Int. J. Adv. Appl. Math. and Mech. 2(2) (2014) 139 - 149 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

Thermal stress analysis due to surface heat source

Research Article

V. S. Kulkarni[1, *], K. C. Deshmukh[2], P. H. Munjankar[3]

[1]*Department of Mathematics, University of Mumbai, Mumbai-400098, Maharashtra, India*

[2]*Department of Mathematics, RTM Nagpur University, Nagpur-440010, Maharashtra, India*

[3]*Department of Mathematics, Dr. Ambedkar College, Chandrapur-442401, Maharashtra, India*

Received 07 November 2014; accepted (in revised version) 30 December 2014

Abstract: The present manuscript deals with the heat transfer and thermal stress analysis of cylinder due to internal heat generation under steady temperature conditions. The internal heat generation is taken as cylindrical surface heat source in annular region of linear length of cylinder and is situated concentrically inside the cylinder. The upper surface of a cylinder is subjected to arbitratory temperature whereas the lower surface and circular boundary surface are thermally insulated. The integral transform methods are used for the solution of non-homogeneous boundary value problem. The theory of linearized thermoelasticity based on solution of Navier's equation in terms of Goodier's thermoelastic displacement potential, Michell's function and the Boussinesq's function for cylindrical co-ordinate system have been used for discussion and analysis of thermal stresses.

MSC: 00A79 • 01A08 • 34B60 • 35K50

Keywords: Heat transfer analysis • Thermal Stress analysis • KSteady-state • Internal heat generation • Surface heat source.

1. Introduction

Thermoelasticity is based on temperature changes induced by expansion and compression of the test part. Although this coupling between mechanical deformation and thermal energy has been known for over a century. After world war second, there was very rapid development of thermoelasticity, stimulated by various engineering sciences. Thermoelasticity contains the generalized theory of heat conductions, the generalized theory of the thermal stresses. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role.

Sipailov et al. [1] studied the effect of heat transfer in the impulse method of measurement on the value of the maximum temperature and on the time for achieving its half-value. Taking this heat transfer effect into account increases the accuracy of measuring the thermophysical properties. Lohle et al. [2], analyze inverse heat conduction problems by the analysis of the system impulse response by the application of the non-integer system identification (NISI) method. Stakhanova et al. [3] carried out heat transfer on model fragments of fuel rod claddings during impulse supply of power. The time taken for reaching steady state for different parameters of impulse and the values of heat-transfer coefficient are determined. Lee [4] obtained three dimensional series solution for elastic thick plate subjected to general temperature distribution. Hata [5] concerned with a method for calculating the thermal-stress distribution in a nonhomogeneous thick elastic plate under steady distribution of the surface temperature whose shear modulus and coefficient of thermal expansion are assumed to be functions of z. Kulkarni et al. [6] determined the temperature changes and thermal stresses due to conduction of heat in the thick circular plate under transient temperature conditions and analyzed his analytical results for heat treatment given in the annular region which is described by Dirac-delta function.

* Corresponding author.
E-mail address: vinayakskulkarni1@rediffmail.com

2. Heat transfer analysis

2.1. Formulation of the Problem

The steady state temperature of the cylinder satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g(r)}{k}=0 \tag{1}$$

with the boundary conditions

$$\frac{\partial T}{\partial r}=0 \text{ at } r=a,\ -h\le z\le h \tag{2}$$

$$T=f(r) \text{ on } z=h,\ a\le r\le b \tag{3}$$

$$\frac{\partial T}{\partial z}=0 \text{ at } z=-h,\ \ a\le r\le b \tag{4}$$

The set of Eqs. (1)-(4) constitute mathematical formulation for temperature change within cylinder due to internal heat source $g(r)$ which is cylindrical surface heat source of radius $r=r_1, 0<r<a$ and of strength g_s of linear length of cylinder and is situated concentrically inside the cylinder at $r=r_1$ given by

$$g(r)=\frac{g_s\delta(r-r_1)}{2\pi r k} \tag{5}$$

2.2. The solution for temperature change

To obtain the expression for temperature $T(r,z)$, introduce the finite Hankel transform over the variable r and its inverse transform defined as in [7]

$$\overline{T}(\alpha_n,z)=\int_a^b rK_0(\alpha_n,r)T(r,z)dr \tag{6}$$

$$T(r,z)=\sum_{n=1}^{\infty}\overline{T}(\alpha_n,z)K_0(\alpha_n,r) \tag{7}$$

where

$$K_0(\alpha_n,r)=\frac{\sqrt{2}J_0(\alpha_n r)}{aJ_0(\alpha_n a)} \tag{8}$$

and a_1, a_2 are roots of the transcendental equation

$$J_1(\alpha a)=0 \tag{9}$$

$Jn(x)$ is Bessel function of the first kind of order n. This transform satisfies the relation

$$H\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]=-\alpha_n^2\overline{T}(\alpha_n,z) \tag{10}$$

and

$$H\left[\frac{\partial^2 T}{\partial z^2}\right]=\frac{d^2\overline{T}}{dz^2} \tag{11}$$

