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New exact solutions for coupled equal width wave equation and (2+1)-dimensional Nizhnik-Novikov-Veselov system using modified Kudryashov method

Research Article

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Abstract: In this paper, the modified Kudryashov method or the rational Exp-function method with the aid of symbolic computation has been proposed to construct exact solutions of both the coupled equal width wave equation and the (2+1)-dimensional Nizhnik-Novikov-Veselov equations. As a result, some new types of exact traveling and solitary wave solutions are obtained, with comparison of the other solution obtained before in literature, which include exponential function, hyperbolic function and trigonometric function. The related results are extend. Obtained results clearly indicate the reliability and efficiency of the modified Kudryashov method.

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Keywords: Exact solutions • Coupled equal width wave equation • (2+1)-dimensional Nizhnik-Novikov-Veselov system • Modified Kudryashov method

1. Introduction

As the mathematical model of complex physics phenomena, nonlinear partial differential equations are involved in many fields from physics to biology, chemistry and engineering, ... etc. In the past decades, great efforts have been made to search for powerful methods to obtain exact solutions. The investigation of exact solutions of nonlinear wave equations plays an important role in the study of nonlinear physical phenomena. There exist some methods such as inverse scattering method [1], Hirota's method [2], homogeneous balance method [3], Jacobi elliptic function method [4], extended tanh-function method [5], Bäcklund transformation method [6], algebraic method [7], sine-cosine method [8], Homotopy perturbation method [9–11], Variational iterative method [12], Homotopy analysis method [13], [14], F-expansion method [15–17] and so on, which proposed to construct periodic wave solutions of nonlinear partial differential equations. For recent developments about the subject, see refs. [18–20]
Here, we aim to shed more light on the coupled equal width wave equation given by [21].

$$u_t + u u_x - u_{xxt} + v v_x = 0 \tag{1}$$
$$v_t + v v_x - v_{xxt} = 0 \tag{2}$$

where the subscripts t and x denoting to the differentiation with respect to time and space respectively. Also, the (2+1)-dimensional Nizhnik-Novikov-Veselov system given by

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$$u_t + k u_{xxx} + r u_{yyy} + s u_x + q u_y - 3k(u v_x + u_x v) - 3r(u w_y + u_y w) = 0 \tag{3}$$
$$u_x = v_y \tag{4}$$
$$u_y = w_x \tag{5}$$

where k, r, s and q are constants, Eqs. (3)-(5) studied using Jacobi elliptic function method, extended hyperbolic function method, further extended tanh-function method, extended mapping method and similarity reduction solutions, and the balancing procedure, see refs. [22–27].
On the other hand, He and Wu [28] developed the exp-function method to seek the solitary, periodic and compaction like solutions of nonlinear differential equations. It is an effective and simple method and is widely used. Based on this method, modified exp-function expansion method is proposed. Hence, in this paper, we shall use the modified Kudryashov method (the rational Exp-function method) [29, 30] to obtain new exact solitary wave solutions of both the coupled equal width wave equation and the (2+1)-dimensional Nizhnik-Novikov-Veselov system. To the best of our knowledge, this study has not been investigated yet.

2. The Modified Kudryashov Method

To illustrate the basic idea of the modified Kudryashov method, we first consider a general form of nonlinear equation

$$p(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \ldots) = 0 \tag{6}$$

where p is a polynomial function with respect to the indicated variables.
Making use of the travelling wave transformation

$$u = u(\xi), \quad \xi = \alpha(x - \beta t) \tag{7}$$

where α and β are arbitrary constants to be determined later, then Eq. (6) reduces to a nonlinear ordinary differential equation

$$p(u, -\alpha\beta u', \alpha u', \alpha^2 u'', \alpha^2\beta^2 u'', -\alpha^2\beta u'', \ldots) = 0 \tag{8}$$

In this section, we shall seek a rational function type of solution for a given partial differential equation, in terms of $\exp(\xi)$, of the following form

$$u(\xi) = \sum_{k=0}^{m} \frac{a_k}{[1+\exp(\xi)]^k} \tag{9}$$

where $a_0, a_1, \ldots, a_m$ are constants to be determined. We can determine m by balance the linear term of the highest order in Eq. (8) with the highest order nonlinear term.
Differentiating Eq. (9) with respect to ξ, introducing the result into Eq. (8), and setting the coefficients of the same power of $\exp(\xi)$ equal to zero, we obtain algebraic equations. The rational function solution of the Eq. (6) can be solved by obtaining $a_0, a_1, \ldots, a_m$ from this system [29].

