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Differential subordination theorems for new classes of meromorphic multivalent Quasi-Convex functions and some applications

Research Article

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Abstract: In the present paper, we study new classes of meromorphic multivalent quasi-convex functions, we obtain some subordination theorems for such classes in punctured unit disk. Also we give some applications of firstâĂŞorder differential subordination.

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1. Introduction

Let $L_p(\lambda)$ denotes the class of all functions f of the form:

$$f(z)=z^{-p}+\sum_{n=0}^{\infty}a_n z^{n-\lambda} \qquad \left(0<\lambda<1, p\in N=\{1,2,\cdots\}\right), \tag{1}$$

which are analytic in the punctured unit disk $U=\{z\in C: 0<|z|<1\}$.
Also, let $T_p(\lambda)$ denotes the class of all functions f of the form:

$$f(z)=z^{-p}-\sum_{n=0}^{\infty}a_n z^{n-\lambda} \qquad \left(a_n>0, 0<\lambda<1, p\in N=\{1,2,\cdots\}\right), \tag{2}$$

which are analytic in the punctured unit disk U.
For two functions f and g analytic in $\Delta=\{z\in C:|z|<1\}$, we say f is subordinate to g in Δ, denote by $f\prec g$ or $f(z)\prec g(z)(z\in\Delta)$, if there exists a Schwarz function w analytic in U with $w(0)=0$ and $|w(z)|<1(z\in\Delta)$ such that $f(z)=g(w(z)),(z\in\Delta)$. In particular, if the function g is univalent in Δ , then $f\prec g$ if and only if $f(0)=g(0)$ and $f(\Delta)\subset g(\Delta)$.
Let $\psi: C^3\times U\to C$. and let h be univalent in Δ. Assume that k, ψ are analytic and univalent in Δ if k satisfies the differential subordination

$$\psi(k(z), zk'(z), z^2k''(z); z)\prec h(z), \tag{3}$$

then k is called a solution of the differential subordination. The univalent function q is called a dominant of the solutions of the differential subordination, or more simply dominant if $k\prec q$ for all k satisfying (3) . A dominant $\check{q}$

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that satisfies $\check{q} \prec q$ for all dominants q of (3) is said to be the best dominant of (3).
Let L_p be the class of all functions Φ of the form:

$$\Phi(z) = z^{-p} + \sum_{n=0}^{\infty} a_n z^n \qquad \left(p \in N = \{1, 2, \cdots\}\right),$$

which are analytic in the punctured unit disk U.
Also, let T_p be the class of all functions Φ of the form:

$$\Phi(z) = z^{-p} - \sum_{n=0}^{\infty} a_n z^n \qquad \left(a_n > 0, p \in N = \{1, 2, \cdots\}\right),$$

which are analytic in the punctured unit disk U.
A function $f \in L_p(\lambda)\left(T_p(\lambda)\right)$ is meromorphic multivalent starlike if $f(z) \neq 0$ and

$$-Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0, z \in U.$$

Similar,$f \in L_p(\lambda)\left(T_p(\lambda)\right)$ is meromorphic multivalent convex if $f'(z) \neq 0$ and

$$-Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0, z \in U.$$

A function $f \in L_p(\lambda)\left(T_p(\lambda)\right)$ is called meromorphic multivalent Quasi-convex function if there exists a meromorphic multivalent convex function g such that $g(z) \neq 0$ and

$$-Re\left\{\frac{\left(zf'(z)\right)'}{g'(z)}\right\} > 0, z \in U.$$

A function $\Phi \in L_p\left(T_p\right)$ is meromorphic multivalent starlike if $\Phi(z) \neq 0$ and

$$-Re\left\{\frac{z\Phi'(z)}{\Phi(z)}\right\} > 0, z \in U.$$

Similar,a function Φ is meromorphic multivalent convex if $\Phi'(z) \neq 0$ and

$$-Re\left\{1 + \frac{z\Phi''(z)}{\Phi'(z)}\right\} > 0, z \in U.$$

Moreover, a function Φ is called meromorphic multivalent Quasi-convex function if there exists a meromorphic multivalent convex function Ψ such that $\Psi'(z) \neq 0$ and