On applying the finite Hankel transform defined in the Eq. (6) to the Eq. (1), one obtain

$$\frac{d^2\overline{T}}{dz^2}-\alpha_n^2\overline{T}+\frac{\overline{g}(r)}{k}=0 \tag{12}$$

where $\overline{T}$ is the Hankel transform of T and $g(r)$ is the Hankel transform of g (r) . On solving (12) under condition given in Eqs. (3) and (4), one obtain

$$\bar{T}(\alpha_n,z)=\left\{\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{cosh[\alpha_n(z+h)]}{cosh(2\alpha_n h)}\right]+\frac{\bar{g}(r)}{\alpha_n^2 k}\right\}$$

On applying inverse Hankel transform defined in (7), one obtain the expression for temperature as

$$T=\sum_{n=1}^{\infty}(\frac{\sqrt{2}\,\overline{f}(\alpha_n)}{a})(\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)})\{[\overline{f}(\alpha_n)-\frac{\overline{g}(r)}{\alpha_n^2 k}][\frac{cosh[\alpha_n(z+h)]}{cosh(2\alpha_n h)}]+\frac{\overline{g}(r)}{\alpha_n^2 k}\} \tag{13}$$

where $\overline{f}(\alpha_n)$ is Hankel transform of $f(r)$ and $\overline{g}(r)$ is Hankel transform of surface heat source which is given by

$$\overline{g}(r)=(\frac{\sqrt{2}\,g_s)}{a})(\frac{J_0(\alpha_n r_1)}{J_0(\alpha_n a)}) \tag{14}$$

. Since initial temperature $Ti=0$, the temperature change

$$\tau=T \tag{15}$$

3. Thermal stress analysis

3.1. Development of thermoelastic equations

Following Noda et al. [8], The Naviers equations for axisymmetric thermoelastic problems can be expressed as

$$\nabla^2 u_r - \frac{u_r}{r^2} + (\frac{1}{1-2\upsilon})\frac{\partial e}{\partial r} - 2\alpha(\frac{1+\upsilon}{1-2\upsilon})\frac{\partial \tau}{\partial r} + \frac{2(1+\upsilon)}{E}F_r = 0 \tag{16}$$

$$\nabla^2 u_z + (\frac{1}{1-2\upsilon})\frac{\partial e}{\partial z} - 2\alpha(\frac{1+\upsilon}{1-2\upsilon})\frac{\partial \tau}{\partial z} + \frac{2(1+\upsilon)}{E}F_z = 0 \tag{17}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{18}$$

e - dilatation, E - YoungŠs modulus a - coefficient of linear thermal expansion υ - Poisson ratio.
The solution of Naviers Eqs. (16) and (17) without body forces can be expressed by Goodiers thermoelastic displacement potential f and Boussinesq harmonic functions j and y under the axisymmetric conditions.
The Goodiers thermoelastic displacement potential f must satisfy the governing equations $\nabla^2\phi = K\tau$,
That is

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = K\tau \tag{19}$$

where K is Restraint coefficient as

$$K = \frac{\beta}{\lambda+2\mu} = (\frac{1+\upsilon}{1-\upsilon})\alpha$$

where β - thermoelastic constant. λ and μ - Lames elastic constants.
Boussinesq harmonic functions ϕ and ψ must satisfy the governing equations

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{20}$$

and

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = 0 \tag{21}$$

when deformation in the cylindrical coordinate system are discussed, Michells function M instead of Boussinesq harmonic functions ϕ and ψ is often used.
Taking

$$M = -\int(\phi + z\psi)dz \tag{22}$$

The Michell's function M must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{23}$$

The component of the displacement and stresses are represented by the thermoelastic displacement potential ϕ and MichellŠs function M as

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2 M}{\partial r\partial z} \tag{24}$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \tag{25}$$

$$\sigma_{rr} = 2G[\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}(\upsilon\nabla^2 M - \frac{\partial^2 M}{\partial r^2})] \tag{26}$$

$$\sigma_{\theta\theta} = 2G[\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}(\upsilon\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r})] \tag{27}$$

$$\sigma_{zz} = 2G[\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}((2-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2})] \tag{28}$$

$$\sigma_{rz} = 2G[\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}((1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2})] \tag{29}$$

For traction free surface the stress functions

$$\sigma_{rr} = \sigma_{rz} = 0 \text{ at } r = 0 \tag{30}$$

The set of Eqs. (16) to (30) constitute mathematical formulation for displacement and thermal stresses developed within solid due to temperature change.

