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Intuitionistic fuzzy $\pi g\beta$ closed sets

Research Article

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Abstract: A new class of functions, called Intuitionistic fuzzy π-generalized π-closed set is introduced. Basic properties of Intuitionistic fuzzy π-generalized π-closed set are studied.

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Keywords: Intuitionistic fuzzy topology • Intuitionistic fuzzy generalized beta closed sets |* Intuitionistic fuzzy generalized beta open sets

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [10] and later Atanasov [3] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets.
Recently many fuzzy topological concepts such as semi closed , α closed, semi pre closed have been generalized for intuitionistic fuzzy topological spaces.
In the literature, β-open sets are being studied by many authors. Abd El-Monsef [1] introduced β-open sets and β-continuous mappings in 1983. Recently, Tahiliani [6] introduced and studied $\pi g\beta$-closed sets in topological spaces.
In the present paper we introduce the concepts of intuitionistic fuzzy π-generalized beta closed set and intuitionistic fuzzy π-generalized beta open set and study some of their properties.

2. Preliminaries

Let X is a nonempty fixed set. An intuitionistic fuzzy set A[1] in X is an object having the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x)+\gamma_A(x) \le 1$ for each $x \in X$. The intuitionistic fuzzy sets $\tilde{0} =< 0, 1 >: x \in X$ and $1 =< x, 1, 0 >: x \in X$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, called a subset of an intuitionitistic fuzzy set $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \le \gamma_B(x)$ for each $x \in X$.
The complement of an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ is the intuitionistic fuzzy set $A^C = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets is given by

$$A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > / x \in X\}$$
$$A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) > / x \in X\}$$

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A family τ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [5] on X if the intuitionistic fuzzy sets $0, 1 \in \tau$, and τ is closed under arbitrary union and finite intersection. The ordered pair (X,τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set.
The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted by $cl(A)$.
The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by $\text{int}(A)$ [4], as follows:

(i) $\text{int}(A) = \cup\{G, G$ is an IFOS in X and $G \subseteq A\}$
(ii) $\text{cl}(A) = \cap\{K, K$ is an IFCS in X and $A \subseteq K\}$

For the sake of simplicity, we use the notation $A =< x,\mu_A, \nu_A >$ instead of $A = \{< x,\mu_A(x), \nu_A(x) > /x \in X\}$ and $A =< x,(\mu_A,\mu_B),(\nu_A, \nu_B) >$ instead of $A =< x,(A/\mu_A, B/\mu_B),(A/\nu_A, B/\nu_B) >$
In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^C of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.
Note that for any IFS A in (X,τ), $\text{cl}(A^C)=(\text{int}(A))^C$ and $\text{int}(A^C)=(\text{cl}(A))^C$.

Definition 2.1 ([5]).
A subset of A of a space (X,τ) is called:
(i) regular open if $A = \text{int}(\text{cl}(A))$.
(ii) π open if A is the union of regular open sets.

Definition 2.2 ([2]).
An IFS $A =< x,\mu_A, \nu_A >$ in an IFTS (X,τ) is said to be an intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$.

Definition 2.3 ([2]).
An IFS $A =< x,\mu_A, \nu_A >$ in an IFTS (X,τ) is said to be an intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$.

Definition 2.4 ([2]).
An IFS A of an IFTS (X,τ) is an
(i) intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq \text{int}(\text{cl}(A))$.

Definition 2.5 ([2]).
An IFS A of an IFTS (X,τ) is an
(i) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Definition 2.6 ([1]).
An IFS A of an IFTS (X,τ) is an
(i) intuitionistic fuzzy β-open set (IFβOS in short) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$,
(ii) intuitionistic fuzzy β-closed set (IFβCS in short) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

Definition 2.7 ([1]).
An IFS A of an IFTS (X,τ) is an
(i) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int}(\text{cl}(A))$,
(ii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \text{cl}(\text{int}(A))$.

Definition 2.8 ([8]).
An IFS A of an IFTS (X,τ) is called an intuitionistic fuzzy w-closed (IFWCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSO in X. and An IFS A of an IFTS(X,τ) is called an intuitionistic fuzzy w-open (IFWOS in short) if A^C is IFWCS.

