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# Unsteady MHD free convective heat transfer flow along a vertical porous flat plate with internal heat generation

**Research Article**

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**Abstract:** In this study, a similarity analysis is made for an unsteady two-dimensional MHD free convective and heat transfer flow of a viscous incompressible fluid along a vertical flat plate with internal heat generation/absorption. The governing time-dependent boundary-layer equations are reduced to non-linear ordinary differential equations by introducing a similarity transformation. The obtained similarity equation for both steady and unsteady cases have been compared with previously published work and excellent agreement is found. The resulting local similarity equations for the unsteady flow have been solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta integration scheme. Numerical results for the dimensionless velocity and temperature profiles as well as the local skin-friction coefficient and local Nusselt number are displayed both by graphically and numerically for different values of the physical parameters entering into the problem.The significance of the physical parameters on the flow field is discussed in detail.The results shows that the velocity profile and the temperature profile is much pronounced with the increasing value of heat generation and unsteadiness parameter but the opposite effect can be found for suction parameter and Prandlt number. Also, the result shows that the values of skin-friction and wall heat transfer are strongly influenced by unstediness and heat generation parameter.

**MSC:** 76M55 • 65L10

**Keywords:** Unsteady flow • Free convection • Porous plate• Similarity solutions • Internal heat generation • h Heat transfer

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## 1. Introduction

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips, and semiconductor wafers. In addition, natural convection with heat generation can be applied to combustion modeling. Heat generation or absorption may be constant, space-dependent or temperature-dependent. Moalem [1] studied the effect of temperature dependent heat sources on the steady convective flow taking place in electrically conducting fluid within a porous medium. Vajravelu and Nayfeh [2] have reported results on the hydromagnetic convection at a cone and a wedge in the presence of temperature-dependent heat generation or absorption effects. Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation was studied by Vajravelu and Hadjinicolaou [3]. They have found that the wall temperature increases as the heat source/sink parameter increases. Molla et al. [4] studied the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation or absorption. The effect of Magnetohydrodynamic natural convection flow on

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a sphere in the presence of heat generation has been studied numerically by Molla et al. [5]. Their results showed that both the velocity and temperature profiles increase significantly when the values of the heat generation parameter increases. Magnetohydrodynamic convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous flat plate in the presence of heat generation or absorption has been analyzed numerically by Rahman and Sattar [6]. They found that for the case of heat generation, the thermal state of the fluid increases, consequently the heat transfer rate decreases. Later, Alam et al. [7] studied numerically the steady combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. The analysis of the obtained results showed that the flow field is significantly influenced by the heat generation parameter.

In recent years the subject of magnetohydrodynamics (MHD) has attracted many authors in view not only of its own interest but also of the applications to geophysics and engineering. When the fluid is a conductor of electricity the free convection-heat transfer flow can be influenced by an imposed magnetic field. MHD phenomenon results from the natural effect of a magnetic field and a conducting fluid flowing across it. Thus, an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluidâĂŹs motion. Examination of flow models will reveal the influence of magnetic fields on the velocity profile, temperature profile and the local heat transfer rate. In light of these various applications, hydromagnetic free convection flow in the stokes problem for a porous vertical limiting surface with constant suction has been analyzed by Nanousis et al. [8]. Singh [9] studied MHD free convection flow in the stokes problem for a porous vertical plate. Numerous scientists works on a similarity solution of an unsteady one-dimensional hydromagnetic heat transfer flow with variable suction or injection. In this study, a time dependent similarity parameter was introduced on the basis of the proposal put forward by Hasimoto [10]. Introducing this similarity parameter, the governing boundary layer equations were reduced to non-linear ordinary differential equations, which are similar in time. Later many articles have been published in this line, the works of which are Sattar and Hossain [11], Sattar [12], Sattar et al. [13], Alam et al. [14] and Rahman and Sattar [15] are worth mentioning.

Non-similar solutions of unsteady two-dimensional boundary layer problems as compared to the above similarity solutions are readily available in literature. However, Wang [16] introduced a similarity transformation to solve the hydrodynamic unsteady two-dimensional boundary layer flows past a thin liquid film on a stretching sheet. This similarity transformation reduces the unsteady Navier-Stokes equations to a non-linear ordinary differential equations governed by a nondimensional unsteady parameter. He obtained the asymptotic as well as numerical solutions to the transformed similarity equation for several values of the unsteadiness parameter. Andersson et al. [17] studied the momentum and heat transfer in a liquid film on an unsteady stretching surface.

