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Nonlinear stability of viscoelastic fluids streaming through porous medium under the influence of vertical electric fields producing surface charges

Research Article

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Abstract: A nonlinear stability analysis of two superposed semi-infinite viscoelastic electrified fluids streaming through porous media in the presence of vertical electric fields admitting surface charges at their interface is investigated in three dimensions. The method of multiple scales is used to obtain a Ginzburg-Landau equation with complex coefficients describing the behavior of the system. The linear and nonlinear stability analysis, in both two- and three-dimensional disturbances, are discussed both analytically and numerically . We found that the surface tension, porosity of porous medium, and kinematic viscosities have stabilizing effects in the linear analysis, and nonlinearity tends to weaken their stabilizing effects. Also, the medium permeability has a stabilizing effect in the linear analysis and a destabilizing effect in the nonlinear case. The fluid velocities are found to have destabilizing effect in the linear case, while they have dual role on the stability in the nonlinear case. It is found also that nonlinearity does not affect the linear behaviors of both electric fields and kinematic viscoelasticities, and that the system in absence of fluid velocities (or porous medium) is more unstable than in their present, while the system is less unstable in absence of electric fields. Comparison between the stability behaviors of all physical parameters in two- and three-dimensional disturbances have been done, and show interesting features.

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Keywords: Nonlinear stability • Viscoelastic fluids • Flows through porous medium• Electrohydrodynamics.

1. Introduction

The flow through porous media has been a subject of great interest for the last several decades. This interest was motivated by numerous engineering applications in various disciplines, such as geophysical thermal and insulation engineering, modeling of packed sphere beds, cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, ceramic processes, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, ground water pollution and filtration processes, to name just a few of these applications. Much of the recent studies on this topic are given by Vafai [1], Pop and Ingham [2], and Nield and Bejan [3]. Also, during the past few decades, non-Newtonian viscoelastic fluids have become more and more important industrially [4]. Among these fluids are the fluids of differential type such as the Walters B′ fluid which has its importance in many industrial applications [5]. The extrusion of plastic sheets, fabrication of adhesive tapes and application of coating layers onto rigid substrates are some of the examples. Study of the flow problems of this class of fluids not only is important technologically, but is also challenging to engineers and applied mathematicians who are interested in obtaining accurate solutions. The instability of the plane interfaces between viscous and viscoelastic fluids through porous medium may find applications in geophysics, technology and biomechanics. The gross effect when the

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fluid slowly percolates through the pores of the rock is given by Darcy's law [6]. As a result, the usual viscous term in the equation of motion of Walters B′ viscoelastic fluid through porous medium is replced by the resistive term $(-\rho/k_1)[\nu-\nu'(\partial/\partial t)]\mathbf{q}$ where ρ is the fluid density, ν and ν' are the kinematic viscosity and viscoelasticity of the Walters B′ viscoelastic fluid, k_1 is the medium permeability, and $\mathbf{q}$ is the Darcian filter velocity of the fluid [7].
On the other hand, electrohydrodynamics study the interplay between an electric field and fluid mechanics, and it has attracted much attention since the last century due to its widely promising applications. Although the literature in the field of interfacial fluid flow is vast, a very small of it has been devoted to the investigation of surface wave phenomena in electrohydrodynamics [8]. A survey on electrohydrodynamics with special reference to many of the developments in this field was given by Melcher [9], Bobbio [10], and recently by Griffiths [11]. El-Sayed and Callebaut [12, 13], and El-Sayed [14, 15] studied the nonlinear electrohydrodynamic instabilities for various physical problems of interest at the interface between two inviscid dielectric fluids using the Maxwell stress tensor due to which the effect of the applied electric field was confined to the interface. Electrohydrodynamic instability studies for flows in porous media has attracted little attention in the scientific literature despite their applications in various fields with great interest [16]. Thus, there is a growing need for original research in the updated electrohydrodynamic phenomena which have some physical and engineering applications. For linear electrohydrodynamic stability of flows through porous media, we refer to the investigations of El-Sayed [17]. The study of nonlinear interfacial electrohydrodynamic instability has received a considerable number of contributions concerning porous media, e.g. Mohamed et al. [18] investigated the nonlinear gravitational electrohydrodynamic stability of streaming fluids through porous medium. Moatimid and El-Dib [19] studied the nonlinear Kelvin-Helmholtz instability of Oldroydian viscoelastic fluids in porous media. The nonlinear electrohydrodynamic stability of capillary-gravity waves on the interface between two semi-infinite dielectric fluids in porous medium under the effect of a vertical electric field in the presence of surface charges was investigated by El-Dib and Moatimid [20]. For recent developments about the topic of this subject, see the recent works of El-Sayed et al. [21–23].
In this article, using the surface coupled model, we shall first formulate the general interfacial problem for two superposed semi-infinite Walters B′ viscoelastic dielectric fluids streaming through porous media in the presence of vertical electric fields admitting surface charges at the interface, then the nonlinear analysis using the multiple time scales method is carried out. In this approach, the electric body force vanishes and the electric problem is decoupled from the fluid problem. The electric field changes the tangential and normal shear stress at the interface, and thus alters the stability of the flow. To the best of our knowledge, this problem has not been investigated yet. The stability criteria are obtained for both the linear and nonlinear problems analytically and discussed numerically. Some limiting cases of previous studies in the literature are recovered. Finally, the obtained results are listed in a concluding remarks section in view of the effects of all physical parameters including in the analysis on the stability of the considered system.

2. Formulation of the problem

Consider two semi-infinite Walters B′ viscoelastic dielectric fluids streaming through a porous medium with uniform densities $\rho^{(1)}$ and $\rho^{(2)}$, dielectric constants $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$, with constant horizontal velocities $U^{(1)}$ and $U^{(2)}$, respectively, separated by an interface at $z=0$, where the superscripts (1) and (2) refer to the lower and upper fluids, respectively. Let the system be influenced by constant electric fields $E_0^{(1)}$ and $E_0^{(2)}$, respectively, acting in the negative z-direction normally to the interface between the two fluids such that there are surface charges present at the interface, i.e. the condition $\varepsilon^{(1)}E_0^{(1)} \neq \varepsilon^{(2)}E_0^{(2)}$ is satisfied in this case [9]. The interface is represented by

$$\Gamma(x,y,z,t)=z-\eta(x,y,t) \tag{1}$$

from which the outward normal vector $\mathbf{n}$ is written as

$$\mathbf{n}=\frac{\nabla\Gamma}{|\nabla\Gamma|}=\left[1+\left(\frac{\partial\eta}{\partial x}\right)^2+\left(\frac{\partial\eta}{\partial y}\right)^2\right]^{-1/2}\left(-\frac{\partial\eta}{\partial x},-\frac{\partial\eta}{\partial y},1\right) \tag{2}$$

The equations of motion and continuity in the bulk of each fluid phase for the flow through a porous medium are

$$\frac{\rho}{m}\left[\frac{\partial\mathbf{q}}{\partial t}+\frac{1}{m}(\mathbf{q}\cdot\nabla)\mathbf{q}\right]=-\nabla p-\rho g\mathbf{e}_z-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\mathbf{q} \tag{3}$$

$$\nabla\cdot\mathbf{q}=0 \tag{4}$$

where $\rho, p, \mathbf{q}, g, m, k_1, \mathbf{e}_z, \nu, \nu'$ denote, respectively, the fluid density, hydrostatic pressure, fluid velocity, acceleration due to gravity, porosity of porous medium, medium permeability, unit vector in the z-direction, kinematic viscosity, and kinematic viscoelasticity.

