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Existence and uniqueness results for impulsive neutral stochastic differential systems with nonlocal conditions

Research Article

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Abstract: This paper is mainly concerned with the existence and uniqueness of mild solutions for impulsive neutral stochastic differential equations with nonlocal conditions in Hilbert spaces. The results are obtained by using fractional powers of operator in the semigroup theory and Sadovskii's fixed point theorem. In the end as an application, an example has been presented to illustrate the results obtained.

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1. Introduction

Stochastic differential equations have been considered extensively through discussion in the finite and infinite dimensional spaces. As a matter of fact, there exist broad literature on the related to the topic and it has played an important role in many ways such as option pricing, forecast of the growth of population, etc., and as an applications which cover the generalizations of stochastic differential equations arising in the fields such as electromagnetic theory, population dynamics, and heat conduction in material with memory. Random differential and integral equations play an important role in characterizing numerous social, physical, biological and engineering problems. For more details reader may refer [1–4] and reference therein.

Byszewski and Lakshmikantham [5] and Byszewski [6] introduced nonlocal initial conditions into initial value problems and argued that the corresponding models are accurately described the phenomena. The nonlocal conditions in the different fields has been discussed in Byszewski and Byszewski and Akca [7, 8], Deng [9] and [10–12] reference therein. The deterministic equations coupled with classical initial conditions has been studied extensively both when A is linear and when A is nonlonear for more details reader may refer [13–16, 25] and references therein.

Impulsive effects exists widely in the various evolutionary process which are characterized by the fact that undergo abrupt change at certain moments of time, involved in such fields as medicine, biology, economics, bioengineering, chemical technology etc. The duration of these changes is often negligible when compared to the total duration of the process. Such changes can be reasonably approximated as being instantaneous changes of the state or as impulses. Thus the theory of impulsive differential equations has seen considerable development. For more details reader may refer [17–20, 23].

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In [21], Balsubramaniam et al. studied the existence of solutions for semilinear neutral stochastic functional differential. In [22] Chang et al. examined the non-densely defined neutral impulsive differential inclusions with nonlocal conditions.
The main aim of this paper is to focus in studying the existence and uniqueness of the impulsive neutral stochastic differential systems with nonlocal conditions,

$$d\Big[x(t)+f_1(t,x(t),x(a_1(t)),\dots,x(a_m(t)))\Big]=\Big[Ax(t)+f_2(t,x(t),x(b_1(t)),\dots,x(b_m(t)))\Big]dt$$
$$+G(t,x(t),x(c_1(t)),\dots,x(c_m(t)))dw(t),\quad t\in J:=[0,b],\, t\neq t_k,\, k=1,2,\dots,m, \tag{1}$$
$$\Delta x(t_k)=x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\quad k=1,2,\dots,m, \tag{2}$$
$$x(0)=x_0+g(x), \tag{3}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators $\{T(t),\ t\geq 0\}$ on a separable Hilbert space on H with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$. Let K be the another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and norm $\|\cdot\|_K$. Suppose $\{w(t)\}_{t\geq 0}$ is a given K- valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q\geq 0$ defined on a filtered complete probability space$(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$. The fixed time t_k, $k=1,\dots,n$, satisfy $0\leq t_1\leq\dots\leq t_n\leq b$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t=t_k$, $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ represents the jump in the state x at time t_k, where $I_k,(k=1,2,\dots,m)$ are bounded which determine the size of the jump. The function f_1, f_2, G, and g are the given functions to be defined later.
The structure of this paper is as follows. In Section 2, we recall some preliminaries. In Section 3, we devoted to the study of existence and uniqueness of mild solution for system (1)-(3). Finally in Section 4, an example is presented, which illustrates the main results.

2. Preliminaries

In this section, we recall a few Stochastic results, Lemmas and notations which are need to establish our main results.

Throughout this paper $(H,\|\cdot\|)$ and $(K,\|\cdot\|)$ denotes the two real separable Hilbert space. Let $\mathscr{L}(K,H)$ be the set of all linear bounded operator from K into H equipped with the usual norm operator $\|\cdot\|$. Let $(\Omega,\mathscr{F},P,H)$ be the complete probability space furnished with a complete family of right continuous increasing σ- algebra $\{\mathscr{F}_t, t\in J\}$ satisfying $\mathscr{F}_t\subset\mathscr{F}$. An H- valued random variable is an $\mathscr{F}$- measurable function $x(t):\Omega\to H$ and a collection of random variables $S=\{x(t,\omega):\Omega\to H\setminus t\in J\}$ is called stochastic process. Usually we write $x(t)$ instead of $x(t,\omega)$ and $x(t): J\to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t):t\geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q\geq 0$, denote $\mathrm{Tr}(Q)=\sum_{i=1}^{\infty}\lambda_i=\lambda<\infty$, which satisfies that $Qe_i=\lambda_i e_i$. So, actually, $\omega(t)=\sum_{i=1}^{\infty}\sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathscr{F}_t=\sigma\{\omega(s):0\leq s\leq t\}$ is the σ-algebra generated by ω and $\mathscr{F}_t=\mathscr{F}$. Let $\Psi\in\mathscr{L}(K,H)$ and define

$$\|\Psi\|_Q^2=\mathrm{Tr}(\Psi Q\Psi^*)=\sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q<\infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathscr{L}_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi:K\to H$. The completion $\mathscr{L}_Q(K,H)$ of $\mathscr{L}(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2=\langle\Psi,\Psi\rangle$ is a Hilbert space with the above norm topology.
Let A be the infinitesimal generator of an analytic semigroup $T(t)$ in H. Suppose that $0\in\rho(A)$ where $\rho(A)$ denote the resolvent set of A and that semigroup $T(\cdot)$ is uniformly bounded that is to say, $\|T(t)\|\leq M_T$ for some constant $M_T\geq 1$ and for every $t\geq 0$. Then for $\alpha\in(0,1]$, it is possible to define the fractional power operator $((-A)^{\alpha})$ as a closed linear invertible operator on its domain $D((-A)^{\alpha})$. Furthermore, the subspace $D((-A)^{\alpha})$ is dense in H and the expression

$$\|x\|_{\alpha}=\|(-A)^{\alpha}x\|,\ x\in D((-A)^{\alpha}),$$

defines the norm on $H_{\alpha}=D((-A)^{\alpha})$. Furthermore of fractional power of operator and semigroup refer [15]. Then the following property are well known [15].

