Int. J. Adv. Appl. Math. and Mech. 2(2) (2014) 7 - 20 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

On convective instability of reaction fronts in porous media using the Darcy-Brinkman formulation

Research Article

Karam Allali, Oussama Khalfi *

Department of Mathematics, Faculty of Science and Technology, University Hassan II, PO-Box 146, Mohammedia, Morocco

Received 28 September 2014; accepted (in revised version) 20 December 2014

Abstract: In this paper, we study the convective instability of reaction fronts in porous media. The model consists of a system of non-linear differential equations describing the heat, the depth of conversion and the motion using the Darcy-Brinkman formulation. Linear stability analysis of the problem is fulfilled. The dispersion relation is obtained by solving the interface problem and the stability boundary is found.

MSC: 34E05 • 76S05

Keywords: Convective instability • Darcy-Brinkman formulation • Porous media

1. Introduction

Within the last decades, a significant number of studies has been devoted to the convection in porous media (see for instance [1, 2] and the references therein). Although Darcy's law is widely used in modelling the fluid flow in porous media; its validity remains questionable. Indeed, when the porous medium is formed by high-porosity matrix; a new viscous term must be added to Darcy's equation; called also Brinkman term [3, 4]. The added term is due to an effective viscosity that must be taken under consideration for special geometry of the porous media; in this case, the fluid flow will be modelled by using Darcy-Brinkman formulation. The formulation is considered as a contribution from the Brinkman term to the viscous dissipation in porous media [5, 6]. The fluid flow problems in porous media with Darcy-Brinkman formulation have many applications such in groundwater, in pollution or in biomedical hydrodynamic studies [7–10].

The influence of convective instability on reaction fronts propagation in porous media is studied in [11]. The aim of this paper is to continue the investigation for the case of fluid flow in porous media with high-porosity structure. For such problem, it is more convenient to use the Boussinesq approximation which state that the change of density is neglected everywhere except in the external force term. The Boussinesq approximation was justified and used to investigate the reaction front instability [12, 13]. For our study, we will consider that the porous matrix is filled by an incompressible fluid and the propagation of reaction front will be in opposite sense of gravity. We will use the Zeldovich-Frank-Kamenetskii method in order to perform the linear stability analysis which is a common approach in reaction front propagation problems [14], and then determine the reduced interface problem. Finally, the boundary threshold is obtained via the dispersion relation.

The paper is organized as follow. We introduce the model in Section 2. The linear stability analysis is performed in Section 3. In Section 4, the dispersion relation is presented and numerical results are discussed. We conclude in the last section.

* Corresponding author.
E-mail address: allali@hotmail.com

2. Governing Equations

2.1. The model

We consider an upward propagating reaction front in a porous medium filled by an incompressible reacting fluid. The porous medium is assumed to be de formed by high-porosity matrix. The model of a such process can be described by a reaction-diffusion system coupled with the hydrodynamics equations using Darcy-Brinkman formulation:

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \Delta T + q K(T)\phi(\alpha), \tag{1a}$$

$$\frac{\partial \alpha}{\partial t} + \mathbf{v} \cdot \nabla \alpha = K(T)\phi(\alpha), \tag{1b}$$

$$\mathbf{v} + \frac{K_p}{\mu}\nabla p - \frac{\mu_{\text{eff}}}{\mu} K_p \Delta \mathbf{v} = \frac{g\beta K_p}{\mu}\rho(T - T_0)\gamma, \tag{1c}$$

$$\nabla \cdot \mathbf{v} = 0. \tag{1d}$$

This system is supplemented by the following boundary conditions

$$T = T_i, \;\; \alpha = 0 \text{ and } \mathbf{v} = 0 \text{ when } y \to +\infty, \tag{2}$$
$$T = T_b, \;\; \alpha = 1 \text{ and } \mathbf{v} = 0 \text{ when } y \to -\infty. \tag{3}$$

Here T is the temperature, α the depth of conversion, $\mathbf{v} = (v_x, v_y)$ the velocity, p the pressure, κ the coefficient of thermal diffusivity, q the adiabatic heat release, μ the viscosity, μ_{eff} the effective viscosity, K_p the permeability of the porous media, g the gravity, β the coefficient of thermal expansion, γ the unit vector in the vertical direction (upward) and ρ is the density.
The gradient, divergence and Laplace operators can be defined as following:

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right), \qquad \nabla \cdot \mathbf{f} = \sum_{i=1}^{2} \frac{\partial \mathbf{f}_i}{\partial x_i}, \qquad \Delta f = \sum_{i=1}^{2} \frac{\partial^2 f}{\partial x_i^2},$$

Moreover, T_0 is the mean value of the temperature, T_i is an initial temperature while T_b is the temperature of the reacted mixture given by $T_b = T_i + q$. The temperature dependence of the reaction rate is given by the Arrhenius law [15]:

$$K(T) = k_0 \exp\left(-\frac{E}{R_0 T}\right), \tag{4}$$

where E is the activation energy, R_0 the universal gas constant and k_0 the pre-exponential factor. We consider zeroth-order reaction

$$\phi(\alpha) = \begin{cases} 1 \text{ if } \alpha < 1, \\ 0 \text{ if } \alpha = 1. \end{cases} \tag{5}$$

The system of equations (1a)-(3) includes the heat equation, the equation of concentration and the equations of motion for an incompressible fluid in a porous medium.

2.2. The dimensionless form of the model

In order to obtain the dimensionless form of the problem (1a)-(3), we first introduce the dimensionless spatial variables xc_1/κ, yc_1/κ, time $tc_1^2/\kappa d$, velocity $\mathbf{v}/c_1$ and pressure $p\kappa\mu/K_p$ with $c_1 = c/\sqrt{2}$. We denote $\theta = (T - T_b)/q$ and we keep, for convenience, the same notation for the other variables, the system of equations (1a)-(1d) can be re-written in the following form

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \Delta \theta + W_Z(\theta)\phi(\alpha), \tag{6}$$

$$\frac{\partial \alpha}{\partial t} + \mathbf{v} \cdot \nabla \alpha = W_Z(\theta)\phi(\alpha), \tag{7}$$

$$\mathbf{v} + \nabla p - \tilde{D}_a \Delta \mathbf{v} = R_p(\theta + \theta_0)\gamma, \tag{8}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{9}$$

with

$$W_Z(\theta) = Z \exp\Big(\frac{\theta}{Z^{-1} + \delta\theta}\Big). \tag{10}$$

Here $Z = \frac{qE}{R_0 T_b^2}$ is the Zeldovich number, $\tilde{D}_a = \frac{\mu_{\text{eff}}}{\mu} D_a$ is the Darcy-Brinkman number where D_a is the Darcy number defined by $D_a = \frac{c^2 K_p}{\kappa^2}$, $R_p = \frac{K_p c_1^2 P^2 R}{\mu^2}$ where R is the Rayleigh number and P is the Prandtl number defined by $R = \frac{g\beta q \kappa^2}{\mu c_1^3}$ and $P = \frac{\mu}{\kappa}$, respectively. The parameters δ and θ_0 are defined by $\delta = \frac{R_0 T_b}{E}$ and $\theta_0 = \frac{T_b - T_0}{q}$. Finally, the boundary conditions (2)-(3) can be re-written as follows:

$$y \to +\infty, \ \ \theta = -1, \ \ \alpha = 0 \text{ and } \mathbf{v} = 0, \tag{11}$$

$$y \to -\infty, \ \ \theta = 0, \ \ \alpha = 1 \text{ and } \mathbf{v} = 0. \tag{12}$$

