Int. J. Adv. Appl. Math. and Mech. 2(3) (2015) 225 - 233 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

Flow of slightly thermo-viscous fluid in a porous slab bounded between two permeable parallel plates

Research Article

N. Pothanna[1, *], N. Ch. Pattabhi Ramacharyulu[2], P. Nageswara Rao[1]

[1]*Department of Mathematics, VNR Vignana Jyothi Institute of Engineering and Technology, Bachupally, 500 090, Hyderabad, India*

[2]*Department of Mathematics, National Institute of Tecnology, Warangal, India*

Received 03 February 2015; accepted (in revised version) 06 March 2015

Abstract: In this paper we examined the problem of steady flow of a second order slightly thermo-viscous incompressible fluid through a porous slab bounded between two infinitely spread permeable parallel plates. The effects of various material parameters like Darcy's porosity parameter, Thermo-mechanical stress interaction coefficient, Strain thermal conductivity coefficient and Suction/injection parameter on the flow field have been discussed with the help of illustrations. It is worth mentioning that the variations of the velocity and temperature of the fluid increase at the faster rate. This effect can be attributed due to the strain thermal conductivity of the fluid.

MSC: 35G30 • 74A15

Keywords: Darcy's Flux • Darcy's Porosity Parameter • Suction/Injection Parameter • Strain Thermal Conductivity Coefficient • Thermo-mechanical Stress Coefficient

1. Introduction

The Non-newtonian fluid flows through porous media have been a subject of both experimental and theoretical research for over one and half centuries, because of its wide range of application in diverse areas of knowledge in Science,Engineering,Space systems,Geophysics,Energy systems,Technology based petroleum industry,Biophysics,Astrophysics,Soil sciences,Agricultural sciences,Biomechanics, Biomedicines, Biomedical Engineering,Artificial dialysis and so on. Some practical problems involving such studies include the percolation of water through solids,the extraction and filtration of oil from wells,the drainage of water for irrigation,the aquifier considered by the ground water hydrologists, the reserve bed used for filtering drinking water,the oil reservoir treated by the reservoir engineer and the seepage through slurries in drains by the sanitary engineer etc. Filters and filter beds are employed in most chemical processes. A chemical reactor would be filled with porous pellets impregnated with a catalyst. Distillation and absorption columns are often filled with beads or porous packing in a variety of shapes. The movement of gases and liquids through porous media is common to many industrial processes. In nuclear industries,porous medium is used for effective and efficient insulation and for emergency cooling of nuclear reactors. Considerable interest has been evinced in the recent years on the study of thermo-viscous flows through porous media because of its natural occurrence and its importance in industrial geophysical and medical applications. The flow of oils through porous rocks,the extraction of energy from geo-thermal regions,the filtration of solids from liquids,the flow of liquids through ion-exchange beds,cleaning of oil-spills are some of the areas in which flows through porous media are noticed. In the physical world, the investigation of the flow of thermo-viscous fluid through a porous medium has become an important topic due to the recovery of crude oil

* Corresponding author.
E-mail address: pothareddy81@gmail.com

Nomenclature

$\alpha_1 = -p$	p =Fluid Pressure
α_3=2μ	μ =Coeffcient of classical (Newtonian) viscocity
$\alpha_5 = 4\mu_c$	μ_c =Coefficient of (Reiner-Rivlin) cross-viscosity
α_6	Thermomechanical stress interaction coefficient (Dimensional form)
a_6	Thermo-mechanical stress interaction coefficient (NonDimensional form)
α_8	Thermo-stress viscosity coefficient(Dimensional form)
$\beta_1 = k$	(Fourier) thermal conductivity coefficient
$\beta_3 = k$	Strain thermal conductivity coefficient(Dimensional form)
b_3	Strain thermal conductivity coefficient(Non-Dimensional form)
k^*	$Darcy's$ porosity parameter (Dimensional form)
S	$Darcy's$ porosity parameter(Non-Dimensional form)
v_0	Suction/injection parameter (Dimensional form)
V_0	Suction/injection parameter (Non-Dimensional form)

from the pores of reservoir rocks.