3.2. The solution for displacement and thermal stresses

Assuming displacement function $\phi(r,z)$ as

$$\phi(r,z)=\sum_{n=1}^{\infty}D_n\{(\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)})[\overline{f}(\alpha_n)-\frac{\overline{g}(r)}{\alpha_n^2 k}][\frac{(z+h)sinh[\alpha_n(z+h)]}{cosh(2\alpha_n h)}]\} \tag{31}$$

Using ϕ in (19), one have

$$D_n=(\frac{K.\overline{f}(\alpha_n)}{a\sqrt{2}\alpha_n})$$

. Thus Eq. (31) becomes

$$\phi(r,z)=\sum_{n=1}^{\infty}(\frac{K.\overline{f}(\alpha_n)}{a\sqrt{2}\alpha_n})(\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)})[\overline{f}(\alpha_n)-\frac{\overline{g}(r)}{\alpha_n^2 k}][\frac{(z+h)sinh[\alpha_n(z+h)]}{cosh(2\alpha_n h)}]\} \tag{32}$$

Now suitable form of Michell function M satisfying (23) is given by

$$M=K\sum_{n=1}^{\infty}(\frac{\sqrt{2}\overline{f}(\alpha_n)}{a})(\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)})[\overline{f}(\alpha_n)-\frac{\overline{g}(r)}{\alpha_n^2 k}]\times\{B_n sinh[\alpha_n(z+h)]+C_n\alpha_n[z+h]cosh[\alpha_n(z+h)]\} \tag{33}$$

where B_n and C_n are arbitrary functions which can be determined by boundary condition on the traction free surface given in the Eq. (30). Now using Eqs. (13), (15), (32), and (33) in (24) to (29), one obtains the expressions for displacements and stresses respectively as

$$\begin{aligned}u_r=&\left(\frac{K}{\sqrt{2}a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\times\\&\left\{\frac{-(z+h)\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)}+B_n\alpha_n^2\cosh[\alpha_n(z+h)]+C_n\alpha_n^2[\cosh[\alpha_n(z+h)]+\alpha_n[z+h]\sinh[\alpha_n(z+h)]\right\}\end{aligned} \tag{34}$$

$$\begin{aligned}u_z=&\left(\frac{K}{\sqrt{2}a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{\alpha_n}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\times\\&\left\{\begin{array}{l}\dfrac{\sinh[\alpha_n(z+h)]+\alpha_n[z+h]\cosh[\alpha_n(z+h)]}{2\alpha_n\cosh(2\alpha_n h)}-B_n\alpha_n^2\sinh[\alpha_n(z+h)]\\+C_n\alpha_n^2[2(1-2\nu)\sinh[\alpha_n(z+h)]-\alpha_n[z+h]\cosh[\alpha_n(z+h)]\end{array}\right\}\end{aligned} \tag{35}$$

$$\begin{aligned}\sigma_{rr}=&\left(\frac{\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left\{\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\right]\left[\frac{-(z+h)\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)}\right]\\&-\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\left(\frac{\bar{g}}{\alpha_n^2 k}\right)+\left(\frac{\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)}\right)\right]+B_n\alpha_n^2\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\cosh[\alpha_n(z+h)]\\&+C_n\alpha_n^2\left[2\nu n\alpha_n\left[\frac{J_0(\alpha_n r)}{J_0\alpha_n a}\right]\cosh[\alpha_n(z+h)]+\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\cosh[\alpha_n(z+h)]+\alpha_n[z+h]\sinh[\alpha_n(z+h)]\right]\right\}\end{aligned} \tag{36}$$

$$\begin{aligned}\sigma_{\theta\theta}=&\left[\frac{\sqrt{2}GK}{a}\right]\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left\{\left(\frac{-1}{r}\right)\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(z+h)\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)}\right]\right.\\&-\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\left(\frac{\bar{g}}{\alpha_n^2 k}\right)+\left(\frac{\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)}\right)\right]+B_n\alpha_n^2\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\cosh[\alpha_n(z+h)]\\&+C_n\alpha_n^2\left[[2\nu n\alpha_n\left[\frac{J_0(\alpha_n r)}{J_0\alpha_n a}\right]\cosh[\alpha_n(z+h)+\frac{1}{r}\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right][[\cosh[\alpha_n(z+h)]+\alpha_n[z+h]\sinh[\alpha_n(z+h)]]\right]\end{aligned} \tag{37}$$

$$\begin{aligned}\sigma_{zz}=&\left(\frac{\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\\&\left\{\begin{array}{l}\left(\dfrac{\bar{g}}{\alpha_n^2 k}\right)+\left(\dfrac{(z+h)\alpha_n\sinh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)}\right)-B_n\alpha_n^3\cosh[\alpha_n(z+h)]\\+C_n\alpha_n^3[(1-2\nu)\cosh[\alpha_n(z+h)]-\alpha_n(z+h)\sinh[\alpha_n(z+h)]]\end{array}\right\}\end{aligned} \tag{38}$$

$$\sigma_{rz}=\left(\frac{\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]$$
$$\left\{\begin{array}{l}-\dfrac{\sinh[\alpha_n(z+h)]+\alpha_n(z+h)\cosh[\alpha_n(z+h)]}{2\cosh(2\alpha_n h)}+B_n\alpha_n^3\sinh[\alpha_n(z+h)]\\ +C_n\alpha_n^3[(2\upsilon\sinh[\alpha_n(z+h)]+\alpha_n(z+h)\cosh[\alpha_n(z+h)]]\end{array}\right\} \tag{39}$$

