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Buckling analysis of functionally graded beams with imperfectly integrated surface piezoelectric layers under low velocity

Article category

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Abstract: This paper focuses on the mechanical buckling analysis of functionally graded beam with piezoelectric layers under low velocity. Simply supported boundary condition considered at both ends of the beam.Fundamental relations, the equilibrium and stability equations derived using the first-order shear deformation theory (FOST) and the Engesser-Timoshenko beam theory (ETB). The influences of periodic loads, dimensionless geometric parameter, functionally graded index and piezoelectric thickness on the critical buckling load of beam presented. To study the precision and accuracy of the present analysis, a compression carried out with a known data and the finite element package ABAQUSE.

MSC: 70H05 • 35Mxx

Keywords: Functionally graded beam • piezoelectric • Mechanical buckling • First-order shear deformation theory • periodic loads • Engesser-Timoshenko beam theory

1. Introduction

FGMs are microscopic inhomogeneous composite made of different phases of material constituents are increasingly used in many engineering fields specially for working high temperature environment applications such as heat exchanger tubes, thermo-electric generators, furnace linings, electrically insulated metal ceramic joints and biomedical. Stability considered one of the most important engineering issues in the design and application of slender structures. Piezoelectric is one of the most popular smart materials which used in vibration control of smart structures. There have been many researches on the field of piezoelectric sensors and actuator applications [1–3]. Piezoelectric actuators are used to enhance the performance of a structural system by inducing a favorable structural deformation. Detailed models on the iteration between piezoelectric sensors or actuators with the structure to which they are bonded or embedded have been developed [4–6]. Many theoretical and mathematical models have been presented for laminated composite structures with piezoelectric sensors and actuators [7–9]. The buckling behavior of composite beams and plates with piezoelectric patches bonded to their surfaces or with piezoelectric layers embedded has been the topic of a number of investigations [10–12]. The aspect of vibration control of plates

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by piezoelectric materials was studied by Yang and Huang [13] and PiÃľfort et al [14]. These models are based on the classical theory of laminated plates which neglects the effects of the transverse shear. Finite element model to predict the vibrations of the piezoelectric actuators is presented by Taleghani and Campbell [15]. Basic actuator and sensor equations of composite shell structures with piezoelectric layers are presented by Tzou and Gadre [16] and Tzou [17]. A mathematical model for beams with partially delaminated layers is presented to investigate their behavior by using Euler-Bernoulli beam theory by Ali Mahieddine et all. [18] . The main advantage of the element is that it permits the modeling of delamination anywhere in the construction. Pradhan and Murmu analyzed the free vibration of FGM sandwich beam including thickness variations in thermal field [19]. Double power-series solutions are developed in thickness and width directions. Wang and Queck [20, 21] analyzed the free vibration problem of a beam integrated with piezoelectric layer(s) based on the classical beam theory. In the field of geometric nonlinear analysis, Simoes Moita et al. [22] presented a finite element model based on the Kirchhoff classical laminated theory for geometric non-linear static analysis of plate/shell laminated structures with piezoelectric sensors and actuators. Huang and Shen [23] investigated the non-linear vibration and dynamic response of FG plates with surface-bonded piezoelectric layers in thermal environments using the higher-order shear deformation theory (HSDT). In recent research, there is no analytical solution for buckling analysis of functionally graded Engesser-Timoshenko beams with piezoelectric layers. In this reasearch, buckling analisys of a functionally graded Engesser-Timoshenko beam with piezoelectric actuators under axial compressive loads is studied. The equilibrium equations of beam for obtaining the critical buckling load are derived using ETB theory and hamilton's principle. The effects of the periodic loads, dimensionless geometrical parameter and functionally graded index on the critical buckling load of beam are presented.

2. Formulation

2.1. Material properties

Consider a functionally graded beam with piezoelectric actuators and rectangular cross-section and subjected to the axial compressive loads (P) that shown in Fig. 1.

Fig. 1. Schematic of the beam studied with boundary condition,under periodic loads.