Definition 2.9 ([7]).
An IFS A of an IFTS (X,τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever

$A \subseteq U$ and U is an IFOS in X. The family of all IFβCSs (respectively, IFβOSs) of an IFTS (X,τ) is denoted by IFβC(X) (respectively IFβO(X)).

Definition 2.10 ([1]).
Let A be an IFS in an IFTS (X,τ). Then the Beta interior and beta closure of A are defined as

$$\beta\text{int}(A) = \cup\{G, G \text{ is an IF}\beta\text{OS in } X \text{ and } G \subseteq A\}$$
$$\beta\text{cl}(A) = \cap\{K, K \text{ is an IF}\beta\text{CS in } X \text{ and } A \subseteq K\}$$

Remark 2.1 ([1]).
Let A of an IFS (X,τ), then
(i) βcl(A)= $A \cup$ int(cl(int(A))), (ii) βint(A)= $A \cap$ cl(int(cl(A)))

Definition 2.11 ([6]).
Let an IFS A of an IFTS (X,τ). Then the semi closure of A(scl(A) in short) is defined as scl$(A) = \cap\{K/K$ is an IFSCS in X and $A \subseteq K$.

Definition 2.12 ([6]).
Let A be an IFS of an IFTS (X,τ). Then the semi interior of A(sint(A) in short) is defined as sint$(A) = \cup\{K/K$ is an IFSOS in X and $K \subseteq A$.

Definition 2.13 ([2]).
An IFS A of an IFTS (X,τ) is an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.14 ([2]).
An IFS A of an IFTS (X,τ) is an intuitionistic fuzzy semi pre open set (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq$cl(B).

Definition 2.15 ([2]).
An IFS A of an IFTS (X,τ) is an intuitionistic fuzzy semi pre closed set (IFSPCS in short) if there exists an IFPCS B such that int$(B) \subseteq A \subseteq B$.

Definition 2.16 ([2]).
An IFS A of an IFTS (X,τ) is an intuitionistic fuzzy generalized beta closed set (IFGβCS in short) if βcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Remark 2.2.
Every IFOS is IFSOS in (X,τ).

Remark 2.3.
Union of two IFROS is IFOS in (X,τ).

Remark 2.4.
Every IFπOS is IFOS in (X,τ).

3. Intuitionistic fuzzy π - generalized beta closed sets

In this section we introduce Intuitionistic fuzzy π generalized beta closed sets and study some of their properties.

Definition 3.1.
An IFS A is said to be an Intuitionistic fuzzy π generalized beta closed sets (IFπGβCS in short) in (X,τ) if βcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFπOS in X. The family of all IFπGβCSs of an IFT(X,τ) is denoted by IFπGβCS(X).

Example 3.1.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X, where $G =< x, (0.3, 0.4), (0.7, 0.5) >$ Then the IFS $A =< x, (0.2, 0.3), (0.7, 0.7) >$ is an IFπGβCS in X.

Theorem 3.1.
Every IFCS is an IFπGβCS but not conversely.

Proof. Let A be an IFCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). Since βcl$(A) \subseteq$ scl$(A) \subseteq$ cl(A) and A is an IFCS in X, βcl$(A) \subseteq$ cl$(A) = A \subseteq U$. Therefore A is an IFπGβCS in X. □

Example 3.2.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X, where $G =< x, (0.2, 0.3), (0.3, 0.4) >$. Then the IFS $A = G$, then βcl$(A) = A$, hence it is an IFπGβCS but cl$(A) = G^C \neq A$, hence it is not an IFCS in X.

Theorem 3.2.
Every IFSCS is an IFπGβCS but not conversely.

Proof. Let A be an IFSCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis, βcl$(A) \subseteq$ scl$(A) \subseteq A \subseteq U$. Hence βcl$(A) \subseteq U$. Therefore A is an IFπGβCS in X. □

Example 3.3.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$, where $G =< x, (0.3, 0.4), (0.4, 0.6) >$. Then the IFS $A =< x, (0.2, 0.3), (0.5, 0.7) >$ is an IFπGβCS because βcl$(A) = A \subseteq U$ but not an IFSCS in X, since int(cl(A)) $= < x, (0.3, 0.4), (0.4, 0.6) > \subset A$.

Theorem 3.3.
Every IFαCS is an IFπGβCS but not conversely.