Now in order to satisfy Eq. (30) solving Eqs. (38) and (39) for B_n and C_n one obtain,

$$B_n=\frac{(1-2\upsilon)}{2\alpha_n^3 cosh(2\alpha_n h)} \tag{40}$$

and

$$C_n=\frac{1}{2\alpha_n^3 cosh(2\alpha_n h)} \tag{41}$$

Using these values of $B_n$and C_n in Eqs. (34) to (39) one obtain the expressions for displacements and stresses as

$$u_r=\left(\frac{K}{\sqrt{2}a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(1-\upsilon)\cosh[\alpha_n(z+h)]}{\alpha_n\cosh(2\alpha_n h)}\right] \tag{42}$$

$$u_z=\left(\frac{K}{\sqrt{2}a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(1-\upsilon)\sinh[\alpha_n(z+h)]}{\alpha_n\cosh(2\alpha_n h)}\right] \tag{43}$$

$$\sigma_{rr}=\left(\frac{\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left\{\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(1-\upsilon)\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)}-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\right\} \tag{44}$$

$$\sigma_{\theta\theta}=\left(\frac{\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]$$
$$\left\{\left[\frac{J_1(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(1-\upsilon)\cosh[\alpha_n(z+h)]}{r\alpha_n\cosh(2\alpha_n h)}\right]-\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\left[\frac{(1-\upsilon)\cosh[\alpha_n(z+h)]}{\cosh(2\alpha_n h)}\right]\}\right\} \tag{45}$$

$$\sigma_{zz}=\left(\frac{-\sqrt{2}GK}{a}\right)\sum_{n=1}^{\infty}\left(\frac{\bar{f}(\alpha_n)}{a}\right)\left[\bar{f}(\alpha_n)-\frac{\bar{g}(r)}{\alpha_n^2 k}\right]\left[\frac{J_0(\alpha_n r)}{J_0(\alpha_n a)}\right]\left(\frac{\bar{g}(r)}{\alpha_n^2 k}\right) \tag{46}$$

and

$$\sigma_{rz}=0 \tag{47}$$

3.3. Numerical conclusions

3.3.1. Special case

Setting

$$f(r)=(r^2-a^2)^2 \tag{48}$$

Applying finite Hankel transform as defined in Eq. (6) to the Eq. (48), one obtain

$$\overline{f}(\alpha_n)=\int_0^a\left(\frac{\sqrt{2}J_o(\alpha_n r)}{J_0(\alpha_n a)}\right)(r(r^2-a^2)^2))dr$$

$$\overline{f}(\alpha_n)=\frac{8\sqrt{2}\{(8-a^2\alpha_n^2)J_1(\alpha_n a)-4a\alpha_n J_0(\alpha_n a)\}}{\alpha_n^5 J_0(\alpha_n a)} \tag{49}$$

3.3.2. Dimensions

Radius of plate $a=l\,m$. Thickness (Height) of plate $h=2m$.

3.3.3. Thermoelastic constants

Poisson ration = 0.35. Lame constant$\mu = 26.67$. Young's modulus $E = 130GPa$. Strength of Internal heat source $g_s = 100^0 C$.

3.3.4. Roots of transcendental equation

Let $\alpha_1 = 3.8317$, $\alpha_2 = 7.0156$, $\alpha_3 = 10.1735$, $\alpha_4 = 13.3237$, $\alpha_5 = 16.470$, $\alpha_6 = 19.6159$, $\alpha_7 = 22.7601$, $\alpha_8 = 25.9037$, $\alpha_9 = 29.0468$, $\alpha_{10} = 32.18$ are the roots of transcendental equation $J_1(\alpha a) = 0$.

3.3.5. Material properties

The numerical calculations has been performed for iron (pure) materials, having properties, Thermal diffusivity $\alpha = 20.3410^6(m^2 s^{-1})$. Thermal conductivity $k = 72.7(W/mk)$. Density $r = 7897kg/m^3$. Specific heat $C_p = 452(J/kgK)$. Coefficient of linear thermal expansion $a_t = 11.810^{-6}(1/K)$. For convenience setting $A = \dfrac{16}{a \times 10^3}, B = \dfrac{8K(1+V)\alpha}{a(1-v)10^3}, C = \dfrac{16GK(1+v)\alpha}{a(1-v)!0^3}$ in the expressions (13), (42) to (46).

