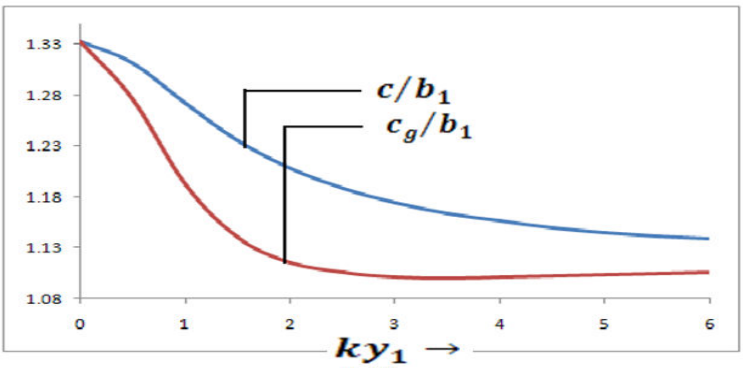
c/b_1
c_g/b_1
ky_1 →

(vii)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c/b_1
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ky_1 →

(viii)

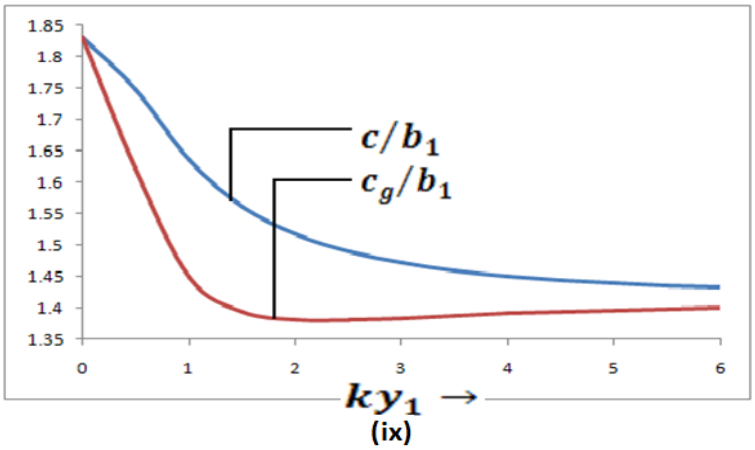


Fig. 1. Variation of $\frac{c}{b_1}$ and $\frac{c_g}{b_1}$ with respect to ky_1. In figure (i) to (iii)with $ky_2 = 1$, $\frac{a_1}{b_1} = 0.0, 0.5, 1.0$ respectively and in figure (iv) to (vi)with $ky_2 = 5$, $\frac{a_1}{b_1} = 0.0, 0.5, 1.0$ respectively and in figure (vii) to (ix)with $ky_2 \rightarrow \infty$, $\frac{a_1}{b_1} = 0.0, 0.5, 1.0$ respectively also with $\frac{\mu_1}{\mu_2} = 0.5$, $\frac{\rho_1}{\rho_2} = 1.05$, $\frac{b_1^2}{b_2^2}$ =0.8 in figure (i) to (ix)

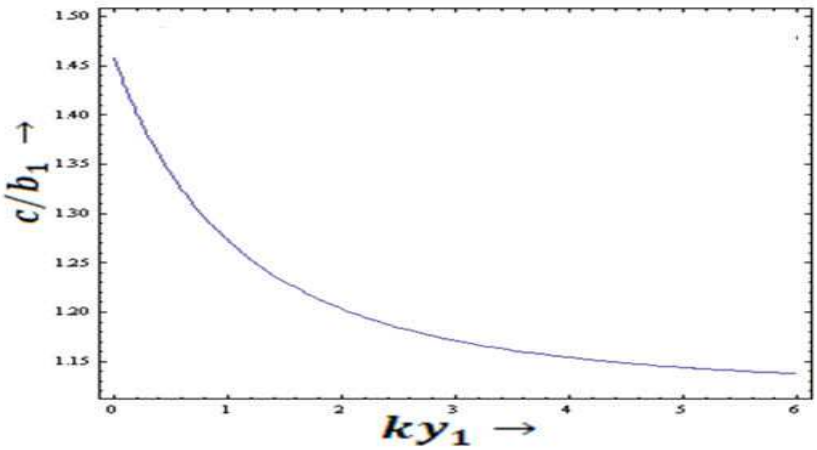
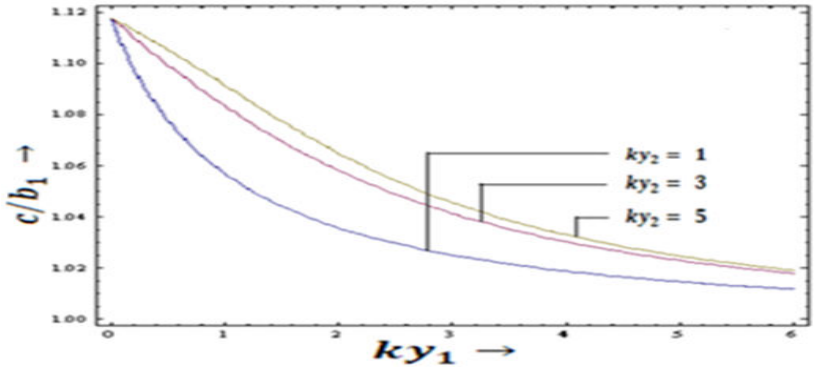