Lemma 2.1.
Suppose the following properties are satisfied.

(i) Let $0\leq\alpha\leq 1$. Then H_{α} is a Banach space.

(ii) If $0<\beta<\alpha\leq 1$, then $H_{\alpha}\subset H_{\beta}$ and the embedding is compact whenever the resolvent operator of A is compact.

(iii) For every $0<\alpha\leq 1$, there exists a positive constant $M_{\alpha}>0$ such that

$$\|(-A)^{\alpha}T(t)\| \le \frac{M_{\alpha}}{t^{\alpha}}, \ for\ all\ 0 < t \le b.$$

The collection of all strongly measurable, square-integrable H- valued random variables, denoted by $\mathscr{PC}(J, L_2(\Omega, \mathscr{F}, P; H)) = \{x : J \to L_2 : x(t)$ is continuous everywhere except for some t_k at which $x(t_k^-)$ and $x(t_k^+)$ exists and $x(t_k^-) = x(t_k),\ k = 1,2,3,\dots,m\}$ is the Banach space of piecewise continuous maps from J into $L_2(\Omega, \mathscr{F}, P; H)$ satisfying the condition $\sup_{t\in J} E\|x(t)\|^2 < \infty$. Let $\mathscr{PC}(J, L_2(\Omega, \mathscr{F}, P; H)) \equiv \mathscr{PC}(J, L_2(\Omega, H))$ be the closed subspace of $\mathscr{PC}(J, L_2(\Omega, \mathscr{F}, P; H))$ consisting of measurable, $\mathscr{F}_t$ - adapted and H-valued processes $x(t)$. Then $\mathscr{PC}(J, L_2(\Omega, H))$ is a Banach space endowed with the norm

$$\|x\|^2_{\mathscr{PC}} = \sup_{t\in J}\left\{E\|x(t)\|^2 : x \in \mathscr{PC}(J, L_2(\Omega, H))\right\}.$$

Here we discuss existence of mild solutions of the system (1)-(3) is studied with the following assumption :

(H1) There exists a constant $\beta \in (0,1)$ such that $f_j : [0,b] \times H^{m+1} \to H$ is a continuous function, and $M_{f_j}, \overline{M}_{f_j} > 0$ such that $(-A)^{\beta} f_j$ satisfies the Lipschitz condition:

$$\begin{aligned}\|(-A)^{\beta} f_j(s_1, x_0, x_1, \dots, x_m) - (-A)^{\beta} f_j(s_2, y_0, y_1, \dots, y_m)\| \\ \le M_{f_j}(|s_1 - s_2| + \max_{i=0,1,\dots,m} \|x_i - y_i\|),\end{aligned}$$

for any $0 \le s_1, s_2 \le b,\ x_i, y_i \in H,\ i = 0,1,\dots,m$. Moreover, the inequality

$$\|(-A)^{\beta} f_j(t, x_0, x_1, \dots,, x_m)\| \le \overline{M}_{f_j}(\max\{\|x_i\| : \ i = 0,1,\dots,m\} + 1), \tag{4}$$

for any $(t, x_0, x_1, \dots, x_m) \in J \times H^{m+1},\ j = 1,2$.

(H2) The function $G : [0,b] \times H^{n+1} \to L_Q(K, H)$ satisfies the following conditions

(i) For each $t \in [0.b]$, the function $G(t, \cdot) : H^{n+1} \to L_Q(K, H)$ is continuous and for each $(x_0, x_1, \dots, x_n) \times H^{n+1}$ the function $G(\cdot, x_0, x_1, \dots, x_n) : J \to L_Q(K, H)$ is $\mathscr{F}_t$- measurable;

(ii) For each positive number $l \in N$, there is a positive function $h_l \in L^2(J)$ such that

$$\sup_{\|x_0\|^2,\dots,\|x_n\|^2 \le l} E\|G(t, x_0, x_1, \dots, x_n)\|^2 \le h_l(t)$$

and

$$\lim_{l\to\infty} \inf \frac{\int_0^b h_l(s)ds}{l} = \mu < +\infty.$$

(H3) $a_k, b_k, c_k \in C(J, J),\ k = 1,2,\dots,m.\ g : \mathscr{PC} \to L_2^0(\Omega, \mathscr{PC})$ satisfies that

(i) There exist positive constants M_g and $\overline{M}_g$ such that,

$$\|g(x)\| \le M_g \|x\|_{\mathscr{PC}} + \overline{M}_g \text{ for all } x \in \mathscr{PC};$$

(ii) g is completely continuous.

(H4) $I_k : \mathscr{PC} \to \mathscr{PC}$ is completely continuous and there exist continuous nondecreasing functions $L_k : R_+ \to R_+$ such that for each $x \in \mathscr{PC}$

$$\|I_k(x)\| \le L_k(\|x\|_{\mathscr{PC}}), \quad \lim_{l\to\infty} \inf \frac{L_k(l)}{l} = \lambda_k < +\infty.$$

Our main results are based upon the following fixed point theorem [24].