Assuming the flow of the fluids to be irrotational, and then there are velocity potentials $\Phi^{(j)}(x,y,z,t)$ such that $\mathbf{q}^{(j)} = U^{(j)}\mathbf{e}_x + \nabla\Phi^{(j)}$, $j = 1,2$, where $\mathbf{q}^{(j)}$ is the total fluid velocities, and $\mathbf{e}_x$ is the unit vector in the x-direction. For incompressible fluids, the potential $\Phi^{(j)}, (j = 12)$ satisfies Laplace's equation, i.e.

$$\nabla^2\Phi^{(j)} = 0,\ j = 1,2 \tag{5}$$

We shall assume that the quasi-static approximation is valid for this problem, and hence the electric field $\mathbf{E}$ is irrotational. Thus the electrical equations are

$$\nabla\cdot(\varepsilon\mathbf{E}) = 0 \quad \text{and} \quad \nabla\times\mathbf{E} = 0 \tag{6}$$

Therefore, the electric field can be expressed in terms of an electrostatic potential $\Psi^{(j)}(x,y,z,t)$, i.e., $\mathbf{E} = -\nabla\Psi$, such that the total electric fields can be written as

$$\mathbf{E}^{(j)} = -E_0^{(j)}\mathbf{e}_z - \nabla\Psi^{(j)},\ j = 1,2 \tag{7}$$

Note that, in the presence of surface charges at the interface, Eqs. (6) should be replaced by

$$\nabla\cdot(\varepsilon\mathbf{E}) = Q \quad \text{and} \quad \nabla\times\mathbf{E} = 0 \tag{8}$$

where free charges Q are present due to different electrophysical properties at the fluids. Since free charges will only present at the interface, then in the bulk, Eqs. (6) are valid, and they have to be solved to obtain the electric field distribution in the analytical domain. It follows from Eqs. (6) and (7) that the electrostatic potentials also satisfy the Laplace's equation

$$\nabla^2\Psi^{(j)} = 0 \tag{9}$$

There are two surface forces that must be accounted for the stress tensor σ_{ik}. The first one results from the effect of the viscoelastic force of the Walters B′ type as given by

$$\sigma_{ik}^{\text{viscoelastic}} = -p\delta_{ik} + \frac{\rho}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\Phi \tag{10}$$

where the pressure p is obtained from Bernoulli's equation as

$$p = -\left[\rho g z + \frac{\rho}{m^2}\left\{m\frac{\partial\Phi}{\partial t} + U^{(j)}\frac{\partial\Phi}{\partial x} + \frac{1}{2}(\nabla\Phi)^2\right\}\right] \tag{11}$$

The other one is due to the electrical forces as given by [24]

$$\sigma_{ik}^{\text{electic}} = \varepsilon E_i E_k - \frac{1}{2}\varepsilon E^2\delta_{ik} \tag{12}$$

Hence the total stress tensor is defined as

$$\sigma_{ik} = \sigma_{ik}^{\text{viscoelastic}} + \sigma_{ik}^{\text{electic}} \tag{13}$$

3. Boundary conditions

The solution for the potentials $\Phi^{(j)}$ and $\Psi^{(j)}$, $(j = 1,2)$ should satisfy the following boundary conditions at the interface $z = \eta(x,y,t)$
1. The kinematic condition that the interface is moving with the fluid, leads to [21]

$$m\frac{\partial\eta}{\partial t} - \frac{\partial\Phi^{(j)}}{\partial z} + \frac{\partial\eta}{\partial x}\left[U^{(j)} + \frac{\partial\Phi^{(j)}}{\partial x}\right] + \frac{\partial\eta}{\partial y}\frac{\partial\Phi^{(j)}}{\partial y} = 0,\ j = 1,2 \tag{14}$$

2. The tangential component of the electric field is continuous at the interface, and thus leads to

$$\left\|\frac{\partial\Psi}{\partial x}\right\| + \frac{\partial\eta}{\partial x}\left\|\frac{\partial\Psi}{\partial z}\right\| + \frac{\partial\eta}{\partial x}\|E_0\| = 0 \tag{15}$$

where $\|*\|$ represents the jump across the interface.

3. The interfacial tangential stress tensor component balanced at the dividing surface of the system, and thus leads to

$$\left[1-\left(\frac{\partial \eta}{\partial x}\right)^2-\left(\frac{\partial \eta}{\partial y}\right)^2\right]\left\|\varepsilon\left[\frac{\partial \eta}{\partial y}\left(E_0+\frac{\partial \Psi}{\partial z}\right)+\frac{\partial \Psi}{\partial y}\right]\left[\frac{\partial \eta}{\partial x}\frac{\partial \Psi}{\partial x}+\frac{\partial \eta}{\partial y}\frac{\partial \Psi}{\partial y}-\left(E_0+\frac{\partial \Psi}{\partial z}\right)\right]\right\|=0 \tag{16}$$

4. The normal component of the stress tensor is discontinuous at the interface by the effective interfacial tension T [22]

$$\begin{aligned}&\frac{1}{m^2}\left\{m\left\|\rho\frac{\partial \Phi}{\partial t}\right\|+\left\|\rho U\frac{\partial \Phi}{\partial x}\right\|+\frac{1}{2}\left\|\rho(\nabla\Phi)^2\right\|\right\}+\frac{1}{k_1}\left\|\rho\nu\Phi\right\|-\frac{1}{k_1}\left\|\rho\nu'\frac{\partial \Phi}{\partial t}\right\|+\frac{1}{2}\left\|\varepsilon\left(\frac{\partial \Psi}{\partial z}\right)^2\right\|\\&+\left\|\varepsilon E_0\frac{\partial \Psi}{\partial z}\right\|-2\frac{\partial \eta}{\partial x}\left\|\varepsilon\left(E_0+\frac{\partial \Psi}{\partial z}\right)\frac{\partial \Psi}{\partial x}\right\|-2\frac{\partial \eta}{\partial y}\left\|\varepsilon\left(E_0+\frac{\partial \Psi}{\partial z}\right)\frac{\partial \Psi}{\partial y}\right\|\\&-\left[\left(\frac{\partial \eta}{\partial x}\right)^2+\left(\frac{\partial \eta}{\partial y}\right)^2\right]\left\|\varepsilon E_0^2+2\varepsilon E_0\frac{\partial \Psi}{\partial z}\right\|-\frac{1}{2}\left\|\varepsilon\left\{\left(\frac{\partial \Psi}{\partial x}\right)^2+\left(\frac{\partial \Psi}{\partial y}\right)^2\right\}\right\|+gz\left\|\rho\right\|\\&+T\left\{\left[1-\frac{3}{2}\left(\left(\frac{\partial \eta}{\partial x}\right)^2+\left(\frac{\partial \eta}{\partial y}\right)^2\right)\right]\left[\frac{\partial^2\eta}{\partial x^2}\left(1+\left(\frac{\partial \eta}{\partial y}\right)^2\right)+\frac{\partial^2\eta}{\partial y^2}\left(1+\left(\frac{\partial \eta}{\partial x}\right)^2\right)-2\frac{\partial^2\eta}{\partial x\partial y}\frac{\partial \eta}{\partial x}\frac{\partial \eta}{\partial y}\right]\right\}=0\end{aligned} \tag{17}$$

4. Multiple scales method and linear stability analysis

In order to describe the nonlinear interactions of small but finite amplitude waves, we use the derivative expansion method with multiple time scales [25]. Following the usual procedure, let us first expand η, $\Phi^{(j)}$ and $\Psi^{(j)}$, $(j=1,2)$ in the following asymptotic series

$$\eta(x,y,t)=\sum_{n=1}^{3}\epsilon^n\eta_n(x_0,x_1,x_2,y_0,y_1,y_2,t_0,t_1,t_2)+O(\epsilon^4) \tag{18}$$

$$\begin{pmatrix}\Phi^{(j)}\\ \Psi^{(j)}\end{pmatrix}(x,y,z,t)=\sum_{n=1}^{3}\epsilon^n\begin{pmatrix}\Phi_n^{(j)}\\ \Psi_n^{(j)}\end{pmatrix}(x_0,x_1,x_2,y_0,y_1,y_2,z,t_0,t_1,t_2)+O(\epsilon^4) \tag{19}$$

where ϵ is a small parameter indicating the weakness of the nonlinearity. The multiple scales $x_n=\epsilon^n x$, $y_n=\epsilon^n y$, and $t_n=\epsilon^n t$ are assumed to satisfy the following derivative expansions

$$\frac{\partial}{\partial \alpha}=\sum_{n=0}^{3}\epsilon^n\frac{\partial}{\partial \alpha_n}+O(\epsilon^4) \tag{20}$$

where α is any of the variables x, y, and t. The short scales x_0 and y_0 and the fast scale t_0 denote respectively to the wavelength and the frequency of the wave. Here t_1 and t_2 represent the slow temporal scales of the phase and the amplitude, respectively, whereas the long scales x_1, y_1 and x_2, y_2 stand for the spatial modulations of the phase and the amplitude. Expanding now the boundary condition (14)-(17) into Taylor series around the undisturbed surface $z=0$, then substituting equations (18)-(20) into equations (5) and (6) and the resulting boundary conditions, and equating the coefficients of the same powers in ϵ, we obtain a sequence of sets of equations for η_n, $\Phi_n^{(j)}$, and $\Psi_n^{(j)}$, $(j=1,2)$. These equations are not given here because they are very lengthy.
We assume that there is a steady flow in the undisturbed state so that we choose the following quasi-monochromatic wave as the starting solutions to the first order problem $(j=1,2)$