Fig. 2. Variation of $\frac{c}{b_1}$ with respect to ky_1 with $\frac{\mu_1}{\mu_2} = 0.3$, $\frac{\rho_1}{\rho_2} = 1.05$, $\frac{b_1^2}{b_2^2}$ =0.8, ky_2 =1, $\frac{a_1}{b_1} = 0.5$ when $\beta_1 < c < \beta_2$



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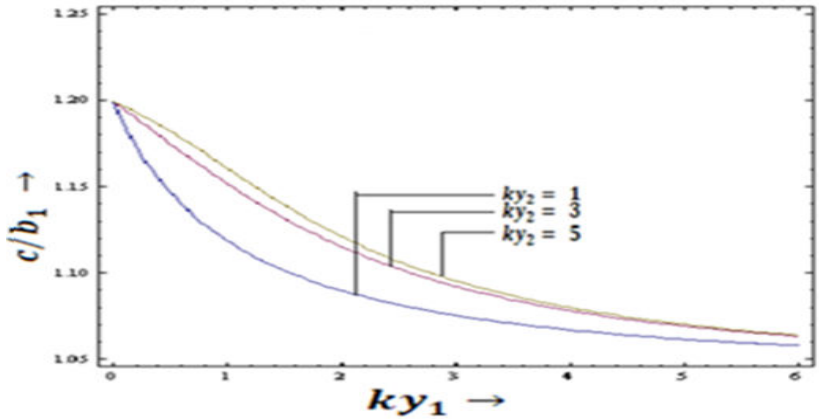

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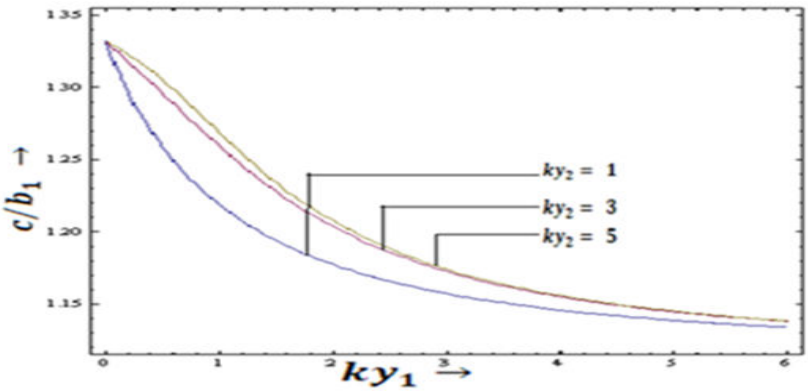
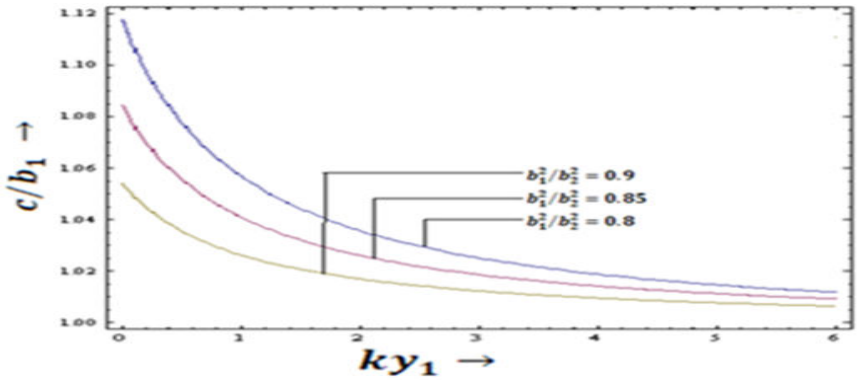


(iii)

Fig. 3. Variation of $\frac{c}{b_1}$ with respect to $k\,y_1$ for $k\,y_2$ =1, 3, 5. In figure (i) to (iii) with $\frac{a_1}{b_1}$ = 0.0, 0.3, 0.5 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $\frac{b_1^2}{b_2^2}$ =0.8



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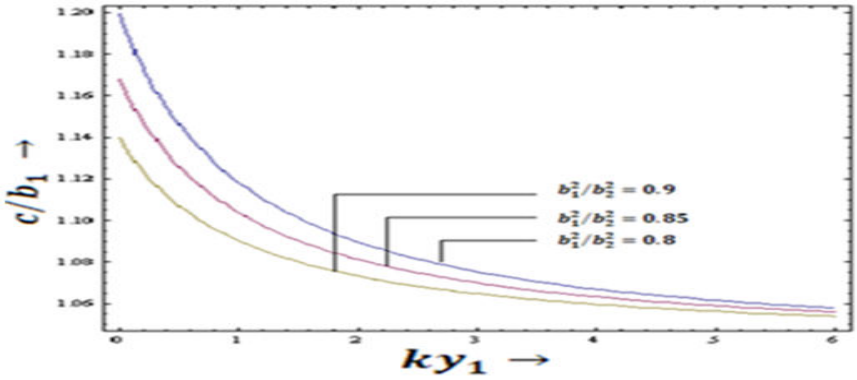

