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Chemical reaction and radiation effects on MHD free convection from an impulsively started infinite vertical plate with viscous dissipation

Research Article

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Abstract: This paper investigates the effects of chemical reaction and radiation on an unsteady transient MHD free convective and heat transfer of a viscous, incompressible and electrically conducting fluid past a suddenly started infinite vertical porous plate taking into account and the viscous dissipation is presented. Assuming the medium to be non scattered and the fluid to be non gray, emitting - absorbing and optically thin radiation limit properties. The dimensionless governing coupled, non linear boundary layer partial differential equations are solved by an efficient finite element method. The results are obtained for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number. The effects of different flow parameters on the flow variables are discussed and presented through graphs and tables.

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Keywords: Heat and Mass Transfer • MHD Flows • Chemical Reaction • Finite Element Method

1. Introduction

Magnetohydrodynamic (MHD) free convection of a viscous incompressible fluid along a vertical wall in porous medium must be studied if we are to understand the behavior of fluid motion in many applications as for example, in MHD electrical power generation, geophysics, astrophysics, etc. The problem of free convection flows of viscous incompressible fluids past a semi-infinite vertical wall has received a great deal of attention in recent years because of its many practical applications, such as in electronic components, chemical processing equipment, etc. Magneto hydrodynamic free convection flow of an electrically conducting fluid in different porous geometries is of considerable interest to the technical field due to its frequent occurrence in industrial, technological and geothermal applications. As an example, the geothermal region gases are electrically conducting and undergo the influence of magnetic field. Also, it has applications in nuclear engineering in connection with reactors cooling. The interest in this field is due to the wide range of applications in engineering and geophysics, such as the optimization of the solidification processes of metals and metal alloys, the study of geothermal sources, the treatment of nuclear fuel debris, the control of underground spreading of chemical wastes and pollutants and the design of MHD power

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Nomenclature

$\bar{B}$	Magnetic induction vector
B_0	Strength of the applied magnetic field
c_p	Specific heat at constant pressure
$C^{'}$	Species concentration
$C^{'}_{\infty}$	Species concentration of the fluid far away from the plate
$C^{'}_{w}$	Species concentration at the plate
D_M	Molecular mass diffusivity
D_T	Thermal diffusivity
E	Electric field
Gm	Grashof number for mass transfer
Gr	Grashof number for heat transfer
$\bar{g}$	Acceleration due to gravity
g	Acceleration due to gravity in magnitude
$\bar{J}$	Electric current density
$\vec{q}$	Fluid velocity vector
$\bar{K}$	Rate of first order homogeneous chemical reaction
$\frac{J^2}{\sigma}$	Joule heat per unit volume
κ	Thermal conductivity
$k^{'}$	Permeability of porous medium
M	Hartmann number
Pr	Prandtl number
p	Fluid pressure
q_r	radiative flux
R	Radiation parameter
Ec	Eckert number
K	Permeability parameter
Kr	Chemical reaction parameter
Sc	Schmidt Number
$T^{'}_{w}$	Temperature at the plate
$T^{'}_{\infty}$	Temperature of the fluid far away from the plate
$t^{'}$	Time
$T^{'}$	Temperature
U_0	Plate velocity
u	non dimensional velocity
T	non dimensional temperature
C	non dimensional species concentration
μ	Coefficient of the viscosity
ρ	Fluid density
ϕ	Dissipation of energy per unit volume due to viscosity
δn	An element of the normal to the surface
σ	Electrical conductivity
υ	Kinematic viscosity
β	Coefficient of volume expansion for heat transfer
$\beta^{'}$	Coefficient of volume expansion for mass transfer

generators. Many papers concerned with the problem of MHD free convection flow in porous media have been published in the literature. In view of its wide applications, Helmy [1] studied MHD unsteady free convection flow past a vertical porous plate. A.Pantokratoras [2] studied Non-Darcian forced convection heat transfer over a flat plate in a porous medium with variable viscosity and variable Prandtl number. Chaudhary et al.[3] studied the effect of free convection effects on magnetohydrodynamic flow past an infinite vertical accelerated plate embedded in porous media with constant heat flux by using Laplace transform technique for finding the analytical solutions. Recently, Das and his co-workers [4] analyzed the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Das et al.[5]investigated numerically the unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux. Das and his associates [6] estimated the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction employing finite difference analysis. Singh et al.[7] studied the heat transfer over stretching surface in porous media with transverse magnetic field. opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et al.[8]. Elbashbeshy, D. M. Yassmin and A. A. Dalia et al.[9] studied heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of heat source or sink. M. S. Alam, M. M. Haque, M. J. Uddin et al.[10] studied unsteady MHD free convective heat transfer flow along a vertical porous flat plate with internal heat generation. In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., Polymer production, manufacturing of ceramics or glassware and food procession. Cramer K. R. and Pai, S. I.et al.[11] taken transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Muthucumaraswamy et al.[12] have studied the effect of homogeneous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Das et al.[13] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. K.Sudhakar and R. Srinivasa Raju et al.[14] have studied chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo. S. Shivaiah and J. Anand Rao et al.[15] studied chemical reaction effect on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Chaudhary and Jha [16] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Anjalidevi et al.[17] have examined the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Moreover, Al-Odat and Al-Azab [18] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy et al.[19]. Ahmed Sahin.et al.[20] have studied influence of chemical reaction on transient MHD free convective flow over a vertical plate in slip-flow regime.