The thickness, length and width of the beam respectively showed by h, L and b. Also, h_T and h_B are the thickness of top and bottom of piezoelectric actuators. The material properties of all three layers are assumed to vary continuously through their local thickness direction according to a power law distribution of the volume fraction of the constituent materials. The *x*-*y* plane coincides with the midplane of the beam and the *z*-axis located along the thickness direction. The properties of the FGM beam are assumed to be vary through the thickness of the plate only, such that the top surface $z = h/2$ is ceramic-rich ani the bottom surface $h= -h/2$ is metcl rich. The Young's modulus E af follows [24]:

$$\begin{aligned} &E(z) = (E_c - E_m)V + E_m \\ &\rho(z) = (\rho_c - \rho_m)V + \rho_m \qquad V = \left(\frac{2z+h}{2h}\right)^k \end{aligned} \tag{1}$$

Where k is the power law index and the subscripts m and c refer to the metal and ceramic constituents, respectively. Fig. 2 shown the changes of Young's modulus in the FG beam.

2.2. Kinematic and constitutive equations

Displacement fields can write as [25]:

$$\begin{aligned} U(x,y,z) &= u_0(x,y,z) - z\phi_x(x,y) \\ V(x,y,z) &= v_0(x,y,z) - z\phi_y(x,y) \\ W(x,y,z) &= w_0(x,y,z) \end{aligned} \tag{2}$$

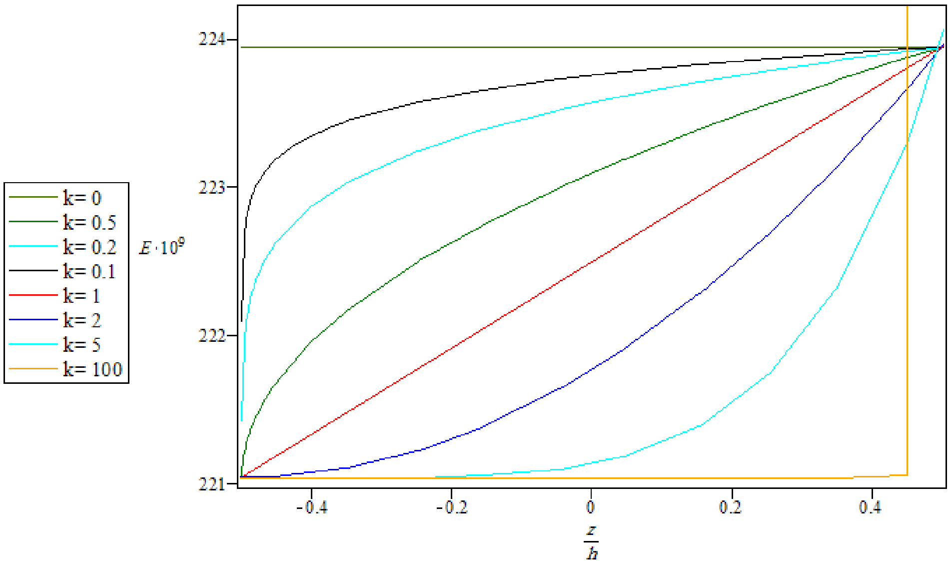


Fig. 2. The changes of Young's modulus along the thickness for various power law indexes.

Where u_0,v_0 and w_0 are the displacement components of a point on the mid-plane of the beam along axial and thickness coordinates, respectively and ϕ_x and ϕ_y are the cross-sectional rotation.
Based on the Engesser-Timoshenko beam theory, the displacement field in cartesian coordinates system can be written as [26]:

$$\begin{aligned} U(x,z) &= z\phi_x(x) \\ W(x,z) &= w_0(x,z) \end{aligned} \tag{3}$$

Hook's law defined as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \vdots \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{16} \\ C_{21} & & & \\ \vdots & & & \vdots \\ C_{61} & \cdots & & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \vdots \\ \varepsilon_{xz} \end{Bmatrix} + \begin{Bmatrix} \sigma^0_{xx} \\ \sigma^0_{yy} \\ \vdots \\ \sigma^0_{xz} \end{Bmatrix} \tag{4}$$

For piezoelectric layers

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{36} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} e_{14} & e_{24} & 0 \\ e_{15} & e_{25} & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \end{Bmatrix} \tag{6}$$