$$-Re\left\{\frac{(z\Phi'(z))'}{\Psi'(z)}\right\} > 0, z \in U.$$

2. Preliminaries

Definition 2.1 (Srivastava and Owa [11]).
The fractional derivative of order $\lambda, (0 < \lambda < 1)$ of a function f is defined by

$$D_z^{\lambda} f(z) = \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z \frac{f(\varepsilon)}{(z-\varepsilon)^{\lambda}} d\varepsilon, \tag{4}$$

where f is an analytic function in a simply-connected region of the z-plane containing the origin, and the multiplicity of $(z-\varepsilon)^{-\lambda}$ is removed by requiring $\log(z-\varepsilon)$ to be real, when $(z-\varepsilon) > 0$.

Let $a, b, c \in C$ with $c \neq 0, -1, -2, \cdots$. The Gaussian hypergeometric function ${}_2F_1$ (see [12]) is defined by

$${}_2F_1(z) = {}_2F_1(a, b, c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \qquad (z \in \Delta),$$

where $(x)_n$ is the Pochhammer symbol defined, in terms of the Gamma function, by

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1 & (n = 0), \\ x(x+1)\cdots(x+n-1) & (n \in N). \end{cases}$$

Definition 2.2 (Goyal and Goyal [4]).
Let $0 \le \lambda < 1$ and $\mu, \nu \in R$. Then, in terms of familiar (Gauss) hypergeometric function ${}_2F_1$, the generalized fractional derivative operator $J_{0,z}^{\lambda,\mu,\nu}$ of a function f is defined by:

$$J_{0,z}^{\lambda,\mu,\nu} f(z) = \begin{cases} \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz}\left\{z^{\lambda-\mu}\int_0^z (z-\varepsilon)^{-\lambda} f(\varepsilon).{}_2F_1(\mu-\lambda,-\nu;1-\lambda;1-\frac{\varepsilon}{z})d\varepsilon\right\}, (0\le\lambda<1) \\ \frac{d^n}{dz^n} J_{0,z}^{\lambda-n,\mu,\nu} f(z) \qquad (n\le\lambda<n+1, n\in N). \end{cases} \tag{5}$$

where the function f is analytic in a simply-connected region of the z-plane containing the origin, with the order

$$f(z) = O(|z|^{\epsilon}), \quad (z\to 0)$$

for $\epsilon > max\{0,\mu-\nu\}-1$, and the multiplicity of $(z-\epsilon)^{-\lambda}$ is removed by requiring $log(z-\epsilon)>0$ to be real, when $(z-\epsilon)>0$.
By comparing (4) with (5), we find

$$J_{0,z}^{\lambda,\lambda,\nu} f(z) = D_z^{\lambda} f(z), (0\le\lambda<1).$$

In terms of gamma function, we have

$$J_{0,z}^{\lambda,\mu,\nu} z^n = \frac{\Gamma(n+1)\Gamma(n-\mu+\nu+1)}{\Gamma(n-\mu+1)\Gamma(n-\lambda+\nu+1)} z^{n-\mu}, (0\le\lambda<1, \mu,\nu\in R, n> max\{0,\mu-\nu\}-1). \tag{6}$$

Lemma 2.1 (Miller and Mocanu [8]).
Let q be univalent in the unit disk $\triangle$ and θ and ϕ be analytic in a domain D containing $q(\triangle)$ with $\phi(w)\neq 0$ when $w\in q(\triangle)$. Set $Q(z) = zq'(z)\phi(q(z))$ and $h(z)=\theta(q(z))+Q(z)$. Suppose that
1- $Q(z)$ is starlike univalent in $\triangle$, and
2- $Re\left\{\frac{zh'(z)}{Q(z)}\right\}>0$ for $z\in\triangle$.
If $\theta(k(z))+zk'(z)\phi(k(z)) \prec \theta(q(z))+zq'(z)\phi(q(z))$, then $k\prec q$ and q is the best dominant.