$$\eta_1=A\exp(i\theta)+c.c. \tag{21}$$

$$\Phi_1^{(j)}=\pm\frac{i}{k}(k_xU^{(j)}-\omega m)A\exp(i\theta\pm kz)+c.c. \tag{22}$$

$$\Psi_1^{(j)}=-E_0^{(j)}A\exp(i\theta\pm kz)+c.c. \tag{23}$$

where $\theta=k_xx_0+k_yy_0-\omega t_0$ is the phase of the carrier wave, $k=\sqrt{k_x^2+k_y^2}$, k_x and k_y are, respectively, the wave number components along the x- and y-directions, ω is the angular frequency, $c.c.$ stands for the complex conjugate of the preceding term (or terms), and i is the imaginary unit. Here, the complex amplitude of the surface

elevation is function of slow scales x_1, x_2, y_1, y_2, t_1 and t_2, i.e. the amplitude A is dependent on the slower time and larger space variables.
In order that the starting solution should not be trivial, the wave number k and the frequency ω must satisfy the following dispersion relation

$$\begin{aligned} S(\omega,k) = &\frac{1}{k}\left[1-\frac{m}{k_1}\left(\alpha^{(2)}\nu'^{(2)}+\alpha^{(1)}\nu'^{(2)}\right)\right]\omega^2+\frac{1}{k}\left[\frac{im}{k_1}(\alpha^{(2)}\nu^{(2)}+\alpha^{(1)}\nu^{(1)})-\frac{2k_x}{m}\left(\alpha^{(2)}U^{(2)}+\alpha^{(1)}U^{(1)}\right)\right.\\ &\left.+\frac{k_x}{k_1}\alpha^{(2)}\nu'^{(1)}U^{(2)}+\alpha^{(1)}\nu'^{(1)}U^{(1)}\right]\omega+\frac{k_x^2}{km^2}\left[\alpha^{(2)}U^{(2)^2}+\alpha^{(1)}U^{(1)^2}\right]-\frac{ik_x}{kk_1}\left[\alpha^{(2)}\nu^{(2)}U^{(2)}+\alpha^{(1)}\nu^{(1)}U^{(1)}\right] \\ &-g\left[(\alpha^{(1)}-\alpha^{(2)})+\frac{k^2T}{g(\rho^{(2)}+\rho^{(1)})}\right]+\frac{kV_E^2}{(\rho^{(2)}+\rho^{(1)})}=0 \end{aligned} \tag{24}$$

where

$$\alpha^{(j)}=\frac{\rho^{(j)}}{(\rho^{(2)}+\rho^{(1)})} \quad \text{and} \quad V_E=\varepsilon^{(1)}E_0^{(1)^2}+\varepsilon^{(2)}E_0^{(2)^2},\ j=1,2 \tag{25}$$

The dispersion relation (24) can be written in the form

$$a_0\omega^2+(a_1+ib_1)\omega+(a_2+ib_2)=0 \tag{26}$$

where

$$\begin{aligned} a_0 &= \frac{1}{k}\left[1-\frac{m}{k_1}\left(\alpha^{(2)}\nu'^{(2)}+\alpha^{(1)}\nu'^{(1)}\right)\right] \\ a_1 &= -\frac{2k_x}{mk}(\alpha^{(2)}U^{(2)}+\alpha^{(1)}U^{(1)})+\frac{k_x}{k_1k}(\alpha^{(2)}\nu'^{(2)}U^{(2)}+\alpha^{(1)}\nu'^{(1)}U^{(1)}) \\ a_2 &= \frac{k_x^2}{km^2}\left(\alpha^{(2)}U^{(2)^2}+\alpha^{(1)}U^{(1)^2}\right)-g\left[(\alpha^{(1)}-\alpha^{(2)})+\frac{k^2T}{g(\rho^{(2)}+\rho^{(1)})}\right]+\frac{kV_E^2}{(\rho^{(2)}+\rho^{(1)})} \\ b_1 &= \frac{m}{kk_1}(\alpha^{(2)}\nu^{(2)}+\alpha^{(1)}\nu^{(1)}) \\ b_2 &= -\frac{k_x}{kk_1}(\alpha^{(2)}\nu^{(2)}U^{(2)}+\alpha^{(1)}\nu^{(1)}U^{(1)}) \end{aligned}$$

Equation (26) represents the linear dispersion relation for surface waves propagation of two streaming dielectric viscoelastic fluids (of Walters B′ type) through porous medium in the presence of surface charges at their common interface. This dispersion relation is satisfied by the values of ω and k for constant values of the other physical parameters. If the real part of ω is positive, the disturbance will grow in time and the basic flow becomes unstable, while if the real part of ω is negative, the disturbance will decay and the basic flow will be stable. Note that, the linear dispersion relation (26), in the limiting case of two dimensional semi-infinite fluids, i.e. when $k_x=k, k_y=0$, reduces to the same equation obtained earlier by El-Sayed [26], and in the limiting case of two dimensional disturbances and absence of electric fields, it reduces to the corresponding equation obtained earlier by Sharma et al. [7] which is a generalization of the result of Chandrasekhar [27] for nonporous medium in which both the kinematic viscosities and viscoelasticities are absent.

Applying the Routh-Hurwitz stability criterion [28] to Eq. (26), we obtain the necessary and sufficient conditions for stability as

$$b_1>0 \quad \text{and} \quad a_2b_1^2-a_1b_1b_2+a_0b_2^2\le 0 \tag{27}$$

Since $\nu^{(1)}$ and $\nu^{(2)}$ are always positive, then the first condition in Eq. (27) is trivially satisfied, while the second condition is satisfied if

$$V_E^2\le V_c \tag{28}$$

where

$$\begin{aligned} V_c = &\frac{g}{k}(\rho^{(1)}-\rho^{(2)})+Tk+\frac{\rho^{(1)}\rho^{(2)}k_x^2}{m^2k^2k_1(\rho^{(1)}\nu^{(1)}+\rho^{(2)}\nu^{(2)})^2} \\ &\times\left\{\begin{array}{l} -k_1\left(\rho^{(1)}\nu^{(1)^2}+\rho^{(2)}\nu^{(2)^2}\right)\left(U^{(1)}-U^{(2)}\right)^2 \\ +m\left(\rho^{(1)}\nu^{(1)}U^{(1)}-\rho^{(2)}\nu^{(2)}U^{(2)}\right)\times\left(\nu^{(1)}\nu'^{(2)}-\nu^{(2)}\nu'^{(1)}\right)\left(U^{(1)}-U^{(2)}\right) \end{array}\right\} \end{aligned} \tag{29}$$

Now, to see the effects of various parameters included in the analysis on the linear stability of the system under consideration, we draw the transition curve $\log V_E^2 = \log V_c$ versus the wave number k_x for various values of the other physical parameters in two-dimensional disturbances $k_y = 0$ (normal curves), and three-dimensional disturbances $k_y = 3$ (curves with $\star$) as shown in Figs. 1-3. Hence, the stability criterion occurs when $V_E^2 \leq V_c$, otherwise, instability holds when $V_E^2 > V_c$. It is clear from Figs. 1-3 that, for all physical parameters included in the analysis, and in both two-, and three-dimensional disturbances, the system is always unstable when $\log V_E^2 > \log V_c$, while it is stable when $\log V_E^2 \leq \log V_c$.

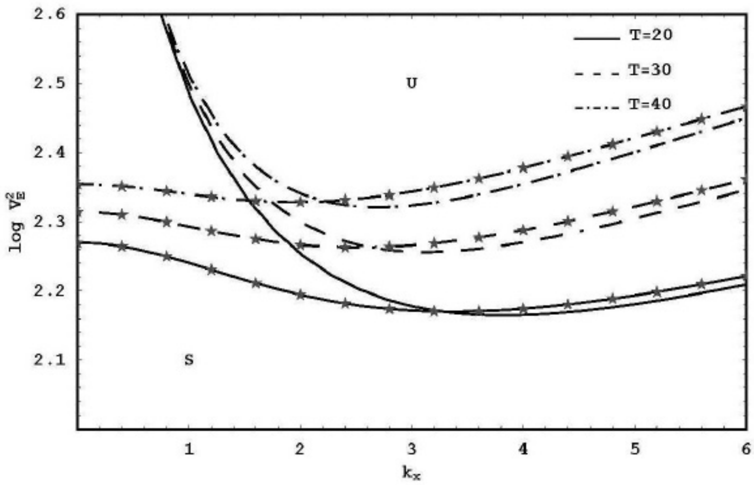

Fig. 1. Variation of $\log V_E^2$ with k_x for various values of T of the system $\rho^{(1)} = 0.987$ g/cm^3, $\rho^{(2)} = 0.689$ g/cm^3, $m = 0.5$ s/cm, $\nu^{(1)} = 0.8$ cm^2/s, $\nu^{(2)} = 0.9$ cm^2/s, $\nu'^{(1)} = 0.6$, $\nu'^{(2)} = 0.7$, $k_1 = 0.6$ cm^2, $U^{(1)} = 5cm/s, U^{(2)} = 7$ cm/s, $g = 981$ cm/s^2, when $k_y = 0$ (normal curves) and $k_y = 2$ (curves with $\star$).

