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Fuzzy logic as a tool in assessing the bridge players' performance

Research Article

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Abstract: In this article Fuzzy Logic is used as a tool in developing a new method for assessing the of bridge-playersâĂŹ performance. For this, the cohorts of playerrs' under assessment are represented as fuzzy subsets of a set of linguistic labels characterizing their performance and the technique of the âĂŸcenter of gravityâĂŹ (COG) is used to convert the fuzzy data collected from the game to a crisp number. Our method could be used informally as a complement of the official bridge-scoring methods (match points or IMPs) for statistical and other obvious reasons. Two real applications related to simultaneous tournaments with pre-dealt boards, organized by the Hellenic Bridge Federation, are also presented, illustrating the importance of our results in practice.

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1. Introduction

There used to be a tradition in science and engineering of turning to probability theory when one is faced with a problem in which uncertainty plays a significant role. This transition was justified when there were no alternative tools for dealing with the uncertainty. Today this is no longer the case. Fuzzy logic, which is based on fuzzy sets theory introduced by Zadeh in 1965, provides a rich and meaningful addition to standard logic.

Let U be the universal set of the discourse and let A be a (crisp) subset of U. Then, according to the standards of the classical logic, for each $x \in U$ is either $x \in A$, or $x \notin A$. However, they often appear situations in real life in which the given definitions have not clear boundaries; this happens for example when we speak about the high mountains of a country, the good players of a football team, the young people of a city, etc. This creates an uncertainty on whether or not a given $x \in U$ belongs or not to A. In fact, this happens if, for example, U is the set of all the mountains of a country, A is the set of the high mountains of this country and x is a mountain of the country which is neither very high, nor very low. In order to overcome such kind of difficulties, Zadeh [1] introduced the notion of a fuzzy subset, say A, of U (or a fuzzy set in U) in terms of a membership function $m_A : U \to [0,1]$. Then A is defined as $A = \{(x, m_A(x)) : x \in U\}$.

The greater is the value of $m_A(x)$, called the membership degree of x in A, the more x satisfies the property characterizing A. The definition of a membership function is more or less arbitrary, based on empirical or statistical data. However its definition must be always compatible to the common logic, in order to be able to model properly the corresponding situation.

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The above Zadeh's definition extends the notion of a crisp set, say A, which can be also considered as a fuzzy subset of U. In fact in this case, given $x \in U$ we take$m_A(x) = 1$, if $x \in A$ and $m_A(x) = 0$, if $x \notin A$. As a result, most notions connected to the crisp sets can be extended to fuzzy sets as well. For example, given the fuzzy subsets A and B of U, we say that A is a subset of B, if $m_A(x) \leqslant m_B(x)$, for all $x \ inU$. In fact, if A and B are crisp subsets of U, then obviously the above definition implies $A \subseteq B$.

Notice that, although probabilities and fuzzy membership degrees are both taking values in the interval $[0,1]$, they are distinct and different to each other notions. For example, the expression "The probability for Mary to be tall is 85%", means that, although Mary is either tall or low in stature, she is very possibly tall. On the contrary the expression "The membership degree of Mary to be tall is 0.85", simply means that Mary can be characterized as rather tall. Another characteristic difference (but not the only one) is that, while the sum of probabilities of all the singleton subsets (events) of U equals 1, this is not necessary to happen for the membership degrees. For general facts on fuzzy sets we refer freely to the book [2].
A real test of the effectiveness of an approach to uncertainty is the capability to solve problems which involve different facets of uncertainty. Fuzzy logic has a much higher problem solving capability than standard probability theory. Most importantly, it opens the door to construction of mathematical solutions of computational problems which are stated in a natural language. The applications which may be generated from or adapted to fuzzy logic are wide-ranging and provide the opportunity for modelling under conditions which are inherently imprecisely defined, despite the concerns of classical logicians (e.g. see Chapter 6 of [2–4] and its relevant references, [5–8]).
The methods of assessing the individuals' performance usually applied in practice are based on principles of the bivalent logic (yes-no). However these methods are not probably the most suitable ones. In fact, fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for this purpose. This gave us several times in the past the impulsion to introduce principles of fuzzy logic in assessing the performance of student groups in learning mathematics and problem solving (e.g. see [4, 5, 8, 9, 18] etc). In this paper we shall use fuzzy logic in assessing the total performance of bridge playersâĂŹ belonging to sets of a special interest (e.g. different bridge clubs during a tournament, men and women, young and old players, etc).