Radiation effects on free convection flow have become very important due to its applications in space technology, processes having high temperature, and design of pertinent equipments. Moreover, heat transfer with thermal radiation on convective flows is very important due its significant role in the surface heat transfer. Recent developments in gas cooled nuclear reactors, nuclear power plants, gas turbines, space vehicles, and hypersonic flights have attracted research in this field. The unsteady convective flows in a moving plate with thermal radiation were examined by Cogley et al.[21] and Mansour[22]. The combined effects of radiation and buoyancy force past a vertical plate were analyzed by Hossain and Takhar [23]. Seddeek [24] explained the importance of thermal radiation and variable viscosity on unsteady forced convection with an align magnetic field. Muthucumaraswamy and Senthil[25] studied the effects of thermal radiation on heat and mass transfer over a moving vertical plate. Pal[26] investigated convective heat and mass transfer in stagnation-point flow towards a stretching sheet with thermal radiation. Palani and Kim [27] analyzed the effect of thermal radiation on convection flow past a vertical cone with surface heat flux. Recently, Mahmoud and Waheed [28] examined thermal radiation on flow over an infinite flat plate with slip velocity. G.Jithender Reddy et al.[29] analyzed the finite element analysis of soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate with heat absorption. S. P. Anjali Devi and D. Vasantha kumari et al.[30] discussed Numerical investigation of slip flow effects on unsteady hydromagnetic flow over a stretching surface with thermal radiation. M. Gnaneswara Reddy et al.[31] studied Heat and mass transfer effects on unsteady MHD flow of a chemically reacting fluid past an impulsively started vertical plate with radiation.

The viscous dissipation effects are important in geophysical flows and also in certain industrial operations are usually characterized by the Eckert number. In the literature, extensive research work is available to examine the effect of natural convection on flow past a plate. Gebhart et al.[32] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf et al.[33] have considered the effects of viscous dissipation for external natural convection flow over a surface. Maharajan and Gebhart et al.[34] have reported the influence of viscous dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Vajravelu, K and Hdjinicolaou et al.[35] studied a Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Anand Rao. J and Srinivasa Raju et al.[36]have studied on applied magnetic field on transient free convective flow of an incompressible viscous dissipative fluid in a vertical channel. Emmanuel Osalusi and his coworkers et al.[37] have studied On the effectiveness of viscous dissipation and joule heating on steady MHD and slip flow of a bingham fluid over a porous rotating disk in the presence of hall and ion-slip currents.

The objective of the present paper is to study the effects of chemical reaction and radiation on an unsteady transient MHD free convective and heat transfer of a viscous, incompressible, electrically conducting and fluid past a suddenly started infinite vertical porous plate taking into account the viscous dissipation is presented. The non dimensional governing coupled partial differential equations are involved in the present analysis and are solved by finite element Galerkin method which is more economical from computational view point.

2. Basic equations

The equations governing the motion of an incompressible, viscous, electrically conducting radiating fluid past a solid surface in presence of a magnetic field are:

The equation of continuity:

$$div\vec{q} = 0$$

The momentum equation:

$$\rho[\frac{\partial \vec{q}}{\partial t'} + (\vec{q}.\vec{\nabla})\vec{q}] = -\vec{\nabla}p + \vec{J} \times \vec{B} + p\vec{g} + \mu\nabla^2\vec{q} - \frac{\mu}{k'}\vec{q}$$

The energy equation:

$$\rho[\frac{\partial T'}{\partial t'} + (\vec{q}.\vec{\nabla})T'] = \kappa\nabla^2 T' + \frac{\vec{J}^2}{\sigma} - \frac{\partial q_r}{\partial \eta'} + \mu\nabla^2\vec{q}$$

The species continuity equation:

$$\frac{\partial C'}{\partial t'} + (\vec{q}.\vec{\nabla})C' - \mu\nabla^2 C' - \overline{K}(C' - C'_\infty)$$

The Ohm's law:

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$$

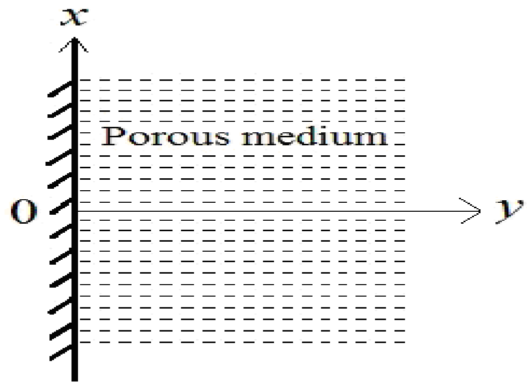

Fig. 1. Geometry of the problem.

We consider effect on unsteady radiative MHD free convection and mass transfer flow of an incompressible, viscous, and electricity conducting fluid past a suddenly moving infinite vertical plate in presence of a uniform transverse magnetic field of strength B_0. Our investigation is restricted to the following assumptions:

i. All the fluid properties except the density in the buoyancy force term are constants.

ii. The magnetic Reynolds number is so small that the induced magnetic field can be neglected.

iii. The chemical reaction is considered to be homogeneous and of first order.

iv. The Permeability of the medium is considered to be constant.

v. The viscous dissipation of energy is considered.

vi. The flow is parallel to the plate.

Initially, the plate and the surrounding fluid were at rest at the same temperature T'_∞ with concentration level C'_∞ at all points in the fluid. At time $t' > 0$, the plate is suddenly moved in its own plane with velocity U_0. The plate temperature and concentration are instantly raised to $T'_w < (T'_\infty)$ and $C'_w < (C'_\infty)$, which are thereafter maintained constant. We introduce a coordinate system (x', y', z') with X'-axis along the plate in the upward vertical direction, Y'-axis normal to the plate and directed into the fluid region and the Z'-axis along the width of the plate. Let $q' = (u', 0, 0)$ be the fluid velocity and $B' = (0, B_0, 0)$ be the magnetic induction vector at a point(x', y', z') in the fluid at

time t', under the above foregoing assumptions and Boussinesq approximation, the equations governing the flow and transport reduce to the following equations:
Continuity Equation:

$$\frac{\partial v'}{\partial t'} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) - \frac{\sigma{\beta_0}^2}{\rho}u' - \frac{v}{k'}u' \tag{2}$$

Energy Equation:

