Int. J. Adv. Appl. Math. and Mech. 2(3) (2015) 87 - 94 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

New analytical solution of MHD fluid flow of fourth grade fluid through the channel with slip condition via collocation method

Research Article

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Received 11 February 2015; accepted (in revised version) 20 March 2015

Abstract: In this study, an analysis has been performed to study the problem of fully developed flow of a fourth grade non-Newtonian fluid between two stationary parallel plates in the presence of an externally applied uniform vertical magnetic field. Slip conditions is taken into account at the wall of the channel. Employing the similarity variables, the governing partial differential equations including continuity and momentum have been reduced to ordinary ones and Collocation Method (CM) has been used to solve the non-linear ordinary differential equation with robin mixed boundary conditions. The influence of the some physical parameters such as the Slip parameter, Non newtonian parameter and magnetic field parameter on non-dimensional velocity profiles is considered. Also the results reveal that the CM can be used for solving nonlinear differential equations with Robin mixed condition easily.

MSC: Code 1 • Code 2

Keywords: Fourth grade fluid • Fully developed • Collocation Method

1. Introduction

In recent years, the study of non-Newtonian fluids has gained great importance, and this is mainly due to their huge range of industrial and technological applications [1]. The classical NavierâĂŞStokes equations have been proved insufficient to describe and illustrate the characteristics of complex rheological fluids such as shampoo, blood, synovial, food stuffs, paints, micro fluidics and polymer solutions. These kinds of fluids are generally known as non-Newtonian fluids. Many empirical and semi-empirical non-Newtonian models or constitutive equations have been proposed [2–4]. Among these, the fluids of differential type [5, 6] have received considerable attention.
The flow of non-Newtonian fluids between parallel plates is also an issue of considerable practical interest. Because of their simplicity and originality, parallel plates are often used to simulate the actual flow domain conditions in some materials processing applications such as continuous casting, plastic forming and extrusion. A number of investigations have been carried out to analyze the flow of different classes of materials in ducts and channels using various constitutive equations such as inelastic and linear/nonlinear viscoelastic models [7–12].
Since it is not easy to get an exact analytical solution of a nonlinear problem, we may go for semi analytic solutions [13–16].
There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation (CM), Galerkin (GM) and Least Square (LSM) are examples of the WRMs. Actually, CM are one of the most effective and convenient solutions for both linear and nonlinear equations and does not require linearization or small perturbation. Motivated by these facts, we used CM to obtain the solutions of the fully developed steady flow of a fourth grade fluid between two stationary parallel plates with slip conditions at walls. This paper deals with the ability of these methods for solving nonlinear equation with Robin mixed condition. What is more, the corresponding results are compared and verified with that found by numerical analysis.

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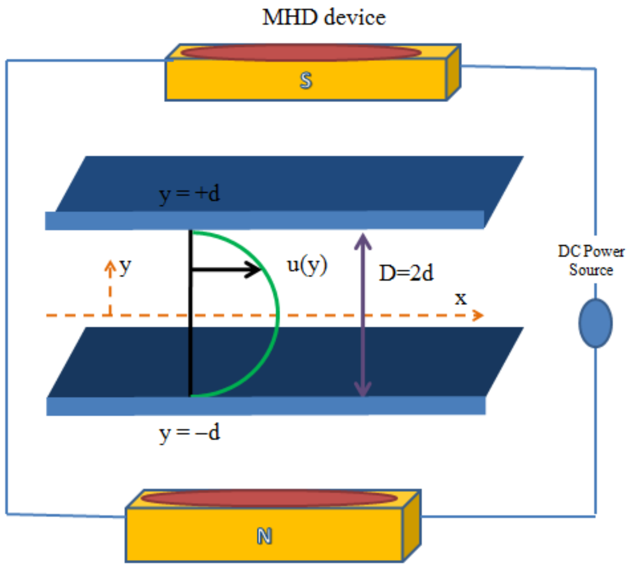