3. Linear stability analysis

3.1. Approximation of infinitely narrow reaction zone

We perform an analytical study by reducing the problem (6)-(12) to a singular perturbation one where the reaction zone is assumed to be narrow over a certain surface, which is the reaction front and the reaction term is neglected ahead of the reaction front because the temperature is not sufficiently high, and behind the reaction front since in this region there are no fresh reactant left. This approach, called Zeldovich-Frank-Kamenetskii approximation, that can leads to an interface problem by applying a formal asymptotic analysis for large Zeldovich number. In other words, for our asymptotic analysis treatment, we will consider $\epsilon = \frac{1}{Z}$ as small parameter.
Denoting by $y = \zeta(t, x)$ the location of the reaction zone, the new independent variable is

$$y_1 = y - \zeta(t, x). \tag{13}$$

Introducing the new unknown functions $\theta_1, \alpha_1, \mathbf{v}_1, p_1$:

$$\begin{aligned} &\theta(t,x,y) = \theta_1(t,x,y_1), \ \ \alpha(t,x,y) = \alpha_1(t,x,y_1), \\ &\mathbf{v}(t,x,y) = \mathbf{v}_1(t,x,y_1), \ \ p(t,x,y) = p_1(t,x,y_1). \end{aligned} \tag{14}$$

The equations can be re-written in the following form (the index 1 for the dependent variables is omitted):

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y_1}\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \tilde{\nabla}\theta = \tilde{\Delta}\theta + W_Z(\theta)\phi(\alpha), \tag{15}$$

$$\frac{\partial \alpha}{\partial t} - \frac{\partial \alpha}{\partial y_1}\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \tilde{\nabla}\alpha = W_Z(\theta)\phi(\alpha), \tag{16}$$

$$\mathbf{v} + \tilde{\nabla} p - \tilde{D}_a \tilde{\Delta} \mathbf{v} = R_p(\theta + \theta_0)\gamma, \tag{17}$$

$$\frac{\partial v_x}{\partial x}-\frac{\partial v_x}{\partial y_1}\frac{\partial \zeta}{\partial x}+\frac{\partial v_y}{\partial y_1}=0, \tag{18}$$

where

$$\tilde{\Delta}=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y_1^2}-2\frac{\partial \zeta}{\partial x}\frac{\partial^2}{\partial x\partial y_1}+\Big(\frac{\partial \zeta}{\partial x}\Big)^2\frac{\partial^2}{\partial y_1^2}-\frac{\partial^2 \zeta}{\partial x^2}\frac{\partial}{\partial y_1}, \tag{19}$$

$$\tilde{\nabla}=\Big(\frac{\partial}{\partial x}-\frac{\partial \zeta}{\partial x}\frac{\partial}{\partial y_1},\frac{\partial}{\partial y_1}\Big). \tag{20}$$

To approximate the jump conditions and then resolve the interface problem, we apply the matched asymptotic expansions. To this end, the outer solution of the problem is sought in the form

$$\begin{aligned}&\theta=\theta^{\check{0}}+\epsilon\theta^{\check{1}}+\ldots,\quad \alpha=\alpha^{\check{0}}+\epsilon\alpha^{\check{1}}+\ldots,\\ &\mathbf{v}=\mathbf{v}^{\check{0}}+\epsilon\mathbf{v}^{\check{1}}+\ldots,\quad p=p^{\check{0}}+\epsilon p^{\check{1}}+\ldots.\end{aligned} \tag{21}$$

For the inner solution, we introduce the stretching coordinate $\eta=y_1/\epsilon$ and then the inner solution is sought in the following form

$$\begin{aligned}&\theta=\epsilon\tilde{\theta}^{\check{1}}+\ldots,\quad \alpha=\tilde{\alpha}^{\check{0}}+\epsilon\tilde{\alpha}^{\check{1}}+\ldots,\\ &\mathbf{v}=\tilde{\mathbf{v}}^{\check{0}}+\epsilon\tilde{\mathbf{v}}^{\check{1}}+\ldots,\quad p=\tilde{p}^{\check{0}}+\epsilon\tilde{p}^{\check{1}}+\ldots,\quad \zeta=\tilde{\zeta}^{\check{0}}+\epsilon\tilde{\zeta}^{\check{1}}+\ldots.\end{aligned} \tag{22}$$

Substituting these expansions into (15)-(18), we obtain the following:

order ϵ^{-2}

$$\tilde{D}_a\left(1+\left(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\right)^2\right)\frac{\partial^2\tilde{v}^{\check{0}}}{\partial\eta^2}=0, \tag{23}$$

order ϵ^{-1}

$$\Big(1+\Big(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\Big)^2\Big)\frac{\partial^2\tilde{\theta}^{\check{1}}}{\partial\eta^2}+\exp\Big(\frac{\tilde{\theta}^{\check{1}}}{1+\delta\tilde{\theta}^{\check{1}}}\Big)\phi(\tilde{\alpha}^{\check{0}})=0, \tag{24}$$

$$-\frac{\partial\tilde{\alpha}^{\check{0}}}{\partial\eta}\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial t}-\frac{\partial\tilde{\alpha}^{\check{0}}}{\partial\eta}\Big(\tilde{v}_x^{\check{0}}\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}-\tilde{v}_y^{\check{0}}\Big)=\exp\Big(\frac{\tilde{\theta}^{\check{1}}}{1+\delta\tilde{\theta}^{\check{1}}}\Big)\phi(\tilde{\alpha}^{\check{0}}), \tag{25}$$

$$\frac{\partial\tilde{p}^{\check{0}}}{\partial\eta}=0, \tag{26}$$

$$-\frac{\partial\tilde{v}_x^{\check{0}}}{\partial\eta}\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}+\frac{\partial\tilde{v}_y^{\check{0}}}{\partial\eta}=0, \tag{27}$$

order ϵ^{0}

$$\tilde{v}_x^{\check{0}}+\frac{\partial\tilde{p}^{\check{0}}}{\partial x}-\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\frac{\partial\tilde{p}^{\check{1}}}{\partial\eta}=0, \tag{28}$$

$$\tilde{v}_y^{\check{0}}+\frac{\partial\tilde{p}^{\check{1}}}{\partial\eta}-\tilde{D}_a\left(\frac{\partial^2\tilde{v}_y^{\check{0}}}{\partial x^2}+\frac{\partial^2\tilde{v}_y^{\check{2}}}{\partial\eta^2}-2\frac{\partial\tilde{\zeta}^{\check{1}}}{\partial x}\frac{\partial^2\tilde{v}_y^{\check{0}}}{\partial x\partial\eta}+\left(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\right)^2\frac{\partial^2\tilde{v}_y^{\check{2}}}{\partial\eta^2}-\frac{\partial^2\tilde{\zeta}^{\check{0}}}{\partial x^2}\frac{\partial\tilde{v}_y^{\check{1}}}{\partial\eta}\right)=R_p\theta_0, \tag{29}$$