Theorem 2.1 (Sadovskii's Fixed Point Theorem).
Let Φ be a condensing operator on a Banach space, that is, Φ is continuous and takes bounded sets into bounded sets, and let $\alpha(\Phi(B)) \le \alpha(B)$ for every bounded set B of H with $\alpha(B) > 0$ of $\Phi(\Omega) \subset \Omega$ for a convex, closed, and bounded set Ω of H, then Φ has fixed point in H.(where $\alpha(\cdot)$ denotes Kuratowski's measure of non-compactness).

3. Main results

In this section we state and prove our existence results, now we define the mild solutions for the system (1)-(3).

Definition 3.1.
An $\mathscr{F}_t$- adapted stochastic process $x(t): J \to H$ is said to be a mild solution of nonlocal Cauchy problem (1)-(3) if

(i) $x_0 \in L_2^0(\Omega, H), g(x) \in L_2^0(\Omega, \mathscr{PC})$;

(ii) $\Delta x|_{t=t_k} = I_k(x(t_k^-)),\ k = 1, 2, \dots, m$;

(iii) for each $t \in J$, $x(t)$ satisfies the following integral equation

$$\begin{aligned}x(t) = & T(t)\big[x_0 + g(x) + f_1(0, x(0), x(a_1(0)), \dots, x(a_m(0)))\big] - f_1(t, x(t), x(a_1(t)), \dots, x(a_m(t)))\\ &+ \int_0^t T(t-s) f_2(s, x(s), x(b_1(s)), \dots, x(b_m(s)))ds\\ &- \int_0^t AT(t-s) f_1(s, x(s), x(a_1(s)), \dots, x(a_m(s)))ds\\ &+ \int_0^t T(t-s)G(s, x(s), x(c_1(s)), \dots, x(c_m(s)))dw(s) + \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)), \quad t \in J.\end{aligned}$$

Theorem 3.1.
Assume the conditions (H1)-(H5) are satisfied and $x(0) \in L_2^0(\Omega, H)$, then then nonlocal Cauchy problem (1)-(3) has a mild solution provided that

$$L_0 = M_{f_1}^2 \left(M_0^2 (M_T^2 + 1) + \frac{(M_{1-\beta} b^{\beta})^2}{2\beta - 1} \right) < 1 \tag{5}$$

and

$$\begin{aligned}36\Big\{ & M_T^2 [6^2 \big((M_g^2 + (M_0 \overline{M} f_1)^2\big) + (2bM_0 \overline{M} f_2)^2 + Tr(Q)\mu + \sum_{k=1}^m \lambda_k] + (2M_0 \overline{M} f_1)^2\\ &+ \frac{1}{2\beta - 1}(2M_{1-\beta} \overline{M}_{f_1} b^{\beta})^2 \Big\} < 1,\end{aligned} \tag{6}$$

where $M_0 = \|(-A)^{-\beta}\|$ and $M_{1-\beta}$ is defined in Lemma 2.1.

Proof. For the sake of brevity, we rewrite that

$$(t, x(t), x(a_1(t)), \dots, x(a_m(t))) = (t, u(t)),$$

$$(t, x(t), x(b_1(t)), \dots, x(b_m(t))) = (t, v(t)),$$

$$(t, x(t), x(c_1(t)), \dots, x(c_m(t))) = (t, p(t)).$$

Consider the operator Φ on $\mathscr{PC}$ defined by

$$\begin{aligned}(\Phi x)(t) = & T(t)\big[x_0 + g(x) + f_1(0, u(0))\big] - f_1(t, u(t)) + \int_0^t T(t-s) f_2(s, v(s))ds\\ &- \int_0^t AT(t-s) f_1(s, u(s))ds + \int_0^t T(t-s)G(s, p(s))dw(s)\\ &+ \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)), \quad t \in J.\\ = & T(t)\big[x_0 + f_1(0, u(0)) + g(x)\big] - f_1(t, u(t)) - I_{f_1}^u(t) + I_{f_2}^v(t) + I_G^p(t)\\ &+ \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)), \quad t \in J.\end{aligned}$$

We shall show that the operator Φ has a fixed point, which is a solution of system (1)-(3). For each positive integer l, let

$$B_l = \{x \in \mathscr{P}\mathscr{C} : E\|x(t)\|^2_{\mathscr{P}\mathscr{C}} \le l, 0 \le t \le b\}.$$

It is clear that for each l, $B_l \subseteq \mathscr{P}\mathscr{C}$ is clearly a bounded closed convex set in $\mathscr{P}\mathscr{C}$. In addition to the familiar Young, Holder and Minkowskii the inequalities of the form $(\sum_{i=1}^n a_i)^m \le n^m \sum_{i=1}^n a_i^m$ where a_i are nonnegative constants $(i = 1,2,\ldots,n)$ and $m,n \in N$ is helpful in establishing various estimates, from Lemma 2.1 and (4) together with Holder inequality, yields the following relation:

$$\begin{aligned} E\|I^u_{f_1}(t)\|^2 &= \|\int_0^t (-A)T(t-s)f_1(s,u(s))ds\|^2 \\ &= \|\int_0^t (-A)^{1-\beta}T(t-s)(-A)^{\beta} f_1(s,u(s))ds\|^2 \\ &= 4\int_0^t \frac{M^2_{1-\beta}}{(t-s)^{2(1-\beta)}}\overline{M}^2_{f_1}(\max\{E\|x_i\|^2 : i=0,1,\ldots,m\}+1)ds \end{aligned} \tag{7}$$