Fig. 1. Schematic diagram of the physical system

2. Formulation of the problem

Consider the steady flow of a fourth grade fluid between two infinite parallel plates distant 2d apart which is imposed a magnetic field. Both plates are stationary and the fluid motion is driven by the constant pressure gradient. We assumed that the flow is fully developed. We used (x, y) coordinate system, where x is in the direction of motion of the fluid between the plates and the y- axis is perpendicular to the plates [See Fig. 1.]
The basic equations governing the motion of an incompressible fluid is: Sample theorem with citation:

$$div\ V = 0 \tag{1}$$

$$\rho \frac{DV}{Dt} = -\nabla p + div\tau \tag{2}$$

where V is the velocity vector, ρ the constant density, ∇ the Nabla operator, p the pressure, τ the stress tensor, and D/Dt denotes the material derivative. As discussed in [1, 2], the stress tensor τ defining a fourth-grade fluid is given by

$$\tau = \sum_{i=1}^{4} S_i, \tag{3}$$

where

$$\begin{aligned} S_0 &= -pI, \quad S_1 = \mu A_1, \quad S_2 = \alpha_1 A_2 + \alpha_2 A_1^2, \\ S_3 &= \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr(A_1)) A_1, \\ S_4 &= \gamma_1 A_4 + \gamma_2 (A_3 A_1 + A_1 A_3) + \gamma_3 A_2^2 + \gamma_4 (A_2 A_1^2 + A_1^2 A_2) + \gamma_5 (trA_2) A_2 \\ &\quad + \gamma_6 (trA_2) A_1^2 + (\gamma_7 (trA_3) + \gamma_8 (A_2 A_1)) A_1, \end{aligned} \tag{4}$$

where I is the identity tensor, $\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7$ and γ_8 are material constants. The Rivlin-Ericksen tensors [1, 12] are defined by

$$\begin{aligned} A_0 &= I \\ A_n &= \frac{DA_{n-1}}{Dt} + A_{n-1}(\nabla V) + (\nabla V)^t A_{n-1}, \qquad n \geq 1, \end{aligned} \tag{5}$$

in which t is the transpose symbol.
For the present model we take the velocity field of the form:

$$V = (u(y), 0, 0) \tag{6}$$

For the velocity field defined in Eq. (6), the equation of continuity (1) is identically satisfied and Eqs. (2)-(6) in component form can be written as:
x-component

$$\mu\left(\frac{d^2}{dy^2}u(y)\right) + 6(\beta_2+\beta_3)\left(\frac{d}{dy}u(y)\right)^2\left(\frac{d^2}{dy^2}u(y)\right) - \sigma\mu^2 u(y) B_y^2 = \left(\frac{\partial}{\partial x}p(x,y)\right) \tag{7}$$

y-component:

$$2\alpha_2\left(\frac{d}{dy}u(y)\right)\left(\frac{d^2}{dy^2}u(y)\right)+8\gamma_6\left(\frac{d}{dy}u(y)\right)^3\left(\frac{d^2}{dy^2}u(y)\right)=\left(\frac{\partial}{\partial y}p(x,y)\right) \tag{8}$$

Eq. (7) has an interesting feature The left side is only a function of y only (because the flow is fully developed); the right side is at most a function of x only (this follows from differentiating both sides of the Eq. (8) with respect to x and integrating with respect to y). Hence, the only way the equation can be valid for all x and y is for each side to in fact be constant:

$$\mu\left(\frac{d^2}{dy^2}u(y)\right)+6(\beta_2+\beta_3)\left(\frac{d}{dy}u(y)\right)^2\left(\frac{d^2}{dy^2}u(y)\right)-\sigma\mu^2u(y)B_y^2=\frac{dp^*}{dx}=A \tag{9}$$

where $\beta=\beta_2+\beta_3$ and A is a constant. Therefore, the problem reduces to solve the second-order nonlinear ordinary differential Eq. (9) subject to the following condition due to symmetry of the flow and similar slip condition at either of the two plates:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}=0,\qquad \left.\frac{\partial u}{\partial y}\right|_{y=d}=-\lambda u(d) \tag{10}$$

By introducing the following non-dimensional parameters

$$\eta=\frac{y}{d},\quad U(\eta)=\frac{\mu u(y)}{Ad^2},\quad N_f=\frac{A^2d^2\beta}{\mu^3},\quad Ha=B_yd\sqrt{\frac{\sigma}{\mu}} \tag{11}$$

By substituting these functions into Eqs. (9) and (10) and then rewritten these equation we finally obtain the following system of nonlinear equations

$$\frac{d^2U}{d\eta^2}+6N_f\left(\frac{dU}{d\eta}\right)^2\frac{d^2U}{d\eta}-Ha^2U-1=0, \tag{12}$$