Q is moved stiffness, E is the electric field intensity and e is the constant stress of piezoelectric shifted.
The geometrically strain-displacement relations can be expressed as:

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} \\ \varepsilon_{xz} &= \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \gamma_{xz} &= 2\varepsilon_{xz} \rightarrow \gamma_{xz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{7}$$

Therefore:

$$\varepsilon_{xx} = z\frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = \phi + \frac{\partial w}{\partial x} \tag{8}$$

The constitutive relations for functionally grade Engesser- Timoshenko beam with piezoelectric layers are given by:

$$\begin{aligned}\sigma_{xx} &= Q_{11}(z)\varepsilon_{xx} + Q_{12}(z)\varepsilon_{yy} + Q_{16}(z)\gamma_{xy} - e_{31}E_z \to \sigma_{xx} = Q_{11}(z)\varepsilon_{xx} - e_{31}E_z \\ \sigma_{xz} &= Q_{45}(z)\gamma_{yz} + Q_{55}(z)\gamma_{xz} - e_{15}E_x - e_{25}E_y \to \sigma_{xz} = Q_{55}(z)\gamma_{xz} - e_{15}E_x\end{aligned} \tag{9}$$

Where

$$E_i = \frac{V}{h_i} \tag{10}$$

Where $\sigma_{xx}, \sigma_{yy}, Q_{11}$ and Q_{55} are the normal shear stresses and plane stress-reduced stiffnesses and e_{11}, e_{15} are piezoelectric elastic stiffnesses, respectively. Also, u and w are displacement components in the x and z directions, respectively.

2.3. Governing equations of motion

To derive the equations of motion of FG beams with piezoelectric layers , the Hamilton principle is employed as:

$$\delta \int_0^t (T - U + W)dt = 0 \tag{11}$$

T, U and W are the kinetic energy, the total potential energy and work done by axial load of smart beam, respectively, the potential energy expressed as:

$$\begin{aligned}U &= \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz})dv \\ &\to U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})dv\end{aligned} \tag{12}$$

Substituting Eqs. (2), (8), (9) into Eq. (12) and neglecting the higher-order terms, we obtain:

$$U = \frac{1}{2}\int_v [(Q_{11}(z)(z\frac{d\phi}{dx}) - e_{31}E_z)(z\frac{d\phi}{dx}) + (Q_{55}(z)(\phi + \frac{dw}{dx}) - e_{15}E_x)(\phi + \frac{dw}{dx})]dv \tag{13}$$

Q_{11}(z) and Q_{55}(z)defined as:

$$Q_{11}(z) = \frac{E(z)}{1-\upsilon^2} \quad , \quad Q_{55}(z) = \frac{E(z)}{2(1+\upsilon)} \tag{14}$$

The width of beam assumed to be constant, which obtained by integrating along y over v, then Eq. (13) becomes:

$$\begin{aligned}U &= \frac{b}{2}\int_0^L [B(\frac{d\phi}{dx})^2 + A(\phi + \frac{dw}{dx})^2]dx \\ -\frac{b}{2}&\int_0^L \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} [e_{31}E_z z\frac{d\phi}{dx} + e_{15}E_x(\phi + \frac{dw}{dx})]dzdx\end{aligned} \tag{15}$$

Where

$$(A, B) = \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} (Q_{55}(z), z^2 Q_{11}(z))dz \tag{16}$$

Where A and B are the shear rigidity and flexural rigidity respectively. Note that, no residual stresses due to the piezoelectric actuator are considered in the present study and the extensional displacement is elected. Thus, the potential energy can written as:

$$U = \frac{b}{2}\int_0^L (B(\frac{d\phi}{dx})^2 + A(\phi^2 + (\frac{dw}{dx})^2 + 2\phi(\frac{dw}{dx})) - e_{31}\frac{d\phi}{dx}(h_T V_T + h_B V_B) - e_{15}(\phi + \frac{dw}{dx})(V_T + V_B))dx \tag{17}$$