and

$$\begin{aligned} E\|I^v_{f_2}(t)\|^2 &= \|\int_0^t T(t-s)f_2(s,v(s))ds\|^2 \\ &= \|\int_0^t (-A)^{-\beta}T(t-s)(-A)^{\beta} f_2(s,v(s))ds\|^2 \\ &= 4\, b^2M_0^2M_T^2\overline{M}^2_{f_2}(\max\{E\|x_i\|^2 : i=0,1,\ldots,m\}+1), \end{aligned} \tag{8}$$

it follows that $(-A)T(t-s)f_1(s,u(s))ds$ and $T(t-s)f_2(s,v(s))$ is integrable on J, so Φ is well defined on B_l. Similarly, from (H2)(ii) and together with the Ito's formula, a computation can be performed to obtain the following:

$$\begin{aligned} E\|I^p_G(t)\|^2 &= E\|\int_0^t T(t-s)G(s,p(s))dw(s)\|^2 \\ &\le Tr(Q)M_T^2\int_0^t \| G(s,p(s))\|^2_Q ds \\ &\le Tr(Q)M_T^2\int_0^t h_l(s)ds. \end{aligned} \tag{9}$$

Step 1. We claim that there exists a positive number l such that $\Phi B_l \subseteq B_l$.
If it is not true, then for each positive number l, there is a function $x_l(\cdot) \in B_l$, but $\Phi x_l(\cdot) \notin B_l$, but $\|\Phi_l(t)\|^2 > l$ for some $t(l) \in J$, where $t(l)$ denotes that t is independent of l. However, on the other hand, we have

$$\begin{aligned} l &< \|\Phi_l x(t)\|^2 \\ &= \|T(t)[x_0+g(x)+f_1(0,u(0))] - f_1(t,u(t)) + \int_0^t T(t-s)f_2(s,v(s))ds \\ &\quad - \int_0^t AT(t-s)f_1(s,u(s))ds + \int_0^t T(t-s)G(s,p(s))dw(s) \\ &\quad + \sum_{0<t_k<t} T(t-t_k)I_k(x_r(t_k^-))\|^2 \\ &\le 36\Big\{(3M_T)^2[\|x_0\|^2_{\mathscr{D}} + 4(M_g^2 l + \overline{M}^2_g) + (2M_0\overline{M}_{f_1})^2(l+1)] + (2M_0\overline{M}_{f_1})^2(l+1) \\ &\quad + (2bM_0M_T\overline{M}_{f_2})^2(l+1) + \frac{1}{2\beta-1}(2M_{1-\beta}\overline{M}_{f_1}b^{\beta})^2(l+1) \\ &\quad + Tr(Q)M_T^2\int_0^t h_l(s)ds + M_T^2\sum_{k=1}^m L_k(l)\Big\} \\ &\le M^* + 36\Big\{(6M_T)^2[(M_g^2 l + (M_0\overline{M}_{f_1})^2(l)] + (2M_0\overline{M}_{f_1})^2(l) \\ &\quad + (2bM_0M_T\overline{M}_{f_2})^2(l) + \frac{1}{2\beta-1}(2M_{1-\beta}\overline{M}_{f_1}b^{\beta})^2(l) \\ &\quad + Tr(Q)M_T^2\, l\frac{1}{l}\int_0^t h_l(s)ds + M_T^2\sum_{k=1}^m L_k(l)\Big\}, \end{aligned}$$

where

$$M^* = 36\Big\{(3M_T)^2[\|x_0\|^2 + 4M_g^2 + (2M_0\overline{M}_{f_1})^2] + (2M_0\overline{M}_{f_1})^2 + (2bM_0M_T\overline{M}_{f_2})^2 + \frac{1}{2\beta-1}(2M_{1-\beta}\overline{M}_{f_1}b^{\beta})^2\Big\}.$$

Dividing on both sides by l and taking the lower limit as $t \to +\infty$, we get

$$36\Big\{(M_T)^2[6^2(M_g^2 + (M_0\overline{M}_{f_1})^2) + (2bM_0\overline{M}_{f_2})^2 + Tr(Q)\mu + \sum_{k=1}^{m}\lambda_k] + (2M_0\overline{M}_{f_1})^2 + \frac{1}{2\beta-1}(2M_{1-\beta}\overline{M}_{f_1}b^{\beta})^2\Big\} \geq 1.$$

This is a contradicts to (6). Hence for positive integer l, $\Phi B_l \subseteq B_l$.

Step 2. Next we will show that the operator Φ has a fixed point on B_l. Now we decompose $\Phi = \Phi_1 + \Phi_2$ (is condensing), Φ_1 is contraction and Φ_2 is compact.

The operators Φ_1, Φ_2 are defined on B_l respectively by

$$(\Phi_1 x)(t) = T(t)f_1(0,u(0)) - f_1(t,u(t)) - \int_0^t AT(t-s)f_1(s,u(s))ds,$$

$$(\Phi_2 x)(t) = T(t)[x_0 + g(x)] + \int_0^t T(t-s)f_2(s,v(s))ds + \int_0^t T(t-s)G(s,p(s))dw(s) + \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-))\}, \quad t \in J.$$

We would verify that Φ_1 is a contraction while Φ_2 is a completely continuous operator.