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

Species Diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D_M\frac{\partial^2 C'}{\partial y'^2} - \overline{K}(C' - C'_\infty) \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\forall t' \le 0 : u', \quad T' = T'_\infty, C' = C'_\infty \quad \forall y' \tag{5}$$

$$\forall t' > 0 : u' = U_0,\ T' = T'_w,\ C' = C'_w \text{ at } y' = 0 \tag{6}$$

$$\forall t' > 0 : u' = 0,\ T' \to T'_\infty,\ C' \to C'_\infty \text{ at } y' \to \infty \tag{7}$$

As mentioned earlier, in the case of optically thin limit, the fluid cannot absorb its own emitted radiation, but it absorbs the radiation emitted by boundaries. Following Cogly et al. [21], the rate of flux of radiation in the optically thin limit for a non gray gas near equilibrium is given by

$$\frac{\partial q_r}{\partial y'} = 4I(T' - T'_\infty) \tag{8}$$

Where I= $\int_0^\infty K_{\lambda w}(\frac{de_{b\lambda}}{dT'})d\lambda$
$K_{\lambda w}$ being the absorption coefficient and $e_{b\lambda}$ is the Plank function.
The equation (8)substitute in (3), we obtain

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - 4I(T' - T'_\infty) + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{9}$$

We now introduce the following non dimensional variables and parameters:

$$y = \frac{U_0^2 y'}{v}, \quad t = \frac{U_0^2 t'}{v}, \quad u = \frac{u'}{U_0}, \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Pr = \frac{\mu c_p}{\kappa}, \quad Sc = \frac{v}{D_M}, \quad Ec = \frac{U_0^2}{c_p(T'_w - T'_\infty)} \quad t = \frac{k' U_0^2}{v^2}, \quad v = \frac{\mu}{\rho},$$

$$M = \frac{\sigma B_0^2 u v}{\rho U_0^2}, \quad Gr = vg\beta\frac{T'_w - T'_\infty}{U_0^3}, \quad Gm = vg\beta'\frac{C'_w - C'_\infty}{U_0^3}, \quad R = \frac{4Iv^2}{U_0^2\kappa}$$

these non dimensional parameters substitute in Eqs. (1), (2), (4)-(7) and (9)
we obtain the following non dimensional form of equations.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr(T) + Gm(C) - \left(M + \frac{1}{K}\right)u \tag{10}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} - RT + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KrC \tag{12}$$

With the initial and boundary conditions

$$\forall t \le 0 : u = 0,\ T = 0,\ C = 0, for\ all\ y \tag{13}$$

$$\forall t > 0 : u = 1,\ T = 1,\ C = 1,\ at\ y = 0 \tag{14}$$

$$\forall t > 0 : u = 0,\ T = 0,\ C = 0,\ at\ y \to \infty \tag{15}$$

3. Method of solution

By applying Galerkin finite element method for (10) over the element (e),($y_j \le y \le y_k$) is

$$\int_{y_j}^{y_k} \left\{ N^{(e)^T} \left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - M^* u^{(e)} + P \right] \right\} dy = 0 \tag{16}$$

where $P = (Gr)T + (Gm)C, M^* = M + \frac{1}{K}$
Integrating the first term in Eq. (16) by parts one obtains

$$\left\{ N^{(e)^T} \frac{\partial u^{(e)}}{\partial y} \right\}_{y_j}^{y_k} - \int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)^T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} - N^{(e)^T} \left[\frac{\partial u^{(e)}}{\partial t} + M^* u^{(e)} - P \right] \right\} dy = 0 \tag{17}$$

Neglecting the first term in Eq. (17), one gets:

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)^T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)^T} \left[\frac{\partial u^{(e)}}{\partial t} + M^* u^{(e)} - P \right] \right\} dy = 0$$

let $u^{(e)} = N^{(e)} \phi^{(e)}$ be the finite element approximation solution over the element ($y_j \le y \le y_k$)
where $N^{(e)} = [N_j\ N_k]$, $\phi^{(e)} = [u_j\ u_k]$ and $N_j = \frac{y_k - y}{y_k - y_j}$, $N_k = \frac{y - y_j}{y_k - y_j}$ are the basis of the functions,One obtains:

$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N'_j N'_j & N'_j N'_k \\ N'_j N'_k & N'_k N'_k \end{bmatrix} \begin{bmatrix} u_j \\ [0.3em] u_k \\ [0.3em] \end{bmatrix} \right\} dy + \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ [0.3em] \dot{u}_k \\ [0.3em] \end{bmatrix} \right\} dy$$
$$+ M^* \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ [0.3em] u_k \\ [0.3em] \end{bmatrix} \right\} dy = P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ [0.3em] N_k \\ [0.3em] \end{bmatrix} dy$$

Simplifying we get

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ [0.3em] u_k \\ [0.3em] \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ [0.3em] \dot{u}_k \\ [0.3em] \end{bmatrix} + \frac{M^*}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \$\$ \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where prime and dot denotes differentiation w.r.to y and time t respectively. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}+\frac{M^*}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \tag{18}$$

Now put row corresponding to the node i to zero, from Eq. (18) the difference schemes with $l^{(e)}$ is:

$$\frac{1}{h^2}[-u_{i-1}+2u_i-u_{i+1}]+\frac{1}{6}[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}]+\frac{M^*}{6}[u_{i-1}+4u_i+u_{i+1}]=P \tag{19}$$

Applying the trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$A_1u_{i-1}^{n+1}+A_2u_i^{n+1}+A_3u_{i+1}^{n+1}=A_4u_{i-1}^{n}+A_5u_i^{n}+A_6u_{i+1}^{n}+P^* \tag{20}$$