Recently, Alam and Huda [18] analyzed a new approach for local similarity solutions of an unsteady hydromagnetic free convective heat transfer flow along a permeable flat surface.But the effect of internal heat generation was absent on the flow field. So, the main objective of the present study is to extend the work of Alam and Huda [18] to free convective heat transfer flow along a vertical porous flat plate with internal heat generation/absorption.This problem has not been introduced in the open literature, despite its fundamental significance. Using similarity transformation, the governing partial differential equations are reduce to a non-linear ordinary differential equation which are solved numerically by applying sixth-order Runge-Kutta method with Nachtsheim-Swigert shooting iteration technique.

## 2. Governing equations and analysis of the flow

We consider an unsteady two-dimensional MHD free convective laminar boundary layer flow of a viscous incompressible fluid along a vertical porous moving plate with velocity $u=U(x,t)=\frac{\nu x}{\delta^2}$ . A uniform magnetic field of strength $B_0$ is considered to acting perpendicular to the plate. The $x$-axis is considered along the plate in the upward direction and the $y$-axis is normal to it. Fluid suction is imposed at the plate surface. In addition a heat source or sink is placed within the flow to allow for possible heat generation or absorption effects. The fluid is assumed to be Newtonian, electrically conducting and heat generating/absorbing. The flow configuration and coordinate system are shown in Fig. 1.

Under the above assumption and using the Boussinesq and boundary layer approximations the governing equations describing the conservation of mass, momentum and energy respectively are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

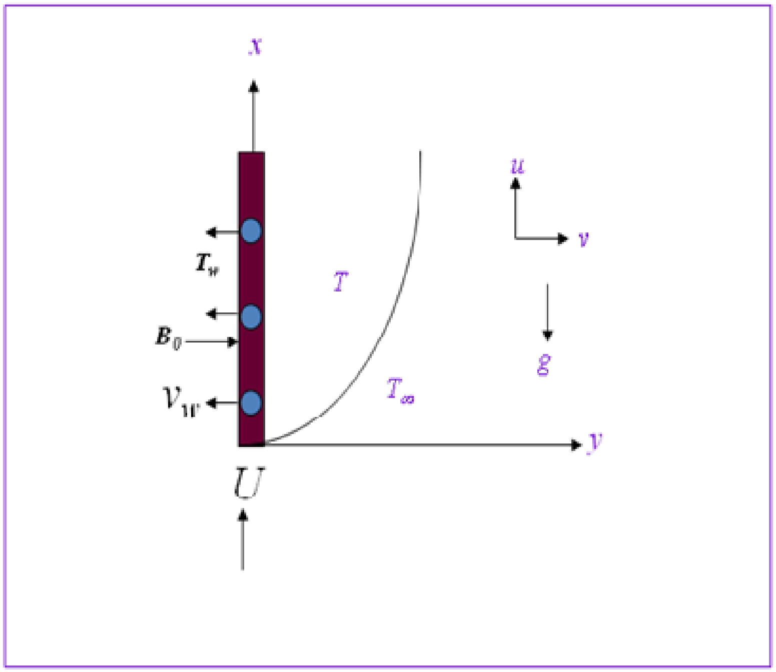


**Fig. 1.** Flow configuration and co-ordinate system

$$\frac{\partial u}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty) \tag{3}$$

where $u$, $v$ are the velocity components along $x$, $y$ co-ordinates respectively, $\nu$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\beta$ is the volumetric coefficient of thermal expansion, $\rho$ is the density of the fluid, $\sigma$ is the magnetic permeability, $B_0$ is the uniform magnetic field acting normal to the plate , $T$ is the temperature of the fluid within the boundary layer, $T_\infty$ is the free stream temperature, $c_p$ is the specific heat of the fluid at constant pressure and $\kappa$ is the thermal conductivity and $Q_0$ is the heat generation/absorption constant. The corresponding boundary conditions to the above model are as follows:

$$\left.\begin{array}{lll} u=U(x,t)=\frac{\nu x}{\delta^2}, v=\pm v_w(x,t), & T=T_w(x,t)=T_\infty+(T_0-T_\infty)\frac{\nu x}{U\delta^2} & at\ y=0 \\ u\to 0, & T\to T_\infty & as\ y\to\infty \end{array}\right\} \tag{4}$$

where $U_0$ is the mean fluid velocity, $T_0$ is the mean fluid temperature and $\delta$ is a time dependent similarity parameter which is taken to be (see [12, 19]) as

$$\delta=\delta(t). \tag{5}$$

In order to obtain similarity solution for the problem under consideration, we may take the following suitable similarity variables

$$\left.\begin{array}{l} \eta=\frac{y}{\delta}, \\ \psi=\frac{\nu x}{\delta}f(\eta), \\ T=T_\infty+(T_0-T_\infty)\frac{\nu x}{U_0\delta^2}\theta(\eta) \end{array}\right\} \tag{6}$$