To prove that Φ_1 is a contraction, we take $x_1, x_2 \in B_l$ arbitrarily. Then for each $t \in J$ and by condition (H1) and (5), we have

$$\begin{aligned}
&E\|(\Phi_1 x_1)(t) - (\Phi_1 x_2)(t)\|^2 \\
&\leq E\|f_1(t,u_1(t)) - f_1(t,u_2(t)) + T(t)[f_1(0,u_1(0)) - f_1(0,u_2(0))] \\
&\quad + \int_0^t (-A)T(t-s)[f_1(s,u_1(s)) - f_1(s,u_2(s))]ds\|^2 \\
&\leq 9\,\Big\{E\|(-A)^{-\beta}T(t)[(-A)^{\beta}f_1(t,u_1(t)) - (-A)^{\beta}f_1(t,u_2(t))]\|^2 \\
&\quad + E\|(-A)^{-\beta}[(-A)^{-\beta}f_1(0,u_1(0)) - (-A)^{\beta}f_1(0,u_2(0))]\|^2 \\
&\quad + E\|\int_0^t (-A)^{1-\beta}T(t-s)[(-A)^{\beta}f_1(s,u_1(s)) - (-A)^{\beta}f_1(s,u_2(s))ds\|^2\Big\} \\
&\leq 9\,\Big\{M_0^2M_T^2M_{f_1}^2 \sup_{0\leq s\leq b} E\|x_1(s) - x_2(s)\|^2 + M_0^2M_{f_1}^2 \sup_{0\leq s\leq b} E\|x_1(s) - x_2(s)\|^2 \\
&\quad + b\int_0^t \frac{M_{1-\beta}^2}{(t-s)^{2(1-\beta)}}M_{f_1}^2 E\|x_1(s) - x_2(s)\|^2\Big\}.
\end{aligned}$$

Hence

$$\begin{aligned}
\|(\Phi_1 x_1)(t) - (\Phi_1 x_2)(t)\|^2 &\leq M_{f_1}^2\Big[M_0^2(M_T^2 + 1) + \frac{(M_{1-\beta}b^{\beta})^2}{2\beta-1}\Big] \sup_{0\leq s\leq b}\|x_1(s) - x_2(s)\|^2 \\
&\leq L_0 \sup_{0\leq s\leq b}\|x_1(s) - x_2(s)\|^2.
\end{aligned}$$

Thus

$$\|(\Phi_1 x_1) - (\Phi_1 x_2)\|^2 \leq L_0\|x_1 - x_2\|^2,$$

and so by assumption $0 \leq L_0 \leq 1$, we see that Φ_1 is a contraction.

To prove that Φ_2 is compact, first we prove that Φ_2 is continuous on B_l. Let $\{x_n\}_{n=0}^{\infty} \subseteq B_l$ with $x_n \to x$ in B_l, then by (H2)(i) and (H4)

(i) I_k, $k = 1,2,\ldots,m$ is continuous.

(ii) $G(s,p_n(s)) \to G(s,p(s))$, $\quad n \to \infty$.

Since

$$E\|G(s,p_n(s))-G(s,p(s))\|\leq 2g_l(s),$$

by the dominated convergence theorem, we have

$$\begin{aligned}
E\|\Phi_2 x_n-\Phi_2 x\|^2 = \sup_{0\leq t\leq b} E\|T(t)[g(x_n)-g(x)]&+\int_0^t T(t-s)f_2(s,v_n(s))-f_2(s,v(s))]ds\\
&+\int_0^t T(t-s)G(s,p_n(s))-G(s,p(s))]dw(s)\\
&+\sum_{0<t_k<t} T(t-t_k)[I_k(x_n(t_k^-))-I_k(x(t_k^-))]\|^2\\
\leq M_T^2\|g(x_n)-g(x)\|^2&+M_T^2\int_0^b \|f_2(s,v_n(s))-f_2(s,v(s))\|^2ds\\
&+M_T^2\int_0^b \|G(s,p_n(s))-G(s,p(s))\|^2dw(s)\\
&+M_T^2\sum_{0<t_k<t} \|I_k(x_n(t_k^-))-I_k(x(t_k^-))\|^2\\
&\to 0,\quad \text{as } n\to\infty.
\end{aligned}$$

Thus, Φ_2 is continuous. Next, we prove that $\{\Phi_2 x : x\in B_l\}$ is a family of equicontinuous functions. Let $x\in B_l$ and $\tau_1,\tau_2\in J$. Then if $0<\tau_1\leq\tau_2\leq b$ and $\varphi\in N_2(x)$, then for each $t\in J$, then

$$\begin{aligned}
\|\Phi_2 x(\tau_2)-\Phi_2 x(\tau_1)\|^2 \leq \|T(\tau_2)-T(\tau_1)\|^2\|x_0+g(x)\|^2&\\
+\int_0^{\tau_1-\epsilon} \|T(\tau_2-s)-T(\tau_1-s)\|^2\|f_2(s,v(s))\|^2ds&\\
+\int_{\tau_1-\epsilon}^{\tau_1} \|T(\tau_2-s)-T(\tau_1-s)\|^2\|f_2(s,v(s))\|^2ds&\\
+\int_{\tau_1}^{\tau_2} \|T(\tau_2-s)\|^2\|f_2(s,v(s))\|^2ds&\\
+\int_0^{\tau_1-\epsilon} \|T(\tau_2-s)-T(\tau_1-s)\|^2\|G(s,p(s))\|^2dw(s)&\\
+\int_{\tau_1-\epsilon}^{\tau_1} \|T(\tau_2-s)-T(\tau_1-s)\|^2\|G(s,p(s))\|^2dw(s)&\\
+\int_{\tau_1}^{\tau_2} \|T(\tau_2-s)\|^2\|G(s,p(s))\|^2dw(s)&\\
+\sum_{0<t_k<\tau_1} \|T(\tau_2-t_k)-T(\tau_1-t_k)\|^2\|I_k(x(t_k^-))\|^2&\\
+\sum_{\tau_1\leq t_k<\tau_2} \|T(\tau_2-t_k)\|^2\|I_k(x(t_k^-))\|^2.&
\end{aligned}$$