Henry Darcy in 1858 observed that, the discharge rate of the fluid percolating in a porous medium is proportional to the hydraulic head and inversely as the distance between the inlet and outlet i.e. proportional to the pressure gradient. Darcy, based on the findings of a large number of flows through porous media, proposed the empirical law known as Darcy's law:

$$Q = -\frac{k^*}{\mu} A.(\nabla P)$$

where Q is the total discharge of the fluid, k^* is the permeability of the medium, A is the cross-sectional area to flow the fluid, μ is the viscosity of the fluid and ∇P is the pressure gradient in the direction of the fluid flow. Dividing both sides of the above equation by the area then the above equation becomes

$$q = -\frac{k^*}{\mu}(\nabla P)$$

where q is known as Darcy's fluid flux and we know that the fluid velocity(u) is proportional to the fluid flux (q) by the porosity(k^*) , then

$$\nabla P = -\frac{\mu}{k^*} q$$

The negative sign indicates that fluids flows from the region of high pressure to low pressure.
The concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences was introduced by Koh and Eringin in 1963. For such a class of fluids, the stress-tensor $'t'$ and heat flux bivector $'h'$ are postulated as polynomial functions of the kinematic tensor, viz., the rate of deformation tensor $'d'$:

$$d_{ij} = \frac{(u_{i,j} + u_{j,i})}{2}$$

and thermal gradient bivector $'b'$

$$b_{ij} = \epsilon_{ijk}\theta_k$$

where u_i is the i^{th} component of velocity and θ is the temperature field.
A second order theory of thermo-viscous fluids is characterized by the pair of thermo-mechanical constitutive relations:

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8(db - bd) \text{ and } h = \beta_1 b + \beta_3(db + bd)$$

with the constitutive parameters α_i, β_i being polynomials in the invariants of d and b in which the coefficients depend on density(rho) and temperature(θ) only. The fluid is Stokesian when the stress tensor depends only on the rate of deformation tensor and Fourier-heat-conducting when the heat flux bivector depends only on the temperature gradient-vector, the constitutive coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient of viscosity respectively and α_5 as that of cross-viscosity.

The development of non-linear theory reflecting the interaction/interrelation between thermal and viscous effects has been preliminarily studied by Koh and Eringen [1] and Coleman and Mizel [2]. A systematic rational approach for such class of fluids has been developed by Green and Nagdhi [3]. In 1965 Kelly [4] examined some simple shear flows of second order thermo-viscous fluids . Nageswara Rao and Pattabhi Ramacharyulu [5] later studied some steady state problems dealing with certain flows of thermo-viscous fluids. Some more problems E.Nagaratnam [6] studied in plane, cylindrical and spherical geometries. The problem of steady flow of a second order thermo-viscous fluid over an infinite plate was studied by Nageswara Rao and Pattabhi Ramacharyulu [7]. The medium of flow in these cases is clear (i.e. non-porous).

None of the theories so proposed takes into account that the porosity effects on the flows of thermo-viscous fluids. Keeping in mind the relevance and growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and industry; the present paper attempts to study the both the thermo-viscous and porosity effects on flows through a porous medium.

2. Basic equations

Flow of incompressible homogeneous thermo-viscous fluids through porous medium satisfies the usual conservation equations: Equation of continuity

$$v_{i,i} = 0$$

Equation of momentum

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i} - \frac{\mu}{k^*} v_i$$

and the energy equation

$$\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma - \frac{\mu}{k^*} v_i v_i$$

where

F_k = Componentofexternalforceperunitmass
$c = S$pecificheat
γ = Thermalenergysourceperunitmass
$q_i = i^{th}$Componentofheatfluxbivector $= \frac{\epsilon_{ijk} h_{jk}}{2}$
t_{ij} = Thecomponentsofstresstensor
d_{ij} = Thecomponentsofrateofdeformationtensor

3. Mathematical analysis and formulation

Consider the slow steady flow of a second order slightly thermo-viscous fluid through a porous medium bounded between two permeable parallel plates [See Fig. 1]. The flow in the absence of external body forces, internal heat source and the pressure gradient in the flow direction is examined. The motion of the upper plate, moving with a given velocity u_0 relative to the lower plate along the flow direction. With reference to a coordinate system O(XYZ) with origin on the fixed plate, the X-axis in the direction of the plate movement, Y-axis perpendicular to plates, the plates are represented by y = 0 and y = h . Further the two plates are maintained at constant temperatures θ_0 and θ_1 respectively.