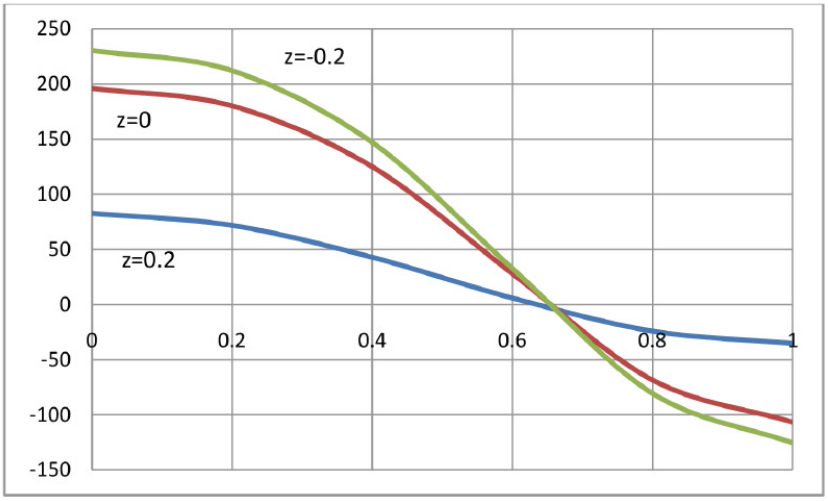

Fig. 1. The temperature function T/A in radial direction

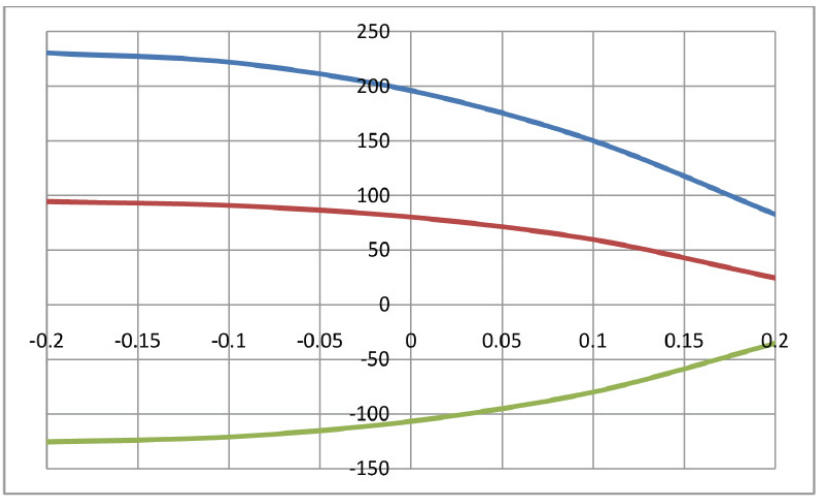

Fig. 2. The temperature function T/A in axial direction

3.4. Concluding remarks

The attempt made for the analysis of the heat transfer and thermal stresses within cylinder due to internal heat generation under steady temperature conditions. The internal heat generation is taken as cylindrical surface heat source in annular region of linear length of cylinder and is situated concentrically inside the cylinder. As a special

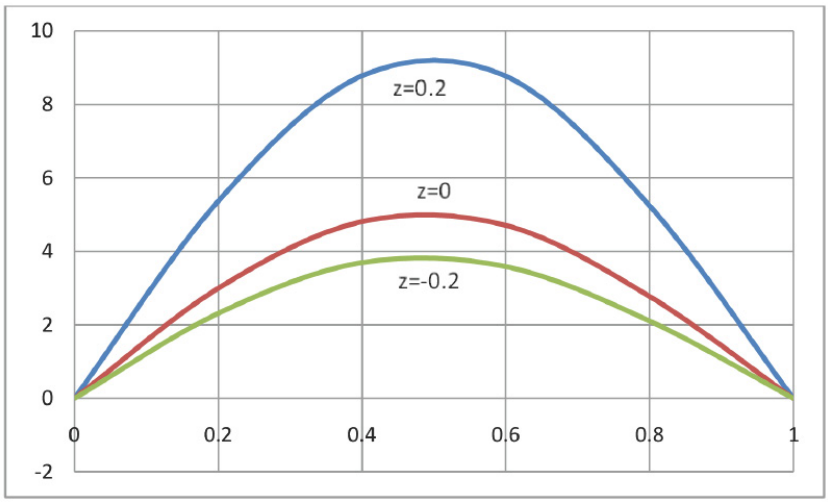

Fig. 3. The radial displacement function u_r/B in radial direction

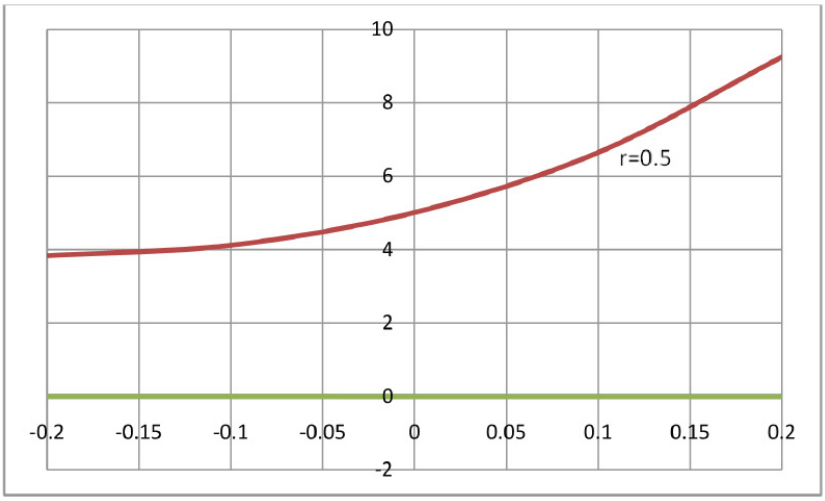

Fig. 4. The axial displacement function u_z/B in radial direction

case mathematical model is constructed for Iron (Pure) thick circular plate with the material properties specified as above.