The right-hand side is independent of $x\in B_l$ and tends to zero as $\tau_2-\tau_1\to 0$, since the compactness of $\{T(t)\}_{t\geq 0}$ implies the continuity in the uniform operator topology. Similarly, using the compactness of the set $g(B_l)$ we can prove that the functions $\Phi_2 x,\ x\in B_l$ are equicontinuous at $t=0$. Hence Φ_2 maps B_l into a family of equicontinuous functions.
It remains to prove that $(\Phi_2 B_l)(t)$ is relatively compact for each $t\in J$, where $V(t)=\{(\Phi_2 x)(t) : x\in B_l\},\ t\in J$. Obviously, by condition (H3), $V(0)$ is relatively compact in B_l. Let $0<t\leq b$ be fixed and $0<\epsilon<t$. For $x\in B_l$, we

have

$$
\begin{aligned}
(\Phi_2^{\epsilon})(t) = & T(t)[x_0 + g(x)] + \int_0^{t-\epsilon} T(t-s) f_2(s, v(s))ds \\
& + \int_0^{t-\epsilon} T(t-s)G(s, p(s))dw(s) + \sum_{0<t_k<t-\epsilon} T(t-t_k)I_k(x(t_k^-)) \\
= & T(t)[x_0 + g(x)] + T(\epsilon)\int_0^{t-\epsilon} T(t-s-\epsilon) f_2(s, v(s))ds \\
& + T(\epsilon)\int_0^{t-\epsilon} T(t-s-\epsilon)G(s, p(s))dw(s) \\
& + T(\epsilon)\sum_{0<t_k<t-\epsilon} T(t-t_k-\epsilon)I_k(x(t_k^-)), \quad t \in J.
\end{aligned}
$$

Since $\{T(t)\}_{t\geq 0}$ is compact, the set $V_\epsilon(t) = \{\Phi_2^{\epsilon}(t) : x \in B_l\}$ is relatively compact in H for every ϵ, $0 < \epsilon < t$. Moreover, for every $x \in B_l$,

$$
\begin{aligned}
\|(\Phi_2 x)(t) - \Phi_2^{\epsilon} x(t)\|^2 \leq & \int_{t-\epsilon}^{t} \|T(t-s) f_2(s, v(s))\|^2 ds + \int_{t-\epsilon}^{t} \|T(t-s)G(s, p(s))\|^2 dw(s) \\
& + \sum_{t-\epsilon<t_k<t} \|T(t-t_k)I_k(x(t_k^-))\|^2 \\
\leq & M_T^2 \int_{t-\epsilon}^{t} h_l(s)dw(s) + M_T^2 \int_{t-\epsilon}^{t} f_2(s, v(s)ds + M_T^2 \sum_{t-\epsilon<t_k<t} L_k(l).
\end{aligned}
$$

Therefore, letting $\epsilon \to 0$, we see that, there are relatively compact sets arbitrarily close to the set $V(t) = \{(\Phi_2)(t) : x \in B_l\}$. Hence the set $V(t)$is relatively compact in B_l.
As a consequence of the above steps and the Arzela-Ascoli theorem, we can conclude that Φ_2 is a compact operator. These arguments enable us to conclude that $\Phi = \Phi_1 + \Phi_2$ is a condensing map on B_l, and by the fixed point theorem of Sadovskii there exists a fixed point $x(\cdot)$ for Φ on B_l. Therefore, the nonlocal system (1)-(3) has a mild solution. The proof is now completed. □

In this part, in order to attain the uniqueness result of (1)-(3), we propose the following assumptions:

(H5) The function $G : J \times H^{n+1} \to L_Q(K, H)$ is continuous and there exists constants $M_G > 0$, for $t \in J$ and $x_i, y_i \in H$ such that

$$E\|G(t, x_i) - G(t, y_i)\|_Q \leq M_G \|x_i - y_i\|, \quad i = 1, 2, \ldots, n.$$

(H6) The nonlocal function $g : \mathscr{PC} \to \mathscr{PC}$ is continuous and there exist constants $M_g > 0$, for $x, y \in \mathscr{PC}$ such that

$$E\|g(x) - g(y)\| \leq M_g \|x - y\|.$$

(H7) $I_k : \mathscr{PC} \to \mathscr{PC}$ is continuous and there exist constants $M_{I_k} > 0$, for $x, y \in \mathscr{PC}$ such that

$$E\|I_k(x) - I_k(y)\| \leq M_{I_k} \|x - y\|, \quad k = 1, 2, \ldots, m.$$

Theorem 3.2.
Assume (H1), (H5)-(H7) hold, then there exists a unique solution to (1)-(3) *if* $\sum_{k=1}^{m} M_{I_k} < 1$.