Proof. Let A be an IFαCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis, cl(int(cl(A)))$\subseteq A$. Therefore int(cl(A))$\subseteq A$. Also, Int$(A) \subseteq A$, cl(int$(A) \subseteq$ cl(A), hence int(cl(int(A)))$\subseteq$ int(cl(A)) $\subseteq A$, This implies βcl$(A) \subseteq A \subseteq U$. Therefore A is an IFπGβCS in X. □

Example 3.4.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X, where $G_1 =< x, (0.2, 0.2), (0.6, 0.7) >$ and $G_2 =< x, (0.4, 0.5), (0.5, 0.5) >$ and IFS $A =< x, (0.3, 0.2), (0.6, 0.5) >$, then βcl$(A) = G_2 \subseteq U$ is an IFπGβCS but not an IFαCS in X, since cl(int(cl(A))) $= <(0.5, 0.5), (0.4, 0.5) > \not\subseteq A$.

Theorem 3.4.
Every IFPCS is an IFπGβCS but not conversely.

Proof. Let A be an IFPCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis, cl(int(A))$\subseteq A$. Therefore int(cl(Int(A)))$\subseteq$Int$(A) \subseteq A$. This implies βcl$(A) \subseteq A \subseteq U$. Therefore A is an IFπGβCS in X. □

Example 3.5.
Let $X = \{x_1, x_2\}$ and $G =< x, (0.3, 0.4), (0.7, 0.5) >$ and let $\{0\sim, G, 1\sim\}$ be an IFT on X. Then the IFS $A = G$, then βcl$(A) = G \subseteq U$ is an IFπGβCS but not an IFPCS in X, because cl(int(A)) $= < x, (0.7, 0.5), (0.3, 0.4) > \not\subseteq A$.

Theorem 3.5.
Every IFβCS is an IFπGβCS but not conversely.

Proof. Let A be an IFβCS in X. By hypothesis, βcl$(A) \subseteq A$ whenever $A \subseteq U$ and U is IFπOS. By (Remark 2.3 βcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS. Therefore A is an IFπGβCS in X. □

Example 3.6.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X, where $G_1 =< x, (0.4, 0.5), (0.3, 0.2) >$, $G_2 =< x, (0.2, 0.3), (0.3, 0.2) >$. Then IFS $A =< x, (0.3, 0.4), (0.3, 0.2) >$ is an IFπGβCS but not an IFβCS in X, since βcl$(A) = 1\sim > \not\subseteq G_1$.

Theorem 3.6.
Every IFRCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFRCS in X. By Definition $A = \text{cl}(\text{int}(A))$. This implies $\text{cl}(A) = \text{cl}(\text{int}(A))$. Therefore $\text{cl}(A) = A$. Therefore A is an IFCS in X. By Theorem 3.1, A is an IFπGβCS in X. □

Example 3.7.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X, where $G =< x, (0.1, 0.2), (0.5, 0.7) >$. Then IFS $A = G$ is an IFπGβCS but not an IFRCS in X, since $\text{cl}(\text{int}(A)) = < x, (0.5, 0.7), (0.1, 0.2) > \neq A$.

Theorem 3.7.
Every IFWCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFWCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis cl$(A)\subseteq U$ whenever $A \subseteq U$, since βcl$(A)\subseteq$ cl(A) and A is an IFWCS in X, βcl$(A)\subseteq$ cl$(A)\subseteq U$, whenever $A \subseteq U$, and U is IFSO. (By remark 2.8). Therefore A is an IFπGβCS in X. □

Example 3.8.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X, where $G =< x, (0.3, 0.2), (0.6, 0.6) >$ and the IFS $A = G$, then βcl$(A) = A$, hence it is an IFπGβCS but not an IFWCS in X since cl(A)= <x, (0.6, 0.6), (0.3, 0.2)>$\not\subseteq G_1$.

Theorem 3.8.
Every IFGCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFGCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis cl$(A)\subseteq U$ whenever $A \subseteq U$, since βcl$(A)\subseteq$ scl$(A)\subseteq$cl$(A)\subseteq U$, whenever $A \subseteq U$. Therefore A is an IFπGβCS in X. □

Example 3.9.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X, where $G =< x, (0.2, 0.3), (0.6, 0.5) >$. Then the IFS $A =< x, (0.2, 0.2), (0.6, 0.6) >$. Then is an IF$\pi$G$\beta$CS but not an IFGCS in X since $A \subseteq G$ but cl(A)= <x, (0.6, 0.5), (0.2, 0.3)>$\not\subseteq G$.