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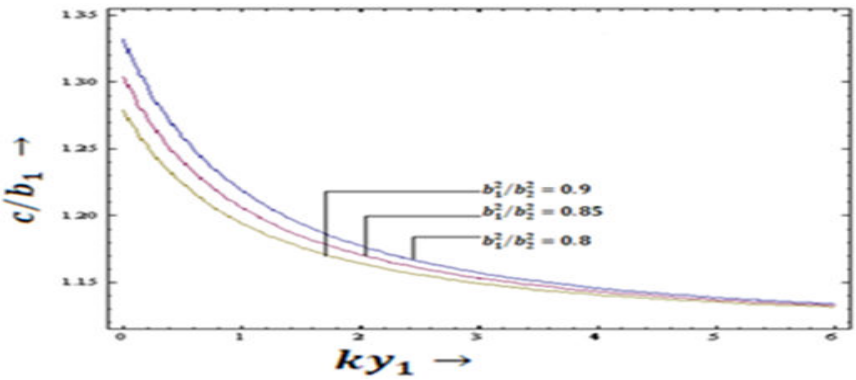
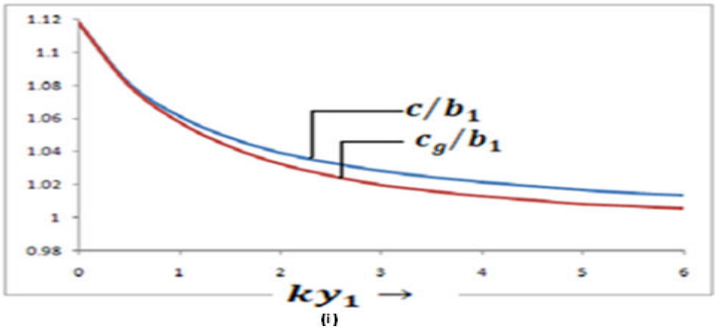


(iii)

Fig. 4. Variation of $\frac{c}{b_1}$ with respect to ky_1 for $\frac{b_1^2}{b_2^2} = 0.80, 0.85, 0.90$. In figure (i) to (iii) with $\frac{a_1}{b_1} = 0.0, 0.3, 0.5$ respectively also with $\frac{\mu_1}{\mu_2} = 0.5$, $\frac{\rho_1}{\rho_2} = 1.05$, $ky_2 = 1$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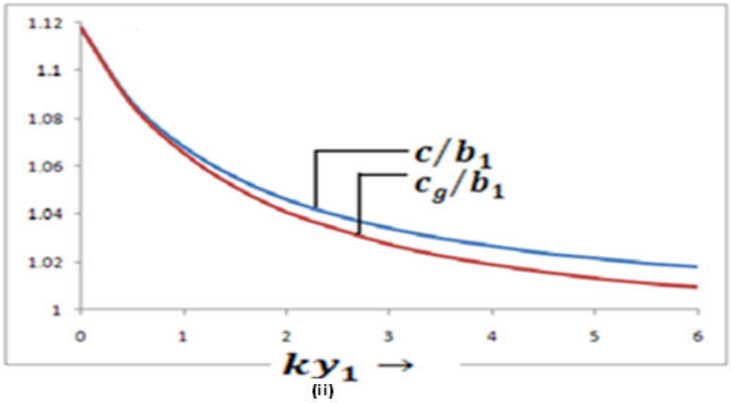
c/b_1
c_g/b_1
$ky_1 \rightarrow$
(i)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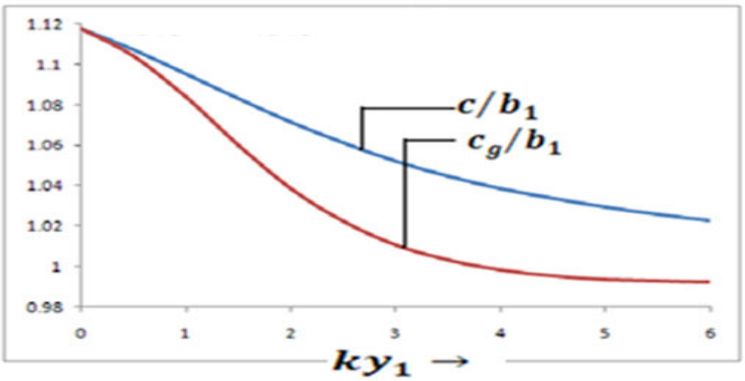
c_g/b_1
$ky_1 \rightarrow$
(ii)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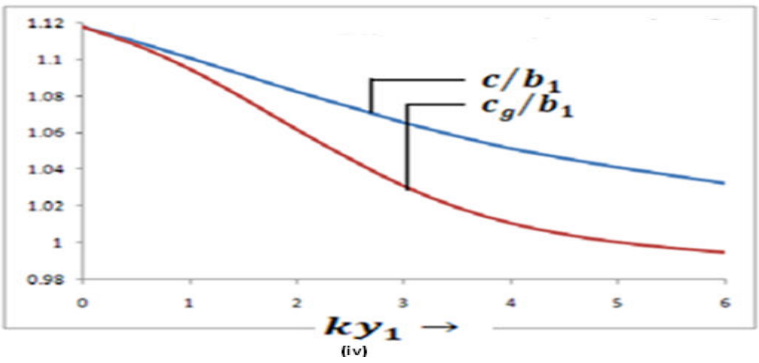
c/b_1
c_g/b_1
$ky_1 \rightarrow$
(iii)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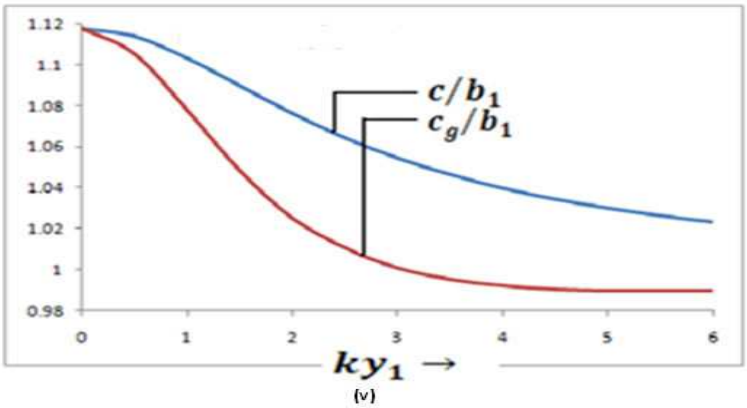
6
c/b_1
c_g/b_1
$ky_1 \rightarrow$
(iv)