Now introducing (5)-(6) in Eqs. (1)-(4) we obtain the following non-linear ordinary differential equations

$$f'''+\frac{\delta}{\nu}\frac{d\delta}{dt}(2f'+\eta f'')+f''f-f'^2+Gr\theta-Mf'=0 \tag{7}$$

$$\theta''+\frac{\delta}{\nu}\frac{d\delta}{dt}\Pr(2\theta+\eta\theta')+\Pr(f\theta'-f'\theta)+Q\Pr\theta=0 \tag{8}$$

The corresponding boundary conditions are

$$\left.\begin{array}{lll} f=f_w, f'=1, & \theta=1 & at\ \eta=0 \\ f'\to 0, & \theta\to 0 & as\ \eta\to\infty \end{array}\right\} \tag{9}$$

where $f_w = -\frac{\delta}{\nu}v_w(x,t)$ is the dimensionless suction parameter, $M = \frac{\sigma B_0^2\delta^2}{\rho\nu}$ is the magnetic parameter, $Gr = \frac{g\beta(T_0-T_\infty)\delta^2}{U_0\nu}$ is the local Grashof number, $\Pr = \frac{\rho\nu C_p}{\kappa}$ is the Prandtl number and $Q = \frac{Q_0\delta^2}{\nu\rho C_p}$ is the heat generation/absorption parameter. Now in order to make the Eqs. (7)-(8) locally similar, let

$$K = \frac{\delta}{\nu}\frac{d\delta}{dt} \tag{10}$$

where K is taken to be a constant and thus can be treated as a dimensionless measure of the unsteadiness. Hence the Eqs. (7)-(8) becomes

$$f''' + K(2f' + \eta f'') + f''f - f'^2 + Gr\theta - Mf' = 0 \tag{11}$$

$$\theta'' + K\Pr(2\theta + \eta\theta') + \Pr(f\theta' - f'\theta) + Q\Pr\theta = 0 \tag{12}$$

Now integrating (10), we obtain

$$\delta = \sqrt{2K\nu t} \tag{13}$$

Choosing $K$ =2 in (13), we get

$$\delta = 2\sqrt{\nu t} \tag{14}$$

The length scale $\delta = 2\sqrt{\nu t}$ for the ordinate similar to one seen in (14) was initially used by Stokes' [20] for an unsteady parallel flow but $\delta(t)$ form of the length was initially developed by Sattar and Hossain [19] in case of a solution of an unsteady one-dimensional boundary layer problem. The characteristics length scale $\delta(t)$ defined particularly in (14) physically related to the boundary layer thickness which can be viewed in Schlichting [21].

## 3. Local Skin-friction coefficient and local Nusselt number

The parameters of engineering interest for the present problem are the local skin-friction coefficient and the local Nusselt number which are given below: Now the equation for wall shear stress is

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu\nu x}{\delta^3}f''(0) \tag{15}$$

Therefore the local skin-friction coefficient is obtained as

$$C_{fx} = \frac{\tau_w}{\rho U_0^2(x,t)} = f''(0), where X = \frac{x}{\delta} \tag{16}$$

Again the wall heat flux is

$$q_w = -\kappa(\frac{\partial T}{\partial y})_{y=0} = -\kappa(T_0 - T_\infty)\frac{\nu x}{U_0\delta^3}\theta'(0) \tag{17}$$

Therefore the local Nusselt number is given by

$$Nu_x = \frac{\delta q_w}{(T_w - T_\infty)\kappa} = -\frac{1}{Re}\theta'(0) \tag{18}$$

where $Re = \frac{U_0\delta^3}{\nu x}$ is the lacal Reynolds number.
Thus from the above definitions we have $C_{fx} \propto f''(0)$, and $Nu_x \propto -\theta'(0)$.

## 4. Numerical Solutions

The locally similar and non-linear ordinary differential Eqs. (11)-(12) with boundary conditions (9) have been solved numerically by using sixth order Runge-Kutta method along with Nachtsheim-Swigert [22] shooting iteration technique [for detailed discussion of this method see Alam et al. [7] and in appendix] with $f_w$, $K$, $Gr$, $Pr$, $M$ and $Q$ as prescribed parameter. The computations were done by a program, which uses a symbolic, and computational computer language FORTRAN LAHEY. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of $10^{-6}$. The value of $\eta_\infty$ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of $\eta_\infty$ was determined when the value of the unknown boundary conditions at $\eta = 0$ does not change in the successful loop with an error less than $10^{-6}$.To assess the accuracy of the present numerical method, we have compared our results with Andersson et al. [23] and Alam et al. [18] in Table 1 and in Table 2 respectively and we see that excellent agreement among the results exists.