Where V_T and V_B are the applied voltages on the top and bottom actuators respectively. The work done by the axial compressive load can expressed as:

$$W = \frac{1}{2}\int_0^L P(t)(\frac{\partial w}{\partial x})^2 dx \quad Abstract18 \tag{18}$$

The kinematic energy of displacements is:

$$T = \frac{1}{2}\int_{0}^{L} \{I_0\left(\frac{\partial w}{\partial t}\right)^2 + I_2\left(\frac{\partial \phi_x}{\partial t}\right)^2\}dx \tag{19}$$

Where

$$\begin{aligned} I_i &= \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} \rho(z)^i dz \\ I_0 &= \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} \rho(z)^0 dz \\ I_2 &= \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} \rho(z)^2 dz \end{aligned} \tag{20}$$

Substitution from Eqs. (17)-(19) into Eq. (11) leads to the following equilibrium equations of the functionally graded Engesser-Timoshenko beam with piezoelectric layers:

$$\begin{aligned} \delta w: \quad & I_0\frac{d^2w}{dt^2} = P(t)(\frac{d^2w}{dx^2}) + bA(\frac{d\phi}{dx} + \frac{d^2w}{dx^2}) \\ \delta\phi: \quad & I_2\frac{d^2\phi}{dt^2} = bB(\frac{d^2\phi}{dx^2}) - bA(\phi + \frac{dw}{dx}) + be_{15}V_T \end{aligned} \tag{21}$$

2.4. Buckling analysis

With putting the left side of Eqs. (20) to zero, stability equations obtained. The values of force are $P(t) = P_0$. Due to compressive load in calculations $P_0 = -P_0$. Also for equal voltages on the upper and lower layers of the beam F_t and F_b (electrical forces induced electrical effects in the upper layer and lower layer) are caused, so that obtained moment of the beam becomes zero.Therefore the Pre-buckling force will be equal:

$$N = p + 2e_{31}V \tag{22}$$

$$\begin{aligned} & bB(\frac{d^3\phi}{dx^3}) - bA(\frac{d\phi}{dx} + \frac{d^2w}{dx^2}) = 0 \\ & Where: \ \frac{d\phi}{dx} = -(1 - \frac{P_0}{bA})(\frac{d^2w}{dx^2}) \\ & \to -bB\frac{d^2}{dx^2}\left((1 - \frac{P_0}{bA})(\frac{d^2w}{dx^2})\right) - bA(\left(-(1 - \frac{P_0}{bA})(\frac{d^2w}{dx^2})\right) + \frac{d^2w}{dx^2}) = 0; \\ & \frac{d^4w}{dx^4} + \frac{P_0}{bB\left(1 - \frac{P_0}{bA}\right)}\frac{d^2w}{dx^2} = 0 \end{aligned} \tag{23}$$

Therefore

$$w(x) = C_1\sin\lambda x + C_2\cos\lambda x + C_3x + C_4 \tag{24}$$

The boundary conditions for the pin-ended Timoshenko column given by:

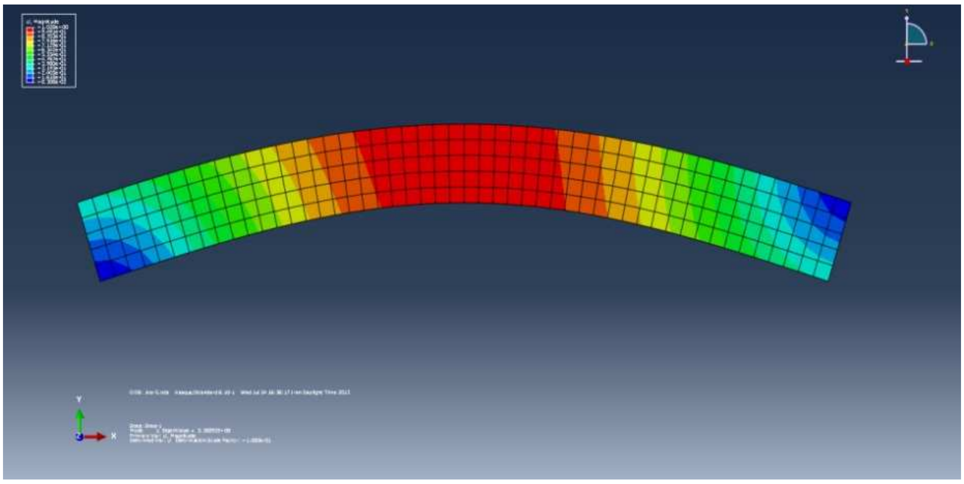
$$\{w(0) = w(L) = \frac{d^2w(0)}{dx^2} = \frac{d^2w(L)}{dx^2} = \frac{d\phi(0)}{dx} = \frac{d\phi(L)}{dx} = 0\} \tag{25}$$