Lemma 2.2 (Shanmugam and et al. [9]).
Let q be convex univalent in the unit disk $\triangle$ and ψ and $\gamma\in C$ with $Re\left\{1+\frac{zq''(z)}{q'(z)}+\frac{\psi}{\gamma}\right\}>0$. If k is analytic in $\triangle$ and $\psi k(z)+\gamma zk'(z)\prec\psi q(z)+\gamma zq'(z)$, then $k(z)\prec q(z)$ and q is the best dominant.

Such type of study was carried out by various authors for another classes, like, Ibrahim and Darus [5–7] , Darus and Ibrahim [3], Singh et al. [10], Billing [2] and Atshan and Wanas [1].

3. Subordination results

In this section, we obtain some sufficient conditions for subordination of analytic functions in the classes $L_p(\lambda)$ and $T_p(\lambda)$.

Theorem 3.1.
Let the function q be univalent in U, $q(z)\neq 0$ and assume that

$$Re\left\{1+pr+\frac{s(1-t)}{t}(q(z))^{s-r}+(r-1)\frac{zq'(z)}{q(z)}+\frac{zq''(z)}{q'(z)}\right\}>0, \tag{7}$$

where $r,s\in C$, $t\in C\setminus\{0\}$. Suppose that $z(q(z))^{r-1}q'(z)$ is starlike univalent in U. If $f\in L_p(\lambda)$ satisfies the subordination

$$(1-t)\left(-\frac{(z^pf'(z))'}{g'(z)}\right)^{\alpha s}+t\left(-\frac{(z^pf'(z))'}{g'(z)}\right)^{\alpha r}\left[p+\alpha\left(\frac{z(z^pf'(z))''}{(z^pf'(z))'}-\frac{zg''(z)}{g'(z)}\right)\right]$$

$$\prec(1-t)(q(z))^s+t(q(z))^r\left(p+\frac{zq'(z)}{q(z)}\right), \tag{8}$$

then

$$\left(-\frac{(z^pf'(z))'}{g'(z)}\right)^{\alpha}\prec q(z), (z\in U, \alpha\in C\setminus\{0\})$$

and q is the best dominant.

Proof. Define the function k by

$$k(z)=\left(-\frac{\left(z^{p}f'(z)\right)'}{g'(z)}\right)^{\alpha},\quad z\in U. \tag{9}$$

Note that

$$(1-t)(k(z))^{s}+t(k(z))^{r}\left(p+\frac{zk'(z)}{k(z)}\right)=(1-t)\left(-\frac{\left(z^{p}f'(z)\right)'}{g'(z)}\right)^{\alpha s}+t\left(-\frac{\left(z^{p}f'(z)\right)'}{g'(z)}\right)^{\alpha r}\times$$

$$\times\left[p+\alpha\left(\frac{z\left(z^{p}f'(z)\right)''}{\left(z^{p}f'(z)\right)'}-\frac{zg''(z)}{g'(z)}\right)\right]. \tag{10}$$

From (8) and (10), we have

$$(1-t)(k(z))^{s}+t(k(z))^{r}\left(p+\frac{zk'(z)}{k(z)}\right)\prec(1-t)\left(q(z)\right)^{s}+t\left(q(z)\right)^{r}\left(p+\frac{zq'(z)}{q(z)}\right). \tag{11}$$

By setting

$$\theta(w)=(1-t)w^{s}+tpw^{r}\ \ and\ \ \phi(w)=tw^{r-1},w\neq 0,$$

we see that $\theta(w)$ is analytic in C, $\phi(w)$ is analytic in $C\setminus\{0\}$ and that $\phi(w)\neq 0, w\in C\setminus\{0\}$.
Also, we get

$$Q(z)=zq'(z)\phi\left(q(z)\right)=tz\left(q(z)\right)^{r-1}q'(z)$$

and

$$h(z)=\theta\left(q(z)\right)+Q(z)=(1-t)\left(q(z)\right)^{s}+t\left(q(z)\right)^{r}\left(p+\frac{zq'(z)}{q(z)}\right).$$