Now from Eqs. (11) and (12), following equations are obtained:

$$B_1T_{i-1}^{n+1}+B_2T_i^{n+1}+B_3T_{i+1}^{n+1}=B_4T_{i-1}^{n}+B_5T_i^{n}+B_6T_{i+1}^{n}+E^* \tag{21}$$

$$C_1C_{i-1}^{n+1}+C_2C_i^{n+1}+C_3C_{i+1}^{n+1}=C_4C_{i-1}^{n}+C_5C_i^{n}+C_6C_{i+1}^{n} \tag{22}$$

where
$A_1=2-6r+M^*k$, $A_2=8+12r+4M^*k$, $A_3=2-6r+M^*k$, $A_4=2+6r-M^*k$, $A_5=8-12r-4M^*k$, $A_6=2+6r-M^*k$, $B_1=-6r+2Pr+Rk$, $B_2=12r+8Pr+4Rk$, $B_3=-6r+2Pr+Rk$, $B_4=6r+2Pr-Rk$, $B_5=-12r+8Pr-4Rk$, $B_6=6r+2Pr-Rk$, $C_1=-6r+2Sc+kKrSc$, $C_2=12r+8Sc+4kKrSc$, $C_3=-6r+2Sc+kKrSc$, $C_4=6r+2Sc-kKrSc$, $C_5=-12r+8Sc-4kKrSc$, $C_6=6r+2Sc-kKrSc$, $P^*=12kP=[(Gr)T_i^n+(Gm)C_i^n]$ $E^*=12k\left(\frac{\partial u_i^n}{\partial y_i^n}\right)^2$
Here $r=\frac{k}{h^2}$ and h, k are mesh sizes along y âĂŞdirection and time-direction respectively. Index i refers to space and j refers to the time. In the Eqs. (20)-(22), taking $i=1$ to n and using initial and boundary conditions (13)-(15), then the following system of equations are obtained:

$$A_iX_i=B_i,\ i=1\ to\ n \tag{23}$$

Where $A_i's$ are matrices of order n and $X_i's$, $B_i's$ are column matrices having n-components. The solutions of above system of Eq. (23) are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-programme. In order to prove the convergence and stability of Galerkin finite element method, the same C-programme was run with smaller values of h and k and no significant change was observed in the values of u, T and C. Hence the Galerkin finite element method is stable and convergent.

4. Skin-Friction, rate of heat and mass Ttransfer

The skin friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin friction at the plate, which in the non-dimensional form is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{24}$$

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number is given by

$$N_u=-\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{25}$$

The rate of mass transfer coefficient, which in the non-dimensional form in terms of the Sherwood number, is given by

$$S_b=-\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{26}$$

5. Results and discussion

Some numerical calculations have been carried out for the non-dimensional velocity (u), temperature (T), concentration (C), skin-friction coefficient and heat and mass transfer coefficients in terms of Nusselt number (N_u) and Sherwood number (S_b) respectively. The effects of material parameters such as Grashof number (Gr) modified Grashof number (Gm), Hartmann number (M), permeability parameter (K) , Prandtl number (Pr), Eckert number (Ec) ,Schmidt number (Sc) and Chemical Reaction parameter (Kr) have been observed. The numerical calculations of these results are presented graphically in Figs. 2 to 15. During the course of numerical calculations of the velocity, temperature and concentration, the values of the Prandtl number are chosen for Mercury ($Pr = 0.025$), air ($Pr = 0.71$), electrolytic solution($Pr = 1.0$), water ($Pr = 7.0$) and water at 4^0C ($Pr = 11.62$), engine oil ($Pr = 100$). To focus out attention on numerical values of the results obtained in the study the values of Sc are chosen for the gases representing diffusing chemical species of most common interest in air namely Hydrogen ($Sc = 0.22$), Helium ($Sc = 0.30$), Water-vapour ($Sc = 0.60$), Oxygen ($Sc = 0.66$), Ammonia ($Sc = 0.78$). Methanol ($Sc = 1.00$) and Propyl-benzene ($Sc = 2.62$) at 20^0C and one atmospheric pressure. For the physical significance, only the real part of complex quantity is invoked for the numerical discussion in the problem and at $t = 1.0$, stable values for velocity, temperature and concentration fields are obtained. To examine the effect of parameters related to the problem on the velocity field and skin-friction numerical computations are carried out at ($Pr = 0.71$) which corresponds to air at 25^0C and one atmospheric pressure.

For various values of Grashof number and modified Grashof number, the velocity profiles are plotted in Fig. 2 and Fig. 3. The Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The modified Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the modified Grashof number.

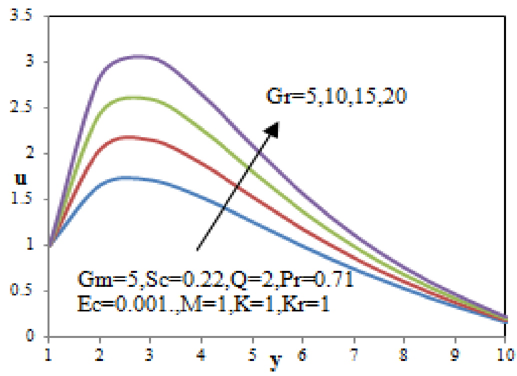

Fig. 2. Velocity Profile with Effect of Gr.

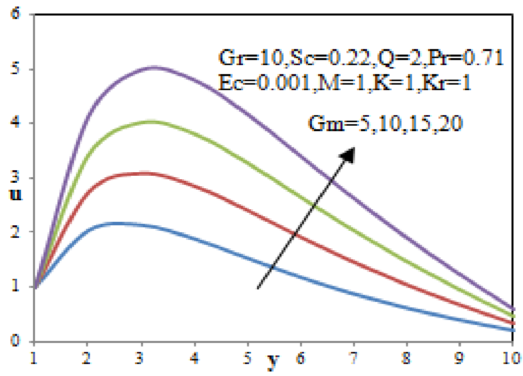

Fig. 3. Velocity Profile with Effect of Gm.