Theorem 3.9.
Every IFGSCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFGSCS in X. By hypothesis, scl$(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFπOS. By (Remark 2.3) βcl$(A)\subseteq U$, whenever $A \subseteq U$ and U is IFOS. Therefore A is an IFπGβCS in X. □

Example 3.10.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X, where $G_1 =< x, (0.5, 0.6), (0.4, 0.3) >$, $G_2 =< x, (0.3, 0.4), (0.4, 0.3) >$. Then IFS $A =< x, (0.4, 0.5), (0.4, 0.3) >$ is an IFπGβCS but not an IFGSCS in X since $A \subseteq G_1$ but βcl(A)= $1 \not\subseteq G$.

Theorem 3.10.
Every IFαGCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFαGCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis cl(int(cl(A)))$\subseteq U$. Therefore int(cl(A))$\subseteq U$. Also int $A \subseteq A$, cl(int(A))$\subseteq$cl(A), therefore int(cl(int(A)))$\subseteq$int(cl(A))$\subseteq U$, which implies βcl$(A)\subseteq U$, whenever $A \subseteq U$. Hence A is an IFπGβCS in X. □

Example 3.11.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X, where $G_1 =< x, (0.1, 0.1), (0.5, 0.6) >$, $G_2 =< x, (0.3, 0.4), (0.4, 0.4) >$. Then IFS $A =< x, (0.2, 0.1), (0.5, 0.4) >$, then βcl$(A) = G_2 \subseteq U$ is an IFπGβCS but not an IFαGCS in X since cl(int(cl(A)))= <(0.4, 0.4), (0.3, 0.4) > $\not\subseteq G_2$.

Theorem 3.11.
Every IFGPCS is an IF$\pi G\beta$ CS but not conversely.

Proof. Let A be an IFGPCS in X and let $A \subseteq U$ and U is an IFπOS in (X, τ). By hypothesis and (Remark 2.3) cl(int(A))$\subseteq U$. Therefore int(cl(int(A)))$\subseteq$ int(U)$\subseteq U$. This implies βcl(A)$\subseteq U$, whenever $A \subseteq U$ and U is IFOS. Therefore A is an IFπGβCS in X. □

Example 3.12.
Let $X = \{x_1, x_2\}$ and $G =< x, (0.1, 0.2), (0.5, 0.6) >$ and let $\tau = \{0\sim, G, 1\sim\}$ be an IFT on X. Then IFS $A = G$ is an IFπGβCS but not an IFGPCS in since cl(int(A))$=< x, (0.5, 0.6), (0.1, 0.2) > \not\subseteq G_2$.

Theorem 3.12.
Every IFGβ CS is an IFπGβ CS but not conversely.

Proof. Let A be an IFGβCS in X. By hypothesis, βcl(A)$\subseteq U$, whenever $A \subseteq U$ and U is IFπOS. By hypothesis and (Remark 2.3) βcl(A)$\subseteq$ scl(A) $\subseteq U$, whenever $A \subseteq U$ and U is IFOS. Therefore A is an IFπGβCS in X. □

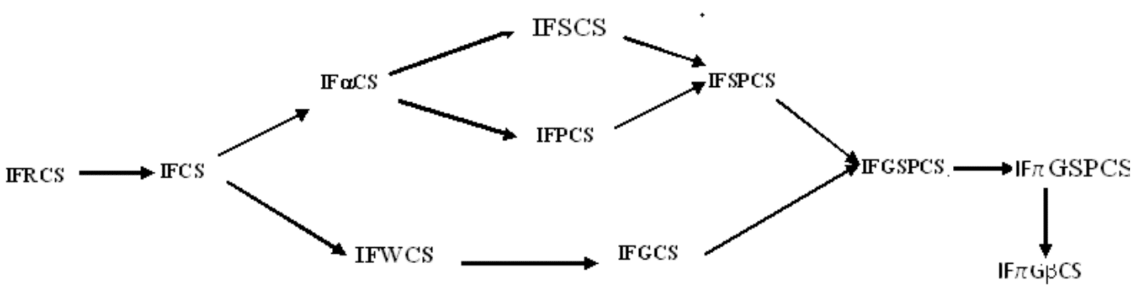
Example 3.13.
Let $X = \{x_1, x_2\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ is an IFT on X, where $G_1 =< x, (0.5, 0.6), (0.3, 0.2) >$, $G_2 =< x, (0.3, 0.4), (0.3, 0.2) >$. Then IFS $A =< x, (0.4, 0.5), (0.3, 0.2) >$ is an IFπGβCS but not an IFGβCS in X since βcl(A)$= 1\sim \not\subseteq G_1$.