It is clear that $Q(z)$ is starlike univalent in U,

$$Re\left\{\frac{zh'(z)}{Q(z)}\right\}=Re\left\{1+pr+\frac{s(1-t)}{t}\left(q(z)\right)^{s-r}+(r-1)\frac{zq'(z)}{q(z)}+\frac{zq''(z)}{q'(z)}\right\}>0. \tag{12}$$

From (7) and (12), we have

$$Re\left\{\frac{zh'(z)}{Q(z)}\right\}>0.$$

Therefore, by Lemma 2.1, we get $k(z)\prec q(z)$. By using (9), we obtain the result. □

By fixing $\alpha=p=1$ in Theorem 3.1, we obtain the following corollary:

Corollary 3.1.
Let the function q be univalent in U, $q(z)\neq 0$ and assume that

$$Re\left\{1+pr+\frac{s(1-t)}{t}\left(q(z)\right)^{s-r}+(r-1)\frac{zq'(z)}{q(z)}+\frac{zq''(z)}{q'(z)}\right\}>0,$$

where $r,s\in C$, $t\in C\setminus\{0\}$. Suppose that $z\left(q(z)\right)^{r-1}q'(z)$ is starlike univalent in U. If $f\in L_{p}(\lambda)$ satisfies the subordination

$$(1-t)\left(-\frac{\left(zf'(z)\right)'}{g'(z)}\right)^{s}+t\left(-\frac{\left(zf'(z)\right)'}{g'(z)}\right)^{r}\left[1+\alpha\left(\frac{z\left(zf'(z)\right)''}{\left(zf'(z)\right)'}-\frac{zg''(z)}{g'(z)}\right)\right]$$

$$\prec(1-t)\left(q(z)\right)^{s}+t\left(q(z)\right)^{r}\left(1+\frac{zq'(z)}{q(z)}\right),$$

then

$$-\frac{\left(zf'(z)\right)'}{g'(z)}\prec q(z)$$

and q is the best dominant.

By taking $q(z) = -\left(\frac{1+Az}{1+Bz}\right)$ $(-1 \le B < A \le 1)$ in Corollary 3.1, we obtain the following corollary:

Corollary 3.2.
Let the function q be convex univalent in U, and assume that

$$Re\left\{1 + pr + \frac{s(1-t)}{t}\left(-\left(\frac{1+Az}{1+Bz}\right)\right)^{s-r} + \frac{1+r(A-B)z - ABz^2}{(1+Az)(1+Bz)}\right\} > 0,$$

where $r, s \in C$, $t \in C\setminus\{0\}$. If $f \in L_p(\lambda)$ satisfies the subordination

$$(1-t)\left(-\frac{(zf'(z))'}{g'(z)}\right)^s + t\left(-\frac{(zf'(z))'}{g'(z)}\right)^r\left[1+\alpha\left(\frac{z(zf'(z))''}{(zf'(z))'} - \frac{zg''(z)}{g'(z)}\right)\right]$$

$$\prec (1-t)\left(-\left(\frac{1+Az}{1+Bz}\right)\right)^s + t\left(-\left(\frac{1+Az}{1+Bz}\right)\right)^r\left(\frac{1+2Az+ABz^2}{(1+Az)(1+Bz)}\right),$$

then

$$-\frac{(zf'(z))'}{g'(z)} \prec -\left(\frac{1+Az}{1+Bz}\right), (-1 \le B < A \le 1)$$

and $q(z) = -\left(\frac{1+Az}{1+Bz}\right)$ is the best dominant.