The rest of the paper is organized as follows: In the next section we give the necessary for the purposes of this article information about the game of contract bridge. In the third section we develop our new assessment method, which is based on principles of fuzzy logic. In section four we present two real applications illustrating the importance of our method in practice. Finally the last section is devoted to conclusions and discussion for the future perspectives of research on this area.

2. The game of contract bridge

Bridge is a card game belonging to the family of trick-taking games. It is a development of Whist, which had become the dominant such game enjoying a loyal following for centuries. In 1904 Auction Bridge was developed, in which the players bid in a competitive auction to decide the contract and declarer. The object became to make at least as many tricks as were contracted for and penalties were introduced for failing to do so. The modern game of Contract Bridge was the result of innovations to the scoring of auction bridge suggested by Harold Stirling Vanderbilt (USA, 1925) and others. Within a few years contract bridge had so supplanted the other forms of the game that "bridge" became synonymous with "contract bridge".

Bridge occupies nowadays a position of great prestige being, together with chess, the only mind sports (i.e. games or skills where the mental component is more significant than the physical one) officially recognized by the International Olympic Committee. Millions of people play bridge worldwide, not only in clubs, tournaments and championships, but also on line (e.g. [10]) and with friends at home, making it one of the world's most popular card games. The World Bridge Federation (WBF) is the international governing body of contract bridge. WBF was formatted in August 1958 by delegates from Europe, North and South America and its membership now comprises 123 National Bridge Organizations, with about 700000 affiliated members.

A match of bridge can be played among teams (two or more) of four players (two partnerships). At the end of the match in this case the result is the difference in International Match Points (IMPs) between the competing teams and then there is a further conversion, in which some fixed number of Victory Points (VPs) is appointed between the teams. It is worth to notice that the table converting IMPs to VPs has been obtained through a rigorous mathematical manipulation [11].

However, the game usually played in tournaments is among fixed partnerships or pairs. For a pairs event a minimum of three tables (6 pairs, 12 players) is needed, but it works better with more players. The usual method of scoring in a pairs' competition is in match points. Each pair is awarded two match points for each pair who scored worse than them on each game's session (hand), and one match point for each pair who scored equally. The total number of match points scored by each pair over all the hands played is calculated and it is converted to a percentage. The pair succeeding the highest percentage wins the game. However, IMPs are also used as a method of scoring in special cases, in which the difference of each pair's IMPs is usually calculated with respect to the mean number of IMPs of all pairs.

For the fundamentals and the rules of the game of bridge, as well as for the conventions usually played between the partners the reader we refer to the famous book [13] of Edgar Kaplan (1925-1997), who was an American bridge player and one of the principal contributors to the game. Kaplan's book was translated in many languages and was reprinted many times since its first edition in 1964 [12].There is also a fair amount of bridge-related information on the Internet, e.g. see web sites [14, 15], etc.

3. The new assessment method

As we have already seen in the previous section, in a game of bridge the performance of each playing element (pair or team) is characterized by using either match points or IMPs. However, apart from the above official scoring methods, it is useful sometimes, for statistical or other reasons, to assess the total performance of certain sets of playing elements (single players, pairs, or teams) appearing to have a special interest. For example, this happens, when one wants to compare the performance of two or more clubs participating in a big tournament, the performance of male and female players or of old and young players, etc.

One way to do this is by calculating the means of the official scores obtained by the elements of the corresponding sets (mean performance). Here, we shall use principles of fuzzy logic in developing an alternative method of assessment, according to which the higher is an element's performance the more its "contribution" to the corresponding set's total performance (weighted performance). For this, we consider as set of the discourse the set $U = \{A, B, C, D, F\}$ of linguistic labels characterizing the playing elements performance, where A characterizes an excellent performance, B a very good, C a good, D a mediocre and F an unsatisfactory performance respectively. Obviously, the above characterizations are fuzzy depending on the user's personal criteria, which however must be compatible to the common logic, in order to be able to model the real situation in a worthy of credit way.