Theorem 3.13.
Every IFπ GSCS is an IFπ Gβ CS but not conversely.

Proof. Let A be an IFπGSCS in X. By hypothesis, scl(A)$\subseteq U$, whenever $A \subseteq U$ and U is IFπOS. βcl(A)$\subseteq$ scl(A) $\subseteq U$, whenever $A \subseteq U$ and U is IFπOS. Therefore A is an IFπGβCS in X. □

Example 3.14.
Let $X = \{x_1, x_2\}$ and let $G_1 =< x, (0.1, 0.3), (0.4, 0.3) >$, $G_2 =< x, (0.0, 0.2), (0.2, 0.3) >$, $G_3 =< x, (0.0, 0.2), (0.4, 0.3) >$, $G_4 =< x, (0.1, 0.3), (0.2, 0.3) >$, $G_5 =< x, (0.3, 0.3), (0.2, 0.3) >$. Then $\tau = \{0\sim, G_1, G_2, G_3, G_4, G_5, 1\sim$ is an IFT in X. Then IFS $A =< x, (0.0, 0.1), (0.3, 0.4) >$ is an IFπGSPCS in X, but $A \cup$int(cl(A))$= G_1 \not\subseteq G_2$. Hence it is not an IFπGSPCS in X.

The following implications are true, none of them is reversible



Remark 3.1.
The intersection of any two IFπGβCS need not be an IFπGβCS in general as seen in the following example.

Example 3.15.
Let $X = \{x_1, x_2\}$ and let $G_1 =< x, (0.1, 0.2), (0.3, 0.3) >$, $G_2 =< x, (0.1, 0.1), (0.2, 0.3) >$, $G_3 =< x, (0.1, 0.2), (0.2, 0.3) >$ $G_4 =< x,, (0.1, 0.1), (0.3, 0.3) >$, $G_5 =< x, (0.3, 0.3), (0.2, 0.3) >$. Then $\tau = \{0, G_1, G_2, G_3, G_4, G_5, 1$ is an IFT on X and the IFSs $A =< x, (0.1, 0.2), (0.3, 0.3) >$ $B =< x, (0.3, 0.1), (0.2, 0.3) >$ are the IFπGβCS but $A \cap B$ is not an IFπGβCS in X.

Theorem 3.14.
Let (X, τ)be an IFTS. Then for every $A \in$IFπGβ C(X) and for every $B \in$IFS(X), $A \subseteq B \subseteq \beta$ cl(A) implies $B \in$IFπGβ C(X).

Proof. Let $B \subseteq U$ and U be an IFπOS. Since $A \subseteq B$, $A \subseteq U$ and A is an IFπGβCS, βcl(A)$\subseteq U$, whenever $A \subseteq U$, By hypothesis, $B \subseteq \beta$cl(A), βcl(B) $\subseteq \beta$cl(A) $\subseteq U$. Therefore βcl(B) $\subseteq U$. Therefore B is an IFπGβCS in X. □

Theorem 3.15.
If A is an IFπOS and an IFπGβCS in (X,τ), then A is an IFβCS in (X,τ).

Proof. Let A be an IFπOS in X. Since $A \subseteq A$, by hypothesis βcl$(A)\subseteq A$ but always $A \subseteq \beta$cl(A). Therefore βcl$(A) = A$. Hence A is an IFβCS in X. □

Theorem 3.16.
Let (X,τ) be an IFTS. If an IFS A is both IFπOS and IFCS of X, then the following statements are equivalent:

(i) A is IFGCS in X

(ii) A is IFπGβCS in X.

Proof. (i) ⇒ (ii): Let A be an IFGCS in X. By Theorem 3.8, A is IFπGβCS in X. (ii) ⇒ (i): Let A be an IFπGβCS in X. Then βcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is IFπOS in X, implies βcl$(A) \subseteq$cl$(A) \subseteq U$, whenever $A \subseteq U$, Since A is both IFπOS and IFCS in X. Therefore A is IFGCS in X. □

Definition 3.2 ([5]).
The π-kernel (π-ker (A)) of A is the intersection of all π-open sets containing A.