Proof. Let $x, y \in B_l$ then we have that, for each $t \in J$

$$
\begin{aligned}
&E\|(\Phi x_1)(t)-(\Phi x_2)(t)\|^2\\
&\leq 36\Big\{E\|T(t)\big[x_0+g(x_1)+f_1(0,u_1(0))\big]-T(t)\big[x_0+g(x_2)+f_1(0,u_2(0))\big]\|^2\\
&\quad+E\|f_1(t,u_1(t))-f_1(t,u_2(t))\|^2+E\|\int_0^t T(t-s)\big[f_2(s,v_1(s))ds-f_2(s,v_2(s))ds\big]\|^2\\
&\quad+E\|\int_0^t(-A)T(t-s)\big[f_1(s,u_1(s))ds-f_1(s,u_2(s))ds\big]\|^2\\
&\quad+E\|\int_0^t T(t-s)\big[G(s,p_1(s))dw(s)-G(s,p_2(s))dw(s)\big]\|^2\\
&\quad+E\|\sum_{0<t_k<t}T(t-t_k)\big[I_k(x_1(t_k^-))-I_k(x_2(t_k^-))\big]\|^2\Big\}\\
&\leq 36\Big\{3M_T^2M_g^2E\|x_1(s)-x_2(s)\|^2+3M_T^2M_{f_1}^2M_0^2E\|x_1(0)-x_2(0)\|^2\\
&\quad+M_{f_1}^2M_0^2E\|x_1(t)-x_2(t)\|^2+b^2M_0^2M_T^2M_{f_2}^2E\|x_1(s)-x_2(s)\|^2\\
&\quad+b\int_0^t\frac{M_{1-\beta}^2}{(t-s)^{2(1-\beta)}}M_{f_1}^2E\|x_1(s)-x_2(s)\|^2+M_G^2Tr(Q)M_T^2E\|x_1(s)-x_2(s)\|^2\\
&\quad+M_T^2\sum_{k=1}^m M_{I_k}^2\sup_{0\leq s\leq b}E\|x_1(s)-x_2(s)\|^2\Big\}\\
&\leq 36\Big\{M_T^2\Big(3M_g^2+M_G^2Tr(Q)+b^2M_0^2M_{f_2}^2+\sum_{k=1}^m M_{I_k}^2\Big)\sup_{0\leq s\leq b}E\|x_1(s)-x_2(s)\|^2\\
&\quad+\Big[\frac{M_{f_1}^2M_{1-\beta}^2b^{2\beta}}{2\beta-1}+\big(M_T^2+3\big)\big(M_0^2M_{f_1}\big)\Big]\sup_{0\leq s\leq b}E\|x_1(s)-x_2(s)\|^2\Big\}\\
&\leq 36\Big\{M_T^2\Big(3M_g^2+M_G^2Tr(Q)+b^2M_0^2M_{f_2}^2+\sum_{k=1}^m M_{I_k}^2\Big)\\
&\qquad+\frac{M_{f_1}^2M_{1-\beta}^2b^{2\beta}}{2\beta-1}+\big(3M_T^2+1\big)\big(M_0^2M_{f_1}\big)\Big\}\sup_{0\leq s\leq b}E\|x_1(s)-x_2(s)\|^2\\
&\leq 36\Big\{M_T^2\Big(3M_g^2+M_G^2Tr(Q)+b^2M_0^2M_{f_2}^2+\sum_{k=1}^m M_{I_k}^2\Big)\\
&\qquad+\frac{M_{f_1}^2M_{1-\beta}^2b^{2\beta}}{2\beta-1}+\big(3M_T^2+1\big)\big(M_0^2M_{f_1}\big)\Big\}\|x_1-x_2\|^2\\
&\leq q\|x_1-x_2\|^2.
\end{aligned}
$$

If we define $q=36\big\{M_T^2\big(3M_g^2+M_G^2Tr(Q)+b^2M_0^2M_{f_2}^2+\sum_{k=1}^m M_{I_k}^2\big)+\frac{M_{f_1}^2M_{1-\beta}^2b^{2\beta}}{2\beta-1}+\big(3M_T^2+1\big)\big(M_0^2M_{f_1}\big)\big\}$ with $0\leq q\leq 1$. This shows that operator Φ is a contraction. Consequently, the operator Φ satisfies all the assumption and it has a unique fixed point which is a solution to (1)-(3). The proof is completed. □

4. Examples

In this section we provide the example to illustrate our previous abstract result.

$$
\begin{aligned}
&d\Big[x(t,z)+\int_0^\pi\mu_1(\xi,z)x(t\sin t,\xi)d\xi\Big]=\Big[\frac{\partial^2}{\partial x^2}x(t,z)-\int_0^\pi\mu_2(\delta,z)x(t\sin t,\delta)d\delta\Big]dt\\
&+h(t,x(t\sin t,z))d\beta(t),\quad 0\leq t\leq b,\quad 0\leq z\leq\pi,\quad t\neq t_k,\ k=1,2,\ldots,m, \qquad (10)\\
&x(t,0)=x(t,\pi)=0, \qquad (11)\\
&x(t_k^+)-x(t_k^-)=I_k(z(t_k^-)),\ k=1,2,\ldots,m, \qquad (12)\\
&x(t,z)+\sum_{i=0}^p\int_0^\pi k(x,\xi)x(t_i,\xi)d\xi=x_0(z),\quad 0\leq z\leq\pi, \qquad (13)
\end{aligned}
$$

where p is a positive integer, $0 \le b \le \pi, 0 < t_0 < \cdots < t_p < 1, and 0 < t_1 < t_2 < \cdots < t_m < b$. The function $x_0(x) \in H = L^2([0,\pi]), k(z,\xi) \in L_2([0,\pi]\times[0,\pi])$. $\beta(t)$ denotes a standard one dimensional Wiener process in H defined on a Probability space (Ω, P) and to rewrite the system (10)-(13) in abstract form of (1)-(3), let $H = L^2([0,\pi])$ and A is defined by $Ay = y''$.

We consider the operator $A : D(A) \subseteq H$ defined by, $D(A) = \{y \in H, y, y'$ are absolutely continuous, $y'' \in H, y(0) = y(\pi) = 0\}$.

Then A infinitesimal generates of a strongly continuous semigroup $T(\cdot)$ which is compact, analytic and self adjoint. Furthermore, A has a discrete spectrum, the eigenvalues are n^2, $n \in N$, with the corresponding normalized eigenvectors $x_n(z) = \sqrt{\frac{2}{\pi}}\sin(nz)$. Then the following properties hold:

(a) If $y \in D(A)$, then

$$Ay = \sum_{n=1}^{\infty} n^2 \langle y, x_n \rangle x_n.$$

(b) For each $y \in H$,

$$A^{-\frac{1}{2}} y = \sum_{n=1}^{\infty} \frac{1}{n} < y, x_n > x_n, \ In Particular, \ \|A^{-\frac{1}{2}}\|^2 = 1.$$

(c) The operator $A^{-\frac{1}{2}}$ is given by

$$A^{\frac{1}{2}} y = \sum_{n=1}^{\infty} n < y, x_n > x_n,$$

on the space $D(A^{\frac{1}{2}}) = \{y(\cdot) \in H, \sum_{n=1}^{\infty} n \langle y, x_n \rangle x_n \in H\}$.