$$\tilde{v}_x^{\check{0}}+\frac{\partial\tilde{p}^{\check{0}}}{\partial x}-\frac{\partial\tilde{\zeta}_y^{\check{0}}}{\partial x}\frac{\partial\tilde{p}^{\check{1}}}{\partial\eta}-\tilde{D}_a\left(\frac{\partial^2\tilde{v}_x^{\check{0}}}{\partial x^2}+\frac{\partial^2\tilde{v}_x^{\check{2}}}{\partial\eta^2}-2\frac{\partial\tilde{\zeta}^{\check{1}}}{\partial x}\frac{\partial^2\tilde{v}_x^{\check{0}}}{\partial x\partial\eta}+\left(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\right)^2\frac{\partial^2\tilde{v}_x^{\check{2}}}{\partial\eta^2}-\frac{\partial^2\tilde{\zeta}^{\check{0}}}{\partial x^2}\frac{\partial\tilde{v}_x^{\check{1}}}{\partial\eta}\right)=R_p\theta_0, \tag{30}$$

order ϵ^1

$$\begin{aligned}&\tilde{v}_x^{\breve{1}} + \frac{\partial \tilde{p}^{\breve{1}}}{\partial x} - \frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\frac{\partial \tilde{p}^{\breve{1}}}{\partial \eta} - \tilde{D}_a\Big(\frac{\partial^2 \tilde{v}_x^{\breve{1}}}{\partial x^2} + \frac{\partial^2 \tilde{v}_x^{\check{3}}}{\partial \eta^2} - 2\Big(\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\frac{\partial^2 \tilde{v}_x^{\breve{1}}}{\partial x \partial \eta} + \frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x}\frac{\partial^2 \tilde{v}_x^{\check{2}}}{\partial x \partial \eta}\Big) + \Big(\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x}\Big)^2 \frac{\partial^2 \tilde{v}_x^{\check{3}}}{\partial \eta^2} + \Big(\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\Big)^2 \frac{\partial^2 \tilde{v}_x^{\check{2}}}{\partial \eta^2}\\ &- \Big(\frac{\partial^2 \tilde{\zeta}^{\check{0}}}{\partial x^2}\frac{\partial \tilde{v}_x^{\check{2}}}{\partial \eta} + \frac{\partial^2 \tilde{\zeta}^{\breve{1}}}{\partial x^2}\frac{\partial \tilde{v}_x^{\breve{1}}}{\partial \eta}\Big)\Big) = R_p \tilde{\theta}^{\breve{1}},\end{aligned} \tag{31}$$

$$\begin{aligned}&\tilde{v}_y^{\breve{1}} + \frac{\partial \tilde{p}^{\check{2}}}{\partial \eta} - \tilde{D}_a\Big(\frac{\partial^2 \tilde{v}_y^{\breve{1}}}{\partial x^2} + \frac{\partial^2 \tilde{v}_y^{\check{3}}}{\partial \eta^2} - 2\Big(\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\frac{\partial^2 \tilde{v}_y^{\breve{1}}}{\partial x \partial \eta} + \frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x}\frac{\partial^2 \tilde{v}_y^{\check{2}}}{\partial x \partial \eta}\Big) + \Big(\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x}\Big)^2 \frac{\partial^2 \tilde{v}_y^{\check{3}}}{\partial \eta^2} + \Big(\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\Big)^2 \frac{\partial^2 \tilde{v}_y^{\check{2}}}{\partial \eta^2}\\ &- \Big(\frac{\partial^2 \tilde{\zeta}^{\check{0}}}{\partial x^2}\frac{\partial \tilde{v}_y^{\check{2}}}{\partial \eta} + \frac{\partial^2 \tilde{\zeta}^{\breve{1}}}{\partial x^2}\frac{\partial \tilde{v}_y^{\breve{1}}}{\partial \eta}\Big)\Big) = R_p \tilde{\theta}^{\breve{1}},\end{aligned} \tag{32}$$

$$\begin{aligned}&-\frac{\partial \tilde{\alpha}^{\check{2}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial t} - \frac{\partial \tilde{\alpha}^{\check{0}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{2}}}{\partial t} - \frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial t} + \frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial t} + \tilde{v}_x^{\check{0}}\Big(\frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial x} - \frac{\partial \tilde{\alpha}^{\check{2}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x} - \frac{\partial \tilde{\alpha}^{\check{0}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{2}}}{\partial x} - \frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\Big)\\ &\tilde{v}_x^{\breve{1}}\Big(\frac{\partial \tilde{\alpha}^{\check{0}}}{\partial x} - \frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x} - \frac{\partial \tilde{\alpha}^{\check{0}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x}\Big) + \tilde{v}_y^{\check{0}}\frac{\partial \tilde{\alpha}^{\check{2}}}{\partial \eta} + \tilde{v}_y^{\breve{1}}\frac{\partial \tilde{\alpha}^{\breve{1}}}{\partial \eta} + \tilde{v}_y^{2}\frac{\partial \tilde{\alpha}^{\check{0}}}{\partial \eta} = 0,\end{aligned} \tag{33}$$

order ϵ^2

$$\frac{\partial \tilde{v}_x^{\check{2}}}{\partial x} - \frac{\partial \tilde{v}_x^{\check{3}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{0}}}{\partial x} - \frac{\partial \tilde{v}_x^{\check{2}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\breve{1}}}{\partial x} - \frac{\partial \tilde{v}_x^{\breve{1}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{2}}}{\partial x} - \frac{\partial \tilde{v}_x^{\check{0}}}{\partial \eta}\frac{\partial \tilde{\zeta}^{\check{3}}}{\partial x} + \frac{\partial \tilde{v}_y^{\check{3}}}{\partial \eta} = 0. \tag{34}$$

The matching conditions as $\eta \to \pm\infty$ are

$$\tilde{v}^{\check{0}} \sim v^{\check{0}}|_{y_1=\pm 0}, \tag{35}$$

$$\tilde{v}^{\breve{1}} \sim \big(\frac{\partial v^{\check{0}}}{\partial y_1}|_{y_1=\pm 0}\big)\eta + v^{\breve{1}}|_{y_1=\pm 0}, \tag{36}$$

$$\tilde{v}^{\check{2}} \sim \frac{1}{2}\big(\frac{\partial^2 v^{\check{0}}}{\partial y_1^2}|_{y_1=\pm 0}\big)\eta^2 + \big(\frac{\partial v^{\breve{1}}}{\partial y_1}|_{y_1=\pm 0}\big)\eta + v^{\check{2}}|_{y_1=\pm 0}, \tag{37}$$

$$\tilde{v}^{\check{3}} \sim \frac{1}{6}\big(\frac{\partial^3 v^{\check{0}}}{\partial y_1^3}|_{y_1=\pm 0}\big)\eta^3 + \frac{1}{2}\big(\frac{\partial^2 v^{\breve{1}}}{\partial y_1^2}|_{y_1=\pm 0}\big)\eta^2 + \big(\frac{\partial v^{\check{2}}}{\partial y_1}|_{y_1=\pm 0}\big)\eta + v^{\check{3}}|_{y_1=\pm 0}. \tag{38}$$

From (23), we have

$$\frac{\partial^2 v^{\check{0}}}{\partial \eta^2} = 0.$$

Thus $\tilde{v}^{\check{0}}(\eta)$ is linear function of η and from the boundedness of velocity it's identically constant. Using (35) we have

$$v^{\check{0}}|_{y_1=0^+} = v^{\check{0}}|_{y_1=0^-}, \tag{39}$$

$$\frac{\partial v^{\check{0}}}{\partial \eta} = 0. \tag{40}$$

Thus the first term of the outer expansion of the velocity, $v^{\check{0}}$, is continuous at the front.