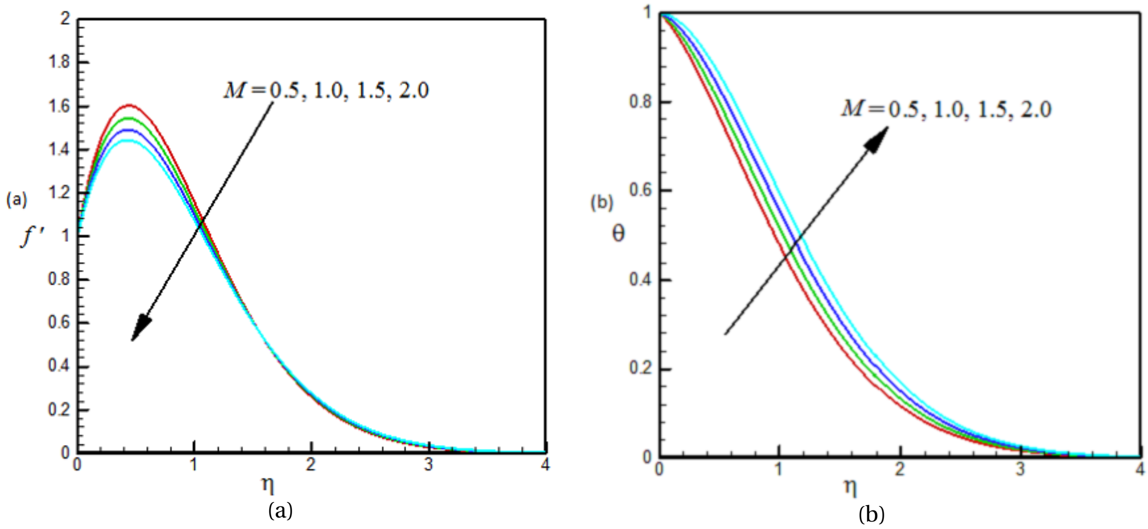


**Fig. 2.** Dimensionless (a) velocity and (b) temperature profiles for different values of $M$ and for Pr=0.70, $f_w = 0.50$, $K = 0.5$, $Gr = 8.0$ and $Q = 2.0$.

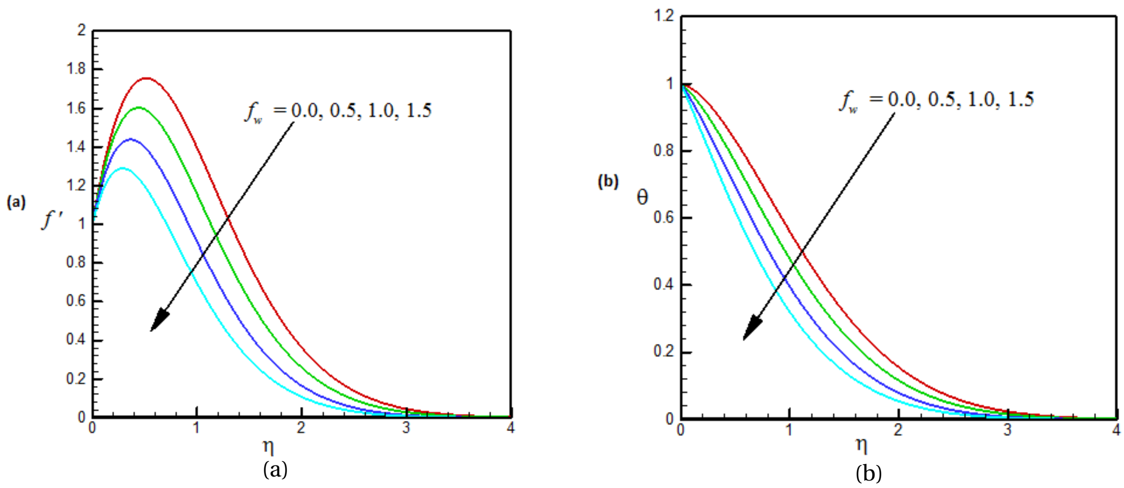


**Fig. 3.** Dimensionless (a) velocity and (b) temperature profiles for different values of $f_w$ and for Pr = 0.70, $M = 0.50$, $K = 0.5$, $Gr = 8.0$ and $Q = 2.0$

## 5. Results and discussion

Numerical calculations have been carried out for different values of $M$, $f_w$, $Gr$, $Pr$, $K$, $Q$ in Figs. 2(a)-7(b) and Table 3 for unsteady case. Figs. 2(a)-2(b) represent typical profiles for the velocity and temperature for various values of the magnetic field parameter$M$ respectively.The presence of a magnetic field normal to the flow in an electrically conducting fluid introduces a Lorentz force, which acts against the flow. This resistive force tends to slow down the flow and hence the fluid velocity decreases with the increase of the magnetic field parameter as observed in Fig. 2(a). From Fig. 2(b) we see that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall.