Remark 3.2 ([5]).
A subset A of a space (X,π) is IFπgβ-closed if βcl$(A) \subseteq \pi$-ker(A).

Theorem 3.17.
A subset A of X is IFπGβ-closed if and only if β cl(A)$\subseteq \pi-$ker(A).

Proof. Since A is IFπgβ-closed, βcl$(A)\subseteq A$ for any π-open set U with $A \subseteq U$ and hence βcl$(A)\subseteq \pi-$ker(A). Conversely, let U be any π-open set such that $A \subseteq U$. By hypothesis, βcl$(A)\subseteq \pi-$ker$(A) \subseteq U$ and hence A is IFπgβ-closed. □

Theorem 3.18.

If A is π-open and IFπgβ-closed, then A is β-closed.

Proof. Since A is π-open and IFπgβ-closed, βCl$(A)\subset A$, but $A \subset \beta$Cl(A) Hence, A is β-closed. □

Theorem 3.19.
Let A be a IFπgβ-closed in (X,τ). Then β Cl(A)$\backslash$A does not contain any nonempty π-closed set.

Proof. Let F be a nonempty π-closed subset of βCl$(A)\backslash A$. Then $A \subset X\backslash F$, where A is IFπgβ-closed and $X\backslash F$ is π-open. Thus βCl$(A) \subset X\backslash F$, or equivalently, F$\subset X\backslash\beta$Cl$(A)$. Since by assumption F $\subset$Cl(A), we get a contradiction. □

Corollary 3.1.
Let A be a IFπgβ-closed in (X,τ). Then A is IFβ-closed if and only if β Cl(A)|A is π-closed.

Proof. **Necessity.** Let A be a IFπgβ-closed. By hypothesis βCl(A)=A and so βCl$(A)\backslash A = \phi$ which is π-closed.

Sufficiency: Suppose βCl(A) $\backslash A$ is π-closed. Then by Proposition 3.19, βCl(A) $\backslash A = \pi$, that is, βCl(A)=A. Hence, A is β closed.

□

4. Intuitionistic fuzzy π - generalized beta open sets:

In this section we introduce Intuitionistic fuzzy π generalized beta open sets and discuss some of its properties.

Definition 4.1.
An intuitionistic fuzzy π- generalized beta open sets (IFπGβOS in short) in (X,τ) if its complement A^C is an IFπGβCS in X. The family of all IFπGβOSs of an IFTS (X,τ) is denoted by IFπGβO(X).

Example 4.1.
Let $X=\{x_1,x_2\}$ and let $\tau=\{0\sim,G,1\sim\}$ is an IFT on X, where $G=<x,(0.3,\ 0.4),\ (0.7,\ 0.5)>$. Then the IFS $A=<x,(0.7,0.7),(0.2,0.3)>$ is an IFπGβOS in X.

Theorem 4.1.
For any IFTS (X,τ), we have the following: Every IFOS,IFSOS,IFαOS,IFGOS,IFPOS,IFβOS is an IFπGβOS. But the converses are not true in general.

Proof. Straight forward. □

Example 4.2.
Let $X=\{x_1,x_2\}$ and $G=<x,(0.2,\ 0.3),\ (0.3,\ 0.4)>$. Then $\tau=\{0\sim,G,1\sim\}$ is an IFT on X. Then IFS $A=<x,(0.3,0.4),(0.2,0.3)>$ is an IFπGβOS in (X,τ) but not an IFOS in X.

Example 4.3.
Let $X=\{x_1,x_2\}$ and $G=<x,(0.3,\ 0.4),\ (0.4,\ 0.6)>$. Then $\tau=\{0\sim,G,1\sim\}$ is an IFT on X. Then IFS $A=<x,(0.5,0.7),(0.2,0.3)>$ is an IFπGβOS but not an IFSOS in X.

Example 4.4.
Let $X=\{x_1,x_2\}$ and let $\tau=\{0\sim,G_1,G_2,1\sim\}$ is an IFT on X, where $G_1=<x,(0.2,\ 0.2),\ (0.6,\ 0.7)>$, $G_2=<x,(0.4,\ 0.5),\ (0.5,\ 0.5)>$ and IFS $A=<x,(0.6,0.5),(0.3,0.2)>$ is an IFπGβOS but not an IFαOS in X.