Therefore

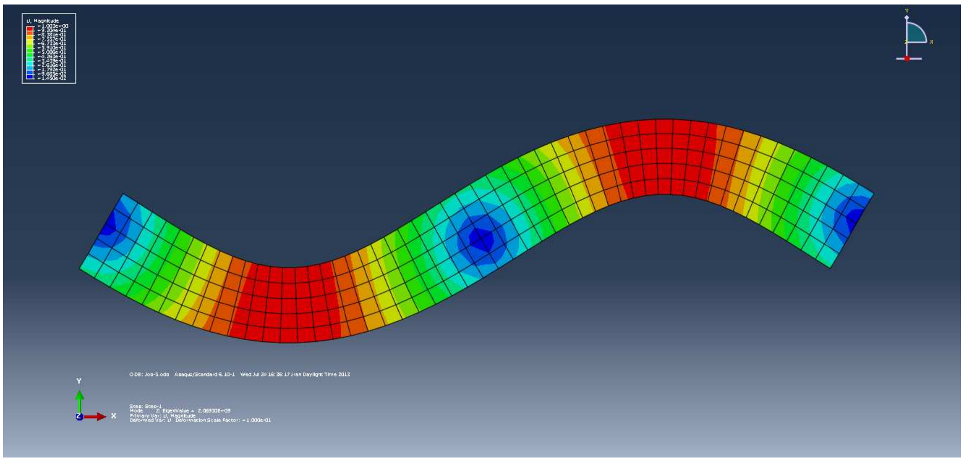
$$\frac{\frac{P_0}{bD}}{\left(1 - \frac{P_0}{bA}\right)} = \lambda^2 \Rightarrow P_0 = \frac{\lambda^2 bD}{1 + \frac{\lambda^2 D}{A}} \Rightarrow P_0 = \lambda^2 bD\left(1 - \frac{\lambda^2 D}{A + \lambda^2 D}\right) \tag{26}$$

Finally the critical Engesser-Timoshenko buckling load of a homogenous beam will derived, that is:

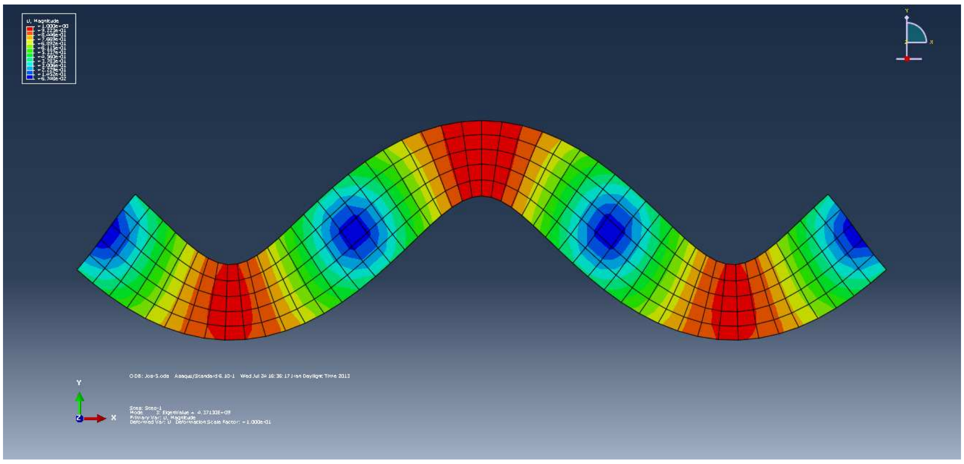
$$P_{cr} = \frac{(\frac{\pi}{L})^2\frac{bh^3Q_{11}}{12}}{1 + (\frac{\pi}{L})^2\frac{h^2Q_{11}}{12Q_{55}}} \tag{27}$$