We assume that the following conditions hold:

(i) The function μ_1, μ_2 is measurable and

$$\int_0^{\pi}\int_0^{\pi} \mu_1^2(\xi,z)\,d\xi\,dz < \infty \ and \ \int_0^{\pi}\int_0^{\pi} \mu_2^2(\delta,z)\,d\delta\,dz < \infty.$$

(ii) The function $\frac{\partial}{\partial z}\mu_1(\xi,z)$ is measurable, $\mu_1(\xi,0) = \mu_1(\xi,\pi) = 0$, and let

$$N_1 = \Big[\int_0^{\pi}\int_0^{\pi}\Big(\frac{\partial}{\partial z}\mu_1(\xi,z)\Big)^2 d\xi\,dz\Big]^{\frac{1}{2}} < \infty.$$

(iii) The function $\frac{\partial}{\partial z}\mu_2(\delta,z)$ is measurable, $\mu_2(\delta,0) = \mu_2(\delta,\pi) = 0$, and let

$$N_2 = \Big[\int_0^{\pi}\int_0^{\pi}\Big(\frac{\partial}{\partial z}\mu_2(\delta,z)\Big)^2 d\delta\,dz\Big]^{\frac{1}{2}} < \infty.$$

(iv) $\beta(t)$ denotes a one dimensional standard Brownian motion.

(v) For the function $h : J \times R \to R$ the following three conditions are satisfied:

  (1) For each $t \in [0,b]$, $h(t,\cdot)$ is continuous.

  (2) For each $x \in \mathscr{PC}$, $h(\cdot,x)$ is measurable.

  (3) There are positive functions $h_1, h_2 \in L^1(J)$ such that

$$|h(t,x)| \le h_1(t)|x| + h_2(t), \quad \forall\,(t,x) \in [0,b]\times H.$$

(vi) The function $I_k(\mathscr{PC},\mathscr{PC})$, $k = 1,2,\ldots,m$ and there exist nondecreasing functions $L_k \in (J, R_+)$, $k = 1,2,\ldots,m$ such that for each $x \in \mathscr{PC}$

$$\|I_k(x)\|^2 \le L_k(\|x\|^2).$$

We define $f_1 : J\times H \to L(K,H)$ and $f_2 : J\times H \to L(K,H)$ $g : \mathscr{PC} \to H$ by

$$f_1(t,x) = Z_1(x),$$
$$f_2(t,x) = Z_2(x),$$
$$G(t,x)(z) = h(t,x(z)), \text{ and}$$
$$g(w(t)) = \sum_{i=0}^{p} K w(t_i), \quad w \in \mathscr{PC},$$

respectively, where

$$Z_1(x)(z) = \int_0^{\pi} \mu_1(\xi,z)x(\xi)d\xi,$$
$$Z_2(x)(z) = \int_0^{\pi} \mu_2(\delta,z)x(\delta)d\delta$$

and

$$K(x)(z) = \int_0^{\pi} k(z,\xi)x(\xi)d\xi.$$

Then G satisfies condition (H2) while g verifies (H3) (noting that $K : \mathscr{PC} \to \mathscr{PC}$ is completely continuous). From (i) it is clear that Z_1 and Z_2 is a bounded linear operators on H. Furthermore, $Z_1(x) \in D[A^{\frac{1}{2}}]$, and $Z_1(x) \in D[A^{\frac{1}{2}}]$, then $\|A^{\frac{1}{2}}Z_1\|^2 \le N_1$, $\|A^{\frac{1}{2}}Z_2\|^2 \le N_2$. In fact, from the definition of Z_1, Z_2 and (ii), (iii) it follows that

$$< Z_1(x), x_n > = \int_0^{\pi} x_n(z)\Big[\int_0^{\pi} \mu_1(\xi,z)x(\xi)d\xi\Big]dz$$
$$= \frac{1}{n}\sqrt{\frac{2}{\pi}} < Z(x), \cos(nz) >,$$

where Z is defined by

$$Z(x)(z) = \int_0^{\pi} \frac{\partial}{\partial x}\mu_1(\xi,z)x(\xi)d\xi$$

and

$$< Z_2(x), x_n > = \int_0^{\pi} x_n(z)\Big[\int_0^{\pi} \mu_2(\delta,z)x(\delta)d\xi\Big]dz$$
$$= \frac{1}{n}\sqrt{\frac{2}{\pi}} < \overline{Z(x)}, \cos(nz) >,$$

where $\overline{Z}$ is defined by

$$\overline{Z(x)}(z) = \int_0^{\pi} \frac{\partial}{\partial x}\mu_2(\delta,z)x(\delta)d\delta.$$

From (ii)and (iii) we know that $Z : H \to H$ is a bounded linear operator with $\|Z\|^2 \le N_1$and $\|\overline{Z}\|^2 \le N_2$. Hence $\|A^{\frac{1}{2}}Z_1(z)\|^2 = \|Z(z)\|^2$ and $\|A^{\frac{1}{2}}Z_2(z)\|^2 = \|\overline{Z}(z)\|^2$, which implies the assertion. Therefore, the conditions (H1)-(H4) are all satisfied. Hence from Theorem 3.1, system (10)-(13) admits a mild solution on J under the above assumptions additionally provided in (5) and (3) hold.

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