The effect of Hartmann number M is shown in the Fig. 4 in case of cooling of the plate. It is observed that the velocity of the fluid decreases with the increase of Hartmann number values. As expected, the velocity decreases with an increase in the Hartmann number. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the Hartmann number. We also see that velocity profiles decrease with the increase of magnetic effect indicating that magnetic field tends to retard the motion of the fluid. Magnetic field may control the flow characteristics. Fig. 5 shows the effect of the permeability of the porous medium parameter K on the velocity distribution. As shown, the velocity is increasing with the increasing dimensionless porous medium parameter. The effect of the dimensionless porous medium K becomes smaller as K increase. Physically, this result can be achieved when the holes of the porous medium may be neglected. Fig. 6 and Fig. 7 show the velocity and temperature profiles for different values of the Radiation parameter, It is found that the temperature profiles T, being as a decreasing function of , decelerate the flow and reduce the fluid velocity. Such an effect may also be expected, as increasing radiation parameter makes the fluid thick and ultimately causes the temperature and the thermal boundary layer thickness to decrease. Velocity and temperature profiles for some realistic values of Prandtl number $Pr = 0.025, 0.71, 1.0, 7.0, 11.62, 100$ which are important in the sense that they physically correspond to mercury, air, electrolytic solution, water, water at 4^0C

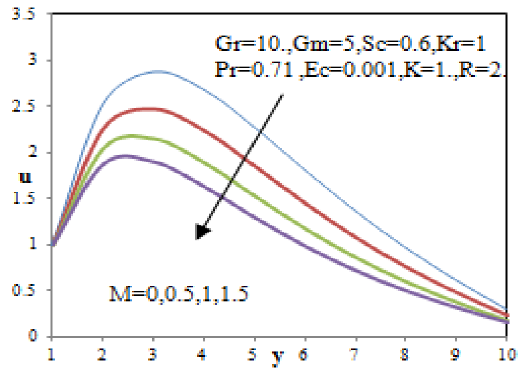

Fig. 4. Velocity Profile with Effect of M.

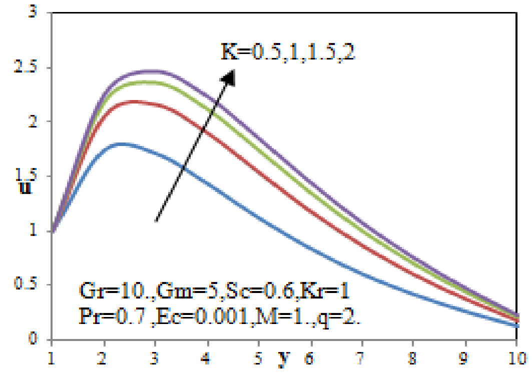

Fig. 5. Velocity Profile with Effect of K.

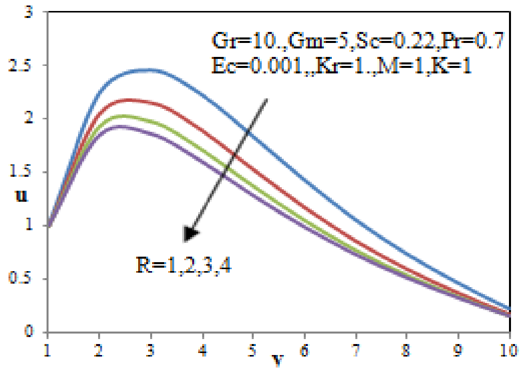

Fig. 6. Velocity Profile with Effect of R .

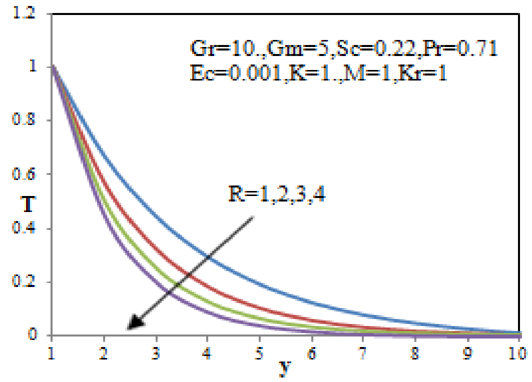

Fig. 7. Temperature Profile with Effect of R.

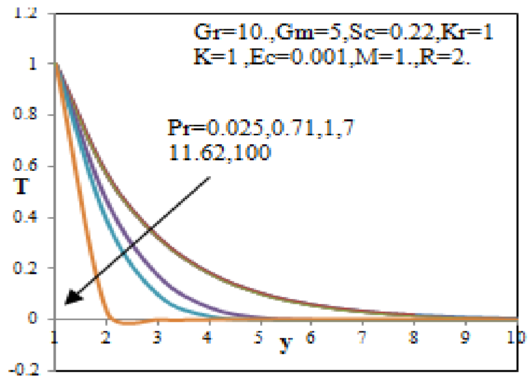

Fig. 8. Velocity Profile with Effect of Pr .

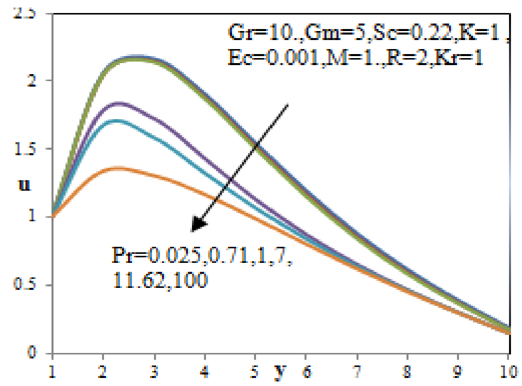

Fig. 9. Temperature Profile with Effect of Pr.

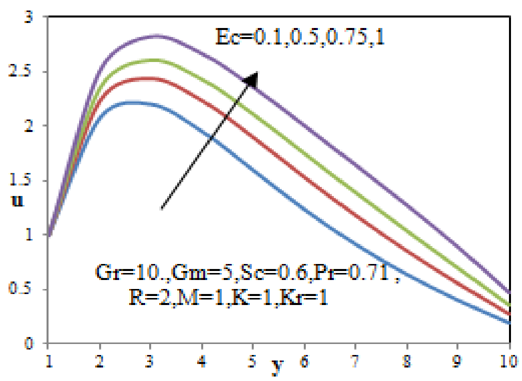

Fig. 10. Velocity Profile with Effect of Ec .