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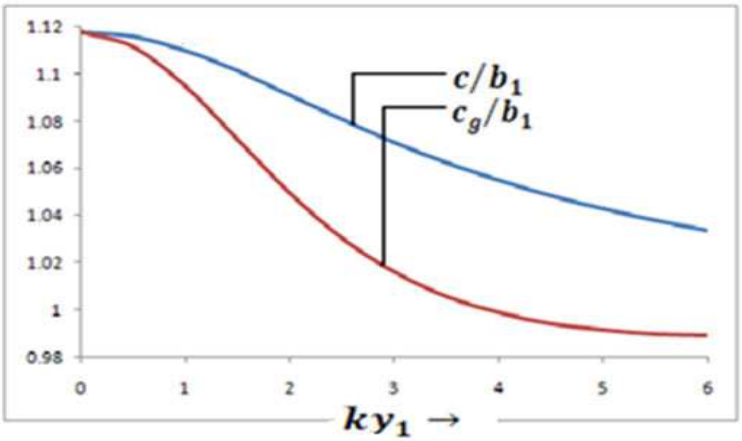
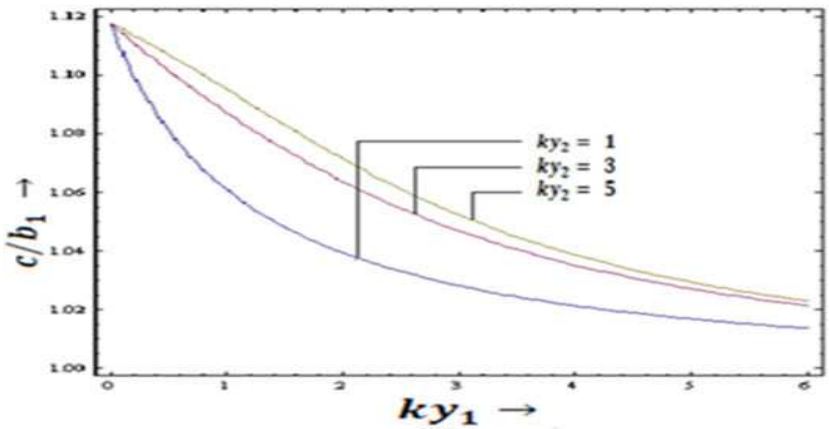


(vi)

Fig. 5. Variation of $\frac{c}{b_1}$ and $\frac{c_g}{b_1}$ with respect to ky_1. In figure (i),(ii)with $ky_2 = 1$, $\frac{a_3}{b_1} = 0.5, 1.0$ respectively and in figure (iii),(iv)with $ky_2 = 5$, $\frac{a_3}{b_1} = 0.5, 1.0$ respectively and in figure (v),(vi)with $ky_2 \rightarrow \infty$, $\frac{a_3}{b_1} = 0.5, 1.0$ respectively also with $\frac{\mu_1}{\mu_2} = 0.5$, $\frac{\rho_1}{\rho_2} = 1.05$, $\frac{b_1^2}{b_2^2} = 0.8$ in figure (i) to (vi)