Subjected to the following dimensionless forms of boundary condition of Eq.(10) are changed as:

$$\begin{aligned}\left.\frac{dU}{d\eta}\right|_{\eta=0}&=0\\ \left.\frac{dU}{d\eta}\right|_{\eta=1}&=-\lambda U(1)\end{aligned} \tag{13}$$

3. Principles of collocation method

Suppose we have a differential operator D acting on a function u to produce a function p :

$$D(u(x))=p(x) \tag{14}$$

We wish to approximate u by a function $\tilde{u}$, which is a linear combination of basis functions chosen from a linearly independent set. That is:

$$u\cong\tilde{x}=\sum_{i=1}^{n}C_i\phi_i \tag{15}$$

Now, when substituted into the differential operator, D , the result of the operations is not, in general, $p(x)$. Hence an error or residual will exist:

$$E(x)=R(x)=D(\tilde{u}(x))-p(x)\neq 0 \tag{16}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)W_i(x)=0\qquad i=1,2,\ldots,n \tag{17}$$

Where the number of weight functions W_i are exactly equal the number of unknown constants C_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants C_i . For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $W_i(x)=\delta(x-x_i)$. The Dirac δ function has the property that:

$$\delta(x-x_i)=\begin{cases}1, & \text{if } x=x_i\\ 0, & \text{otherwise}\end{cases} \tag{18}$$

And residual function in Eq. (**??**) must force to be zero at specific points.

Table 1. Determined values of unknown constants C_i at various Nf and Ha for λ

Method	Constant	$Nf=0.1$	$Nf=1$
		$Ha=0.1$ $\lambda=0.1$	$Ha=2$ $\lambda=0.9$
CM	C_0	9.15886202100	0.0976611706200
	C_0	9.15886202100	0.0976611706200
	C_1	−0.02124813622	0.0002592917211
	C_2	−1.66559390900	0.0592753897500
	C_3	0.631027785400	0.0114375519000

3.1. Application of collocation method (CM)

Consider the trial function as:

$$
\begin{aligned}
u(\eta)=&\frac{1}{2}c_0\lambda\eta^2+\frac{1}{3}c_1\lambda\eta^3+\frac{1}{4}c_2\lambda\eta^4+\frac{1}{5}c_3\lambda\eta^5+c_0\left(-\frac{1}{2}\lambda-1\right)+c_1\left(-\frac{1}{3}\lambda-1\right)\\
&+c_2\left(-\frac{1}{4}\lambda-1\right)+c_3\left(-\frac{1}{5}\lambda-1\right)
\end{aligned}
\tag{19}
$$

Witch satisfies the boundary condition in Eq. (13) and set it into Eq. (12), residual function, $R(c_0, c_1, c_2, c_3, \eta)$, is found as:

$$
\begin{aligned}
R(\eta)=&1+24N_f c_0^2\lambda^3+\eta^3 c_1+30N_f c_0^2\lambda^3\eta^4 c_2+36N_f c_0^2\lambda^3\eta^5 c_3+30N_f c_0^2\lambda^3\eta^4 c_1^2\\
&+42N_f c_0^2\lambda^3\eta^6 c_2^3+54N_f c_0^2\lambda^3\eta^8 c_3^2 42N_f c_1^2\lambda^3\eta^6 c_2+48N_f c_1^2\lambda^3\eta^7 c_3+48N_f c_1\lambda^3\eta^7 c_2^2\\
&+60N_f c_1\lambda^3\eta^9 c_3+60N_f c_2^2\lambda^3\eta^9 c_3 36N_f c_2\lambda^3\eta^{10} c_3^2+12N_f c_1^2\lambda^3\eta^5+18N_f c_2^3\lambda^3\eta^8\\
&+6N_f c_0^3\lambda^3\eta^2+24N_f c_3^3\lambda^3\eta^{11}-\frac{1}{2}Ha^2 c_0\lambda\eta^2-\frac{1}{3}Ha^2 c_1\lambda\eta^3+84N_f c_0\lambda^3\eta^6 c_1 c_3\\
&-\frac{1}{5}Ha^2 c_3\lambda\eta^5 Ha^2 c_0+Ha^2 c_1+Ha^2 c_2+Ha^2 c_3+96N_f c_0\lambda^3\eta^7 c_2 c_3-\frac{1}{4}Ha^2 c_2\lambda\eta^4\\
&+108N_f c_1\lambda^3\eta^8 c_2 c_3 72N_f c_0\lambda^3\eta^5 c_1 c_2+c_0\lambda+\frac{1}{3}H2^2 c_0\lambda+\frac{1}{3}Ha^2 c_1\lambda+\frac{1}{4}Ha^2 c_2\lambda\\
&+2c_1\lambda\eta+4c_3\lambda\eta^3+\frac{1}{5}Ha^2 c_3\lambda+3c_2\lambda\eta^2=0
\end{aligned}
\tag{20}
$$