Let the steady flow between the two permeable parallel plates is characterized by the velocity field $[u(y), v_0, 0]$ and temperature field $\theta(y)$. This choice of the velocity evidently satisfies the continuity equation.
Adopting the notation given in nomenclature, the basic equations characterizing the flow are the following:

Equation of motion in X-direction :

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial^2 \theta}{\partial y^2} - \frac{\mu}{k^*} u \tag{1}$$

Equation of motion in Y-direction :

$$0 = \mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho f_y \tag{2}$$

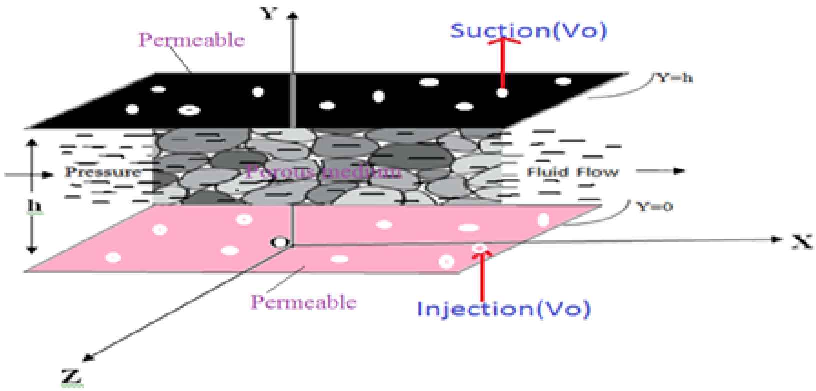

Fig. 1. Flow configuration

Equation of motion in Z-direction :

$$0=\alpha_8\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}\right)+\rho f_z \tag{3}$$

and the energy equation:

$$\rho c\left(u\frac{\partial \theta}{\partial x}+v_0\frac{\partial \theta}{\partial y}\right)=\mu\left(\frac{\partial u}{\partial y}\right)^2-\alpha_8\frac{\partial \theta}{\partial x}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}+k\frac{\partial^2\theta}{\partial y^2}+\beta_3\frac{\partial \theta}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k*}u^2 \tag{4}$$

The boundary conditions are :

$$\mathrm{u}=0,\theta=\theta_0 \text{ at } \mathrm{y}=0 \; and \; \mathrm{u}=u_0,\theta=\theta_1 \text{ at } \mathrm{y}=\mathrm{h} \tag{5}$$

It can be noted from the Eqs. (2) and (3) that a rectilinear motion between two parallel plates of a thermo-viscous fluid generates forces $\rho f_y=-\mu_c\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2$ in the Y-direction and $\rho f_z=-\alpha_8\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}\right)$ in the Z-direction, such a force of ρf_y is the normal stress noted by Reiner-Rivlin(1954) for visco-inelastic fluids, the force ρf_z is thermo-mechanical viscous stress force generated perpendicular to both the flow direction and the Reiner-Rivlin normal stress. This is observed by Koh and Eringen(1963) for a second order thermo-viscous fluids. The generation of normal stress ρf_y (proportional to the cross-viscosity coefficient μ_c) is also observed by Lakshmana Rao and Bhatnagar(1957) who investigated the rectilinear flow of visco-inelastic fluid through non-circular ducts.