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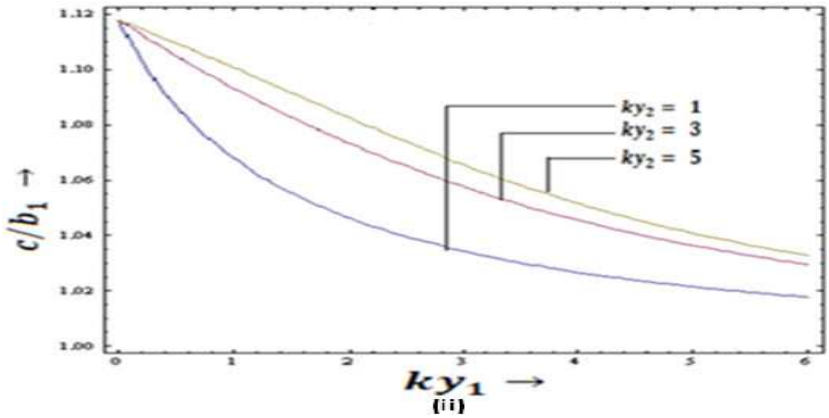

Fig. 6. Variation of $\frac{c}{b_1}$ with respect to $k\,y_1$ for $k\,y_2$ =1, 3, 5. In figure (i),(ii) with $\frac{a_1}{b_1}$ = 0.3, 0.5 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $\frac{b_1^2}{b_2^2}$ =0.8

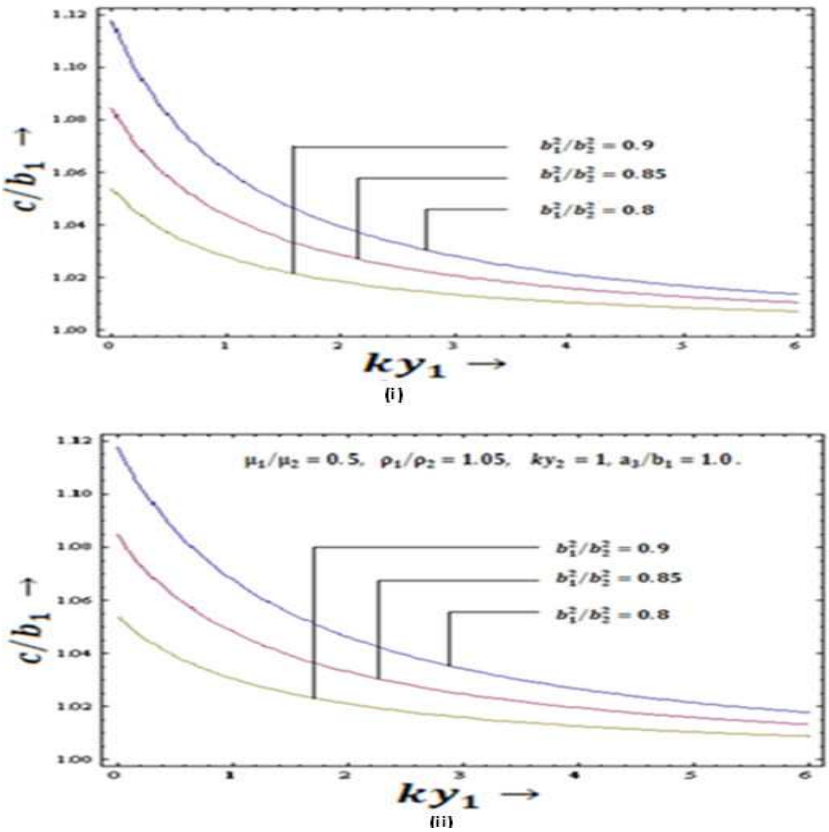

Fig. 7. Variation of $\frac{c}{b_1}$ with respect to $k\,y_1$ for $\frac{b_1^2}{b_2^2}$ = 0.80, 0.85, 0.90. In figure (i) to (iii) with $\frac{a_3}{b_1}$ = 0.3, 0.5,1.0 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $k\,y_2$ =1