On the other hands, the residual function must be close to zero. For reaching this importance, for specific points in the domain $\eta\in[0,1]$ should be chosen. These points are selected as:

$$
R\left(\frac{1}{5}\right)=0,\quad R\left(\frac{2}{5}\right)=0,\quad R\left(\frac{3}{5}\right)=0,\quad R\left(\frac{4}{5}\right)=0
\tag{21}
$$

For example the first equation is written as:

$$
\begin{aligned}
a_1=&-1+\frac{12}{25}Ha^2 c_0\lambda+\frac{18}{390625}N_f c_2^3\lambda^3+\frac{3124}{15625}Ha^2 c_3\lambda+\frac{12}{3125}N_f c_1^3\lambda^3+\frac{124}{375}Ha^2 c_1\lambda\\
&+\frac{24}{125}N_f c_0^2\lambda^3 c_1++\frac{6}{125}N_f c_0^2\lambda^3 c_2+\frac{36}{3125}N_f c_0^2\lambda^3 c_3++\frac{6}{125}N_f c_0\lambda^3 c_1^2++\frac{42}{15625}N_f c_0\lambda^3 c_2^2\\
&+\frac{54}{390625}N_f c_0\lambda^3 c_3^2+\frac{42}{15625}N_f c_1^2\lambda^3 c_2+\frac{48}{78125}N_f c_1^2\lambda^3 c_3+\frac{48}{78125}N_f c_1\lambda^3 c_2^2+Ha^2 c_0\\
&+\frac{156}{625}Ha^2 c_2\lambda+\frac{12}{390625}N_f c_1\lambda^3 c_3^2+\frac{12}{390625}N_f c_2^2\lambda^3 c_3+\frac{66}{9765625}N_f c_2\lambda^3 c_3^2+Ha^2 c_1\\
&+\frac{72}{3125}N_f c_0\lambda^3 c_1 c_2+\frac{96}{78125}N_f c_0\lambda^3 c_2 c_3+\frac{84}{15625}N_f c_0\lambda^3 c_1 c_3+\frac{108}{390625}N_f c_1\lambda^3 c_2 c_3\\
&+\frac{4}{125}c_3\lambda+\frac{2}{5}c_1\lambda+\frac{3}{25}c_2\lambda+Ha^2 c_3+Ha^2 c_2++\frac{24}{48828125}N_f c_3^3\lambda^3++\frac{6}{25}N_f c_0^3\lambda^3+c_0\lambda
\end{aligned}
\tag{22}
$$

The rest of equations are written similarity. Finally by substitutions the λ, N_f and Ha into the residual function, $R(c_0, c_1, c_2, c_3, \eta)$, a set of four equations and four unknown coefficients are obtained. After solving these equations for unknown parameters (c_0, c_1, c_2, c_3) , the velocity distribution equation will be determined that shows in Table 1.

4. Results and Discussion

In this study, we employed CM to find the velocity field for fully developed steady flow of a fourth grade fluid between two stationary parallel plates under transverse magnetic filed. The solutions are shown graphically, because they

Table 2. The results of CM and Numerical methods for $U(\eta)$ for $\lambda = 0.4, Ha = 1$ and $N_f = 1$

η	CM	NUM	Error of CM
0.00	−0.755435424	−0.756099597	0.000664173
0.05	−0.755135498	−0.755798622	0.000663124
0.10	−0.754233549	−0.754893640	0.000660091
0.15	−0.755135498	−0.753381563	0.000655243
0.20	−0.750610552	−0.751259302	0.000648750
0.25	−0.747882989	−0.748523770	0.000640781
0.30	−0.744540374	−0.745171879	0.000631505
0.35	−0.740579449	−0.741200541	0.000621093
0.40	−0.735996956	−0.736606669	0.000609712
0.45	−0.730789639	−0.731387173	0.000597534
0.50	−0.724954241	−0.725538967	0.000584726
0.55	−0.718487503	−0.719058962	0.000571460
0.60	−0.711386168	−0.711944071	0.000557903
0.65	−0.703646980	−0.704191206	0.000544226
0.70	−0.695266681	−0.695797279	0.000530598
0.75	−0.686242013	−0.686759201	0.000517188
0.80	−0.676569720	−0.677073886	0.000504166
0.85	−0.666246544	−0.666738245	0.000491701
0.90	−0.655269227	−0.655749190	0.000479963
0.95	−0.643634512	−0.644103634	0.000469122
1.00	−0.631339143	−0.631798488	0.000459346