Let us consider both the outer and the inner expansion of the temperature (we use the same technique for the variables α and $\mathbf{v}$) and recall that $\eta = y_1/\epsilon$:

$$\theta(x,y_1) = \theta^{\check{0}}(x,y_1) + \epsilon\theta^{\check{1}}(x,y_1) + \epsilon^2\theta^{\check{2}}(x,y_1) + ...,$$

$$\theta(x,\epsilon\eta) = \epsilon\tilde{\theta}^{\check{1}}(x,\eta) + \epsilon^2\tilde{\theta}^{\check{2}}(x,\eta) + ...$$

We write the outer solution in terms of the inner variable η and we use the Taylor expansion:

$$\theta(x,y_1) = \theta^{\check{0}}(x,0) + \epsilon\Big(\frac{\partial\theta^{\check{0}}}{\partial\eta}(x,0)\eta + \theta^{\check{1}}(x,0)\Big) + \epsilon^2\Big(\frac{1}{2}\frac{\partial^2\theta^{\check{0}}}{\partial\eta^2}(x,0)\eta^2 + \frac{\partial\theta^{\check{1}}}{\partial\eta}(x,0)\eta + \theta^{\check{2}}(x,0)\Big) + ...$$

The zero order terms with respect to ϵ correspond to the stationary solution. Equating the first order terms and taking into account that $\dfrac{\partial\theta^{\check{0}}}{\partial\eta}(x,0^-) = 0$, we obtain, using the matching principle [16], the following matching conditions:

$$\eta \to +\infty: \ \tilde{\theta}^{\check{1}} \sim \theta^{\check{1}}|_{y_1=0^+} + \eta\frac{\partial\theta^{\check{0}}}{\partial y_1}|_{y_1=0^+}, \ \tilde{\alpha}^{\check{0}} \to 0, \ \tilde{\mathbf{v}}^{\check{0}} \to \mathbf{v}^{\check{0}}|_{y_1=0^+}, \tag{41}$$

$$\eta \to -\infty: \ \tilde{\theta}^{\check{1}} \to \theta^{\check{1}}|_{y_1=0^-}, \ \tilde{\alpha}^{\check{0}} \to 1, \ \tilde{\mathbf{v}}^{\check{0}} \to \mathbf{v}^{\check{0}}|_{y_1=0^-}. \tag{42}$$

From (26) we obtain that $\tilde{p}^{\check{0}}$ does not depend on η, which implies that at the leading order the pressure is continuous through the interface. Next, denoting by s the quantity

$$s = \tilde{v}_x^{\check{0}}\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x} - \tilde{v}_y^{\check{0}}, \tag{43}$$

we obtain from (27) that s does not depend on η. Finally from (28), (29) and (43) we easily obtain that $\tilde{v}_x^{\check{0}}$ and $\tilde{v}_y^{\check{0}}$ do not depend on η, which provides the continuity of the velocity across the interface.
We next derive the jump conditions for the temperature from (24), in the same way as it is usually done for combustion problems. From (25) it follows that $\tilde{\alpha}^{\check{0}}$ is a monotone function of η and $0 < \tilde{\alpha}^{\check{0}} < 1$. Since we consider zero-order reaction, we have $\phi(\tilde{\alpha}^{\check{0}}) \equiv 1$. From (24) we conclude that $\dfrac{\partial^2\tilde{\theta}^{\check{1}}}{\partial\eta^2} \le 0$. From (42) we have $\dfrac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta} = 0$ at $\eta = -\infty$. Then $\dfrac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta} \le 0$ and $\tilde{\theta}^{\check{1}}$ is a monotone function.

We multiply (24) by $\dfrac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta}$ and integrate with respect to η from $-\infty$ to $+\infty$ taking into account (41), (42) ($\tilde{\theta}^{\check{1}}$ changes from $\theta^{\check{1}}|_{y_1=0^-}$ to $-\infty$ when η changes from $-\infty$ to $+\infty$):

$$\Big(\frac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta}\Big)^2\Big|_{\eta=+\infty} - \Big(\frac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta}\Big)^2\Big|_{\eta=-\infty} = \frac{2}{A}\int_{-\infty}^{\theta^{\check{1}}|_{y_1=0^-}} \exp\Big(\frac{\tau}{1+\delta\tau}\Big)d\tau, \tag{44}$$

where we have set

$$A = 1 + \Big(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial x}\Big)^2. \tag{45}$$

Next, adding (24) and (25) and integrating we obtain

$$\frac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta}\Big|_{\eta=+\infty} - \frac{\partial\tilde{\theta}^{\check{1}}}{\partial\eta}\Big|_{\eta=-\infty} = -\frac{1}{A}\Big(\frac{\partial\tilde{\zeta}^{\check{0}}}{\partial t} + s\Big). \tag{46}$$

Truncating the outer expansion (21), we have $\theta \approx \theta^{\check{0}}$, $\zeta^{\check{0}} \approx \zeta$, $\mathbf{v} \approx \mathbf{v}^{\check{0}}$. From the inner expansion (22) we obtain $Z\theta \approx \tilde{\theta}^{\check{1}}$, and from the matching condition (42), $\tilde{\theta}^{\check{1}}|_{\eta=-\infty} \approx \theta^{\check{1}}|_{y_1=0^-} \approx Z\theta|_{y_1=0^-}$. Thus,

$$\theta^{\check{0}} \approx \theta, \ \theta^{\check{1}}|_{y_1=0^-} \approx Z\theta|_{y_1=0}, \ \zeta^{\check{0}} \approx \zeta, \ \mathbf{v} \approx \mathbf{v}^{\check{0}}, \tag{47}$$

We finally obtain the jump conditions (with the change of variables $\tau \to Z\tau$ under the integral)

$$\Big(\frac{\partial\theta}{\partial y_1}\Big)^2\Big|_{y_1=0^+} - \Big(\frac{\partial\theta}{\partial y_1}\Big)^2\Big|_{y_1=0^-} = 2Z\Big(1 + \Big(\frac{\partial\zeta}{\partial x}\Big)^2\Big)^{-1}\int_{-\infty}^{\theta|_{y_1=0}} \exp\Big(\frac{\tau}{Z^{-1}+\delta\tau}\Big)d\tau, \tag{48}$$

$$\frac{\partial \theta}{\partial y_1}\Big|_{y_1=0^+} - \frac{\partial \theta}{\partial y_1}\Big|_{y_1=0^-} = -\Big(1+\Big(\frac{\partial \zeta}{\partial x}\Big)^2\Big)^{-1}\Big(\frac{\partial \zeta}{\partial t} + \Big(v_x \frac{\partial \zeta}{\partial x} - v_y\Big)\Big|_{y_1=0}\Big), \tag{49}$$

$$v_z\big|_{z_1=0^+} = v_z\big|_{z_1=0^-}, \tag{50}$$

$$\frac{\partial v_y}{\partial y_1}\Bigg|_{y_1=0^+} = \frac{\partial v_y}{\partial y_1}\Bigg|_{y_1=0^-}, \tag{51}$$

$$\frac{\partial^2 v_y}{\partial y_1^2}\Bigg|_{y_1=0^+} = \frac{\partial^2 v_y}{\partial y_1^2}\Bigg|_{y_1=0^-} \tag{52}$$

and

$$\frac{\partial^3 v_y}{\partial y_1^3}\Bigg|_{y_1=0^-} - \frac{\partial^3 v_y}{\partial y_1^3}\Bigg|_{y_1=0^+} = -R_p\left(1+\left(\frac{\partial \zeta}{\partial x}\right)^2\right)^{-1}\left(\left(1+\left(\frac{\partial \zeta}{\partial x}\right)^2\right)^{-1} - 1\right)\frac{\partial \theta}{\partial y}\Bigg|_{y_1=0^+}. \tag{53}$$

Note that we need only consider the matching condition for the component v_y of the velocity since the other component are not used below.