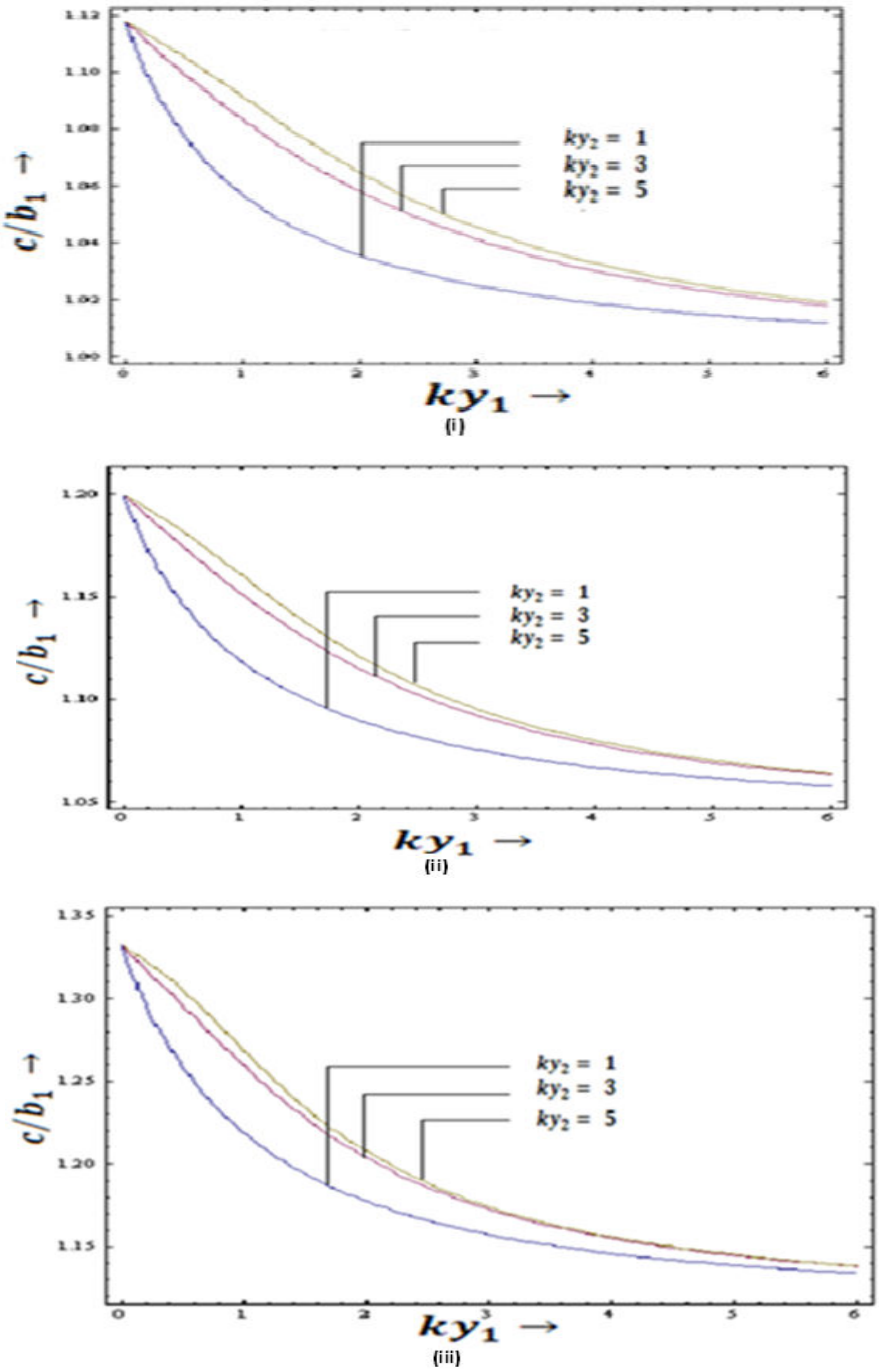

Fig. 8. Variation of $\frac{c}{b_1}$ with respect to $k\,y_1$ for $k\,y_2$ =1, 3, 5. In figure (i) to (iii) with $\frac{a_1}{b_1}$ = 0.0, 0.3, 0.5 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $\frac{b_1^2}{b_2^2}$ =0.8