Fig. 1 shows the variation of $\log V_E^2$ with k_x when $k_y = 0$ and 3, for different values of surface tension coefficient T. It is seen from this figure that, in the two-dimensional disturbances case, $k_y = 0$, and for all values of T, the system always stable for small wave number values after which instability sets in. By increasing the surface tension coefficient value, it is found that the unstable region decreases. Therefore, we conclude in the two-dimensional disturbances case that the surface tension coefficient has a stabilizing effect. It should be noted also from Fig. 1 that in the three-dimensional disturbances case, $k_y = 3$, the unstable regions increase (or decrease) after critical wave numbers values k_x in comparison with the corresponding two-dimensional disturbances case. Hence, the surface tension coefficient is found to has a stabilizing effect in both two-, and three-dimensional disturbances, separately, and the system in the three-dimensional disturbances case is more unstable (or stable) than the corresponding two-dimensional disturbances case according to whether the wave number value is lower (or higher) than a critical wave number value k_x. The effects of both the porosity of porous medium m and the kinematic viscosities $\nu^{(1)}$ and $\nu^{(2)}$ on the stability of the considered system are found to be exactly similar to the effect of surface tension coefficient illustrated in Fig. 1, but figures are not given here to save space. Therefore, they have also stabilizing effects in two-, and three-dimensional disturbances.

Fig. 2 shows the variation of $\log V_E^2$ with k_x when $k_y = 0$ and 3, for different values of fluid velocities $U^{(1)}$ and $U^{(2)}$. It is seen from this figure that, in both two-, and three-dimensional disturbances case (when $k_y = 0$, 3, respectively), and for all values of $U^{(1)}$ and $U^{(2)}$, that the unstable region increases by increasing the fluid velocities values. Therefore, we conclude in the two-, and three-dimensional disturbances cases that the fluid velocities have destabilizing effects. It should be noted also from Fig. 2 that the system in the three-dimensional disturbances case is more unstable (or stable) than the corresponding two-dimensional disturbances case according to the wave number value is lower (or higher) than a critical wave number value k_x. The effect of kinematic viscoelasticities $\nu'^{(1)}$ and $\nu'^{(2)}$ on the stability of the considered system is found to be exactly similar to the effect of fluid velocities illustrated in Fig. 2, but the corresponding figure excluded. Therefore, they have also destabilizing effects in both two-, and three-dimensional disturbances.

Fig. 3 shows the variation of $\log V_E^2$ with k_x when $k_y = 0$ and 3, for different values of medium permeability k_1. It is clear from this figure, and in the presence of porous medium, that the stable region increases by increasing the medium permeability values. Therefore, we conclude that the medium permeability has a stabilizing effect in both two-, and three-dimensional disturbances cases. Note also that the system is more unstable in absence of porous medium than in its presence. Finally, from Figs. 1-3, for small electric fields values, the system is always stable, and for higher electric fields, the instability of the system sets in and it increases by increasing the electric fields for all

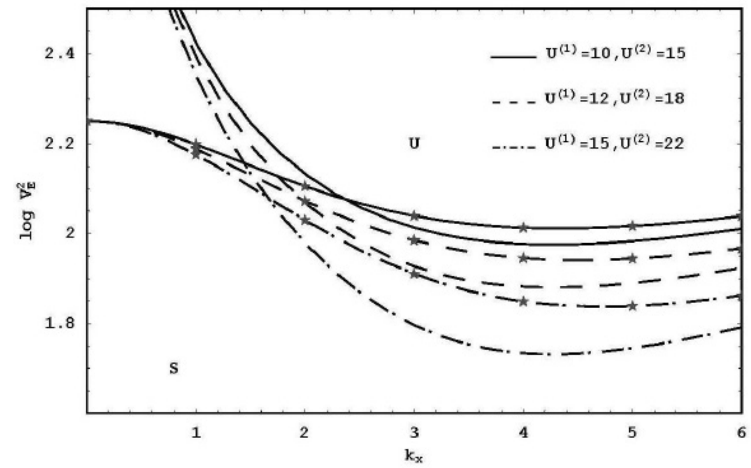

Fig. 2. Variation of $\log V_E^2$ with k_x for various values of $U^{(1)}$ and $U^{(2)}$ of the system $\rho^{(1)} = 0.987$ g/cm^3, $\rho^{(2)} = 0.689$ g/cm^3, $\nu'^{(1)} = 0.6$, $\nu'^{(2)} = 0.7$, $k_1 = 0.05$ cm^2, $\nu^{(1)} = 0.8$ cm^2/s, $\nu^{(2)} = 0.9$ cm^2/s, $T = 16$ dyn/cm, $m = 0.5$ s/cm, $g = 981$ cm/s^2, when $k_y = 0$ (normal curves) and $k_y = 3$ (curves with $\star$).

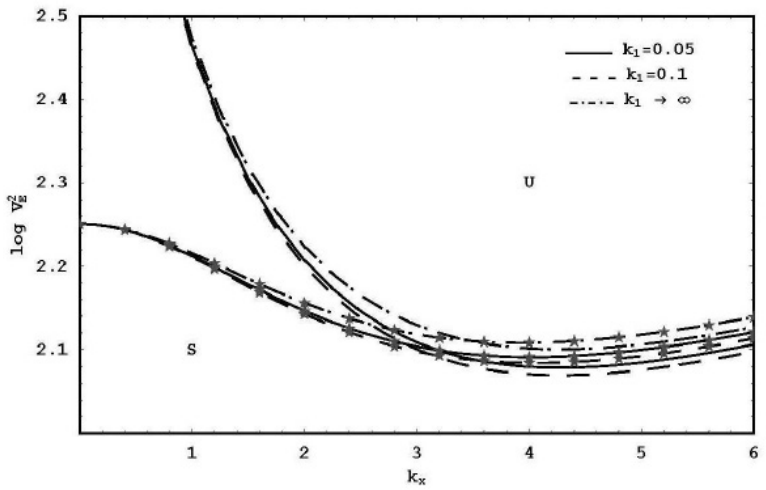

Fig. 3. Variation of $\log V_E^2$ with k_x for various values of k_1 of the system $\rho^{(1)} = 0.987$ g/cm^3, $\rho^{(2)} = 0.689$ g/cm^3, $m = 0.5$ s/cm, $\nu^{(1)} = 0.8$ cm^2/s, $\nu^{(2)} = 0.9$ cm^2/s, $\nu'^{(1)} = 0.6$, $\nu'^{(2)} = 0.7$, $T = 76$ dyn/cm, $U^{(1)} = 5$ cm/s, $U^{(2)} = 10$ cm/s, $g = 981$ cm/s^2, when $k_y = 0$ (normal curves) and $k_y = 3$ (curves with $\star$).

physical parameters included in the analysis. Hence, we conclude that, the electric fields have usually destabilizing effects.

5. Higher-order problems and evolution equation

Since our aim is to study the amplitude modulation for traveling waves, we shall substitute the linear solutions given by Eqs. (21)-(23) into the second-order problem. The resulting equations give the following solutions for $\eta_2, \Phi_2^{(j)}$ and $\Psi_2^{(j)}$ $(j = 1, 2)$

$$\eta_2 = \Lambda A^2 \exp(2i\theta) + c.c. \tag{30}$$

$$\begin{aligned}\Phi_2^{(j)} = \pm\Big[& \frac{1}{k^2}\left(k_x U^{(j)} - \omega m\right)\left(k_x \frac{\partial A}{\partial x_1} + k_y \frac{\partial A}{\partial y_1}\right) z \\ & + \frac{1}{k}\left\{\pm m \frac{\partial A}{\partial t_1} \pm U^{(j)} \frac{\partial A}{\partial x_1}\right\} \mp \frac{1}{k^2}\left(k_x U^{(j)} - \omega m\right)\left(k_x \frac{\partial A}{\partial x_1} + k_y \frac{\partial A}{\partial y_1}\right)\Big] A \exp(i\theta \pm kz) \\ & - \frac{i}{k}\left(k_x U^{(j)} - \omega m\right)(k \mp \Lambda) A^2 \exp 2(i\theta \pm kz) + c.c.\end{aligned} \tag{31}$$