The following non-dimensional quantities are introduced to convert the above basic equations to the non-dimensional form.

$$\begin{array}{llllll} Y=\frac{y}{h}, & U=\frac{\rho h}{\mu}u, & u_0=\frac{\mu}{\rho h}, & U_0=\frac{\rho h}{\mu}u_0, & T=\frac{\theta-\theta_0}{\theta_1-\theta_0}, & C_2=\frac{h}{\theta_1-\theta_0}\frac{\partial \theta}{\partial x} \\ S=\frac{h^2}{k*}, & V=\frac{\rho h}{\mu}v_0, & p_r=\frac{\mu_c}{k}, & b_3=\frac{\beta_3}{\rho h^2 c}, & a_6=\frac{\alpha_6\rho(\theta_1-\theta_0)^2}{\mu^2}, & a_1=\frac{\mu^2}{\rho h^2 c(\theta_1-\theta_0)} \end{array}$$

where $C_2$is non-dimensional temperature gradient which is assumed to be constant. Further , S is the non-dimensional Darcy's porosity parameter and is the non-dimensional Suction/injection parameter.

In terms of these non-dimensional quantities , the equations of motion and energy reduce to the following:

Equation of motion in the X-direction:

$$V_0\frac{dU}{dY}=\frac{d^2U}{dY^2}-a_6C_2\frac{d^2T}{dY^2}-SU \tag{6}$$

Equation of motion in the Y-direction :

$$0=\mu_c\frac{d}{dY}\left(\frac{dU}{dY}\right)^2+\rho F_Y \tag{7}$$

Equation of motion in the Z-direction :

$$0=\alpha_8\frac{d}{dY}\left(\frac{dT}{dY}\frac{dU}{dY}\right)+\rho F_Z \tag{8}$$

and the energy equation :

$$UC_2+V_0\frac{dT}{dY}=a_1\left[\left(\frac{dU}{dY}\right)^2-a_6C_2\frac{dT}{dY}\frac{dU}{dY}-SU^2\right]+b_3C_2\frac{d^2U}{dY^2}+\frac{1}{p_r}\frac{d^2T}{dY^2} \tag{9}$$

The Eqs. (6) and (9) are non linear and coupled differential equations in terms of the velocity field $U(Y)$ and the temperature field $T(Y)$. The perturbation technique is most powerful and elegant method to get the solutions of these type of equations. Hence this technique is employed to obtain the solutions of equation of motion and energy together with the boundary conditions :

$$U(0)=0, U(1)=1 \tag{10}$$

and

$$T(0)=0, T(1)=1 \tag{11}$$

4. Method of solution

The fluid is assumed to be slightly thermo-viscous in the sense that the interaction between the mechanical stress and thermal gradients (characterized by the coefficient a_6) is of a lower order in magnitude compared to the magnitude of viscous dissipation $\left(\frac{dU}{dY}\right)^2$, the non-Fourier heat transfer coefficient b_3 and the Darcy'sporosity parameter S (i.e. terms containing a_6 in the momentum and energy balance equations taken to be smaller than the other terms in the energy equation).
For a slightly thermo-viscous fluid (in the sense that is small) , the thermo-mechanical stress interaction coefficient is very small (i.e.$a_6 << 1$) and so the flow of the thermo-viscous fluid is a perturbation over a non-thermo viscous fluid. In such a case the velocity and temperature fields can be expressed as

$$U(Y)=U^{(0)}(Y)+a_6U^{(1)}(Y)+a_6^2U^{(2)}(Y)+\ldots\ldots \tag{12}$$

and

$$T(Y)=T^{(0)}(Y)+a_6T^{(1)}(Y)+a_6^2T^{(2)}(Y)+\ldots\ldots \tag{13}$$

with a_6 is the perturbation parameter. Substituting the Eqs. (12) and (13) in Eqs. (6) and (9) and collecting terms of like powers of a_6 , we obtain the equations in the successive approximations.