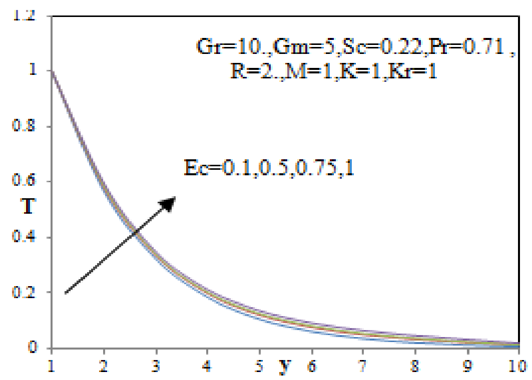

Fig. 11. Temperature Profile with Effect of Ec.

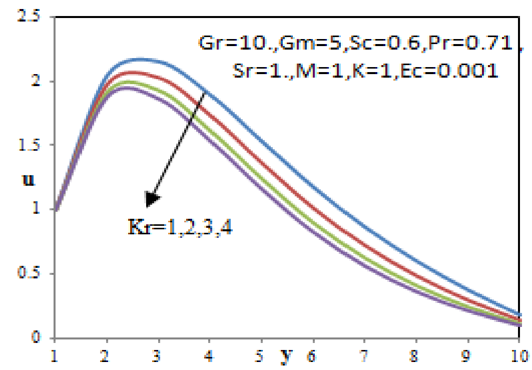

Fig. 12. Velocity Profile with Effect of Kr .

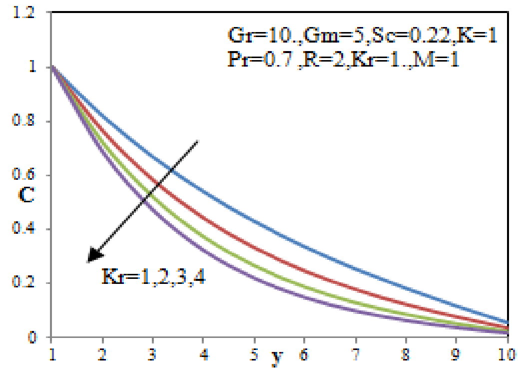

Fig. 13. Concentration Profile with Effect of Kr.

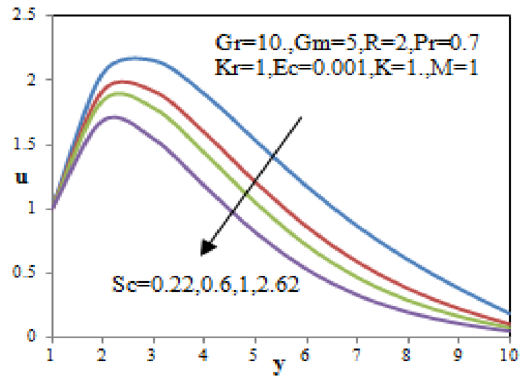

Fig. 14. Velocity Profile with Effect of Sc .

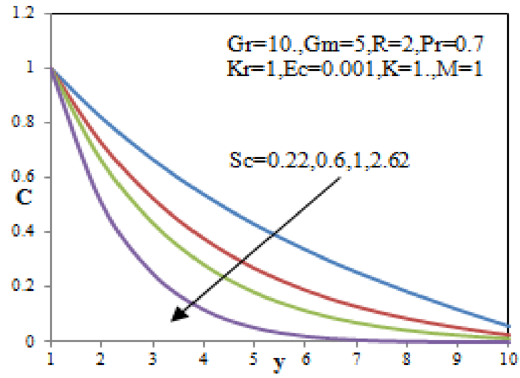

Fig. 15. Concentration Profile with Effect of Sc.

and engine oil, are shown in Fig. 8,Fig. 9 respectively. From Fig. 8, it is found that the momentum boundary layer thickness increases for the fluids with $Pr < 1$ and decreases for $Pr > 1$. The Prandtl number actually describes the relationship between momentum diffusivity and thermal diffusivity and hence controls the relative thickness of the momentum and thermal boundary layers. When Pr is small, that is, $Pr = 0.025$, it is noticed that the heat diffuses very quickly compared to the velocity (momentum).This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer. In Fig. 9, we observe that the temperature decreases with increasing values of Prandtl number Pr. It is also observed that the thermal boundary layer thickness is maximum near the plate and decreases with increasing distances from the leading edge and finally approaches to zero. Furthermore, it is noticed that the thermal boundary layer for mercury which corresponds to $Pr = 0.025$ is greater than those for air, electrolytic solution, water, and engine oil. It is justified due to the fact that thermal conductivity of fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer thickness and the temperature profiles.The effects of the viscous dissipation parameter i.e., Eckert number on the velocity and temperature are shown in Fig. 10 and Fig. 11 respectively. The Eckert number (Ec) expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity. This behavior is evident from Fig. 10 and Fig. 11. Fig. 12 displays the effect of the chemical reaction parameter Kr on the velocity profiles. As expected, the presence of the chemical

Table 1. Numerical values of Heat transfer coefficient for different values of R, Pr and Ec

R	Pr	Ec	N_u
1	0.71	0.001	1.0042
2	0.71	0.001	1.4159
3	0.71	0.001	1.7303
2	0.71	0.001	1.416
2	1	0.001	1.4196
2	7	0.001	1.6886
2	0.71	0.1	1.4109
2	0.71	2	1.2955
2	0.71	3	1.2311

Table 2. Numerical values of Mass transfer coefficient for different values of Sc and Kr

Sc	Kr	S_b
0.22	1	0.5012
0.6	1	0.7912
1	1	1.0081
2.62	1	1.6491
0.22	1	0.5012
0.22	2	0.6683
0.22	3	0.8105
0.22	4	0.9308

Table 3. Numerical values of Skin-friction coefficient for varies values of physical parameters