From Figs. 1 and 2, as a result of cylindrical surface heat source the heat flow can be observed from centre to outer circular boundary in radial direction whereas due to temperature prescribed on upper surface heat flows from upper surface to lower surface in axial direction.

From Figs. 3 and 4, radial displacement function u_r shows sinusoidal fluctuations from centre to outer circular surface in radial direction where as in axial direction it is zero at centre and on circular boundary of cylinder. The displacement takes place at annular region $r = r_1 = 0.5$ where heat is generated.

From Figs. 5 and 6, axial displacement function u_z is maximum on upper surface and decreases towards lower surface in both radial and axial direction.

From Figs. 7 and 8, the radial stress function σ_{rr} develops tensile stresses in radial direction within circular region $0.2 \le r \le 1$ and it develops at annular region $r = r_1 = 0.5$ where heat is generated and decreases from upper surface to lower surface in axial direction. Also it can be observed that the radial stress function σ_{rr} satisfies the equilibrium conditions i.e. $\sigma_{rr} = 0$ at outer traction free surface $r = 1$.

From Figs. 9 and 10, the angular stress function $\sigma_{\theta\theta}$ develops compressive stresses near centre in radial and axial direction it develops tensile stresses on the upper surface where as it is almost zero at the lower surface.

From Figs. 11 and 12, the axial stress function σ_{zz} develops on the upper surface, it is almost zero on the lower surface, also it develops compressive stresses near centre in radial and axial direction it develops tensile stresses on the upper surface where as it is almost zero at the lower surface. Due to applying the cylindrical surface heat source in annular region, the radial and axial displacements occurs near the centre and plate expands towards the lower direction. It means we may find out that displacement and stress components occurs near heat source. The results presented here will be more useful in engineering problem particularly in the engineering systems subjected

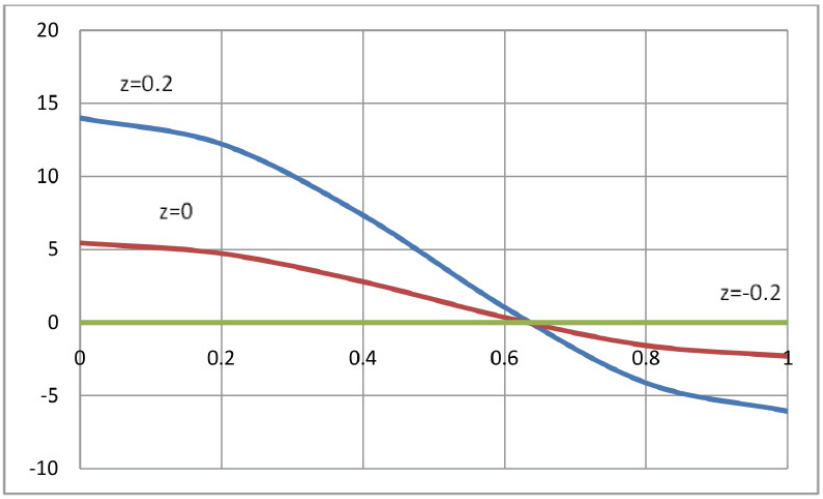

Fig. 5. The axial displacement function u_z/B in radial direction

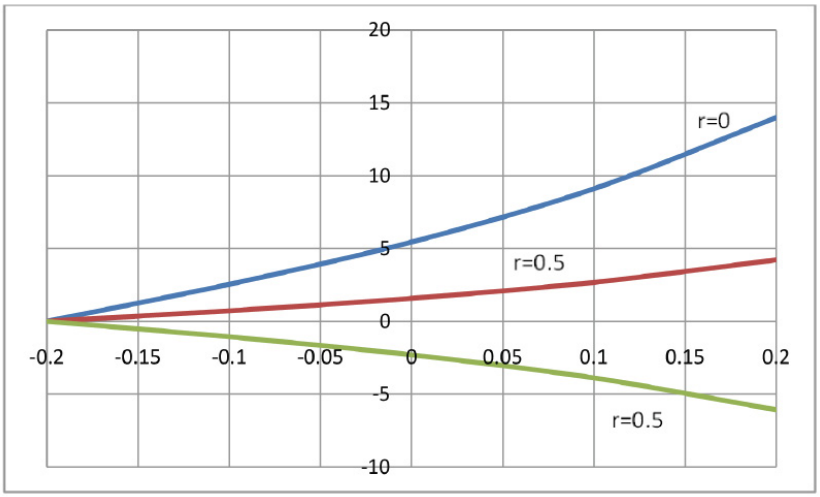

Fig. 6. The axial displacement function u_z/B in axial direction

to cylindrical surface heat source.