3.2. Formulation of the interface problem

The interface problem will be written as a system of equations for the reactant and a system of equations for the product as well as the jump conditions derived from the free boundary problem of the last subsection.
We have for $y > \zeta$ (in the unburnt medium)

$$\frac{\partial \theta}{\partial t} + \mathbf{v}\cdot\nabla\theta = \Delta\theta, \tag{54}$$

$$\alpha \equiv 0, \tag{55}$$

$$\mathbf{v} + \nabla p - \tilde{D}_a \Delta \mathbf{v} = R_p(\theta+\theta_0)\gamma, \tag{56}$$

$$\nabla\cdot\mathbf{v} = 0. \tag{57}$$

In the burnt medium ($y < \zeta$), we have the system

$$\frac{\partial \theta}{\partial t} + \mathbf{v}\cdot\nabla\theta = \Delta\theta, \tag{58}$$

$$\alpha \equiv 1, \tag{59}$$

$$\mathbf{v} + \nabla p - \tilde{D}_a \Delta \mathbf{v} = R_p(\theta+\theta_0)\gamma, \tag{60}$$

$$\nabla\cdot\mathbf{v} = 0. \tag{61}$$

We complete these systems by the following jump conditions at the interface $y = \zeta$:

$$[\theta] = 0,\ \Big[\frac{\partial \theta}{\partial y}\Big] = \frac{\frac{\partial \zeta}{\partial t}}{1+\left(\frac{\partial \zeta}{\partial x}\right)^2}, \tag{62}$$

$$\Big[\Big(\frac{\partial\theta}{\partial y}\Big)^2\Big] = -\frac{2Z}{1+(\frac{\partial\zeta}{\partial x})^2}\int_{-\infty}^{\theta(t,x,\zeta)} \exp\Big(\frac{s}{1/Z+\delta s}\Big)ds, \tag{63}$$

$$[\mathbf{v}] = 0, \tag{64}$$

$$\left[\frac{\partial v}{\partial y}\right] = 0, \tag{65}$$

$$\left[\frac{\partial^2 v}{\partial y^2}\right] = 0, \tag{66}$$

$$\left[\frac{\partial^3 v}{\partial y^3}\right] = -R_p\left(1+\frac{\partial\zeta}{\partial x}\right)^{-1}\left(\left(1+\left(\frac{\partial\zeta}{\partial x}\right)^2\right)^{-1}-1\right)\frac{\partial\theta}{\partial y}\Big|_{\zeta_{0+}}. \tag{67}$$

With [ ] denotes the jump at the interface,

$$[f] = f|_{\zeta_{0+}} - f|_{\zeta_{0-}}.$$

This free boundary problem is completed by the following conditions

$$y\to+\infty,\ \ \theta=-1 \text{ and } \mathbf{v}=0, \tag{68}$$

$$y\to-\infty,\ \ \theta=0 \text{ and } \mathbf{v}=0. \tag{69}$$

3.3. Travelling wave solution

We perform in this subsection the linear stability analysis of the steady-state solution for the interface problem. Indeed, this interface problem has a travelling wave solution $(\theta,\alpha,\mathbf{v})$,

$$\theta(t,x,y)=\theta_s(y-ut),\ \ \alpha(t,x,y)=\alpha_s(y-ut),\ \ \mathbf{v}=0,\ \ \zeta=ut, \tag{70}$$

with

$$\theta_s(y_1)=\begin{cases}0 & \text{if } y_1<0\\ e^{-uy_1}-1 & \text{if } y_1>0\end{cases} \tag{71}$$

and

$$\alpha_s(y_1)=\begin{cases}1 & \text{if } y_1<0\\ 0 & \text{if } y_1>0\end{cases}, \tag{72}$$

here $y_1 = y-ut$ denotes the moving coordinate frame, where u is the wave speed. With these coordinates the travelling wave is a stationary solution of the problem

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial y_1}+\mathbf{v}\cdot\nabla\theta=\Delta\theta, \tag{73}$$

$$\mathbf{v}+\nabla p-\tilde{D}_a\Delta\mathbf{v}=R_p(\theta+\theta_0)\gamma, \tag{74}$$

$$\nabla\cdot\mathbf{v}=0, \tag{75}$$

supplemented with the jump conditions (62)-(67).

We consider now a small time-dependent perturbation of the stationary solution of the form. To this end, we chose the perturbation of the temperature and of the velocity in the following form:

$$\zeta(t,x) = ut + \xi(t,x), \tag{76}$$

$$\begin{aligned} &\theta(t,x,y) = \theta_s(y-ut) + \theta_j(z)e^{\omega t+ikx}, \ j = 1,2, \\ &v_y(t,x,y) = v_j(z)e^{\omega t+ikx}, \end{aligned} \tag{77}$$

where

$$z = y - \zeta(t,x) = y - ut - \xi(t,x), \ \ \xi(t,x) = \epsilon_1 e^{\omega t+ikx}, \tag{78}$$

$j = 1$ corresponds to $z < 0$ and $j = 2$ to $z > 0$.
We seek to linearize system (73)-(75) around the stationary solution. For simplicity, we eliminate the pressure p and the component v_x of the velocity from the interface problem by applying two times the operator $curl$. Thus, we obtain the following fourth-order differential system of equations
for $z < 0$:

$$\theta_1'' + u\theta_1' - (\omega + k^2)\theta_1 = 0, \tag{79}$$

$$-\tilde{D}_a v_1^{(4)} + (1 + 2k^2\tilde{D}_a)v_1'' - k^2(1 + k^2\tilde{D}_a)v_1 = -R_p k^2\theta_1 \tag{80}$$

for $z > 0$:

$$\theta_2'' + u\theta_2' - (\omega + k^2)\theta_2 = -v_2 u e^{-uz}, \tag{81}$$

$$-\tilde{D}_a v_2^{(4)} + (1 + 2k^2\tilde{D}_a)v_2'' - k^2(1 + k^2\tilde{D}_a)v_2 = -R_p k^2\theta_2. \tag{82}$$

These systems is supplemented with the following linearized jump conditions:

$$\theta_2(0) - \theta_1(0) = u\epsilon_1, \tag{83}$$

$$\theta_2'(0) - \theta_1'(0) = -\epsilon_1(u^2 + \omega), \tag{84}$$

$$-u(u^2\epsilon_1 + \theta_2'(0)) = Z\theta_1(0), \tag{85}$$

$$v_1^{(i)}(0) = v_2^{(i)}(0) \ , \ i = 0,1,2,3, \tag{86}$$

where

$$v^{(i)} = \frac{\partial^i v}{\partial z^i}, \ \ \theta' = \frac{\partial \theta}{\partial y_1}.$$