Fig. 3(a) shows the velocity profiles for different values of suction parameter $f_w$ for a cooling plate. It can be seen that for cooling of the plate the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. For a fixed suction velocity $f_w$, velocity is found to increase and reaches a maximum value in a region close to the plate, then gradually decreases to zero.Fig. 3(b) indicates the temperature profiles showing the effect of suction parameter $f_w$. It can be seen that temperature decreases with the increase of suction parameter. Decelerated fluid particles close to the heated wall absorb more heat from the plate and as a consequence the temperature of the fluid within the boundary layer increases. But

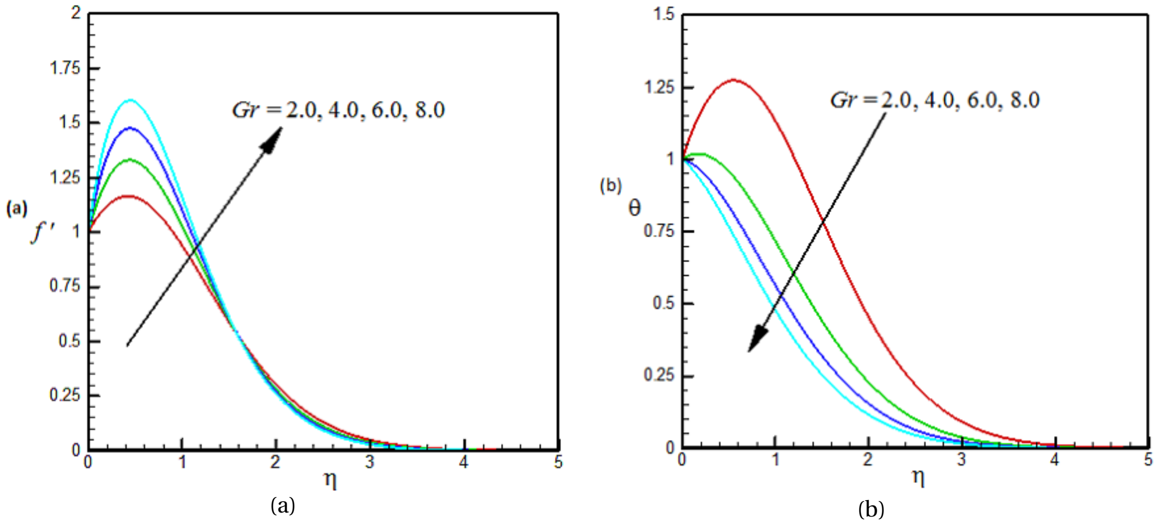


**Fig. 4.** Dimensionless (a) velocity and (b) temperature profiles for different values of $Gr$ and for Pr $= 0.70$, $f_w = 0.50$, $K = 0.5$, $M = 0.50$ and $Q = 2.0$.

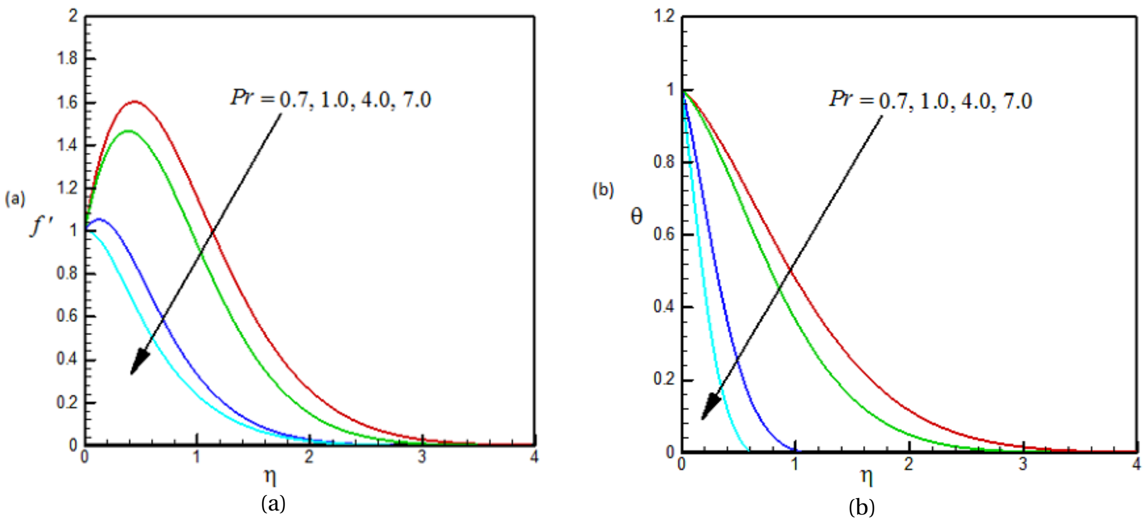


**Fig. 5.** Dimensionless (a) velocity and (b) temperature profiles for different values of Pr and for $M = 0.50$, $f_w = 0.50$, $K = 0.5$, $Gr = 8.0$ and $Q = 2.0$.