Example 4.5.
Let $X=\{x_1,x_2\}$ and $G=<x,(0.2,\ 0.3),\ (0.6,\ 0.5)>$. Then $\tau=\{0\sim,G,1\sim\}$ is an IFT in X. The IFS $A=<x,(0.6,0.6),(0.2,0.2)>$ is an IFπGβOS but not an IFGOS in X.

Example 4.6.
Let $X=\{x_1,x_2\}$ and $G=<x,(0.3,\ 0.4),\ (0.7,\ 0.5)>$ and let $\tau=\{0\sim,G,1\sim\}$ is an IFT on X. The IFS $A=<x,(0.7,0.5),(0.3,0.4)>$ is an IFπGOS but not an IFPOS.

Example 4.7.
Let $X=\{x_1,x_2\}$ and let $\tau=\{0\sim,G_1,G_2,1\sim\}$ is an IFT on X, where $G_1=<x,(0.4,\ 0.5),\ (0.3,\ 0.2)>$, $G_2=<x,(0.2,\ 0.3),\ (0.3,\ 0.2)>$ and IFS $A=<x,(0.3,0.2),(0.3,0.4)>$ is an IFπGβOS but not an IFβOS in X.

Theorem 4.2.
Let (X,τ) be an IFTS. If $A\in$ IFπGO(X) then $V\subseteq$ cl(int(cl(A))) whenever $V\subseteq A$ and V is IFCS in X.

Proof. Let $A\in$ IFπGO(X). Then A^C is an IFπgβCS in X. Therefore βcl$(A^C)\subseteq U$, whenever $A^C\subseteq U$ and U is an IFπOS in X. This implies that int(cl(int(A^C)))$\subseteq U$. Therefore $U^C\subseteq$ cl(int(cl(A))) whenever $U^C\subseteq A$, and U^C is IFCS in X. Replacing U^C by V, we get $V\subseteq$ cl(int(cl(A))) whenever $V\subseteq A$ and V is IFCS in X. □

Theorem 4.3.
Let (X,τ) be an IFTS. Then, $\forall\ A\in$ IFπGβO(X) and $\forall\ B\in$ IFS(X), βint$(A)\subseteq B\subseteq A$ implies $B\in$ IFπGβO(X).

Proof. By hypothesis $A^C\subseteq B^C\subseteq(\beta\text{int}(A))^C$. Let $B^C\subseteq U$ and U be an IFπOS. Since $A^C\subseteq B^C$, $A^C\subseteq U$. But A^C is an IFπGβCS, βcl$(A^C)\subseteq U$. Also $B^C\subseteq(\beta\text{int}(A))^C=\beta\text{cl}(A^C)$, hence βcl$(B^C)\subseteq$ spcl$(A^C)\subseteq U$. Hence B^C is an IFπGβCS, which implies B is an IFπGβOS of X. □

Remark 4.1.
The union of any two IFπGβOSs need not be an IFπGOS in general.

Example 4.8.
Let $X = \{x_1, x_2\}$ be an IFTS and let $G_1 =< x,(0.1, 0.2), (0.3, 0.3) >$, $G_2 =< x,(0.1, 0.1), (0.2, 0.3) >$, $G_3 =< x,(0.1, 0.2), (0.2, 0.3) >$, $G_4 =< x,,(0.1, 0.1), (0.3, 0.3) >$ $G_5 =< x,(0.3, 0.3), (0.2, 0.3) >$. Then $\tau = \{0, G_1, G_2, G_3, G_4, G_5, 1$ is an IFT on X and the IFSs $A =< x,(0.3,0.3),(0.1,0.2) >$, $B =< x,(0.2,0.3),(0.3,0.1) >$ are IFπGβOS but $A \cup B$ is not an IFπGβOS in X.

Theorem 4.4.
An IFS A of an IFTS (X,τ) is an IFπGβOS if and only if $F \subseteq \beta$ int(A) whenever F is an IFπCS and $F \subseteq A$.