Gr	Gm	M	K	R	Pr	Ec	Sc	Kr	τ
5	5	1	1	2	0.71	0.001	0.22	1	2.8787
10	5	1	1	2	0.71	0.001	0.22	1	4.6341
10	10	1	1	2	0.71	0.001	0.22	1	7.1936
10	15	1	1	2	0.71	0.001	0.22	1	9.7449
10	5	0	1	2	0.71	0.001	0.22	1	6.0756
10	5	0.5	1	2	0.71	0.001	0.22	1	5.2917
10	5	1	0.5	2	0.71	0.001	0.22	1	3.6475
10	5	1	1.5	2	0.71	0.001	0.22	1	5.0482
10	5	1	1	1	0.71	0.001	0.22	1	5.1778
10	5	1	1	3	0.71	0.001	0.22	1	4.2902
10	5	1	1	2	1	0.001	0.22	1	4.6133
10	5	1	1	2	7	0.001	0.22	1	3.9335
10	5	1	1	2	0.71	0.1	0.22	1	4.74
10	5	1	1	2	0.71	0.5	0.22	1	5.2552
10	5	1	1	2	0.71	0.001	0.66	1	4.2834
10	5	1	1	2	0.71	0.001	2.62	1	3.6473
10	5	1	1	2	0.71	0.001	0.22	1	4.6341
10	5	1	1	2	0.71	0.001	0.22	2	4.4469
10	5	1	1	2	0.71	0.001	0.22	3	4.3013

reaction significantly affects the velocity profiles. It should be mentioned that the studied case is for a destructive chemical reaction Kr. In fact, as chemical reaction Kr increases, the considerable reduction in the velocity profiles is predicted, and the presence of the peak indicates that the maximum value of the velocity occurs in the body of the fluid close to the surface but not at the surface. Fig. 13 shows the effect of the chemical reaction parameter Kr on concentration profiles. As expected, the presence of the chemical reaction significantly affects the concentration profiles. It should be mentioned that the studied case is for a destructive chemical reaction Kr. In fact, as chemical reaction Kr increases, the concentration decreases. It is evident that the increase in the chemical reaction Kr significantly alters the concentration boundary layer thickness but does not alter the momentum boundary layers. The effect of the Schmidt number on the velocity and concentration are shown in Fig. 14 and Fig. 15. As the Schmidt number increases, the velocity and concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers. Fig. 15 shows the concentration field due to variation in Schmidt number for the gasses Hydrogen, Water-vapour, Oxygen, Ammonia and Methanol. It is observed that concentration field is steady for Hydrogen and falls rapidly for Oxygen and Ammonia in comparison to Water-vapour. Thus, Hydrogen can be used for maintaining effective concentration field and Water-vapour can be used for maintaining normal concentration field.

Table 1 shows the numerical values of heat transfer coefficient (N_u) for different values of Prandtl number R, Pr and Ec respectively. An increase in R or Pr leads to an increase in heat transfer coefficient while increasing in Ec leads to decrease in heat transfer coefficient.Table 2 Shows the numerical values of mass transfer coefficient (S_b) for different values of Schmidt number Sc and Kr respectively. An increase in Sc or Kr leads to an increase in mass transfer coefficient.Table 3 Shows the numerical values of skin-friction coefficient (τ) for variations in Gr, Gm, M, K, R, Pr, Ec, Sc and Kr respectively, corresponding to cooling of the plate. An increase in Gr or Gm or K or Ec leads to an increase in the value of skin-friction coefficient while in increase in M or R or Pr or Sc or Kr leads to a decrease in the value of skin-friction coefficient.

6. Conclusions

We have formulated the problem of two-dimensional fluid flow in the presence of Radiative heat transfer, Dissipative flow and Chemical reaction Parameter. A finite element technique is employed to solve the resulting coupled partial differential equations. The following conclusions are drawn from the study.

- The velocity increases with the increase in thermal Grashof, Modified Grashof, Eckert numbers, and Permeability parameter while the velocity decreases with the increase of Hartman number, Radiation parameter, Prandtl number, Schmidt number and Chemical reaction parameter.
- Concentration decreases with the increasing of Schmidt number and chemical reaction parameter.
- The temperature increase with increasing of Eckert number while temperature decreases with an increase in the Radiation parameter and Prandtl number.
- An increase in Grashof, modified Grashof, Eckert numbers and Permeability parameter leads to an increase in the value of skin-friction coefficient while in increase in Hartman, Schmidt, Prandtl numbers, Radiation and Chemical reaction parameters leads to a decrease in the value of skin-friction coefficient.
- An increase in Radiation parameter and Prandtl number leads to an increase in heat transfer coefficient while increasing Eckert number leads to decrease in heat transfer coefficient.
- An increase in Schmidt number and chemical reaction parameter leads to an increase in mass transfer coefficient.