4. The dispersion relation and numerical results

4.1. The dispersion relation

By introducing the two linear differential operators:

$$L_1\theta=\theta''+u\theta'-(\omega+k^2)\theta,\ \ L_2v=-\tilde{D}_a v^{(4)}+(1+2k^2\tilde{D}_a)v''-k^2(1+k^2\tilde{D}_a)v,$$

the system of equations (79)-(82) can be re-written as follows:

$$L_1\theta_1=0,\ \ L_2v_1=-R_pk^2\theta_1, \tag{87}$$

$$L_1\theta_2=-ue^{-uz}v_2,\ \ L_2v_2=-R_pk^2\theta_2. \tag{88}$$

Due to the fact that the perturbations decay at infinity ($v_i(\pm\infty)=0$, $\theta_i(\pm\infty)=0$, $i=1,2$), the general solutions of (87)-(88) have the following form:

$$v_1(z)=a_1\frac{-R_pk^2}{(1+\tilde{D}_a(k^2-\mu_1^2))(k^2-\mu_1^2)}\tilde{w}_1(z)+a_2\frac{1}{r_1^2-k^2}\tilde{w}_2(z)+a_3\tilde{w}_3(z),\ \ \theta_1(z)=a_1\tilde{w}_1(z), \tag{89}$$

$$v_2(z)=b_1w_1(z)+b_2w_2(z)+b_3w_3,\ \ \theta_2(z)=b_1s_1(z)+b_2s_2(z)+b_3s_3(z), \tag{90}$$

where b_1, b_2, b_3, a_1, a_2, a_3 are arbitrary constants and the functions w_i, $\tilde{w}_i$, s_i for $i=1,2,3$ have the form

$$\tilde{w}_1(z)=e^{\mu_1 z},\ \ \tilde{w}_2(z)=e^{r_1 z},\ \ \tilde{w}_3(z)=e^{kz},\ \ w_i(z)=\sum_{j=1}^{\infty}a_{i,j}e^{\sigma_{i,j}z},\ \ s_i(z)=\sum_{j=1}^{\infty}c_{i,j}e^{\sigma_{i,j}z}, \tag{91}$$

with

$$\begin{aligned}
&\mu_1=\frac{-u+\sqrt{u^2+4(\omega+k^2)}}{2},\ \ r_1=\sqrt{\frac{1+k^2\tilde{D}_a}{\tilde{D}_a}},\\
&\sigma_{1,1}=\frac{-u-\sqrt{u^2+4(\omega+k^2)}}{2},\ \ \sigma_{2,1}=-\sqrt{\frac{1+k^2\tilde{D}_a}{\tilde{D}_a}},\ \ \sigma_{3,1}=-k,\\
&\sigma_{i,j+1}=\sigma_{i,j}-u,\ \ i=1,2,3\ \ j=1,2,...\\
&c_{11}=1,\ \ c_{21}=0,\ \ c_{31}=0,\\
&a_{11}=0,\ \ a_{21}=1,\ \ a_{31}=1,\\
&a_{i,j}=\frac{-R_pk^2c_{i,j}}{(1+\tilde{D}_a(k^2-\sigma_{i,j}^2))(\sigma_{i,j}^2-k^2)}\ \text{for}\ (i,j)\neq(2,1),\ (i,j)\neq(3,1)\ \text{and}\ i=1,2,3,\ \ j=1,2,...\\
&c_{i,j+1}=\frac{-ua_{i,j}}{\sigma_{i,j+1}^2+u\sigma_{i,j+1}-(\omega+k^2)}\ \text{for}\ \ i=1,2,3,\ \ j=1,2,....
\end{aligned} \tag{92}$$

We assume here that

$$1+\tilde{D}_a(k^2-\sigma_{i,j}^2)\neq0,\ \ k^2-\sigma_{i,j}^2\neq0,\ \ \text{for}\ \ i=1,2,3,\ \ j=1,2,...,$$

$$1+\tilde{D}_a(k^2-\mu_1^2)\neq0,\ \ k^2-r_1^2\neq0,\ \ k^2-\mu_1^2\neq0,\ \ \tilde{D}_a\neq0$$

and

$$\sigma_{i,j+1}^2+u\sigma_{i,j+1}-(\omega+k^2)\neq0,\ \ \text{for}\ \ i=1,2,3,\ \ j=1,2,....$$

We note that the series (91) converge uniformly in z.
We show now that the functions (w_i,s_i) satisfy (87)-(88). Indeed, we denote

$$w_i^n(z)=\sum_{j=1}^{n}a_{i,j}e^{\sigma_{i,j}z},\ \ s_i^n(z)=\sum_{j=1}^{n}c_{i,j}e^{\sigma_{i,j}z},\ \ i=1,2,3.$$

Substituting these partial sums into (87)-(88), we have

$$L_1 s_i^n = -ue^{-uz}(w_i^n - a_{in}e^{\sigma_{in}z}),\ \ L_2 w_i^n = -R_p k^2 s_i^n,\ \ i = 1,2,3.$$

Passing to the limit as $n \to \infty$ we obtain that series (91) satisfy the system (87)-(88).
The constants a_1, a_2 , a_3, b_1, b_2 and b_3 are sought from the jump conditions (83)-(86). Simple computations lead us to the following linear system of equations

$$\begin{cases} b_1 s_1(0) + b_2 s_2(0) + b_3 s_3(0) - a_1 = u\epsilon_1 \\ b_1 s_1'(0) + b_2 s_2'(0) + b_3 s_3'(0) - a_1\mu_1 = -\epsilon_1(u^2 + \omega) \\ b_1 s_1'(0) + b_2 s_2'(0) + b_3 s_3'(0) + a_1 \frac{Z}{u} = -u^2\epsilon_1 \\ b_1\omega_1(0) + b_2\omega_2(0) + b_3\omega_3(0) - a_3 = \alpha_1 a_2 + \beta_1 a_1 \\ b_1\omega_1'(0) + b_2\omega_2'(0) + b_3\omega_3'(0) - a_3 k = \alpha_1 r_1 a_2 + \beta_1\mu_1 a_1 \\ b_1\omega_1''(0) + b_2\omega_2''(0) + b_3\omega_3''(0) - a_3 k^2 = \alpha_1 r_1^2 a_2 + \beta_1\mu_1^2 a_1 \\ b_1\omega_1'''(0) + b_2\omega_2'''(0) + b_3\omega_3'''(0) - a_3 k^3 = \alpha_1 r_1^3 a_2 + \beta_1\mu_1^3 a_1 \end{cases} \tag{93}$$

We look for a non-trivial solution of the linear system (93). The coefficient matrix $\mathscr{A}$ of such system is given as follows

$$\mathscr{A} = \begin{pmatrix} s_1(0) & s_2(0) & s_3(0) & -1 & 0 & 0 & -u \\ s_1'(0) & s_2'(0) & s_3'(0) & -\mu_1 & 0 & 0 & (u^2+\omega) \\ s_1'(0) & s_2'(0) & s_3'(0) & \frac{Z}{u} & 0 & 0 & u^2 \\ \omega_1(0) & \omega_2(0) & \omega_3(0) & -\beta_1 & -\alpha_1 & -1 & 0 \\ \omega_1'(0) & \omega_2'(0) & \omega_3'(0) & -\beta_1\mu_1 & -\alpha_1 r_1 & -k & 0 \\ \omega_1''(0) & \omega_2''(0) & \omega_3''(0) & -\beta_1\mu_1^2 & -\alpha_1 r_1^2 & -k^2 & 0 \\ \omega_1'''(0) & \omega_2'''(0) & \omega_3'''(0) & -\beta_1\mu_1^3 & -\alpha_1 r_1^3 & -k^3 & 0 \end{pmatrix} \tag{94}$$

The general condition that the system (93) has a non-trivial solution is the requirement that the determinant of the coefficient matrix vanish ($det(\mathscr{A}) = 0$), which allows us to obtain the dispersion relation.