In case of a pairs' competition, for example, with match points as the scoring method and according to the usual standards of contract bridge, we can characterize the pairs' (or the players' individually) performance, according to the percentage of success, say p, achieved by them, as follows:

- Excellent (A), if $p > 65\%$.
- Very good (B), if $55\% < p \leqslant 65\%$
- Good (C), if $48\% < p \leqslant 55\%$.
- Mediocre (D), if $40\% \leqslant p \leqslant 48\%$.
- Unsatisfactory (F), if $p < 40\%$.

In an analogous way one could characterize the teamsâĂŹ (or pairs') performance with respect to the VPs, gained in bridge games played with IMPs.

Assume now that one wants to assess the total performance of a special set, say S, of n playing pairs (or players'), where n is an integer, $n \geqslant 2$. We are going to represent S as a fuzzy subset of U. For this, if n_A, n_B, n_C, n_D and n_E denote the number of pairs/players of S that had demonstrated an excellent, very good, good, mediocre and unsatisfactory performance respectively at the game, we define the membership function $m : U \to [0,1]$ in terms of the frequencies, i.e. by $m(x) = \frac{n_s}{n}$, for each x in U. Then S can be written as a fuzzy subset of U in the form: $S = \left\{\left(x, \frac{n_s}{n}\right) : x \in U\right\}$.

In converting the fuzzy data collected from the game we shall make use of the defuzzification technique known as the method of the centre of gravity (COG). According to this method, the centre of gravity of the graph of the membership function involved provides an alternative measure of the systemâĂŹs performance. The application of the COG method in practice is simple and evident and, in contrast to other defuzzification techniques in use, like the measures of uncertainty (for example see [4] and its relevant references, or [7]), needs no complicated calculations in its final step. The techniques that we shall apply here have been also used earlier in [6, 9, 17], etc.

The first step in applying the COG method is to correspond to each $x \in U$ an interval of values from a prefixed numerical distribution, which actually means that we replace U with a set of real intervals. Then, we construct the graph, say G, of the membership function $y = m(x)$. There is a commonly used in fuzzy logic approach to measure performance with the coordinates $(x_c,\ y_c)$ of the centre of gravity, say F_e, of the graph G, which we can calculate using the following well-known from Mechanics formulas:

$$x_c = \frac{\iint\limits_F x\,dx\,dy}{\iint\limits_F dx\,dy},\ y_c = \frac{\iint\limits_F y\,dx\,dy}{\iint\limits_F dx\,dy} \tag{1}$$

In our case we characterize a pair's performance as unsatisfactory (F), if $x \in [0,1)$, as mediocre (D), if $x \in [1,2)$, as good (C), if $x \in [2,3)$, as very good (B), if $x \in [3,4)$ and as excellent (A), if $x \in [4,5]$ respectively. In other words, if $x \in [0,1)$, then $y_1 = m(x) = m(F) = \frac{n_F}{n}$, if $x \in [1,2)$, then $y_2 = m(x) = m(D) = \frac{n_D}{n}$ etc.

Therefore in our case the graph G of the membership function attached to S is the bar graph of Fig. 1 consisting of five rectangles, say $G_i, i = 1,2,3,4,5$, whose sides lying on the X axis have length 1. In this case $\iint\limits_F dx\,dy$ is the area of G which is equal to

$$\sum_{i=1}^{5} y_i = \frac{n_F + n_D + n_C + n_B + n_A}{n} = 1.$$

Also,

$$\begin{aligned}\iint\limits_F x\,dx\,dy &= \sum_{i=1}^{5} \iint\limits_{F_i} x\,dx\,dy \\ &= \sum_{i=1}^{5} \int_0^{y_i} dy \int_{i-1}^{i} x\,dx \\ &= \sum_{i=1}^{5} y_i \int_{i-1}^{i} x\,dx \\ &= \frac{1}{2}\sum_{i=1}^{5}(2i-1)y_i\end{aligned}$$

and

$$\begin{aligned}\iint\limits_F y\,dx\,dy &= \sum_{i=1}^{5} \iint\limits_{F_i} y\,dx\,dy \\ &= \sum_{i=1}^{5} \int_0^{y_i} y\,dy \int_{i-1}^{i} dx \\ &= \sum_{i=1}^{n} \int_0^{y_i} y\,dy \\ &= \frac{1}{2}\sum_{i=1}^{n} {y_i}^2\end{aligned}$$