3. Solutions of coupled equal width wave equation

we consider the coupled equal width wave equations, in the normalized form

$$u_t + u u_x - u_{xxt} + v v_x = 0 \tag{10}$$
$$v_t + v v_x - v_{xxt} = 0 \tag{11}$$

By using the transformation

$$u(x,t) = U(\xi), \; v(x,t) = V(\xi), \; \xi = \alpha(x - \beta t) \tag{12}$$

where α and β are arbitrary constant, then Eqs. (10) and (11) become

$$-\alpha\beta U' + \alpha U U' + \alpha^3 \beta U''' + \alpha V V' = 0 \tag{13}$$
$$-\alpha\beta V' + \alpha V V' + \alpha^3 \beta V''' = 0 \tag{14}$$

In order to determine values of m and n, we balance the linear term of the highest order partial derivative terms and the highest order nonlinear terms in Eqs. (13) and (14), then we get $m = n = 2$.
By using the rational function in $\exp(\xi)$, we may choose the solutions of Eqs. (13) and (14) in the form

$$U(\xi) = a_0 + \frac{a_1}{[1+\exp(\xi)]} + \frac{a_2}{[1+\exp(\xi)]^2} \tag{15}$$
$$V(\xi) = b_0 + \frac{b_1}{[1+\exp(\xi)]} + \frac{b_2}{[1+\exp(\xi)]^2} \tag{16}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are arbitrary constants to be determined later.
Differentiating Eqs. (15) and (16) with respect to ξ, introducing the result into Eqs. (13) and (14), and setting the coefficients of the same power of $\exp(\xi)$ equal to zero, we obtain the following algebraic equations

$$\begin{aligned}&-2a_0a_2 - a_1^2 - 3a_1a_2 - 2a_2^2 - b_0b_1 - 2b_0b_2 - b_1^2 - 3b_1b_2\\&-2b_2^2 - \alpha^2\beta a_1 + \beta a_1 + 2\beta a_2 - a_0a_1 - 2\alpha^2\beta a_2 = 0\end{aligned} \tag{17}$$

$$\begin{aligned}&4\beta a_2 - 4b_0b_2 - 3a_0a_1 + 3\alpha^2\beta a_1 - 3b_1b_2 - 3b_0b_1 - 4a_0a_2\\&-3a_1a_2 + 14\alpha^2\beta a_2 - 2a_1^2 - 2b_1^2 + 3\beta a_1 = 0\end{aligned} \tag{18}$$

$$\begin{aligned}&-2a_0a_2 - b_1^2 - 2b_0b_2 - a_1^2 + 3\beta a_1 + 2\beta a_2 - 3a_0a_1\\&+3\alpha^2\beta a_1 - 8\alpha^2\beta a_2 - 3b_0b_1 = 0\end{aligned} \tag{19}$$

$$-b_0b_1 - \alpha^2\beta a_1 + \beta a_1 - a_0a_1 = 0 \tag{20}$$

$$2\alpha^2\beta b_2 - 2\beta b_2 + b_0b_1 + 2b_0b_2 + b_1^2 + 3b_1b_2 + 2b_2^2 + \alpha^2\beta b_1 - \beta b_1 = 0 \tag{21}$$

$$-4\beta b_2 + 4b_0b_2 + 2b_1^2 + 3b_1b_2 - 3\alpha^2\beta b_1 - 3\beta b_1 - 14\alpha^2\beta b_2 + 3b_0b_1 = 0 \tag{22}$$

$$b_1^2 + 8\alpha^2\beta b_2 + 3b_0b_1 - 2\beta b_2 - 3\beta b_1 - 3\alpha^2\beta b_1 + 2b_0b_2 = 0 \tag{23}$$

$$\alpha^2\beta b_1 - \beta b_1 + b_0b_1 = 0 \tag{24}$$

Solving the system of algebraic Eqs. (17)-(24) with the aid of Maple, we obtain two cases of solutions