4.2. Numerical results

The neutral instability boundary of the reaction front can be determined from the fact whether the perturbation (77) remains bounded in time or not. This situation holds when ω lies on the imaginary axis; we find the cellular instability boundary if $\omega = 0$ and the oscillatory one if $\omega = i\phi$ for $\phi \neq 0$ (ω is a pure imaginary number). It is clear from the form of the matrix $\mathscr{A}$ (94) that when $\omega = 0$ its determinant doesn't depend on Z (linear dependence of matrix elements in the seventh column); the cellular stability boundary doesn't depend on Zeldovich number.

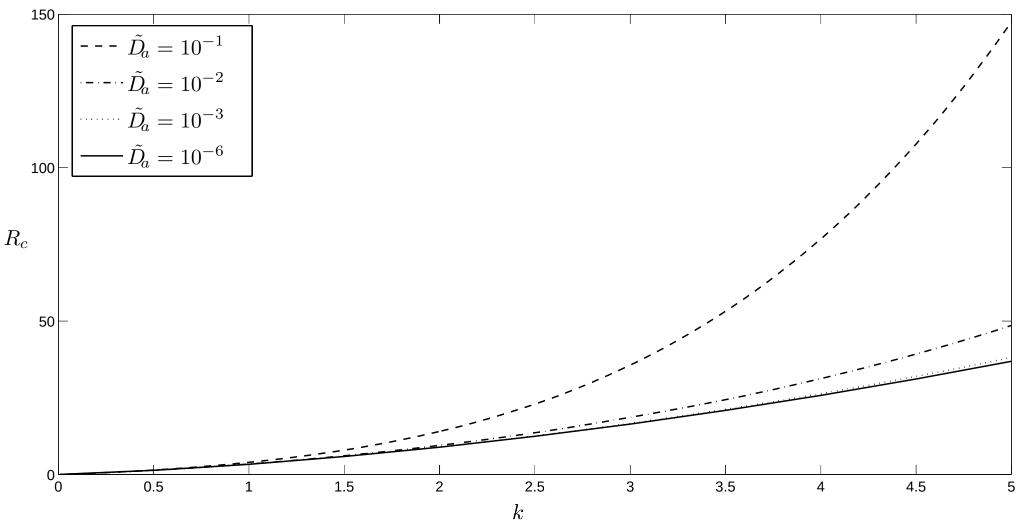

Fig. 1. Critical Rayleigh number versus wavenumber for $u = \sqrt{2}$ (cellular threshold).

Fig. 1 shows, for $u = \sqrt{2}$ and for different values of the Darcy-Brinkman number, the critical Rayleigh number as function of wavenumber k (cellular threshold). The plots indicate that when the wavenumber tends toward zero, Darcy-Brinkman number has no effect on the convective instability of the reaction front, however a stabilizing effect is observed for increased values of the wavenumber. It is also seen in this figure that a decrease of the value of

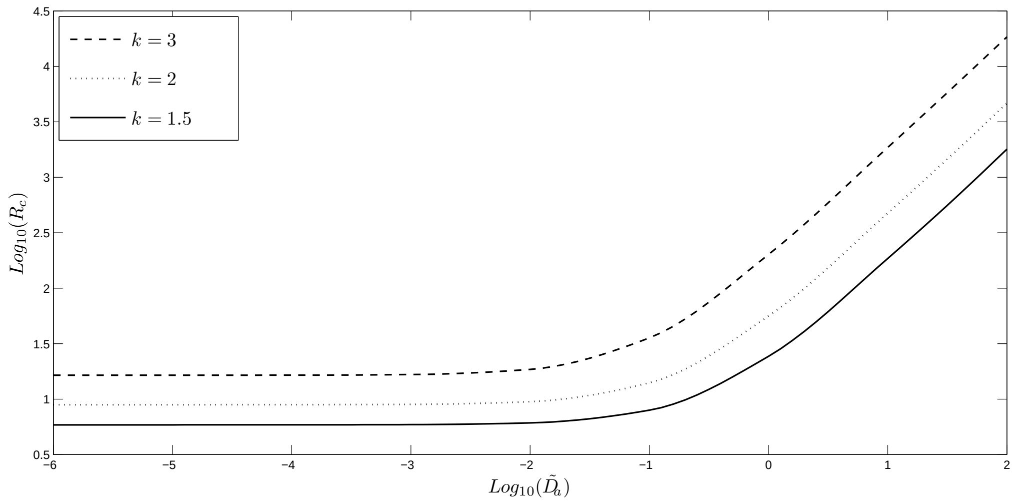

Fig. 2. Critical Rayleigh number versus Darcy-Brinkman number for $u = \sqrt{2}$ (cellular threshold).

the Darcy-Brinkman number destabilizes the reaction front and when its values are less than $\approx 10^{-3.5}$ we find the same result as in [11]. The critical Rayleigh number as function of Darcy-Brinkman number (cellular threshold) is illustrated in Fig. 2 for $u = \sqrt{2}$ and for different values of the wavenumber. It can be seen that the viscous effects lead to monotonic increase of the Rayleigh number and the reaction front gains stability monotonically. The figure shows the transition between the darcian porous fluid and the clear fluid limits with a sudden significative increase from a certain value of Darcy-Brinkman number ($\approx 10^{-1}$), it is also shown that an increase of the wavenumber stabilizes the reaction front.

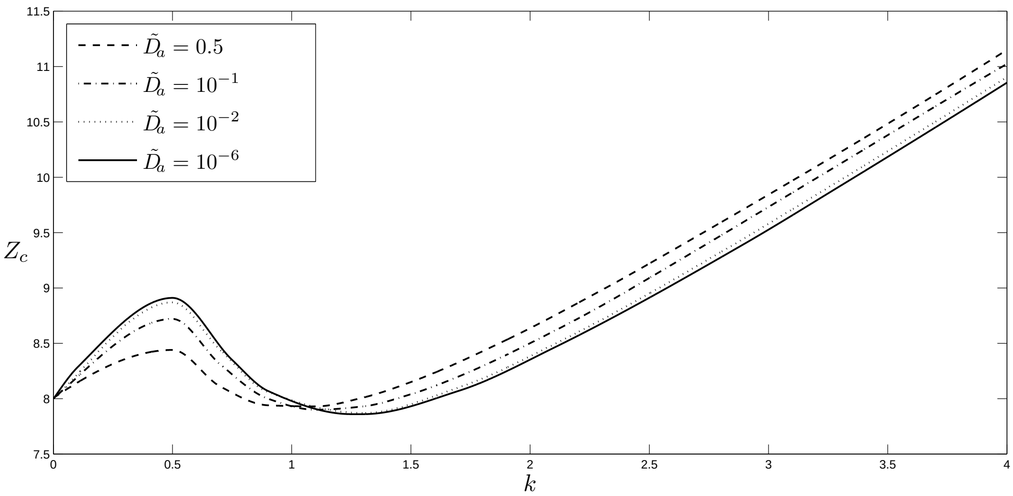

Fig. 3. Critical Zeldovich number versus the wavenumber for $u = \sqrt{2}$ and $R_p = 2.5$ (oscillatory threshold).