4.1. Basics or Zero order approximation

The Equations in this approximation

$$V_0\frac{dU^{(0)}}{dY}=\frac{d^2U^{(0)}}{dY^2}-SU^{(0)} \tag{14}$$

and

$$U^{(0)}C_2+V_0\frac{dT^{(0)}}{dY}=b_3C_2\frac{d^2U^{(0)}}{dY^2}+\frac{1}{p_r}\frac{d^2T^{(0)}}{dY^2} \tag{15}$$

together with the boundary conditions :

$$U^{(0)}(0)=0, U^{(0)}(1)=1 \tag{16}$$

and

$$T^{(0)}(0)=0, T^{(0)}(1)=1 \tag{17}$$

From the Eq. (14) and using the boundary conditions in (16) the velocity distribution is obtained as

$$U^{(0)}(Y)=e^{m_1(Y-1)}\frac{sinhm_2(Y)}{sinhm_2} \tag{18}$$

The Eq. (15) together with theEq. (18) and the boundary conditions (17) yields the temperature distribution

$$T^{(0)}(Y)=\frac{e^{V_0p_rY}-1}{e^{V_0p_r}-1}+\frac{(e^{V_0p_rY}-e^{V_0p_r})(q_2-q_3)}{e^{V_0p_r}-1}+\frac{(1-e^{V_0p_rY})e^{m_1}(q_2e^{m_2}-q_3e^{-m_2})}{e^{V_0p_r}-1}+\frac{(e^{V_0p_r}-1)e^{m_1Y}(q_2e^{m_2Y}-q_3e^{-m_2Y})}{e^{V_0p_r}-1} \tag{19}$$

where

$$m_1=\frac{V_0}{\upsilon}, m_2=\sqrt{\frac{V_0^2}{\upsilon}+\frac{4S}{\mu}}, q_2=\frac{p_re^{-m_1}C_2(m_1+m_2)\left(C\rho-b_3(m_1+m_2)^2\right)}{d_1sinhm_2} \text{ and } q_3=\frac{p_re^{-m_1}C_2(m_1-m_2)\left(C\rho-b_3(m_1-m_2)^2\right)}{d_2sinhm_2}$$

here

$$d_1=(m_1+m_2)(m_1+m_2-V_0p_r) \text{ and } d_2=(m_1-m_2)(m_1-m_2-V_0p_r)$$

4.2. First order approximation

The Equations in this approximation

$$V_0\frac{dU^{(1)}}{dY}=\frac{d^2U^{(1)}}{dY^2}+C_2\frac{d^2T^{(0)}}{dY^2}-SU^{(1)} \tag{20}$$

and

$$U^{(1)}C_2+V_0\frac{dT^{(1)}}{dY}=b_3C_2\frac{d^2U^{(1)}}{dY^2}+\frac{1}{p_r}\frac{d^2T^{(1)}}{dY^2} \tag{21}$$

together with the boundary conditions :

$$U^{(1)}(0)=0, U^{(1)}(1)=1 \tag{22}$$

and

$$T^{(1)}(0)=0, T^{(1)}(1)=1 \tag{23}$$

In these equations, $U^{(0)}(Y)$ and $T^{(0)}(Y)$ are given by (18) and (19). The Eq. (20), (21) and the boundary conditions (21) yield the velocity distribution

$$U^{(1)}(Y)=\frac{C_2e^{m_1Y}q_1}{\mu sinhm_2(V_0p_r-m_1)^2}\{e^{(V_0p_r-m_1)Y}sinhm_2-e^{(V_0p_r-m_1)}sinhm_2Y-sinhm_2Y-1\}$$
$$+\frac{C_2e^{m_1Y}}{2m_2\mu sinhm_2}\{(q_2e^{m_2}-q_3e^{-m_2})(Y-1)sinhm_2Y\} \tag{24}$$

where

$$q_1=\frac{(V_0p_r)^2}{(V_0p_r-1)sinhm_2}[sinhm_2+h_1+h_2]$$

here

$$h_1=\frac{p_re^{-m_1}(m_1^2+m_2^2)}{d_1} \text{ and } h_2=\frac{2p_re^{m_2}m_1m_2}{d_2}$$