3.1. Case (1)

$$\begin{aligned}&a_0 = \frac{\beta}{2}(1+i\sqrt{3})(1-\alpha^2),\ \mathrm{b}_0 = \beta(1 - \alpha^2)\\&a_1 = 6\alpha^2\beta(1+i\sqrt{3}),\\&\mathrm{b}_1 = 12\alpha^2\beta\mathrm{a}_2 = -\ \mathrm{a}_1, \qquad \mathrm{b}_2 = -\ \mathrm{b}_1\end{aligned} \tag{25}$$

By back substitution in Eqs. (15) and (16) with Eq. (12), new exact solution for the coupled equal width wave equation is obtained

$$u_1(x,t) = 6\beta(1+i\sqrt{3})\left[\frac{(1-\alpha^2)}{12} + \frac{\alpha^2\exp\alpha(x-\beta t)}{[1+\exp\alpha(x-\beta t)]^2}\right] \tag{26}$$

$$v_1(x,t) = \beta\left\{(1-\alpha^2) + \frac{12\alpha^2\exp\alpha(x-\beta t)}{[1+\exp\alpha(x-\beta t)]^2}\right\} \tag{27}$$

3.2. Case (2)

$$
\begin{aligned}
&a_0=\frac{\beta}{2}(1-i\sqrt{3})(1-\alpha^2),\ \ b_0=\beta(1-\alpha^2)\\
&a_1=6\alpha^2\beta(1-i\sqrt{3}),\ \ b_1=12\alpha^2\beta\\
&a_2=-a_1,\ \ b_2=-b_1
\end{aligned}
\tag{28}
$$

The following new exact solution for the coupled equal width wave equation is given by

$$
u_2(x,t)=\beta(1-i\sqrt{3})\left\{\frac{(1-\alpha^2)}{2}+\frac{6\alpha^2\exp\alpha(x-\beta t)}{\left[1+\exp\alpha(x-\beta t)\right]^2}\right\}
\tag{29}
$$

$$
v_2(x,t)=\beta\left\{(1-\alpha^2)+\frac{12\alpha^2\exp\alpha(x-\beta t)}{\left[1+\exp\alpha(x-\beta t)\right]^2}\right\}
\tag{30}
$$

4. Solutions of (2+1)-dimensional Nizhnik-Novikov-Veselov system

Eqs. (3)-(5) can be rewritten as

$$
u_t+ku_{xxx}+ru_{yyy}+su_x+qu_y=3k(uv_x+u_xv)+3r(uw_y+u_yw)
\tag{31}
$$

$$
u_x=v_y
\tag{32}
$$

$$
u_y=w_x
\tag{33}
$$

By using the transformation

$$
u(x,y,t)=U(\xi),\ v(x,y,t)=V(\xi),\ w(x,y,t)=W(\xi),\ \xi=\alpha(x+\gamma y-\beta t)
\tag{34}
$$

where α, β and γ are arbitrary constant, then Eqs. (31)-(33) become

$$
\begin{aligned}
&-\alpha\beta U'+k\alpha^3U'''+r\alpha^3\gamma^3U'''+s\alpha U'+q\alpha\gamma U'\\
&-3k\alpha(UV'+U'V)-3r\alpha\gamma(UW'+U'W)=0
\end{aligned}
\tag{35}
$$

$$
\alpha U'-\alpha\gamma V'=0
\tag{36}
$$

$$
\alpha\gamma U'-\alpha W'=0
\tag{37}
$$

In order to determine values of m, n and l , we balance the linear term of the highest order partial derivative terms and the highest order nonlinear terms in Eqs. (35)-(37), then we get $m=n=l=2$.
By using the rational function in $\exp(\xi)$, the solutions of Eqs. (35)-(37) can be written in the form