when these decelerated fluid particles are sucked through the porous plate there is a decrease to the temperature profile. Thus suction can be used for controlling the temperature function, which is required in many engineering applications like nuclear reactors, generators etc.

The influence of local Grashof number $Gr$ on the dimensionless velocity and temperature are displayed in Figs. 4(a)-4(b) respectively. Fig. 4(a) shows the free convection effects on the velocity profiles. From this figure we see that velocity increases as $Gr$ increases. This figure also shows that for large Gr $\geq 1$ velocity profiles overshoot near the surface of the plate. From Fig. 4(b) we observe that the thermal boundary layer thickness decrease with an increasing value of local Grashof number.

The effect of Prandtl number $Pr$ on the dimensionless velocity and temperature profiles are shown in Figs. 5(a) 5(b) respectively. Fig. 5(a) shows that for small Prandtl number $Pr$ values, the velocity overshoots the free stream velocity and thus there is a larger growth of the boundary layer. But for larger values of $Pr$, the velocity is found to decrease monotonically and hence there appears a thin boundary layer indicating the decrease of the free convection. From Fig. 5(b) we also see that the effects of Prandtl number on the thermal boundary layer are similar to those of velocity boundary layer.

The effects of the unsteadiness parameter $K$ on the dimensionless velocity and temperature profiles are shown in

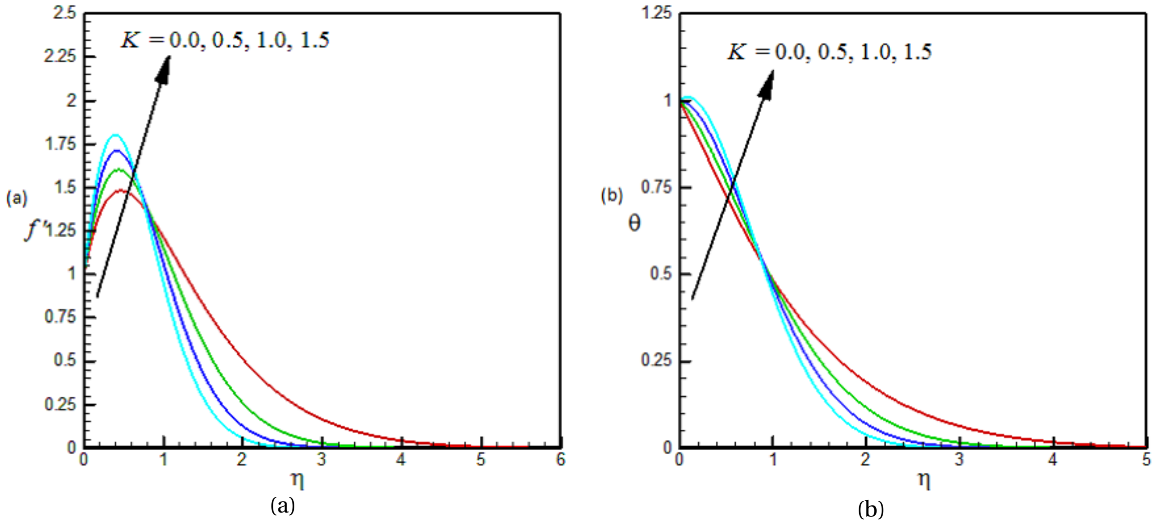


**Fig. 6.** Dimensionless (a) velocity and (b) temperature profiles for different values of $K$ and for $\Pr = 0.70$, $M = 0.50$, $f_w = 0.50$, $Gr = 8.0$ and $Q = 2.0$.