Theorem 3.2.
Let the function q be convex univalent in U, $q'(z) \neq 0$ and assume that

$$Re\left\{1 + \frac{zq''(z)}{q'(z)} + \frac{1}{\gamma}\right\} > 0, \tag{13}$$

where $\gamma \in C\setminus\{0\}$.
Suppose that $\left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha$ is analytic in U. If $f \in T_p(\lambda)$ satisfies the subordination

$$\left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha + \alpha\gamma\left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha\left(\frac{z(z^p f'(z))''}{(z^p f'(z))'} - \frac{zg''(z)}{g'(z)}\right) \prec q(z) + \gamma z q'(z), \tag{14}$$

then

$$\left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha \prec q(z), (z \in U, \alpha \in C\setminus\{0\})$$

and q is the best dominant.

Proof. Define the function k by

$$k(z) = \left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha, \quad z \in U. \tag{15}$$

Note that

$$k(z) + \gamma z k'(z) = \left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha + \alpha\gamma\left(-\frac{(z^p f'(z))'}{g'(z)}\right)^\alpha\left(\frac{z(z^p f'(z))''}{(z^p f'(z))'} - \frac{zg''(z)}{g'(z)}\right). \tag{16}$$

From (14) and (16), we have

$$k(z) + \gamma z k'(z) \prec q(z) + \gamma z q'(z). \tag{17}$$

By setting $\psi = 1$ in Lemma 2.2, we get $k(z) \prec q(z)$. By using (15), we obtain the result. □

By fixing $\alpha = p = 1$ in Theorem 3.2, we obtain the following corollary:

Corollary 3.3.

Let the function q be convex univalent in U, $q'(z)\neq 0$ and assume that (3.2). Suppose that $-\frac{\left(z^p f'(z)\right)'}{g'(z)}$ is analytic in U. If $f \in T_p(\lambda)$ satisfies the subordination

$$-\frac{\left(zf'(z)\right)'}{g'(z)}+\gamma\left(-\frac{\left(zf'(z)\right)'}{g'(z)}\right)\left(\frac{z\left(zf'(z)\right)''}{\left(zf'(z)\right)'}-\frac{zg''(z)}{g'(z)}\right)\prec q(z)+\gamma z q'(z),$$

then

$$-\frac{\left(zf'(z)\right)'}{g'(z)}\prec q(z), \quad (z\in U).$$

and q is the best dominant.

By taking $q(z)=-\left(\frac{1+z}{1-z}\right)$ in Corollary 3.3, we obtain the following corollary:

Corollary 3.4.
Let the function q be convex univalent in U and assume that

$$Re\left\{\frac{z^2+1}{(1-z)(1+z)}+\frac{1}{\gamma}\right\}>0.$$

If $f \in T_p(\lambda)$ satisfies the subordination

$$-\frac{\left(zf'(z)\right)'}{g'(z)}+\gamma\left(-\frac{\left(zf'(z)\right)'}{g'(z)}\right)\left(\frac{z\left(zf'(z)\right)''}{\left(zf'(z)\right)'}-\frac{zg''(z)}{g'(z)}\right)\prec -\left(\frac{1+z}{1-z}\right)-\frac{2\gamma z}{(1-z)(1+z)},$$

then

$$-\frac{\left(zf'(z)\right)'}{g'(z)}\prec -\left(\frac{1+z}{1-z}\right), \quad (z\in U)$$

and q is the best dominant.

4. Applications of fractional derivative operator

In this section, we introduce some applications of section 3 containing fractional derivative operators. Assume that

$$\Phi(z)=\sum_{n=0}^{\infty}\sigma_n z^n.$$

By Definition 2.1, we have

$$D_z^{\lambda}\Phi(z)=\sum_{n=0}^{\infty}\frac{\Gamma(n+1)}{\Gamma(n+1-\lambda)}\sigma_n z^{n-\lambda}=\sum_{n=0}^{\infty}a_n z^{n-\lambda},$$

where

$$a_n=\frac{\Gamma(n+1)}{\Gamma(n+1-\lambda)}\sigma_n, \quad n=0,1,2,\ldots .$$

Thus $z^{-p}+D_z^{\lambda}\Phi(z)\in L_p(\lambda)$ and $z^{-p}-D_z^{\lambda}\Phi(z)\in T_p(\lambda)(\sigma_n\geq 0)$, then we have the following results:

Theorem 4.1.
Let the assumptions of Theorem 3.1 hold. Then

$$\left[-\frac{\left(z^p\left(z^{-p}+D_z^{\lambda}\Phi(z)\right)'\right)'}{\left(z^{-p}+D_z^{\lambda}\Psi(z)\right)'}\right]^{\alpha}\prec q(z), \quad z\in U$$

and q is the best dominant.