$$\Psi_2^{(j)} = \frac{i\varepsilon^{(j\pm1)}\left(E_0^{(1)}-E_0^{(2)}\right)}{k\left(\varepsilon^{(1)}+\varepsilon^{(2)}\right)}\left(k_x\frac{\partial A}{\partial x_1}+k_y\frac{\partial A}{\partial y_1}\right)z\exp(i\theta\pm kz) + \frac{\varepsilon^{(j\pm1)}\left(E_0^{(1)}-E_0^{(2)}\right)}{k\left(\varepsilon^{(1)}+\varepsilon^{(2)}\right)}(k\mp\Lambda)A^2\exp 2(i\theta\pm kz)+c.c. \tag{32}$$

where Λ is given by

$$\Lambda=\frac{1}{S(2\omega,2k)}\left\{-\frac{1}{m^2}\left[\alpha^{(2)}(k_xU^{(2)}-\omega m)^2-\alpha^{(1)}(k_xU^{(1)}-\omega m)^2\right] -\frac{\omega}{k_1}\left[\alpha^{(2)}\nu'^{(2)}(k_xU^{(2)}-\omega m)-\alpha^{(1)}\nu'^{(1)}(k_xU^{(1)}-\omega m)\right]-\frac{k^2}{(\rho^{(2)}+\rho^{(1)})}\left(\varepsilon^{(2)}E_0^{(2)^2}-\varepsilon^{(1)}E_0^{(1)^2}\right)\right\} \tag{33}$$

The case when $S(2\omega,2k)=0$ for which η_2, $\Phi_2^{(j)}$, and $\Psi_2^{(j)}$, $(j=1,2)$ become infinite, corresponds to the case of second harmonic resonance which can be dealt with along the same lines outlined by Singla *et al.* [29]. Hence, we have assumed this quantity to be different from zero in Eqs. (30)-(32).

The non-secularity condition for the second-order perturbation can be obtained from the last boundary condition of the second-order equations by equating to zero the coefficient of $\exp(i\theta)$, and leads to

$$\frac{\partial A}{\partial t_1}+\upsilon_{k_x}\frac{\partial A}{\partial x_1}+\upsilon_{k_y}\frac{\partial A}{\partial y_1}=0 \tag{34}$$

Together with its c.c., where υ_{k_x} and υ_{k_y} are the group velocities of the wave train in the x-and y-directions, respectively, expressed as

$$\upsilon_{k_x}=-\left(\frac{\partial S}{\partial k_x}\right)\left(\frac{\partial S}{\partial\omega}\right)^{-1} \quad\text{and}\quad \upsilon_{k_y}=-\left(\frac{\partial S}{\partial k_y}\right)\left(\frac{\partial S}{\partial\omega}\right)^{-1} \tag{35}$$

Eq. (34) shows the modulations on the time scale ϵ^{-1} propagate without change of shape with the group velocities. Equation (34) indicates that A depends on x_1, y_1 and t_1 through the transformation $\gamma_1=k^{-1}\{\left(k_xx_1+k_yy_1\right)-\left(k_x\upsilon_{k_x}+k_y\upsilon_{k_y}\right)t_1\}$.

Now, we proceed to the third-order problem. By using the first- and second-order solutions and simplifying the right-hand side of the third-order equations and after some straightforward reductions, we can express the particular solutions for η_3, $\Phi_3^{(j)}$ and $\Psi_3^{(j)}$, $(j=1,2)$, in the forms

$$\eta_3=\frac{1}{2}k^2A^2\bar{A}\exp(i\theta)+c.c. \tag{36}$$

$$\begin{aligned}\Phi_3^{(j)}=&\left\{\pm\frac{1}{k}\left(m\frac{\partial A}{\partial t_2}+U^{(j)}\frac{\partial A}{\partial x_2}\right)-i(k_xU^{(j)}-\omega m)(3\Lambda\mp k)A^2\bar{A}\pm\frac{i}{k^3}(1\right.\\ &\mp kz)\left(k_xm\frac{\partial^2A}{\partial x_1\partial t_1}+k_yU^{(j)}\frac{\partial^2A}{\partial x_1\partial y_1}+k_xU^{(j)}\frac{\partial^2A}{\partial x_1^2}+k_ym\frac{\partial^2A}{\partial y_1\partial t_1}\right)\\ &\mp\frac{i(k_xU^{(j)}-\omega m)}{2k^5}\left(3\mp3kz+k^2z^2\right)\left(k_x^2\frac{\partial^2A}{\partial x_1^2}+k_y^2\frac{\partial^2A}{\partial y_1^2}\right.\\ &\left.+2k_xk_y\frac{\partial^2A}{\partial x_1\partial y_1}\right)\mp\frac{(k_xU^{(j)}-\omega m)}{k^3}(1\mp kz)\left(k_x\frac{\partial A}{\partial x_2}+k_y\frac{\partial A}{\partial y_2}\right)\\ &\left.\pm\frac{i(k_xU^{(j)}-\omega m)}{2k^3}(1\mp kz)\left(\frac{\partial^2A}{\partial x_1^2}+\frac{\partial^2A}{\partial y_1^2}\right)\right\}\exp(i\theta\pm kz)\\ &+\frac{1}{k}\left\{\pm\frac{1}{2}(\Lambda\mp k)\left(m\frac{\partial A^2}{\partial t_1}+U^{(j)}\frac{\partial A^2}{\partial x_1}\right)+\frac{(k_xU^{(j)}-\omega m)}{k}\right.\\ &\left.\times\left(z(\Lambda\mp k)\mp\frac{\Lambda}{2k}\right)\left(k_x\frac{\partial A^2}{\partial x_1}+k_y\frac{\partial A^2}{\partial y_1}\right)\right\}\exp 2(i\theta\pm kz)\\ &-3(k_xU^{(j)}-\omega m)\left(\Lambda\mp\frac{k}{2}\right)A^3\exp 3(i\theta\pm kz)+c.c.\end{aligned} \tag{37}$$

$$\Psi_3^{(j)} = E_0^{(j)} \left\{ \pm k(3\Lambda \mp k) A^2 \bar{A} + \frac{i}{2k^3} \left[\mp i k^2 z \left(\frac{\partial^2 A}{\partial x_1^2} + \frac{\partial^2 A}{\partial y_1^2} \right) \right. \right.$$
$$\pm 2k^2 z \left(k_x \frac{\partial A}{\partial x_2} + k_y \frac{\partial A}{\partial y_2} \right) - i \left(k z^2 \mp z \right) \left(k_x^2 \frac{\partial^2 A}{\partial x_1^2} + k_y^2 \frac{\partial^2 A}{\partial y_1^2} \right.$$
$$\left. \left. \left. + 2 k_x k_y \frac{\partial^2 A}{\partial x_1 \partial y_1} \right) \right] \right\} \exp(i\theta \pm kz)$$
$$\pm \frac{i E_0^{(j)}}{k} \left(-\frac{1}{2} + (\Lambda \mp k) z \right) \left(k_x \frac{\partial A^2}{\partial x_1} + k_y \frac{\partial A^2}{\partial y_1} \right) \exp 2(i\theta \pm kz)$$
$$\pm 3k E_0^{(j)} \left(\Lambda \mp \frac{k}{2} \right) A^3 \exp 3(i\theta \pm kz) + c.c. \tag{38}$$

Finally, substituting from the third-order solution given by Eqs. (36)-(38) into the last boundary condition in the third-order problem, we obtain the non-secularity condition from the coefficient of $\exp(i\theta)$, in the form

$$i \left\{ \frac{\partial S}{\partial \omega} \frac{\partial A}{\partial t_2} - \frac{\partial S}{\partial k_x} \frac{\partial A}{\partial x_2} - \frac{\partial S}{\partial k_y} \frac{\partial A}{\partial y_2} \right\} + \left\{ \frac{\partial^2 S}{\partial \omega \partial k_x} \frac{\partial^2 A}{\partial t_1 \partial x_1} + \frac{\partial^2 S}{\partial \omega \partial k_y} \frac{\partial^2 A}{\partial t_1 \partial y_1} - \frac{\partial^2 S}{\partial k_x \partial k_y} \frac{\partial^2 A}{\partial x_1 \partial y_1} \right\}$$
$$- \frac{1}{2} \left\{ \frac{\partial^2 S}{\partial \omega^2} \frac{\partial^2 A}{\partial t_1^2} + \frac{\partial^2 S}{\partial k_x^2} \frac{\partial^2 A}{\partial x_1^2} + \frac{\partial^2 S}{\partial k_y^2} \frac{\partial^2 A}{\partial y_1^2} \right\} + G A^2 \overline{A} = 0 \tag{39}$$