(a) The first mode



(b) The second mode



(c) The third mode

Fig. 3. Three mode of critical buckling load

3. Numerical results and discussions

The mechanical buckling behaviors of simply supported functionally graded Engesser-Timoshenko beams with piezoelectric actuators are studied in this paper. It is assumed that the top and bottom piezoelectric layers have the same thickness; $h_B = h_T$ and the same voltages to both actuators. The material properties of the beam are listed in Table 1.The Poisson's ratio is chosen to be 0.3 for both materials.

Table 1. Material properties of FGM layer and Piezoelectric layer

Property	FGM layer		Piezoelectric layer
	Stainless steel	Nike	
Young's modulus (GPa)	$E = 221.04$	$E = 223.95$	$E = 63$
Poission's ratio	$\upsilon = 0.3$	$\upsilon = 0.3$	$E = 223.95$
Length (m)	$L = 0.3$	$L = 0.3$	$L = 0.3$
Thickness (m)	$h = 0.1$	$h = 0.1$	$h = 0.00005$
Density (kg/m^3)	$\rho = 8166$	$\rho = 8900$	$\rho = 7600$
Piezoelectric constant ($1/cm2$)	–	–	$e_31 = e_15 = 17.6$

Table 2. Comparing the first critical buckling load with simply-simply boundary condition

h/L		P_{cr1}	P_{cr2}	P_{cr1}
0.1	Present	0.9683	3.5441	7.0158
	[27]	0.9683	3.5442	7.0159
	Abaqus	0.9580	3.5364	7.2114
0.2	Present	0.8860	2.6840	4.3864
	[27]	0.8860	2.6841	4.3865
	Abaqus	0.8976	3.0211	5.0652

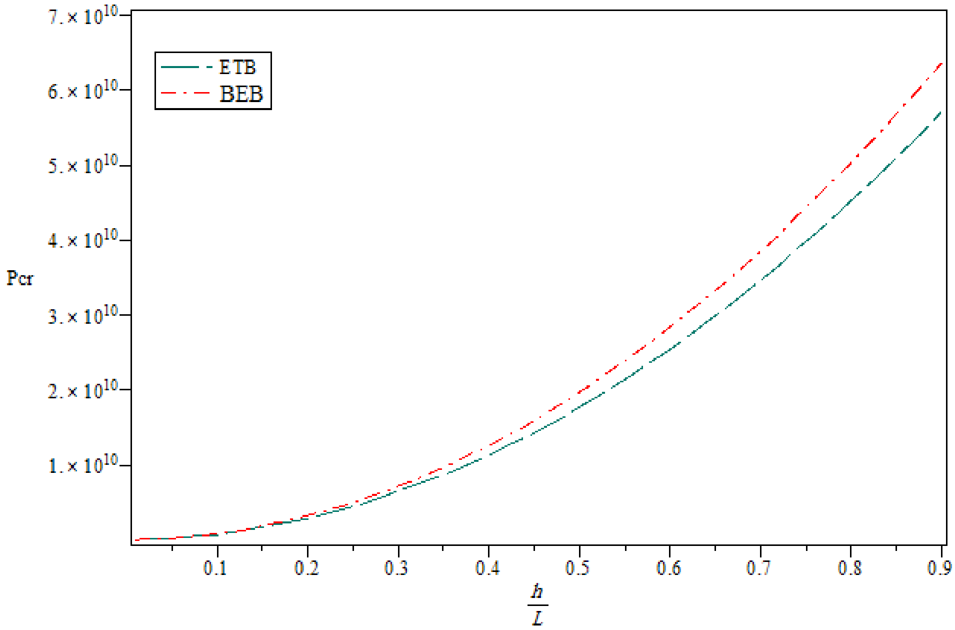


Fig. 4. Comparison of the critical buckling load of FG beam with piezoelectric actuators versus h/L

Fig. 3 shown three modes of critical buckling load using ABAQUS software. (The boundary condition is*s-s* and $h/L = 0.1$).
Comparing the first critical buckling load between presenting, reference and ABAQUS software are listed in Table 2.