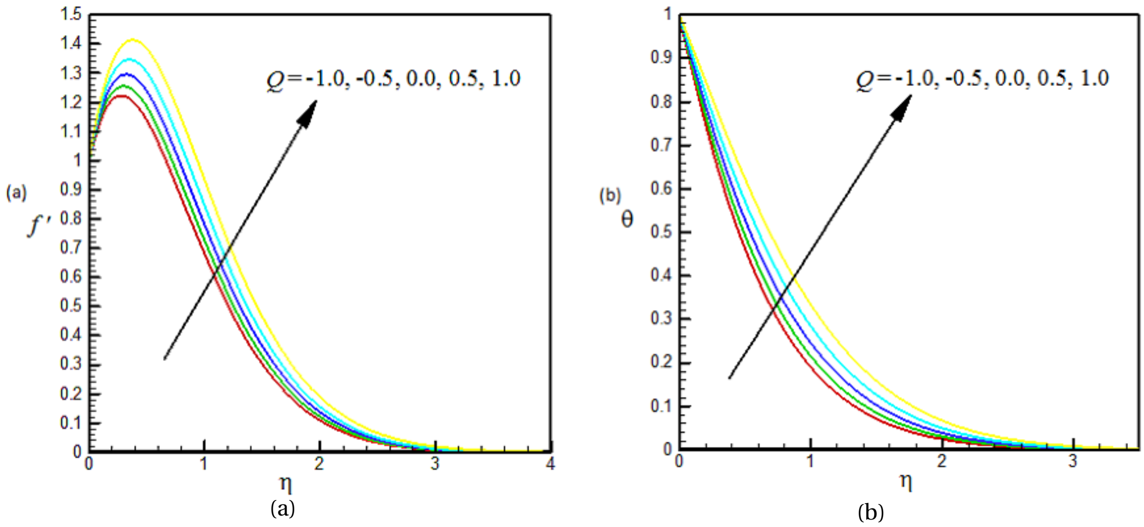


**Fig. 7.** Dimensionless (a) velocity and (b) temperature profiles for different values of $Q$ and for $\Pr = 0.70$, $M = 0.50$, $f_w = 0.50$, $Gr = 8.0$ and $K = 0.5$.

Figs. 6(a)-6(b) respectively. From these figures we observe that both the velocity and temperature are found to increase with the increase of the parameter $K$ within some domain $\eta \leq \eta_{critical}$ and then for $\eta \geq \eta_{critical}$ the tendency is reversed in the lower portion of the boundary layer. The effects of heat generation (or absorption) parameter $Q$ on the velocity profiles are shown on Fig. 7(a). It is seen from this figure that when the heat is generated ($Q > 0$) the buoyancy force increases, which induces the flow rate to increase giving, rise to the increase in the velocity profiles. Again when the heat absorption ($Q < 0$) intensifies the velocity is found to decrease due to the decrease in the buoyancy force. From Fig. 7(b), we observe that when the value of heat generation parameter ($Q > 0$) increases, the temperature distribution also increases significantly. An opposite effects is observed for $Q < 0$. Finally, the effects of the above-mentioned parameters on the local skin-friction coefficients and rate of heat transfer are shown in Table 3. From these tables we observed that Skin friction coefficients increases with the increasing values of Grashof number $Gr$, unsteadiness parameter $K$ and heat generation parameter $Q$ whereas it decreases with the increasing values of magnetic field parameter $M$, Prandlt number $Pr$, and suction parameter $f_w$. It is also found from these tables that the rate of heat transfer increases with the increasing values of a Prandlt number $Pr$, suction parameter $f_w$ and Grashof number $Gr$ whereas it decreases with an increasing values of magnetic field parameter $M$, unsteadiness parameter $K$ and heat generation parameter $Q$.

**Table 1.** Comparison of $-f''(0)$ with Andersson et al. (1992) for their Newtonian fluid case and for our Pr=0.70 and $f_w = Gr = K = Q = 0.0$ case with different $M$.

| $M$ | Andersson et al.(1992) | Present Study |
|---|---|---|
| 0.0 | 1.000 | 1.0000092 |
| 0.5 | 1.225 | 1.2247449 |
| 1.0 | 1.414 | 1.4142136 |
| 1.5 | 1.581 | 1.5811388 |
| 2.0 | 1.732 | 1.7320508 |

**Table 2.** Comparison of $f''(0)$ and $-\theta'(0)$ with Alam et al. (2013) for $f_w = 0.5$, $M = 0.50$, Pr=0.70, $Gr = 8.0$ and $Q = 0.0$ case with different $K$.

| Case | Alam et al.(2013) | | Present study | |
|---|---|---|---|---|
| $K$ | $f''(0)$ | $-\theta'(0)$ | $f''(0)$ | $-\theta'(0)$ |
| 0.0 | 1.5057596 | 1.2048581 | 1.5056151 | 1.2048786 |
| 0.2 | 1.7299524 | 1.1283679 | 1.7295829 | 1.1284248 |
| 0.5 | 2.0661789 | 1.0121884 | 2.0650883 | 1.0117619 |
| 0.8 | 2.4070362 | 0.8937310 | 2.4064813 | 0.8939260 |
| 1.0 | 2.6330982 | 0.8140027 | 2.6327829 | 0.8141178 |
| 1.5 | 3.1921880 | 0.6126888 | 3.1910554 | 0.6134376 |
| 2.0 | 3.7519991 | 0.4069908 | 3.7517397 | 0.4072427 |