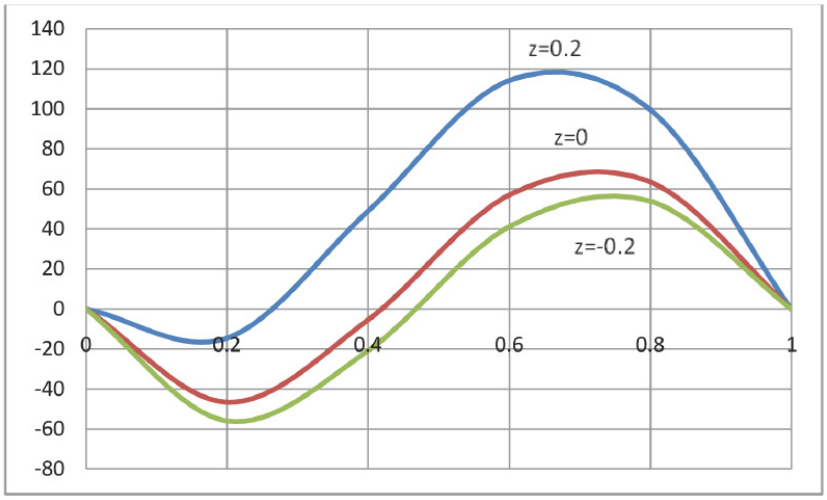

Fig. 7. The radial stress function σ_{rr}/C in radial direction

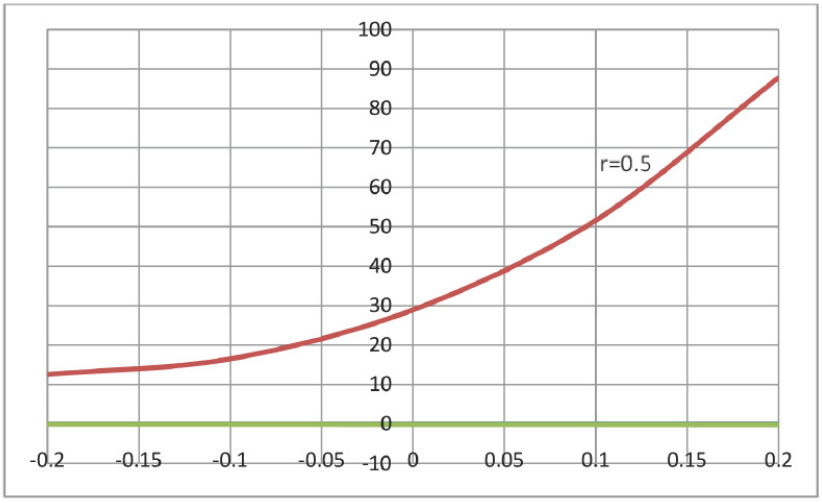

Fig. 8. The radial stress function σ_{rr}/C in axial direction

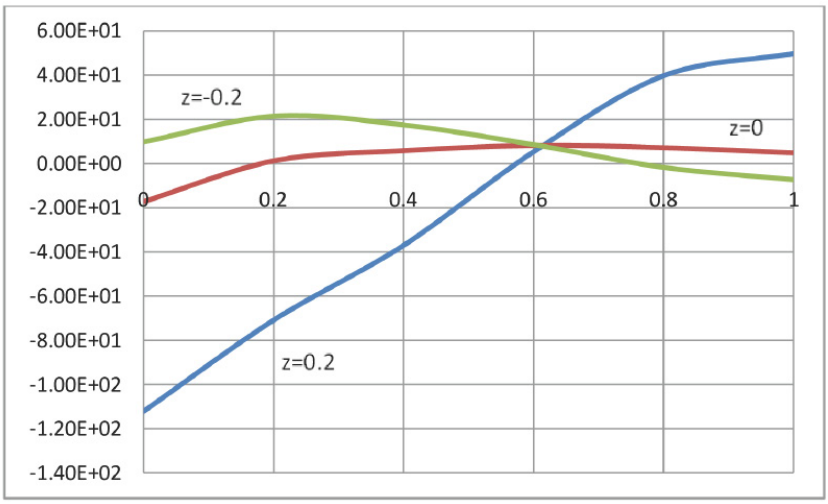

Fig. 9. The stress function $\sigma_{\theta\theta}/C$ in radial direction

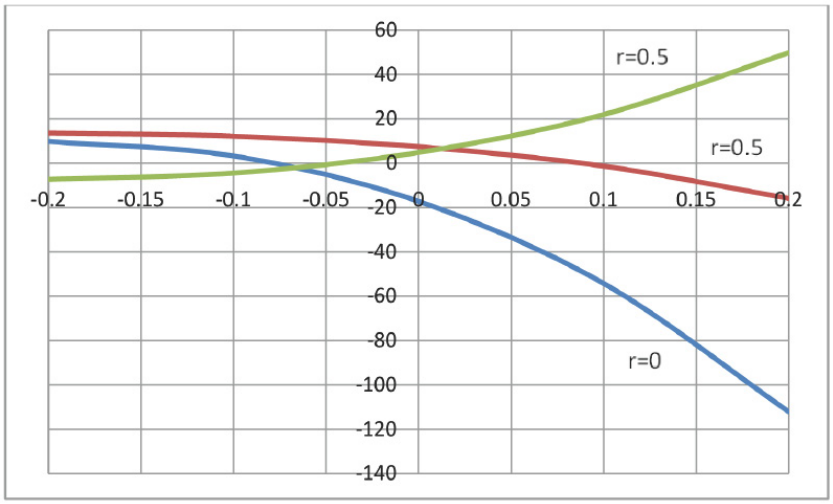

Fig. 10. The stress function $\sigma_{\theta\theta}/C$ in axial direction

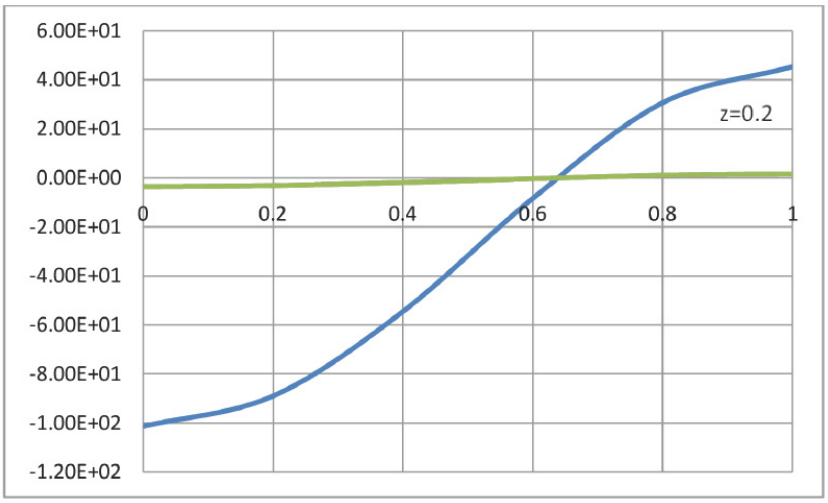

Fig. 11. The stress function $\sigma_{zz}C$ in radial direction

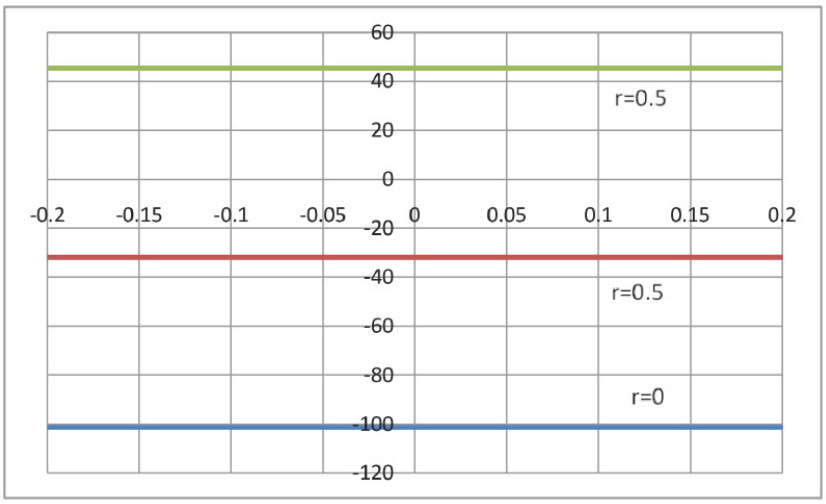

Fig. 12. The stress function $\sigma_{zz}C$ in axial direction

References

[1] V. A. Sipailov , L. D. Zagrebin, O. F. Kasatkina, V. E. Zinov'ev, Effect of heat transfer in the impulse method on the measurement of thermophysical properties, Journal of Engineering Physics and Thermophysics 51 (1986) 964-966.