Therefore formulas (1) are transformed into the following form:

$$\begin{aligned} x_c &= \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right) \\ y_c &= \frac{1}{2}\left(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2\right)\end{aligned} \tag{2}$$

But, $0 \leqslant (y_1 - y_2)^2 = {y_1}^2 + {y_2}^2 - 2y_1y_2$, therefore ${y_1}^2 + {y_2}^2 \geqslant 2y_1y_2$, with the equality holding if, and only if $y_1 = y_2$. In the same way one finds that ${y_1}^2 + {y_3}^2 \geqslant 2y_1y_3$, with the equality holding if, and only if, $y_1 = y_3$ and so on. Hence

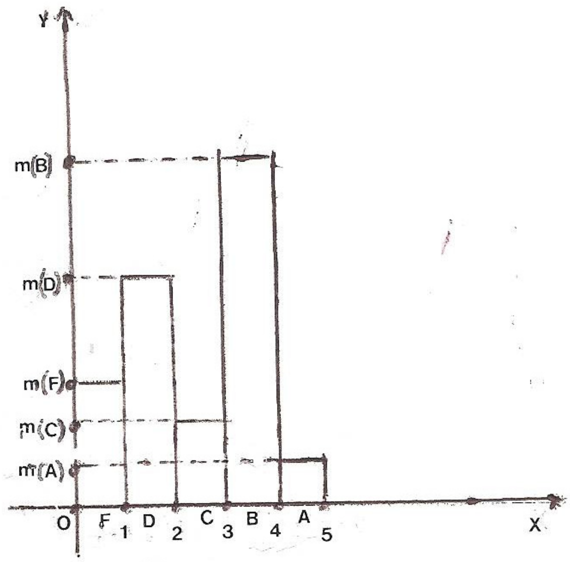

Fig. 1. Bar graphical data representation

it is easy to check that $\left(y_1+y_2+y_3+y_4+y_5\right)^2 \leqslant 5\left(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2\right)$ with the equality holding if, and only if, $y_1=y_2=y_3=y_4=y_5$. But $y_1+y_2+y_3+y_4+y_5=1$, therefore

$$1 \leqslant 5\left(y_1^2+y_2^2+y_3^2+y_4^2+y_5\right), \tag{3}$$

with the equality holding if, and only if, $y_1+y_2+y_3+y_4+y_5=\frac{1}{5}$.

Then the first of (2) gives that $x_c=\frac{5}{2}$. Further, combining the inequality (3) with the second of formulas (2), one finds that $1 \leqslant 10y_c$, or $y_c \geqslant \frac{1}{10}$. Therefore the unique minimum for y_c corresponds to the centre of gravity$F_m\left(\frac{5}{2},\frac{1}{10}\right)$.

The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (2) we get that $x_c=\frac{9}{2}$ and $y_c=\frac{1}{2}$. Therefore the centre of gravity in this case is the point $F_i\left(\frac{9}{2},\frac{1}{2}\right)$.
On the other hand, in the worst case $y_1=1$ and $y_2=y_3=y_4=\ y_5=0$. Then by formulas (2), we find that the centre of gravity is the point $F_w\left(\frac{1}{2},\frac{1}{2}\right)$.

Therefore the "area" where the centre of gravity F_c lies is represented by the triangle F_w, F_m, F_i of Fig. 2.

Then from elementary geometric considerations it follows that the greater is the value of x_c the better is the corresponding group's performance. Also, for two groups with the same $x_c \geqslant 2,5$, the group having the centre of gravity which is situated closer to F_i is the group with the higher y_c; and for two groups with the same $x_c < 2.5$ the group having the centre of gravity which is situated farther to F_w is the group with the lower y_c. Based on the above considerations it is logical to formulate our criterion for comparing the groups performances in the following form:

- Among two or more groups the group with the higher x_c performs better.
- If two or more groups have the same $x_c \geqslant 2.5$, then the group with the higher y_c performs better.
- If two or more groups have the same $x_c < 2.5$, then the group with the lower y_c performs better.