$$
U(\xi)=a_0+\frac{a_1}{[1+\exp(\xi)]}+\frac{a_2}{[1+\exp(\xi)]^2}
\tag{38}
$$

$$
V(\xi)=b_0+\frac{b_1}{[1+\exp(\xi)]}+\frac{b_2}{[1+\exp(\xi)]^2}
\tag{39}
$$

$$
W(\xi)=c_0+\frac{c_1}{[1+\exp(\xi)]}+\frac{c_2}{[1+\exp(\xi)]^2}
\tag{40}
$$

where a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1 and c_2 are arbitrary constants to be determined.
Differentiating Eqs. (38)-(40) with respect to ξ, introducing the result into Eqs. (35)-(37), and setting the coefficients of the same power of $\exp(\xi)$ equal to zero, we obtain the following algebraic equations

$$
\begin{aligned}
&-\beta a_1-9ka_2b_1+2k\alpha^2a_2-12ka_2b_2-6ka_1b_1-6ka_0b_2+r\alpha^2a_1\\
&+k\alpha^2a_1-9ka_1b_2-3ra_0c_1-6ka_2b_0-3ka_0b_1+2r\alpha^2a_2-6ra_0c_2\\
&-6ra_1c_1-9ra_1c_2+qa_1+sa_1-12ra_2c_2+2sa_2-2\beta a_2-3ra_1c_0\\
&+2qa_2-3ka_1b_0-9ra_2c_1-6ra_2c_0=0
\end{aligned}
\tag{41}
$$

$$\begin{aligned}&-9ka_2b_1+3sa_1-12ra_2c_0-9ka_1b_2-4\beta a_2-9ra_0c_1-9ka_0b_1\\&-3\beta a_1+4qa_2+3qa_1-9ra_1c_0-12ka_1b_1-9ka_1b_0+4sa_2\\&-9ra_2c_1-14r\alpha^2a_2-12ra_1c_1-9ra_1c_2-3k\alpha^2a_1-12ra_0c_2\\&-12ka_2b_0-3r\alpha^2a_1-14k\alpha^2a_2-12ka_0b_2=0\end{aligned} \tag{42}$$

$$\begin{aligned}&-2\beta a_2+3sa_1-3k\alpha^2a_1-6ka_2b_0+2qa_2+8k\alpha^2a_2-6ra_2c_0\\&+2sa_2-6ra_1c_1-3\beta a_1-9ka_0b_1+8r\alpha^2a_2-9ra_1c_0-9ra_0c_1\\&-3r\alpha^2a_1+3qa_1-6ka_1b_1-9ka_1b_0-6ra_0c_2-6ka_0b_2=0\end{aligned} \tag{43}$$

$$qa_1-\beta a_1-3ka_1b_0-3ra_1c_0-3ka_0b_1+k\alpha^2a_1+r\alpha^2a_1-3ra_0c_1+sa_1=0 \tag{44}$$

$$\gamma b_1-a_1=0 \tag{45}$$

$$-a_1+2\gamma b_2-2a_2+\gamma b_1=0 \tag{46}$$

$$\gamma a_1-2c_2+2\gamma a_2-c_1=0 \tag{47}$$

$$-c_1+\gamma a_1=0 \tag{48}$$

Solving the system of algebraic Eqs. (41)-(48) with the aid of Maple, we obtain two cases of solutions

4.1. Case 1

$$\begin{aligned}&a_0=a_0,\ b_0=\frac{\beta-q\gamma+3r\gamma c_0-s}{3r\gamma^3},\ c_0=c_0,\ \gamma^3=-\frac{k}{r}\\&(a_1,a_2)=\gamma(b_1,b_2),\ b_1=b_1,\ b_2=b_2,\ (c_1,c_2)=\gamma^2(b_1,b_2)\end{aligned} \tag{49}$$