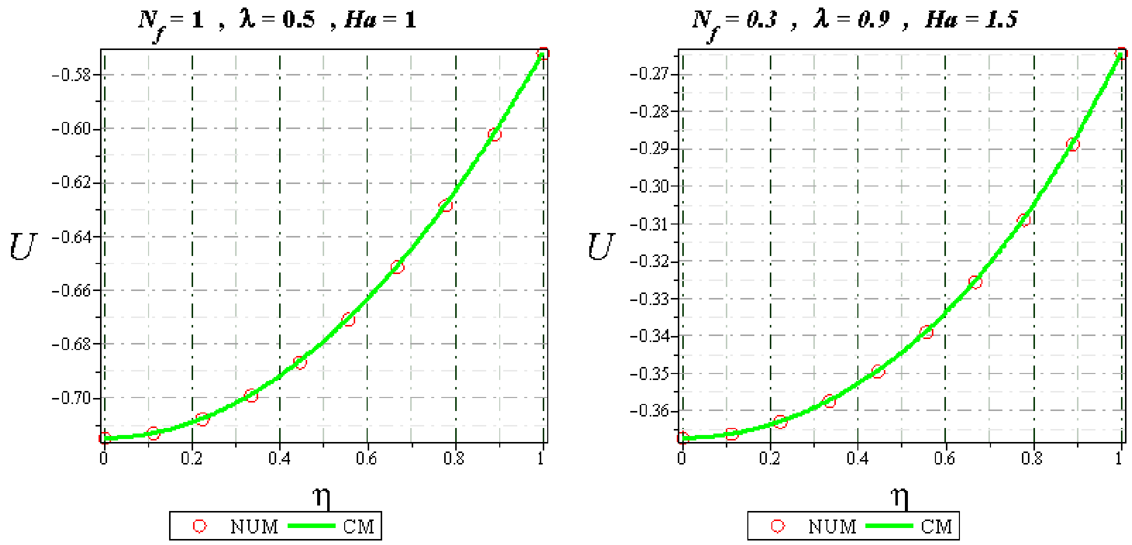

Fig. 2. Comparison of the Numeric Method with the result obtained by CM

were too long to be mentioned here. the comparison of results between the applied methods, CM and Numerical Methods, for different values of active parameters is shown in Fig. 2. The numerical solution is performed using the algebra package Maple 16.0, to solve the present case. The package uses a fourth order RungeâĂŞKutta procedure for solving nonlinear boundary value (B-V) problem [17, 18]. Validity of CM is shown in Table 2. In these tables, the Error is defined as:

$$Error = \left| U(\eta)_{NUM} - U(\eta)_{C} \right| \tag{23}$$

The results are proved to be precise and This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters influence on this fluid. AS expected, an increase in the magnetic parameter leads to decrease in the velocity components at given point as can be seen from Fig. 3. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity.

In addition, Fig. 4 shows the effect of non-Newtonian non Newtonian parameter N_f on the velocity components for $\lambda = 0.9, Ha = 1$. It is noticed that an increase in dimensionless parameters N_f tends to decrease the velocity profile $U(\eta)$. It is worth mention that, the same effect is observed for the slip parameter which is depicted by the Fig. 5.

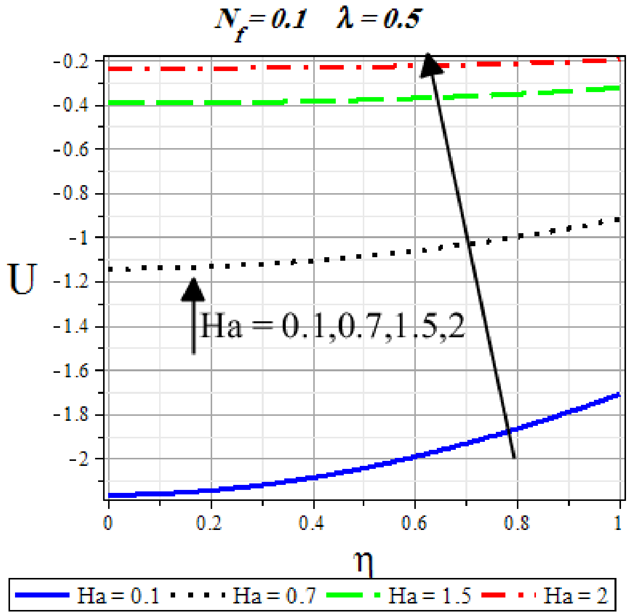

Fig. 3. Dimensionless velocities predicted by analytical solution in different Ha number when $N_f = 0.1, \lambda = 0.5$