The critical buckling loads for Bernoulli-Euler beam (BEB) and Engesser-Timoshenko beam (ETB) evaluated considering of $h_a/h = 0.1$, $b/h=1$, $L=1$, $V=10v$ and several values of dimensionless geometrical parameter h/L are shown in Fig. 4.
It seen that the critical buckling loads for Engesser-Timoshenko beam are generally lower than the corresponding values of the Bernoulli - Euler beam. Demonstrates of the buckling loads for functionally graded Engesser-Timoshenko beam shown in Fig. 5.
It is seen that the critical buckling loads for Engesser-Timoshenko beam increased with an increase of the ratio h/L and decreased with an increase of power-law index of constituent volume fractions.The variation of critical buckling loads for Engesser- Timoshenko beam versus h/L for different applied voltage is recorded in Fig. 6.
According to Fig. 6 the critical buckling loads for Engesser-Timoshenko beam decreased with increase of applied voltage.

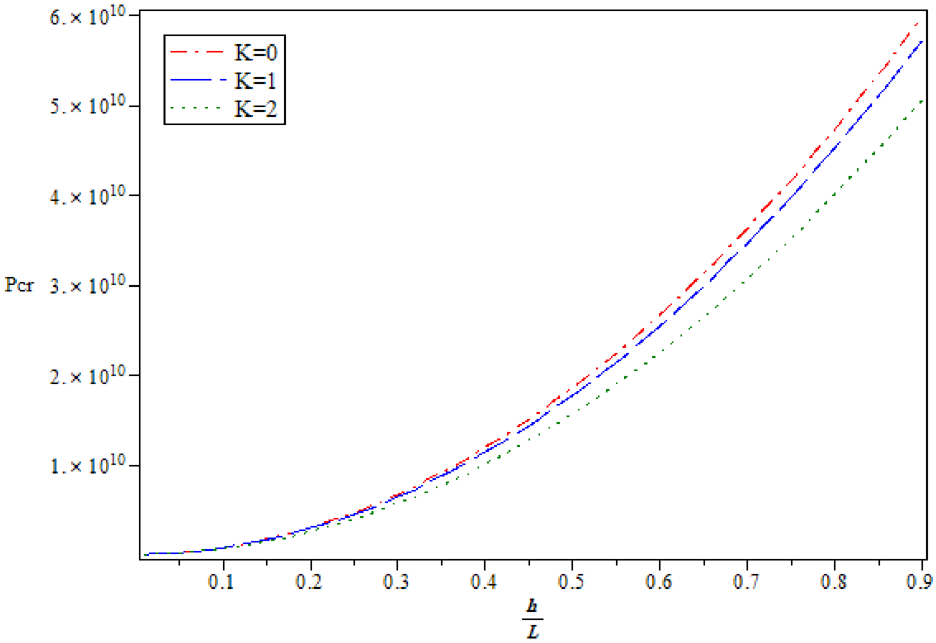


Fig. 5. Comparison of the critical buckling load of FG beam with piezoelectric actuators versus h/L for v=10v