Proof. Define the function f by

$$f(z) = z^{-p} + D_z^{\lambda}\Phi(z) \ (z \in U),$$

it can easily observed that $f \in L_p(\lambda)$. Thus by using Theorem 3.1 , we obtain the result. □

Theorem 4.2.
Let the assumptions of Theorem 3.2 hold. Then

$$\left[-\frac{\left(z^p\left(z^{-p} - D_z^{\lambda}\Phi(z)\right)'\right)'}{\left(z^{-p} - D_z^{\lambda}\Psi(z)\right)'}\right]^{\alpha} \prec q(z), \ z \in U$$

and q is the best dominant.

Proof. Define the function f by

$$f(z) = z^{-p} - D_z^{\lambda}\Phi(z) \ (z \in U),$$

it can easily observed that $f \in T_p(\lambda)$. Thus by using Theorem 3.2 , we obtain the result. □

By using (6), we have

$$J_{0,z}^{\lambda,\mu,\nu}\Phi(z) = \sum_{n=0}^{\infty} \frac{\Gamma(n+1)\Gamma(n-\mu+\nu+1)}{\Gamma(n-\mu+1)\Gamma(n-\lambda+\nu+1)}\sigma_n z^{n-\mu} = \sum_{n=0}^{\infty} a_n z^{n-\mu},$$

where

$$a_n = \frac{\Gamma(n+1)\Gamma(n-\mu+\nu+1)}{\Gamma(n-\mu+1)\Gamma(n-\lambda+\nu+1)}\sigma_n, \ n = 0, 1, 2, \dots \ .$$

Let $\mu = \lambda$. Then $z^{-p} + J_{0,z}^{\lambda,\mu,\nu}\Phi(z) \in L_p(\lambda)$ and $z^{-p} - J_{0,z}^{\lambda,\mu,\nu}\Phi(z) \in T_p(\lambda)$ $(\sigma_n \geq 0)$, then we have the following results:

Theorem 4.3.
Let the assumptions of Theorem 3.1 hold. Then

$$\left[-\frac{\left(z^p\left(z^{-p} + J_{0,z}^{\lambda,\mu,\nu}\Phi(z)\right)'\right)'}{\left(z^{-p} + J_{0,z}^{\lambda,\mu,\nu}\Psi(z)\right)'}\right]^{\alpha} \prec q(z), \ z \in U$$

and q is the best dominant.

Proof. Define the function f by

$$f(z) = z^{-p} + J_{0,z}^{\lambda,\mu,\nu}\Phi(z) \ (z \in U),$$

it can easily observed that $f \in L_p(\lambda)$. Thus by using Theorem 3.1 , we obtain the result. □

Theorem 4.4.
Let the assumptions of Theorem 3.2 hold. Then

$$\left[-\frac{\left(z^p\left(z^{-p} - J_{0,z}^{\lambda,\mu,\nu}\Phi(z)\right)'\right)'}{\left(z^{-p} - J_{0,z}^{\lambda,\mu,\nu}\Psi(z)\right)'}\right]^{\alpha} \prec q(z), \ z \in U$$

and q is the best dominant.

Proof. Define the function f by

$$f(z) = z^{-p} - J_{0,z}^{\lambda,\mu,\nu}\Phi(z) \ (z \in U),$$

it can easily observed that $f \in T_p(\lambda)$. Thus by using Theorem 3.2 , we obtain the result. □

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