4. Real applications

The Hellenic Bridge Federation (HBF) organizes, on a regular basis, simultaneous bridge tournaments (pair events) with pre-dealt boards, played by the local clubs in several cities of Greece. Each of these tournaments consists of six in total events, played in a particular day of the week (e.g. Wednesday), for six successive weeks. In each of these events there is a local scoring table (match points) for each participating club, as well as a central scoring table, based on the local results of all participating clubs, which are compared to each other. At the end of the tournament

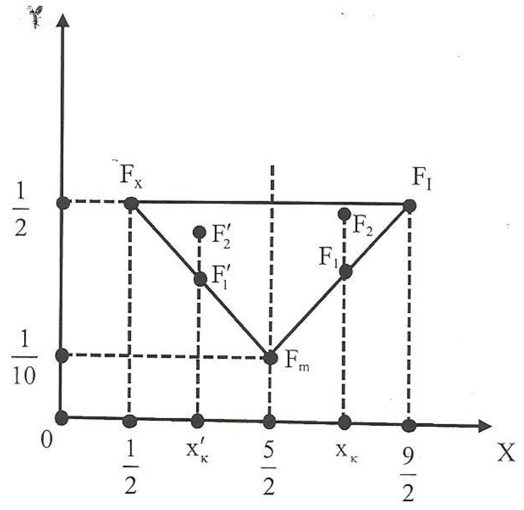

Fig. 2. Graphical representation of the "area" of the centre of gravity

it is also formed a total scoring table in each club, for each player individually. In this table each player's score equals to the mean of the scores obtained by him/her in the five of the six in total events of the tournament. If a player has participated in all the events, then his/her worst score is dropped out. On the contrary, if he/she has participated in less than five events, his/her name is not included in this table and no possible extra bonuses are awarded to him/her.
In this section and in order to illustrate the importance of our results obtained in the previous section, we shall present two real applications connected to the above simultaneous tournaments.

The first application concerns the third event of such a simultaneous tournament played on Wednesday, March 12, 2014, in which participated 17 in total clubs from several cities of Greece (see results in [16]. Among those clubs were included the two bridge clubs, lets call them C1 and C2 respectively, of the city of Patras. Nine in total pairs from club C_1 played in this event obtaining the following scores in the central scoring table: 62.67%, 57.94%, 56.04%, 55.28%, 50.43%, 46%, 44.75%, 39.91% and 36.16%. Eight in total pairs from club C_2 played also in the same event obtaining the following scores: 63.14%, 57.64%, 56.86%, 50.17%, 50.13%, 43.28%, 42.11% and 36.63%. The above scores give an average percentage 49.909% for the first and 49.995% for the second club. This means that the second club demonstrated a slightly better mean performance than the first one, but the difference was marginal; only 0.086%. The above results are summarized in Table 1.

Table 1. Results of the two bridge clubs of Patras

% Scale	First club (C_1)			Second club (C_2)		
	Performance	Amount of pairs	$m(x)$	Performance	Amount of pairs	$M(x)$
$> 65\%$	A	0	0	A	0	0
$55-65\%$	B	4	4/9	B	3	3/8
$48-55\%$	C	1	1/9	C	2	2/8
$40-48\%$	D	2	2/9	D	2	2/8
$< 40\%$	F	2	2/9	F	1	1/8
Total		9			8	

Then, using the first of formulas (2) of the previous section one finds that $x_c = \frac{1}{2}\left(\frac{2}{9} + 3.\frac{2}{9} + 5.\frac{1}{9} + 7.\frac{4}{9}\right) = 2.278$ for the first club, and $x_c = \frac{1}{2}\left(\frac{1}{8} + 3.\frac{2}{8} + 5.\frac{2}{8} + 7.\frac{3}{8}\right) = \frac{38}{16} = 2.375$ for the second club. Therefore, according to our criterion (first case) stated in the previous section, the second club demonstrated a better weighted performance than the first one, but the difference is small again; just 0.097 units.