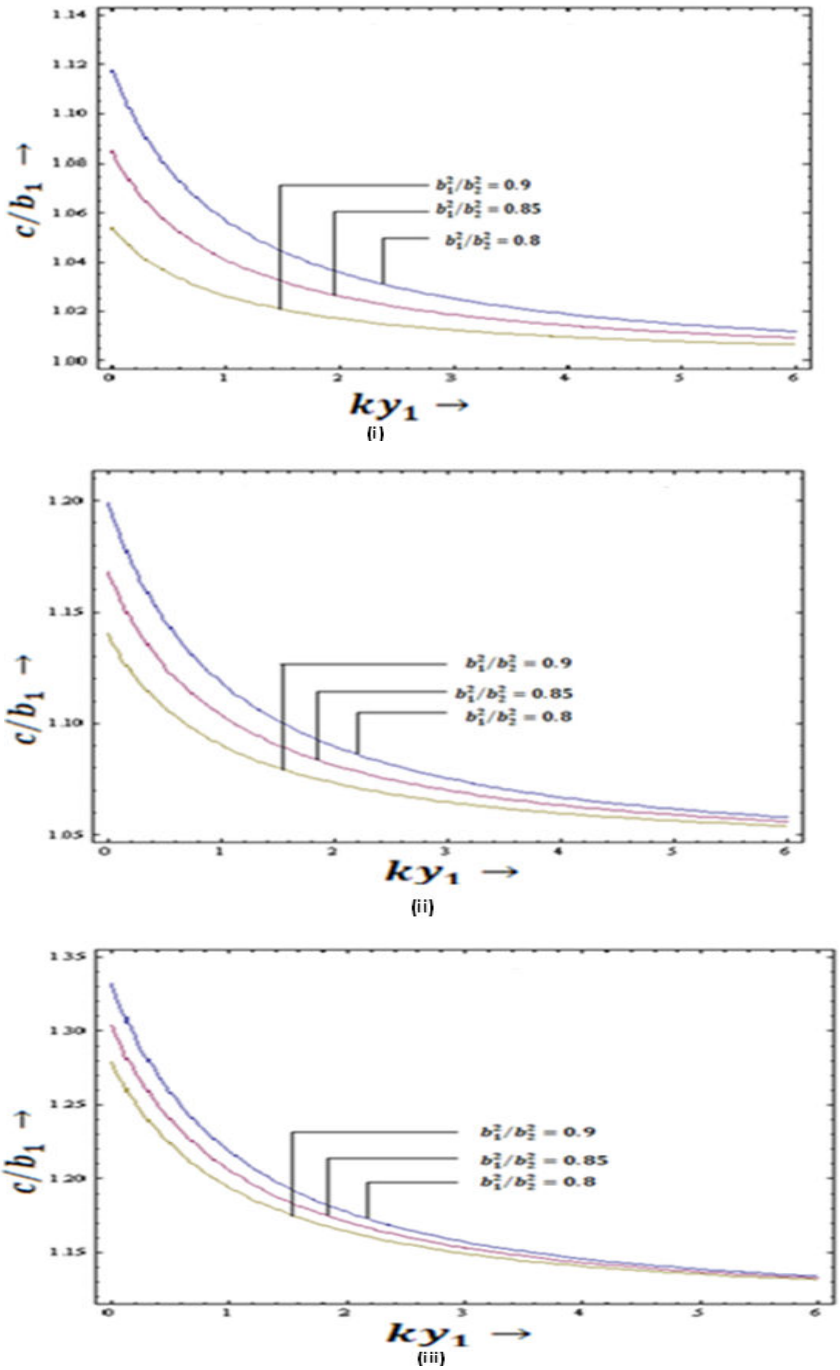

Fig. 9. Variation of $\frac{c}{b_1}$ with respect to ky_1 for $\frac{b_1^2}{b_2^2} = 0.80, 0.85, 0.90$. In figure (i) to (iii) with $\frac{a_1}{b_1} = 0.0, 0.3, 0.5$ respectively also with $\frac{\mu_1}{\mu_2} = 0.5$, $\frac{\rho_1}{\rho_2} = 1.05$, $ky_2 = 1$

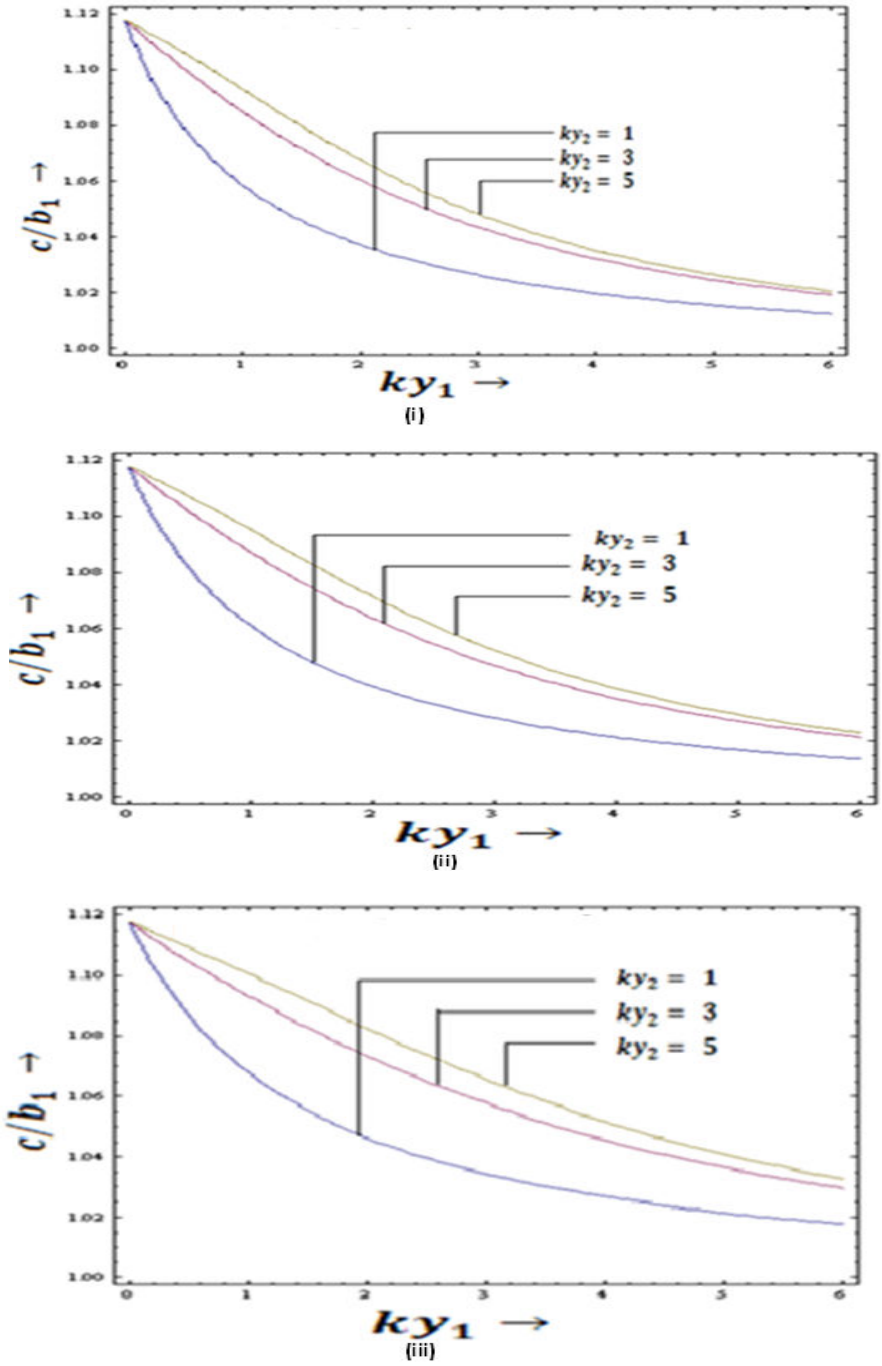

Fig. 10. Variation of $\frac{c}{b_1}$ with respect to ky_1 for ky_2 =1, 3, 5. In figure (i) to (iii) with $\frac{a_3}{b_1}$ = 0.3, 0.5, 1.0 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $\frac{b_1^2}{b_2^2}$ =0.8