By back substitution new exact solution for the (2+1)-dimensional Nizhnik-Novikov-Veselov equations is obtained

$$u_1(x,y,t)=a_0+\frac{\sqrt[3]{-\frac{k}{r}}\,b_1}{\left[1+\exp\left\{\alpha\left(x+\sqrt[3]{-\frac{k}{r}}\,y-\beta t\right)\right\}\right]}+\frac{\sqrt[3]{-\frac{k}{r}}\,b_2}{\left[1+\exp\left\{\alpha\left(x+\%\sqrt[3]{-\frac{k}{r}}\,y-\beta t\right)\right\}\right]^2} \tag{50}$$

$$\begin{aligned}v_1(x,y,t)=&-\frac{(\beta-\mathrm{s})+\sqrt[3]{-\frac{\mathrm{k}}{\mathrm{r}}}(3\mathrm{rc}_0-\mathrm{q})\%}{3k}+\frac{b_1}{\left[1+\exp\left\{\alpha\left(x+\sqrt[3]{-\frac{k}{r}}\%y-\beta t\right)\right\}\right]}\\&+\frac{b_2}{\left[1+\exp\left\{\alpha\left(x+\sqrt[3]{-\frac{k}{r}}\,y-\beta t\right)\right\}\right]^2}\end{aligned} \tag{51}$$

$$w_1(x,y,t)=c_0+\frac{\left(-\frac{k}{r}\right)^{\frac{2}{3}}b_1}{\left[1+\exp\left\{\alpha\left(x+\sqrt[3]{-\frac{k}{r}}\,y-\beta t\right)\right\}\right]}+\frac{\left(-\frac{k}{r}\right)^{\frac{2}{3}}b_2}{\left[1+\exp\left\{\alpha\left(x+\sqrt[3]{-\frac{k}{r}}\,y-\beta t\right)\right\}\right]^2} \tag{52}$$

4.2. Case 2

$$\begin{aligned}
&a_0=\frac{(s-\beta+k\alpha^2-3kb_0)\gamma+(q-3rc_0)\gamma^2+r\alpha^2\gamma^4}{3(r\gamma^3+k)},\\
&b_0=b_0,\ c_0=c_0,\\
&(a_1,\ a_2)=2\alpha^2\gamma(-1,\ 1),\ (b_1,\ b_2)=2\alpha^2(-1,\ 1),\ (c_1,\ c_2)=2\alpha^2\gamma^2(-1,\ 1)
\end{aligned} \tag{53}$$

By back substitution we get the following new exact solution for the (2+1)-dimensional Nizhnik-Novikov-Veselov equations

$$u_2(x,\ y,\ t)\ =\frac{(s-\beta\gamma+k\alpha^2-3kb_0)\gamma+(q-3rc_0)\gamma^2+r\alpha^2\gamma^4}{3(r\gamma^3+k)}-\frac{2\alpha^2\gamma\exp\{\alpha(x+\gamma y-\beta t)\}}{\left[1+\exp\{\alpha(x+\gamma y-\beta t)\}\right]^2} \tag{54}$$

$$v_2(x,\ y,\ t)\ =b_0-\frac{2\alpha^2\exp\{\alpha(x+\gamma y-\beta t)\}}{\left[1+\exp\{\alpha(x+\gamma y-\beta t)\}\right]^2} \tag{55}$$

$$w_2(x,\ y,\ t)\ =c_0-\frac{2\alpha^2\gamma^2\exp\{\alpha(x+\gamma y-\beta t)\}}{\left[1+\exp\{\alpha(x+\gamma y-\beta t)\}\right]^2} \tag{56}$$

5. Conclusions

In this paper, we have applied the modified Kudryashov method or the rational Exp-function method on both the coupled equal width wave equation and (2+1)-dimensional Nizhnik-Novikov-Veselov system, respectively. Some new exact wave solutions of both equations under consideration are successfully found. Compared to the methods used before, one can see that this method is concise and effective, and it can be applied to other nonlinear problems of physical interest. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas described by nonlinear evolution equations.

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