[2] S. Löhle, U. Fuchs, P. Digel, T. Hermann, J. L. Battaglia, Analysing inverse heat conduction problems by the analysis of the system impulse response, Inverse Problems in Science and Engineering 22 (2013) 297-308.
[3] A. A. Stakhanova, A. N. Varava, A. V. Dedov, A. T. Komov, Studying heat transfer during impulse heating of model fragments of fuel rod claddings 58 (2011) 602-609.
[4] C. W. Lee, Thermal stresses in thick plate, Journal of Solids and Structures 6 (1970) 605-615.
[5] h. Toshiaki, Thermal stresses in a nonhomogeneous thick plate under steady distribution of temperature, Journal of Thermal Stresses 5 (1982) 1-11.
[6] V. S. Kulkarni, K. C. Deshmukh, Quasi-static thermal stresses in a thick circular plate Applied Mathematical Modelling 31 (2007) 1479-1488.
[7] I. N. Sneddon, The Use of Integral Transform, McGraw Hill, New York, 1972.
[8] N. Naotake, R. B. Hetnarski, y. Yoshinobu, Thermal Stresses 2nd edn., Taylor and Francis New York, 2003.
[9] M. M. Ozisik, Boundary value problem of heat conduction, International Text book Company, Scranton, Pennsylvania 1968.