where G is given by

$$G = \frac{g k^2}{2} (\alpha^{(2)} - \alpha^{(1)}) + \frac{T k^4}{(\rho^{(2)} + \rho^{(1)})} - \frac{2k^2}{(\rho^{(2)} + \rho^{(1)})} \left\{ \varepsilon^{(2)} E_0^{(2)2} \left(\frac{3k}{4} - \Lambda \right) + \varepsilon^{(1)} E_0^{(1)2} \left(\frac{3k}{4} + \Lambda \right) \right\}$$
$$- \frac{2}{m^2} \left\{ \alpha^{(2)} (k_x U^{(2)} - \omega m)^2 \left(\frac{3k}{4} - \Lambda \right) + \alpha^{(1)} (k_x U^{(1)} - \omega m)^2 \right.$$
$$\left. \times \left(\frac{3k}{4} + \Lambda \right) \right\} - \frac{2\omega}{k_1} \left\{ \alpha^{(2)} \nu'^{(2)} (k_x U^{(2)} - \omega m) \left(\frac{3k}{4} + \Lambda \right) + \alpha^{(1)} \nu'^{(1)} (k_x U^{(1)} - \omega m) \left(\frac{3k}{4} - \Lambda \right) \right\}$$
$$+ \frac{2i}{k_1} \left\{ \alpha^{(2)} \nu^{(2)} (k_x U^{(2)} - \omega m) \left(\frac{3k}{4} - \Lambda \right) + \alpha^{(1)} \nu^{(1)} (k_x U^{(1)} - \omega m) \left(\frac{3k}{4} + \Lambda \right) \right\} \tag{40}$$

Assuming that A depends on x_2, y_2 and t_2 through the transformation $\gamma_2 = k^{-1}\{(k_x x_2 + k_y y_2) - (k_x \upsilon_{k_x} + k_y \upsilon_{k_y}) t_2\}$ and $\tau = t_2$ [23], we obtain finally from Eq. (39) the following nonlinear Ginzburg-Landau equation with complex coefficients

$$i \frac{\partial A}{\partial \tau} + \gamma \frac{\partial^2 A}{\partial \gamma_1^2} = \beta |A|^2 A \tag{41}$$

in which the coefficients γ and β are defined as

$$\gamma = \frac{1}{2k^2} \left[k_x^2 \upsilon_{k_x k_x} + 2 k_x k_y \upsilon_{k_x k_y} + k_y^2 \upsilon_{k_y k_y} \right] \tag{42}$$

and

$$\beta = -G \left(\frac{\partial S}{\partial \omega} \right)^{-1} \tag{43}$$

In Eq. (41), the complex coefficients γ and β can be written in the form

$$\gamma = \gamma_r + i\gamma_i \quad \text{and} \quad \beta = \beta_r + i\beta_i \tag{44}$$

Note that, in the limiting case of absence of fluid velocities, kinematic viscosities and kinematic viscoelasticities, Eq. (41) reduces to the same equations obtained earlier by Mohamed and Elshehawey [30, 31], in absence and presence of fluid velocities, respectively, then their results are therefore recovered. It is interesting also to note that the two coefficients γ and β are responsible for the modulational instability of the nonlinear plane wave solution of the Ginzburg-Landau equation, as described in the next section.

6. Nonlinear stability analysis and discussion

We study the stability of the capillary-gravity waves for Walters B′ viscoelastic dielectric fluids streaming through porous medium in the presence of surface charges at their common interface, when the modulation of the wave packet amplitude takes place in the direction of the carrier wave propagation. We consider the dynamic solution of the complex Ginzburg-Landau equation (41). Accordingly, we separate the amplitude A into two parts

$$A=\left[A_0+\delta A\left(\gamma_1,\tau\right)\right]\exp\left(i\beta|A_0|^2\tau\right) \tag{45}$$

where A_0 is the constant amplitude perturbation, $\delta A(\delta A \ll A_0)$ is the small amplitude perturbation, and the non-linear frequency shift is (-$\beta|A_0|^2$). Substituting Eq. (45) into Eq. (41), and linearizing the resulting equation with respect to $\delta A\left(\gamma_1,\tau\right)$, we obtain the evolution equation for the perturbation in the form

$$i\frac{\partial(\delta A)}{\partial\tau}+\gamma\frac{\partial^2(\delta A)}{\partial\gamma_1^2}+\beta\,|A_0|^2(\delta A+\delta A^*)=0 \tag{46}$$

where δA^* is the complex conjugate of δA. We introduce [32]

$$\delta A\left(\gamma_1,\tau\right)=\tilde{U}\exp\left[i\left(\tilde{K}\gamma_1-\Omega\tau\right)\right]+\tilde{V}\exp\left[i\left(\tilde{K}\gamma_1-\Omega^*\tau\right)\right] \tag{47}$$

where $\tilde{U}$ and $\tilde{V}$ are complex constant amplitudes, with $(\tilde{K}\gamma_1-\Omega\tau)$ as modulation phase, and $\tilde{K}(\ll K)$ and $\Omega(\ll\omega)$ are the wave number and frequency of the modulated waves, respectively. Using Eq. (47) into Eq. (46) given a linear homogeneous system of equations for $\tilde{U}$ and $\tilde{V}$ in the form

$$\left[\Omega+\gamma\tilde{K}^2-\beta\,|A_0|^2\right]\tilde{U}-\beta\,|A_0|^2\,\tilde{V}=0 \tag{48}$$

and

$$\beta^*\,|A_0|^2\,\tilde{U}+\left[\Omega-\gamma^*\tilde{K}^2+\beta^*\,|A_0|^2\right]\tilde{V}=0 \tag{49}$$

Using Eq. (44), then the coupled system of equations (48) and (49) gives the following nonlinear dispersion relation

$$\Omega^2+2i\left[\tilde{K}^2\gamma_i-\beta_i\,|A_0|^2\right]\Omega-\tilde{K}^2\left[\tilde{K}^2\left(\gamma_i^2+\gamma_r^2\right)-2\left(\gamma_i\beta_i+\gamma_r\beta_r\right)|A_0|^2\right]=0 \tag{50}$$

Solving Eq. (50), we obtain

$$\Omega_{\pm}=-i\left[\tilde{K}^2\gamma_i-\beta_i\,|A_0|^2\right]\pm\sqrt{\gamma_r\tilde{K}^2\left(\gamma_r\tilde{K}^2-2\beta_r\,|A_0|^2\right)-\beta_i^2\,|A_0|^4} \tag{51}$$

Substituting Eq. (51) into Eq. (48), and after mathematical manipulation, we arrive at the modulational instability criterion for the nonlinear problem as

$$\gamma_r\beta_r+\gamma_i\beta_i>0 \tag{52}$$

The inequality (52) fulfils the well known Lange and Newell's criterion [33]. Consequently, the system is stable or unstable for $\gamma_r\beta_r+\gamma_i\beta_i \lessgtr 0$, respectively.

Now, we shall discuss numerically the nonlinear stability of the system under consideration by drawing the transition curves $\gamma_r\beta_r+\gamma_i\beta_i$ versus the wave number k_x for different parameters, namely $U^{(1)}, U^{(2)}, m, k_1, \nu'^{(1)}, \nu'^{(2)}, \nu^{(1)}, \nu^{(2)}, T, E_0^{(1)}$, and $E_0^{(2)}$, included in the analysis in the case of two-dimensional disturbances (normal curves), and three-dimensional curves (curves with $\star$).