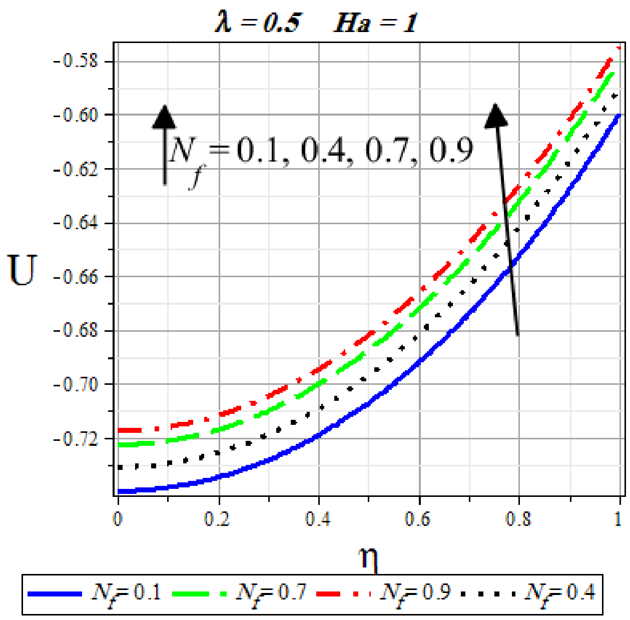

Fig. 4. Dimensionless velocities predicted by analytical solution in different N_f number when $\lambda = 0.9, Ha = 1$

5. Conclusions

In this study, a fully developed steady flow of a fourth grade fluid between two stationary parallel plates was analyzed using Collocation Method(CM). CM does not require small parameters in the equation so that the limitations of the traditional perturbation methods can be eliminated and thereby the calculations are straightforward. Effects of different physical parameters, such as,λ ,the Slip number, N_f, the Non newtonian number,Ha, the magnetic field parameter on the velocity profiles of the problem have been investigated. As an important outcome from the present study, it can be observed that the results of LSM are more accurate than CM and they are in excellent agreement with numerical ones, so LSM can be used for finding analytical solutions of non-Newtonian problems easily. Also it can be concluded that increasing the magnetic parameter leads to decrease in velocity values in whole domain. In addition, increasing in slip parameter caused a decrease in velocity components too. It was shown that proposed methods provide simple, accurate and appropriate techniques for solving nonlinear differential equations with Robin mixed.

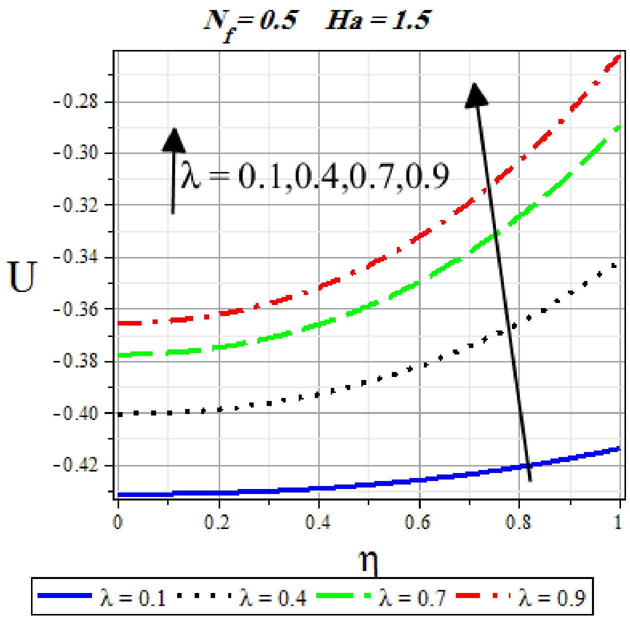

Fig. 5. Dimensionless velocities predicted by analytical solution in different λ number when $N_f = 0.5, Ha = 1.5$

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