References

[1] K. A. Helmy, MHD Unsteady free convection flow past a vertical porous plate. ZAAM,78(1998) 255-270.
[2] A. Pantokratoras, Non-Darcian forced convection heat transfer over a flat plate in a porous medium with variable viscosity and variable Prandtl number. J. Porous Media,10 (2007) 201 - 208.
[3] R. C. Chaudhary, M. C. Goyal, A. Jain, Free convection effects on MHD flow past an infinite vertical accelerated plate embedded in porous media with constant heat flux, Matematicas Ensenanza Universitaria, XVII (2009) 73-82.
[4] S. S. Das, A. Satapathy, J. K. Das and J. P. Panda, Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source, Int. J. Heat Mass Transfer, 52(2009) 5962-5969.
[5] S. S. Das, A. Satapathy, J. K. Das and S. K. Sahoo, Numerical solution of unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux, J. Ultra Sci. Phys. Sci., 19(1)(2007) 105-112.
[6] S. S. Das, S. K. Sahoo and G. C. Dash, Numerical solution of mass transfer effects on an unsteady flow past an accelerated vertical porous plate with suction, Bull.Malays. Math. Sci. Soc., 29(1) (2006) 33-42.
[7] P. Singh, N. S. Tomer, D. Sinha, Numerical study of heat transfer over stretching surface in porous media with transverse magnetic field, Proceeding of International Conference on Challenges and application of Mathematics in Sciences and Technology (2010) 422-430.
[8] D. Angirasa, G. P. Peterson, I. Pop, Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium, Int. J. Heat Mass Trans. 40(12)(1997).
[9] E. M. A. Elbashbeshy, D. M. Yassmin, A. A. Dalia, Heat Transfer Over an Unsteady Porous Stretching Surface Embedded in a Porous Medium with Variable Heat Flux in the Presence of Heat Source or Sink, African Journal of Mathematics and Computer Science Research 3(5) (2010) 68-73.
[10] M. S. Alam, M. M. Haque, M. J. Uddin, Unsteady MHD free convective heat transfer flow along a vertical porous flat plate with internal heat generation,Int. J. Adv. Appl. Math. and Mech. 2(2) (2014) 52 - 61.
[11] K. R. Cramer, S. I. Pai, Magneto fluid Dynamics for Engineering and Applied Physicists, Mc Graw Hill, New York, 1973.
[12] R. Muthucumaraswamy, S. Meenakshisundaram. Theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature. Theoret. Appl. Mech. 33(3) (2006) 245-257.
[13] U. N. Das, R. K. Deka, V. M. Soundalgekar, Effects of Mass Transfer on Flow Past an Impulsively started Infinite Vertical Plate with Constant Heat Flux and Chemical reaction. Forschung in Inge-Engg Reseach 60 (1994) 284-287.
[14] K. Sudhakar, R. Srinivasa Raju, M. Rangamma, Chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo, International Journal of Modern Engineering Research 2(5) (2012) 3329-3339.
[15] S. Shivaiah, J. Anand Rao, studied Chemical reaction effect on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection, Theoret. Appl. Mech., 39(2) (2012) 185âĂŞ208.

[16] R. C. Chaudhary, A. K. Jha, Effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime, Applied Mathematics and Mechanics, 29(9) (2008) 1179-1194.
[17] S. P. Anjalidevi, R. Kandasamy, Effects of a chemical reaction heat and mass transfer on MHD flow past a semi infinite plate, Z. Angew. Math. Mech. 80 (2000) 697-701.
[18] M. Q. Al-Odat, Al-Azab, Influence of chemical reaction on transient MHD free convection over a moving vertical plate, Emirates J. Engg. Res. 12(3) (2007) 15-21.
[19] R. Kandasamy, K. Periasamy, K. K. Sivagnana Prabhu, Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects, Int. J. Heat and Mass Transfer 48 (2005) 21-22.
[20] Ahmed Sahin, Influence of chemical reaction on transient mhd free Convective flow over a vertical plate in slip-flow Regime, Emirates Journal for Engineering Research 15(1)(2010) 25-34.
[21] A. C. Cogley, W. C. Vincenti, S. E. Gilles, Differential approximation for radiation transfer in a non-gray gas near equilibrium, American Institute of Aeronautics and Astronautics Journal 6 (1968) 551-555.
[22] M. A. Mansour, Radiative and free convection effects on the oscillatory flow past a vertical plate, Astrophysics and Space Science 166(2) (1990) 269-275.
[23] M. A. Hossain, H. S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer 31(4) (1996) 243-248.
[24] M. A. Seddeek, Effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow, International Journal of Heat and Mass Transfer 45(4) (2002) 931-935.
[25] R. Muthucumaraswamy, G. K. Senthil,The effect of heat and mass transfer on moving vertical plate in the presence of thermal radiation, Journal of Theoretical And Applied Mechanics 31(1) (2004) 35-46.
[26] D. Pal, Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation, Meccanica 44(2) (2009) 145-158.
[27] G. Palani, K. Y. Kim, Influence of magnetic field and thermal radiation by natural convection past vertical cone subjected to variable surface heat flux, Applied Mathematics and Mechanics (English Edition) 33 (2012) 605-620.
[28] M. A. A. Mahmoud, S. A. Waheed, Variable fluid properties and thermal radiation effects on flow and heat transfer in micropolar fluid film past moving permeable infinite flat plate with slip velocity, Applied Mathematics and Mechanics (English Edition) 33 (2012) 663-678.
[29] G. Jithender Reddy, R. Srinivasa raju, Siva Reddy Sheri, Finite element analysis of soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate with heat absorption, International Journal of Mathematical Archive 5(4) (2014) 211-220.
[30] S. P. Anjali Devi1, D. Vasantha kumari, Numerical investigation of slip flow effects on unsteadyhydromagnetic flow over a stretching surface with thermal radiation, Int. J. Adv. Appl. Math. and Mech. 1(4) (2014) 20-32.
[31] M. Gnaneswara Reddy, Heat and mass transfer effects on unsteady MHD flow of a chemically reacting fluid past an impulsively started vertical plate with radiation,Int. J. Adv. Appl. Math. and Mech 1(2)(2013) 1-15.
[32] B. Gebhart, Effect of viscous dissipation in natural convection, J. Fluid Mech. 14(2) (1962) 225-232.
[33] B. Gebhart, J. Mollendraf, Viscous dissipation in external natural convection flows, J. Fluid. Mech. 38 (1969) 97-107.
[34] R.L. Maharajan, B. B. Gebhart, Influence of viscous heating dissipation effects in natural convective flows, Int. J. Heat Mass Transfer 32(7) (1989) 1380-1382.
[35] K. Vajravelu, A. Hdjinicolaou, Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation, Int. Comm. Heat Mass Transfer 20 (1993) 417-430.
[36] J. Anand Rao, R. Srinivasa Raju, Applied Magnetic Field on Transient Free Convective Flow of an Incompressible Viscous Dissipative Fluid in a Vertical Channel, Journal of Energy, Heat and Mass Transfer 32 (2010) 265-277.
[37] Emmanuel Osalusi, Jonathan Side, Robert Harris, Barry Johnston, On the Effectiveness of Viscous Dissipation and Joule Heating on Steady MHD and Slip flow of a Bingham fluid over a porous rotating disk in the presence of Hall and ion-slip Currents, Romanian Reports in Physics 61(1) (2009) 71-93.