Fig. 4 shows the variation $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for different values of the fluid velocities $U^{(1)}$ and $U^{(2)}$ when $k_y=0$ (two-dimensional disturbances) and $k_y=3$ (three-dimensional disturbances). It is clear from Fig. 4 that, in two-dimensional disturbances and absence of fluid velocities, i.e. when $k_y=0$ and $U^{(1)}=U^{(2)}=0$ (Rayleigh-Taylor instability case), the system is always unstable since the condition (52) is satisfied, while for the three-dimensional disturbances, the system is found to be more unstable than the corresponding two-dimensional disturbances case. Thus, we conclude that the system is unstable in absence of fluid velocities in both two- and three-dimensional disturbances, and this instability increases in the presence of the third dimension. In presence of fluid velocities (Kelvin-Helmholtz instability case), the system is stable, and the stability effect increases by increasing the fluid velocities values. Therefore, the fluid velocities have stabilizing effects in this case. Fig. 4 shows also that, in the three-dimensional disturbances ($k_y=3$) and for any values of fluid velocities, the system is unstable for a small wave number k_x range, and then it is stable, and this instability range decreases by increasing the fluid velocities. Hence, we conclude, in the three-dimensional disturbances, that the fluid velocities have dual role on the stability of the system, destabilizing for small wave number values and then stabilizing for higher wave

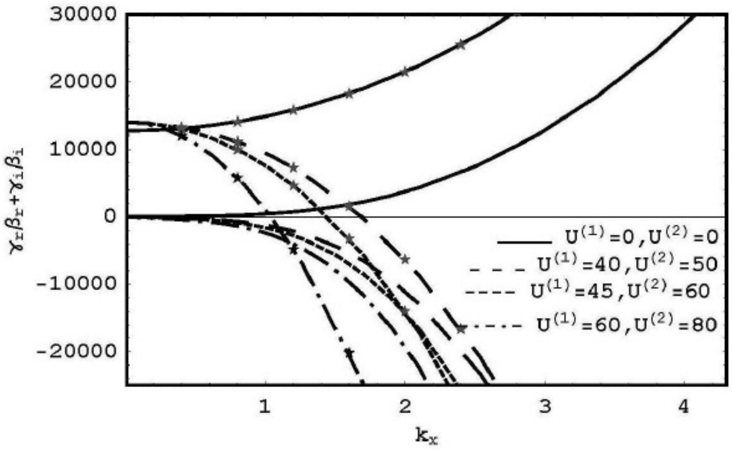

Fig. 4. Variation of $\gamma_r\beta_r+\gamma_i beta_i$ with k_x for various values of $U^{(1)}$ and $U^{(2)}$ of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=32$ dyn/cm, $k_1=0.3$ cm^2, $\nu^{(1)}=0.1$ cm^2/s, $\nu^{(2)}=0.3$ cm^2/s, $\nu'^{(1)}=0.2$, $\nu'^{(2)}=0.4$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=50$ V/cm, $E_0^{(2)}=100$ V/cm, $m=0.5$ s/cm, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

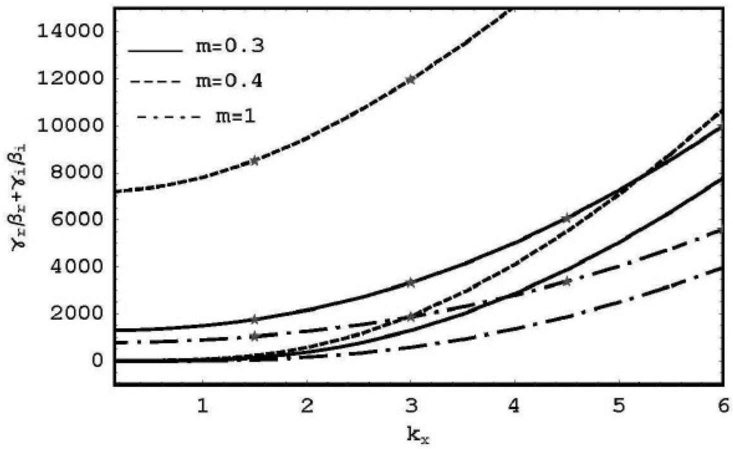

Fig. 5. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of m of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=32$ dyn/cm, $k_1=0.5$ cm^2, $\nu^{(1)}=0.1$ cm^2/s, $\nu^{(2)}=0.3$ cm^2/s, $\nu'^{(1)}=0.4$, $\nu'^{(2)}=0.6$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=100$ V/cm, $E_0^{(2)}=200$ V/cm, $U^{(1)}=40$ cm/s, $U^{(2)}=60$ cm/s, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

numbers values, separated by a critical wave number value which decreases by increasing the fluid velocities values.

Fig. 5 shows the variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for various values of the porosity of porous medium m, and it indicates that, for the two-dimensional disturbances and all values of the porosity of porous medium, the system is always unstable, while for the three-dimensional disturbances, the system is found to be more unstable than the corresponding two-dimensional disturbances case. It should be noted that the system in the presence of porous medium is more unstable than in its absence. Thus, we conclude that the porosity of porous medium has a destabilizing effect in both two- and three-dimensional disturbances, and this instability increases in the presence of a third dimension.

Fig. 6 shows the variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for various values of medium permeability k_1, and it indicates that for two-dimensional disturbances, that the system is always stable, while for the three-dimensional disturbances, we notice that the system is unstable for small wave number k_x range after which the system is a stable one. Note also that in absence of porous medium, the system is always unstable since the condition (52) is always satisfied, and this instability increases by increasing the third dimension. Therefore, the medium permeability has been found to has a stabilizing effect, and the third dimension destabilizes a small wave numbers range which is stable in the corresponding two-dimensional disturbances case.

Fig. 7 shows the variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for various values of kinematic viscoelasticities

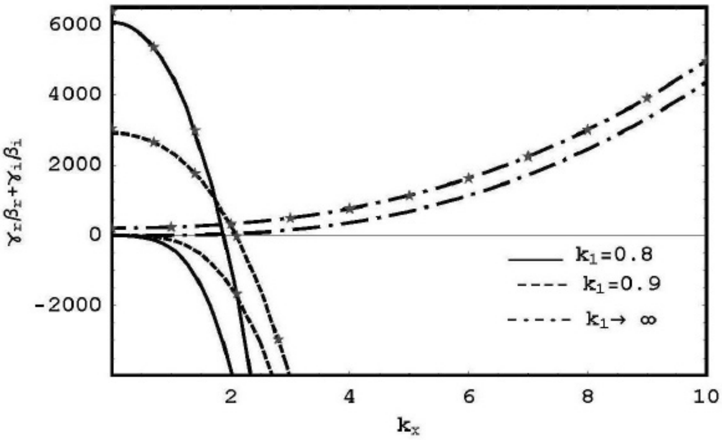

Fig. 6. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of k_1 of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=32$ dyn/cm, $\nu^{(1)}=0.2$ cm^2/s, $\nu^{(2)}=0.5$ cm^2/s, $\nu'^{(1)}=0.7$, $\nu'^{(2)}=0.9$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=50$ V/cm, $E_0^{(2)}=100$ V/cm, $m=0.5$ s/cm, $U^{(1)}=40$ cm/s, $U^{(2)}=50$ cm/s, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

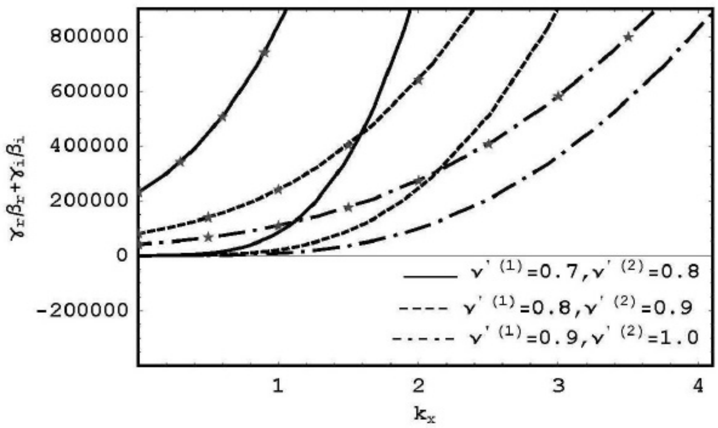

Fig. 7. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of $\nu'^{(1)}$ and $\nu'^{(2)}$ of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=32$ dyn/cm, $k_1=0.3$ cm^2, $U^{(1)}=40$ cm/s, $U^{(2)}=50$ cm/s, $\nu^{(1)}=0.1$ cm^2/s, $\nu^{(2)}=0.3$ cm^2/s, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=50$ V/cm, $E_0^{(2)}=100$ V/cm, $m=0.5$ s/cm, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

$\nu'^{(1)}$, $\nu'^{(2)}$, and it indicates that, for the two-dimensional disturbances and all values of the kinematic viscoelasticities, the system is always unstable since the condition (52) is satisfied, and this instability decreases by increasing kinematic viscoelasticities, while for the three-dimensional disturbances, the system is found to be more unstable than the corresponding two-dimensional disturbances case. Thus, we conclude that kinematic viscoelasticities have destabilizing effects in both two- and three-dimensional disturbances, and this instability increases in the presence of a third dimension.

Figs. 8 and 9 show the variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for various values of kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$, and surface tension coefficient T, respectively. Figure (8) indicates that. for the two-dimensional disturbances and all values of the kinematic viscosities, the system is always stable, and this stability decreases by increasing the kinematic viscosities, while for the three-dimensional disturbances, the system is found to be unstable for small wave number values, after which it a stable one. Thus, we conclude that kinematic viscosities have have dual role on the stability of the system destabilizing (for a small wabenumbers range), and then stabilizing afterwards in three-dimensional disturbances, i.e. the kinematic viscosities in the three-dimensional disturbances case destabilize a certain wave numbers range which is stable in the two-dimensional disturbances case.