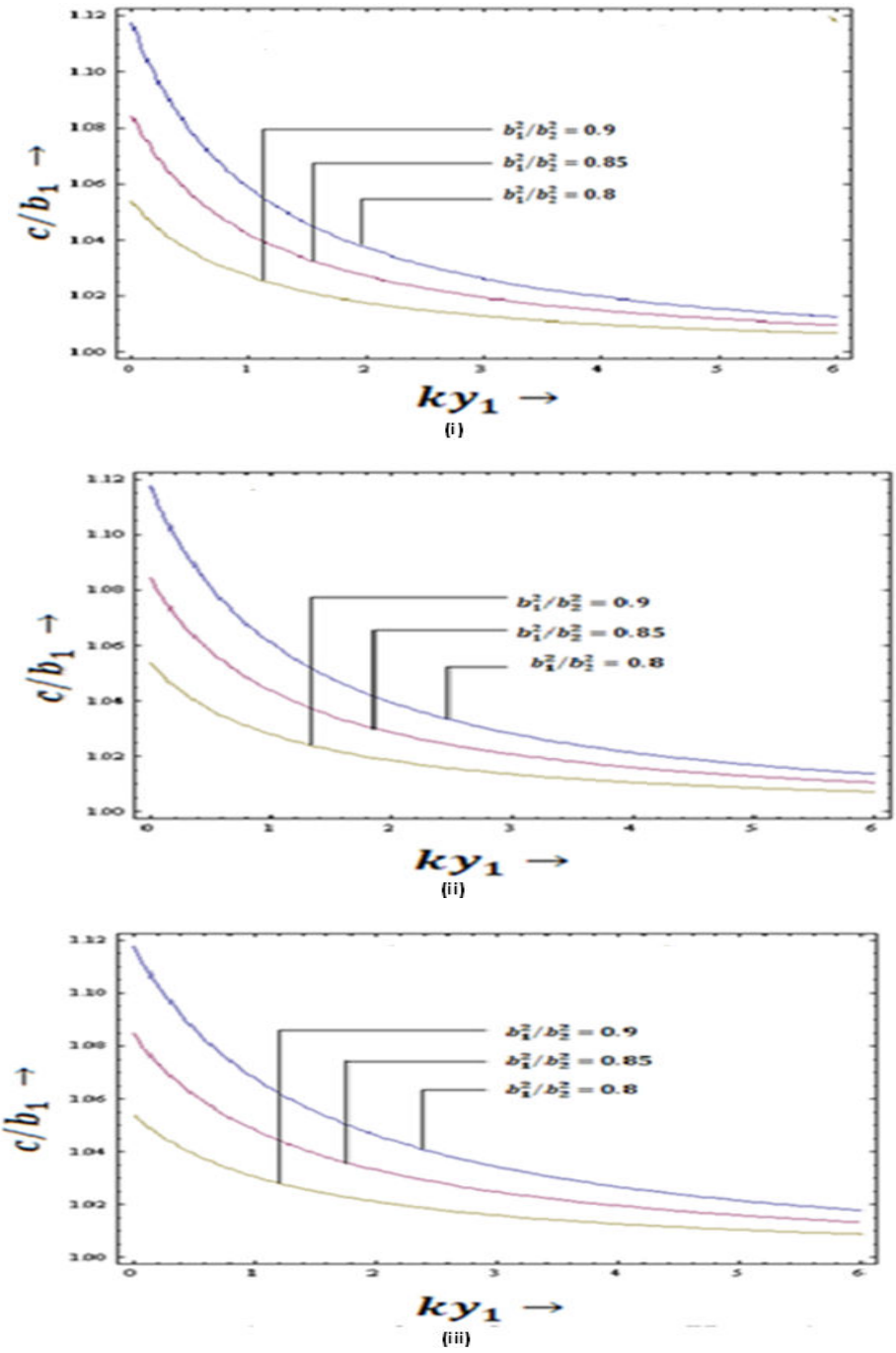

Fig. 11. Variation of $\frac{c}{b_1}$ with respect to ky_1 for ky_2 =1, 3, 5. In figure (i) to (iii) with $\frac{a_3}{b_1}$ = 0.3, 0.5, 1.0 respectively also with $\frac{\mu_1}{\mu_2}$ = 0.5, $\frac{\rho_1}{\rho_2}$ = 1.05, $\frac{b_1^2}{b_2^2}$ =0.8