Proof. **Necessity:** Suppose A is an IFπGβOS in X. Let F be an IFπCS and $F \subseteq A$. Then F^c is an IFπOS in X such that $A^c \subseteq F^c$. Since A^c is an IFπGSPCS, βcl$(A^c) \subseteq F^C$. Hence $(\beta\text{int(A)})^c \subseteq F^C$. Therefore F$\subseteq \beta$int$(A)$.

Sufficiency: Let A be an IFS of X and let F $\subseteq \beta$int(A) whenever F is an IFCS and F$\subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFπOS. By hypothesis, $(\beta\ \text{int}(A))^c \subseteq F^c$, which implies βcl$(A^c) \subseteq F^c$. Therefore A^c is an IFπGβCS of X. Hence A is an IFπGβOS of X. □

Theorem 4.5.
Let (X,τ) be an IFTS, and $A, B \subset X$, If B is IFπGO(X) and β int$(B) \subset A$ then $A \cap B$ is IFπGβO(X).

Proof. Since B is IFπGβO(X) and βint$(B) \subset A$, βint$(B) \subset A \cap B \subset B$, by Theorem 4.3, $A \cap B$ is IFπGβO(X). □

Theorem 4.6.
If a set A is $\pi g\beta$-open in a IFTS (X,τ), then $G = X$ Whenever G is π-open in (X,τ) and β int$(A) \cup A^C \subset G$.

Proof. Suppose that G is π-open and βInt$(A) \cup A^c \subset G$. Now $G^c \subset \beta$Cl$(A^c \backslash A^c)$. Since G^c is π-closed and A^c is IFπgβ-closed, by Proposition 3.19, $G^c = \varphi$ and hence $G = X$. □

Theorem 4.7.
Let A be $\pi g\beta$-open in IFTS(X,τ) and B be IFα-open. Then $A \cap B$ is $\pi g\beta$-open in (X,τ).

Proof. Let F be any π-closed subset of X such that $F \subset A \cap B$. Hence $F \subset A$ and by Theorem 4.4, $F \subset \beta$Int $(A) = \{U : U$ is β open and $U \subset A\}$. Obviously, $F \subset (U \cap B)$, where U is a open set in X contained in A. Since $U \cap B$ is a β open set contained in $A \cap B$ for each open set U contained in A, F $\subset \beta$Int$(A \cap B)$, and by Theorem 4.5, $(A \cap B)$ is πgβ-open in X. □

Theorem 4.8.
Let (X,τ) be IFTS. If $A \subset X$ is $\pi g\beta$-closed, then β Cl$(A) \backslash A$ is $\pi g\beta$-open.

Proof. Let A be IFπgβ-closed and let F be a π-closed set such that $F \subset \beta$ Cl$(A) \backslash A$. Then by Proposition 3.19, $F = \varphi$. So, $F \subset \beta$Int$(\beta$cl$(A) \backslash A)$. By Theorem 4.4 βCl$(A) \backslash A$ is IFπgβ-open. The following Lemma can be easily verified. □

Lemma 4.1.
For every subset A of a IFTS (X,τ), β Int(β Cl(A)$\backslash A = \varphi)$

Theorem 4.9.
Let $A \subset B \subset X$ and let β Cl$(A) \backslash A$ is $\pi g\beta$-open. Then β Cl$(A) \backslash B$ is also $\pi g\beta$-open.

Proof. Suppose βCl$(A) \backslash A$ is πgβ-open and let F be a pi-closed subset of (X,τ) with $F \subset \beta$cl$(A) \backslash B$. Then $F \subset \beta$ Cl$(A) \backslash A$. By Theorem 4.4 and Lemma 4.1, $F \subset \beta$ Cl$(A) \backslash A = \varphi$. Thus, $F = \varphi$ and hence, $F \subset \beta$ Cl$(A) \backslash B$. □

Remark 4.2.
Let (X,τ) be IFTS. For any $A \subset X$, βint$(\beta$cl$(A) - A) = \varphi$.

Theorem 4.10.
Let (X,τ) be IFTS. If $A \subset X$, $\beta\, int(\beta\, cl(A) - A) = \varphi$.

Proof. Let A be IFπgβ-closed let F be pi-closed set. $F \subset \beta$ Cl$(A) - A$. By Theorem 3.19, $F = \varphi$, by Remark 4.2, βint$(\beta$cl$(A) - A) = \varphi$. Thus $F \subset (\beta$ Cl$(A) - A)$. Thus β Cl$(A) - A$ is πgβ-open. □

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