Fig. 9 indicated that, in two-dimensional disturbances, the system is also stable for all surface tension coefficient T values, and this stability increases by increasing the surface tensions, while for the three-dimensional disturbances, the system is found to be unstable for small wave numbers values after which it is a stable one, and the instability wave numbers range increases by increasing the surface tension values. Thus, we conclude that the surface tensions have dual roles on the stability of the system destabilizing (for small wave numbers range), and then stabilizing

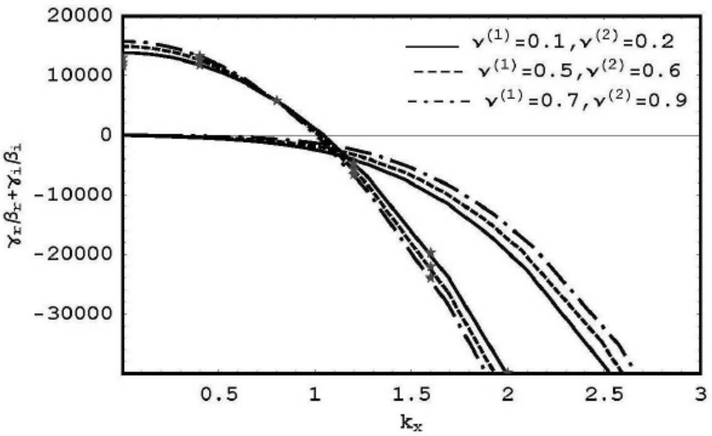

Fig. 8. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of $\nu^{(1)}$ and $\nu^{(2)}$ of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=32$ dyn/cm, $k_1=0.3$ cm^2, $U^{(1)}=60$ cm/s, $U^{(2)}=80$ cm/s, $\nu'^{(1)}=0.2$, $\nu'^{(2)}=0.4$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=50$ V/cm, $E_0^{(2)}=100$ V/cm, $m=0.5$s/cm, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

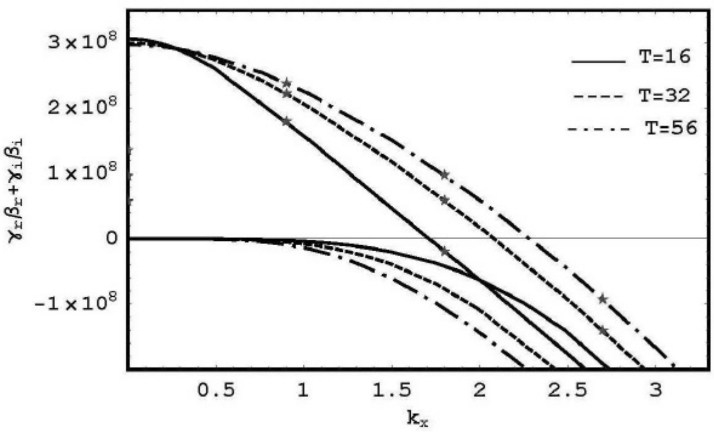

Fig. 9. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of T of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $U^{(1)}=40$ cm/s, $U^{(2)}=60$ cm/s, $k_1=0.3$ cm^2, $\nu^{(1)}=0.1$ cm^2/s, $\nu^{(2)}=0.3$ cm^2/s, $\nu'^{(1)}=0.4$, $\nu'^{(2)}=0.6$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $E_0^{(1)}=200$ V/cm, $E_0^{(2)}=400$ V/cm, $m=0.5$ s/cm, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

afterwards in the three-dimensional disturbances, i.e. the surface tensions in the three-dimensional disturbances case destabilize a certain wave numbers range which is stable in the two-dimensional disturbances case.

Fig. 10 shows the variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with the wave number k_x for various values of the electric fields $E_0^{(1)}$ and $E_0^{(2)}$ in both two-, and three-dimensional disturbances (when $k_y=0$ and 3) including the case of pure hydrodynamical model. It is clear from the figure that, in the case of absence of electric fields, i.e. when $E_0^{(1)}=E_0^{(2)}=0$, then the system is unstable for all wave numbers values, and it is more unstable in the three-dimensional disturbances than in the corresponding case of two-dimensional disturbances. In the presence of electric fields, i.e. when $E_0^{(1)}\neq 0$ and $E_0^{(2)}\neq 0$, we found that instability increases, since the quantity $\gamma_r\beta_r+\gamma_i\beta_i$ is always positive, and it increases by increasing the electric fields values. Therefore, we conclude that the electric fields have destabilizing effects for both two- and three-dimensional disturbances, and that instabilities in the three-dimensional disturbances case occur usually faster than their destabilizing effects in the corresponding two-dimensional disturbances case.

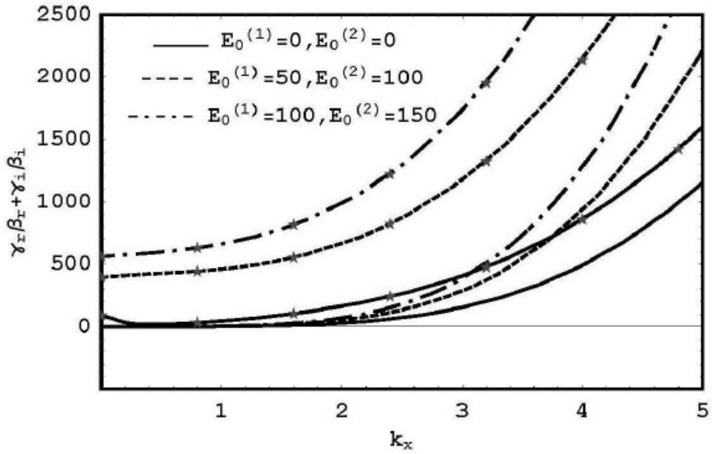

Fig. 10. Variation of $\gamma_r\beta_r+\gamma_i\beta_i$ with k_x for various values of $E_0^{(1)}$ and $E_0^{(2)}$ of the system $\rho^{(1)}=0.2$ g/cm^3, $\rho^{(2)}=0.8$ g/cm^3, $T=16$ dyn/cm, $k_1=0.05$ cm^2, $\nu^{(1)}=0.7$ cm^2/s, $\nu^{(2)}=0.8$ cm^2/s, $\nu'^{(1)}=0.7$, $\nu'^{(2)}=0.9$, $\varepsilon^{(1)}=0.4$, $\varepsilon^{(2)}=0.7$, $U^{(1)}=30$ cm/s, $U^{(2)}=70$ cm/s, $m=0.6$ s/cm, $g=981$ cm/s^2, when $k_y=0$ (normal curves) and $k_y=3$ (curves with $\star$).

7. Concluding remarks

The nonlinear electrohydrodynamic Kelvin-Helmholtz instability of two superposed semi-infinite Walters B′ viscoelastic dielectric fluids streaming through porous media under the effect of applied normal electric fields to their interface in absence of surface charges is investigated. The method of multiple scales is used to obtain a dispersion relation for the linear problem, and to derive a nonlinear Ginzburg-Landau equation with complex coefficients for the nonlinear problem. The linear and nonlinear stability conditions are obtained and discussed both analytically and numerically. In the linear stability analysis, and for both two- and three-dimensional disturbances cases, we found that:

1. The surface tension, porosity of porous medium, kinematic viscosities, and medium permeability have stabilizing effects.
2. T he fluid velocities, electric fields and kinematic viscoelasticities have destabilizing effects.
3. The system in the three-dimensional disturbances is more unstable than in three-dimensional disturbances, and then it is more stable of it after a critical wave number.

While in the nonlinear stability analysis case, we found that:

1. In absence of fluid velocities, the system is unstable in two-dimensional disturbances, and it is more unstable in three-dimensional disturbances.
2. The fluid velocities have dual roles on the stability of the system, i.e. stabilizing as well as destabilizing.
3. The porosity of porous medium has a destabilizing effect, and this instability in three-dimensional disturbances is higher than its effect in two-dimensional disturbances case. Also, the system is more unstable in presence of porous medium than in its absence.
4. In absence of porous medium, the system has been found to be unstable, and this instability in three-dimensional disturbances is higher than its effect in two-dimensional disturbances.
5. The kinematic viscoelasticities and electric fields have destabilizing effects, and this instability in three-dimensional disturbances is higher than its effect in two-dimensional disturbances.
6. The kinematic viscosities, medium permeability, and surface tension have stabilizing effects in two-dimensional disturbances, while in three-dimensional disturbances, they have dual roles on the stability destabilizing and then stabilizing.

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