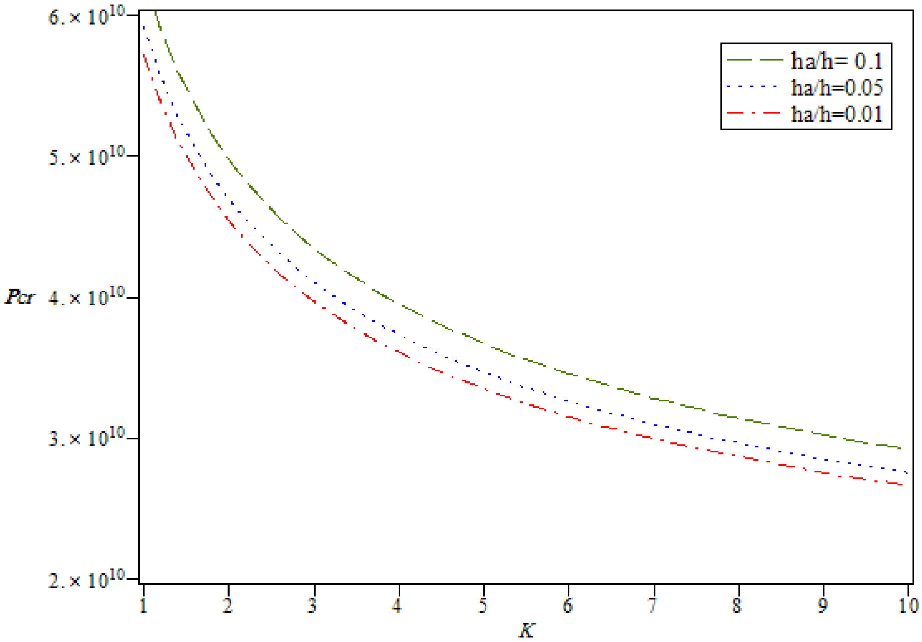


Fig. 6. Comparison of the critical buckling load of FG beam with piezoelectric actuators versus h/L for V=10v

The variation of critical buckling loads for Engesser-Timoshenko beam versus h_a/h for applied voltage is shown in Fig. 7.

4. Conclusion

Buckling analysis of a functionally graded Engesser- Timoshenko beam with piezoelectric actuators subjected to axial compressive loads is studied. The conclusions can explain briefly as follows:

a) The piezoelectric actuators induce tensile piezoelectric force produced by applying negative voltages that significantly affect the stability of the functionally graded Engesser-Timoshenko beam with piezoelectric actua-

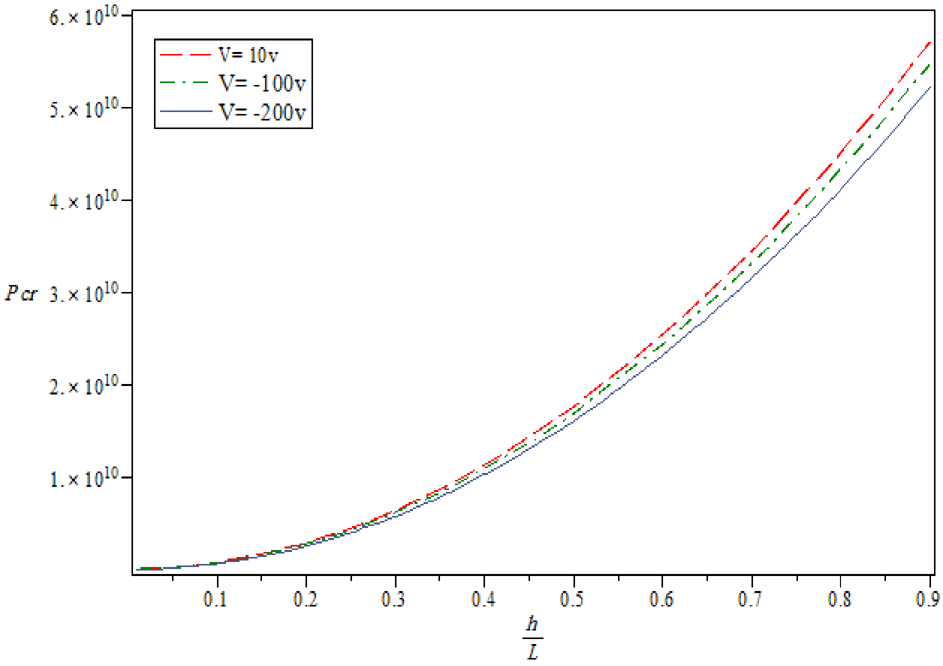


Fig. 7. Effect of applied voltage on the critical buckling load of FG beam with piezoelectric actuators

tors.

b) Critical buckling load decreases with increasing the power law index that for the reduction of K between 0 and 3 being greater.

c) The critical buckling loads of FG Engesser- Timoshenko beam under axial compressive load generally increases with the increase of relative thickness h/L.

d) The accuracy of Engesser-Timoshenko beam theory is more than Bernoulli-Euler beam theory.

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