## 6. Concluding Remarks

In this paper, we have presented a similarity transformation in order to obtain a local similarity solutions for the unsteady hydromagnetic boundary layer flow of a viscous incompressible fluid along a vertical porous flat plate with the internal heat generation/absorption. With the help of this similarity transformation, the governing boundary-layer equations are reduced to ordinary differential equations, which are then solved numerically by applying Nachtsheim-Swigert shooting iteration method. Comparison with previously published work is performed and excellent agreement among the results is obtained. From this study the major findings are listed below:

1. A similarity transformation has been introduced to reduce the unsteady hydromagnetic boundary-layer equations into the ordinary differential equations.This similarity approach has the advantage that one can obtain both the steady and unsteady solutions.

2. Velocity profiles increases with the increasing values of the Grashof number, unsteadiness parameter and heat generation parameter whereas it decreases with an increasing values of the magnetic field parameter, suction parameter and Prandlt number.

3. Temperature profiles increases with the increasing values of magnetic field parameter, unsteadiness parameter and heat generation parameter whereas it decreases with an increasing value of suction parameter, Prandlt number and Grashof number.

4. Local skin friction coefficients increases with the increasing values of Grashof number, unsteadiness parameter and heat generation parameter whereas it decreases with the increasing values of magnetic field parameter, Prandlt number, and suction parameter.

5. The rate of heat transfer increases with the increasing values of Prandlt number, suction parameter and Grashof number whereas it decreases with an increasing values of magnetic field parameter, unsteadiness parameter and heat generation parameter.

**Table 3.** Effects of different parameters on Local Skin-friction coefficient and Nusselt number

| Parameter | Different values of Parameter | Fixed values | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|
| | 0.2 | $f_w = 0.5$ | 3.2436260 | 0.3054834 |
| | 0.4 | $Gr = 8.0$ | 3.1282578 | 0.2732270 |
| $M$ | 0.6 | Pr=0.70 | 3.0166369 | 0.2408591 |
| | 0.8 | $K = 0.50$ | 2.9085666 | 0.2083827 |
| | 1.0 | $Q = 2.0$ | 2.8038611 | 0.1758009 |
| | 0.7 | $f_w = 0.5$ | 3.0719915 | 0.2570568 |
| | 1.0 | $Gr = 8.0$ | 2.6869865 | 0.3090036 |
| $Pr$ | 2.0 | $M = 0.50$ | 1.8666643 | 0.5239468 |
| | 4.0 | $K = 0.50$ | 0.8827019 | 1.2615165 |
| | 7.0 | $Q = 2.0$ | 0.1094686 | 2.7878525 |
| | 0.0 | $Pr = 0.70$ | 3.2239427 | 0.0838158 |
| | 0.5 | $Gr = 8.0$ | 3.0719915 | 0.2570568 |
| $f_w$ | 1.0 | $M = 0.50$ | 2.7442981 | 0.4738836 |
| | 1.5 | $K = 0.50$ | 2.2852821 | 0.7257848 |
| | 2.0 | $Q = 2.0$ | 1.6725407 | 1.0181015 |
| | 2.0 | $f_w = 0.5$ | 0.8016919 | -1.0054873 |
| | 4.0 | $Pr = 0.70$ | 1.6177248 | -0.2335854 |
| $Gr$ | 6.0 | $M = 0.50$ | 2.3707473 | 0.0734859 |
| | 8.0 | $K = 0.50$ | 3.0719915 | 0.2570568 |
| | 10.0 | $Q = 2.0$ | 3.7353599 | 0.3860959 |
| | 0.0 | $f_w = 0.5$ | 2.4334249 | 0.4990743 |
| | 0.5 | $Gr = 8.0$ | 3.0719915 | 0.2570568 |
| $K$ | 1.0 | $M = 0.50$ | 3.7111898 | 0.0089410 |
| | 1.5 | $Pr = 0.70$ | 4.3233323 | -0.2377329 |
| | 2.0 | $Q = 2.0$ | 4.9661731 | -0.4963626 |
| | -2.0 | $f_w = 0.5$ | 1.5558457 | 1.6322088 |
| | -1.0 | $Gr = 8.0$ | 1.7751381 | 1.3901224 |
| $Q$ | 0.0 | $M = 0.50$ | 2.0650883 | 1.1037401 |
| | 1.0 | $K = 0.50$ | 2.4784091 | 0.7441964 |
| | 2.0 | $Pr = 0.70$ | 3.0719915 | 0.2570568 |


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