The second application is related to the total scoring table of the players of club C_1, who participated in at least five of the six in total events of another simultaneous tournament organized by the HBF, which ended on February 19, 2014 (see results in [5]). Nine men and five women players are included in this table, who obtained the following scores. Men: 57.22%, 54.77%, 54.77%, 54.35%, 54.08%, 50.82%, 50.82%, 49.61%, 47.82%. Women: 59.48%, 54.08%, 53.45%, 53.45%, 47.39%. The above results give a mean percentage of approximately 52.696% for the men and 53.57% for the women players. Therefore the women demonstrated a slightly better mean performance than the men players, their difference being 0.874%. The above results are summarized in Table 2.

Table 2. Results of the two bridge clubs of Patras

% Scale	Men			Women		
	Performance	Amount of pairs	$m(x)$	Performance	Amount of pairs	$M(x)$
$>65\%$	A	0	0	A	0	0
$55-65\%$	B	1	1/9	B	1	1/5
$48-55\%$	C	7	7/9	C	3	3/5
$40-48\%$	D	1	1/9	D	1	1/5
$<40\%$	F	0	0	F	0	0
Total		9			5	

Thus, according to the first of formulas (2) of the previous section, we find that $x_c = \frac{1}{2}\left(3.\frac{1}{9}+5.\frac{7}{9}+7.\frac{1}{9}\right) = \frac{45}{18} = 2.5$ for the men players, and $x_c = \frac{1}{2}\left(3.\frac{1}{5}+5.\frac{3}{5}+7.\frac{1}{5}\right) = \frac{25}{10} = 2.5$ for the women players.

Further, the second of formulas (2) gives $y_c = \frac{1}{2}\left[\left(\frac{1}{9}\right)^2+\left(\frac{7}{9}\right)^2+\left(\frac{1}{9}\right)^2\right] = \frac{51}{162} \simeq 0.315$ for the men and $y_c = \frac{1}{2}\left[\left(\frac{1}{5}\right)^2+\left(\frac{3}{5}\right)^2+\left(\frac{1}{5}\right)^2\right] = \frac{11}{50} = 0.22$ for the women players. Thus, according to our criterion (second case) and in contrast to the mean performance, the men demonstrated a higher weighted performance than the women players.

5. Conclusions and discussion

In the present paper we developed a new method for assessing the total performance of certain groups of pairs or teams or of bridge players individually, appearing to have a special interest. In developing the above method we represented each of the groups of players' under assessment as a fuzzy subset of a set U of linguistic labels characterizing the bridge players' performance and we used the COG defuzzification technique in converting the fuzzy data collected from the game to a crisp number. According to the above assessment method the higher is an element's performance the more its "contribution" to the corresponding groupâĂŹs total performance (weighted performance). Thus, in contrast to the mean of the scores of all the groupâĂŹs elements, which is connected to the mean groupâĂŹs performance, our method is connected somehow to the groupâĂŹs quality performance. As a result, when the above two different assessment methods are used in comparing the performance of two or more groups of bridge players, the results obtained may differ to each other in certain cases, where there are marginal differences in the groups' performance.

Two real applications were also presented, related to simultaneous tournaments (pair events) organized by the HBF. In the first of these applications we compared the total performance of the two bridge clubs of the city of Patras in a particular event of a recent such tournament, while in the second one we compared the performance of the men and women players of one of the above clubs, based on their total scoring in the six events of another simultaneous tournament. In general, our method is suitable to be applied in parallel with the official bridge scoring methods (match points or IMPs) for statistical and other obvious reasons.

Our future plans for further research on the subject aim at applying our new assessment method in more real situations, including also bridge games (pairs or teams) played with IMPs, in order to get statistically safer and more solid conclusions about its applicability and usefulness. In a wider basis, since our method is actually a general assessment method, it could be extended to cover other sectors of the human activity as well, apart from the studentsâĂŹ (e.g. see [4, 5, 8, 9, 18], etc) and the bridge players' assessment (in this paper), where we have already applied it.

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