Fig. 3 depicts the influence of the Darcy-Brinkman number on the oscillatory stability of the reaction front. First, all the curves start at the value $Z_c \simeq 8$, which is in good agreement with [17, 18]. This figure also depicts an interesting phenomenon, that is in a certain interval of wavenumber $k \lesssim 1.1$ the oscillatory stability of the reaction front is gained for a decreased values of the Darcy-Brinkman number while in opposite range of the interval $k \gtrsim 1.1$ the oscillatory stability of the reaction front is gained for a increased values of $\tilde{D}_a$. It is worthy to notice that for sufficiently small values of Darcy-Brinkman number ($\lesssim 10^{-3.5}$) we find the Darcian limit [11]. Finally, Fig. 4 shows the critical Rayleigh number versus the wavenumber for the pulsating case and for $R_p = 0$. When we vary the Darcy-Brinkman number the oscillatory threshold remains unchanged. The Darcy-Brinkman number has no effect in the case of absence of buoyancy.

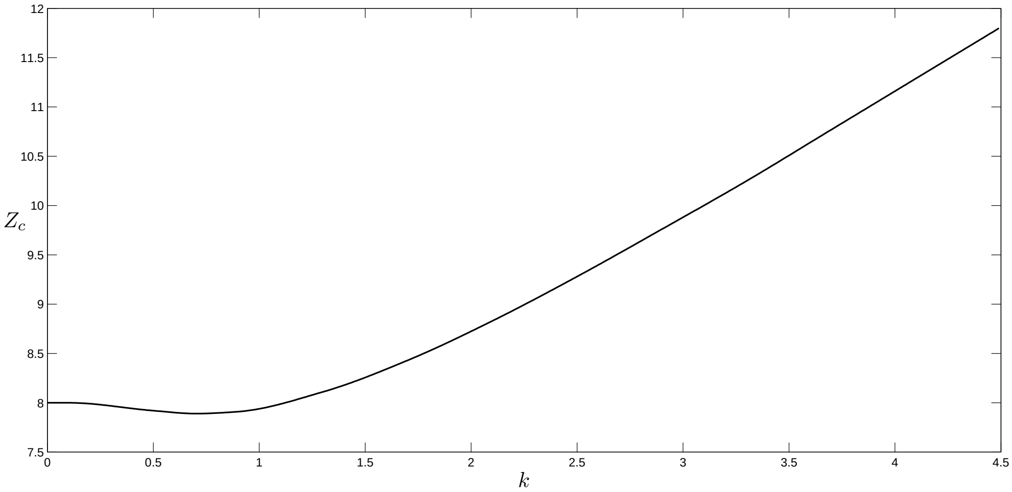

Fig. 4. Critical Zeldovich number versus the wavenumber for $u = \sqrt{2}$ and $R_p = 0$ (oscillatory threshold).

5. Conclusion

In this work we have studied the convective instability of reaction fronts in porous media with Darcy-Brinkman formulation. The considered model includes the heat equation, the equation for the depth of conversion and the equations of motion using the Darcy-Brinkman formulation. We have assumed that the fluid is incompressible and the reaction front propagation is in the opposite sense to the gravity. The linear stability analysis of the original problem, by using the narrow-zone method, is performed in order to find the interface problem. The convective instability threshold is determined via the dispersion relation.
It was shown that for relatively high values of the wavenumber k the Darcy-Brinkman number has a significant effects on the cellular convective instability of the reaction front than for small ones. A more pronounced stabilizing effect can be gained for increased values of Darcy-Brinkman number. The oscillatory instability threshold is also studied, an increase of the Darcy-Brinkman number destabilazes the reaction front for a certain interval of the wavenumber while a stabilizing effect is observed for the other values not belonging to this interval. For both cellular and oscillatory convective instability studies, a convergence of solutions toward Darcian fluid limit is also observed for very small values of the Darcy-Brinkman number.
The results of this work show that in the presence of high porosity matrix, the convection instability of reaction fronts in porous media can be controlled and the reaction front may remain stable for certain values of the Darcy-Brinkman number.

References

[1] D. A. Nield, A. Bejan, Convection in porous media. springer, 2006.
[2] K. Vafai, Handbook of porous media. Crc Press, 2010.
[3] A. A. Hill, B. Straughan, Global stability for thermal convection in a fluid overlying a highly porous material, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science 465(2101) (2009) 207-217.
[4] J. P. Kelliher, R. Temam, Xiaoming Wang, Boundary layer associated with the Darcy-Brinkman-Boussinesq model for convection in porous media, Physica D: Nonlinear Phenomena 240 (2011) 619-628.
[5] H. C. Brinkman, A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, Applied Scientific Research 1 (1949) 27-34.
[6] J. Sheela-Francisca, C. P. Tso, Viscous dissipation effects on parallel plates with constant heat flux boundary conditions, International communications in heat and mass transfer 36 (2009) 249-254.
[7] A. S. Joodi, S. Sizaret, S. Binet, A. Bruand, P. Alberic, M. Lepiller, Development of a Darcy-Brinkman model to simulate water flow and tracer transport in a heterogeneous karstic aquifer (Val d'Orleans, France). Hydrogeology journal 18 (2010) 295-309.
[8] H. Liu, P. R. Patil, U. Narusawa, On Darcy-Brinkman equation: viscous flow between two parallel plates packed with regular square arrays of cylinders, Entropy 9 (2007) 118-131.
[9] A. R. Khaled, K. Vafai, The role of porous media in modeling flow and heat transfer in biological tissues, International Journal of Heat and Mass Transfer 46 (2003) 4989-5003.

[10] P. Nithiarasu, K. N. Seetharamu, T. Sundararajan, Double-diffusive natural convection in an enclosure filled with fluid-saturated porous medium: a generalized non-Darcy approach, Numerical Heat Transfer, Part A Applications 30 (1996) 413-426.

[11] K. Allali, A. Ducrot, A. Taik, V. Volpert, Convective instability of reaction fronts in porous media, Mathematical Modelling of Natural Phenomena 2 (2007) 20-39.

[12] B. J. Matkowsky, G. I. Sivashinsky, Propagation of a pulsating reaction front in solid fuel combustion, SIAM Journal on Applied Mathematics 35 (1978) 465-478.

[13] B. J. Matkowsky, G. I. Sivashinsky, An asymptotic derivation of two models in flame theory associated with the constant density approximation, SIAM Journal on Applied Mathematics 37 (1979) 686-699.

[14] Y. B. Zeldovich, Y. B., D. A. Frank-Kamenetskii, A theory of thermal propagation of flame, Zh. Fiz. Khim. 12 (1938) 100–105.

[15] M. Menzinger, R. Wolfgang, The meaning and use of the Arrhenius activation energy, Angewandte Chemie International Edition in English 8 (1969) 438–444.

[16] A. H. Nayfeh, Perturbation methods. John Wiley & Sons, 2008.

[17] M. Garbey, A. Taik, V. Volpert, Influence of natural convection on stability of reaction fronts in liquids, Quarterly of Applied Mathematics 56 (1998) 1–35.

[18] M. Garbey, A. Taik, V. Volpert, Linear stability analysis of reaction fronts in liquids, Quarterly of applied mathematics 54 (1996) 225–247.