and the temperature distribution

$$T^{(1)}(Y)=\frac{1}{d_1(e^{V_0p_r}-1)}\{(e^{V_0p_rY}-e^{V_0p_r})T_1+e^{m_1-m_2}(1-e^{V_0p_rY})(T_1+T_2)+e^{(m_1-m_2)Y}(e^{V_0p_r}-1)(T_1+T_2Y)\}$$
$$+\frac{T_2[2(m_1-m_2)-V_0p_r]}{d_1^2(e^{V_0p_r}-1)}\{(e^{V_0p_rY}-e^{V_0p_r})+e^{m_1-m_2}(1-e^{V_0p_rY})+e^{(m_1-m_2)Y}(e^{V_0p_r}-1)\}$$
$$+\frac{1}{d_2(e^{V_0p_r}-1)}\{(e^{V_0p_rY}-e^{V_0p_r})T_3+e^{m_1+m_2}(1-e^{V_0p_rY})(T_3+T_4)+e^{(m_1+m_2)Y}(e^{V_0p_r}-1)(T_3+T_4Y)\}$$
$$+\frac{T_4[2(m_1+m_2)-V_0p_r]}{d_2^2(e^{V_0p_r}-1)}\{(e^{V_0p_rY}-e^{V_0p_r})+e^{m_1+m_2}(1-e^{V_0p_rY})+e^{(m_1+m_2)Y}(e^{V_0p_r}-1)\}$$
$$+\frac{T_5}{V_0p_r}\{e^{V_0p_r}(1-e^{V_0p_rY})-Ye^{V_0p_rY}(1-e^{V_0p_r})\} \tag{25}$$

which satisfy the boundary conditions in (23)
where

$$T_1=\frac{p_rC_2^2(C\rho-(m_1-m_2)^2)}{2\mu e^{m_1}sinhm_2}\left\{\frac{q_1(e^{V_0p_r}-e^{m_1+m_2})}{(V_0p_r-m_1)^2-m_2^2}+\frac{q_2e^{m_1+m_2}+q_3e^{m_1-m_2}}{2m_2}\right\}+\frac{q_3C_2(m_1-m_2)}{\mu m_2}$$

$$T_2=\frac{q_3C_2}{2\mu m_2}[\rho cp_rC_2+(m_1-m_2)^2]$$

$$T_3=\frac{p_rC_2^2(C\rho-(m_1+m_2)^2)}{2\mu e^{m_1}sinhm_2}\left\{\frac{q_1(e^{V_0p_r}-e^{m_1-m_2})}{(V_0p_r-m_1)^2-m_2^2}+\frac{q_2e^{m_1+m_2}+q_3e^{m_1-m_2}}{2m_2}\right\}+\frac{q_2C_2(m_1+m_2)}{\mu m_2}$$

$$T_4=\frac{q_2C_2}{2\mu m_2}[\rho cp_rC_2+(m_1+m_2)^2] \text{ and } T_5=\frac{q_1C_2p_r(\rho cC_2+V_0^2p_r)}{\mu[(V_0p_r-m_1)^2-m_2^2]}$$

The velocity and temperature distribution up to the first order approximation is as follows:

The velocity distribution :

$$U(Y)=U^{(0)}(Y)+a_6U^{(1)}(Y)$$

and the temperature distribution :

$$T(Y)=T^{(0)}(Y)+a_6T^{(1)}(Y)$$

Note: The variations of the velocity and temperature profiles for different values of the flow parameters are shown in the illustrations Fig. 2 - Fig. 13.

5. Results and discussion

The flow presented in this paper is investigated under the assumption of slow steady motion of a fluid so that the viscous dissipation , Darcy'sdissipation and thermo-mechanical stress interaction coefficient terms are neglected. The effects of various material parameters such as Strain thermal conductivity coefficient(b_3), thermo-mechanical stress interaction coefficient(a_6), Darcy'sporosity parameter(S) and the Suction/injection parameter(V_0) on the velocity and temperature distributions have been illustrated for the fixed values $C_2 = 1$,$p_r = 1$ and $a_1 = 1$.
The variations of velocity and temperature profiles for small values of Darcy'sporosity parameter(S) and the Suction/injection parameter(V_0) are shown in Fig. 2, Fig. 3, Fig. 4, Fig. 8, Fig. 9 and Fig. 10 and for large values of Darcy'sporosity parameter(S) and the Suction/injection parameter((V_0)) are shown in Fig. 5, Fig. 6, Fig. 7, Fig. 11, Fig. 12 and Fig. 13.

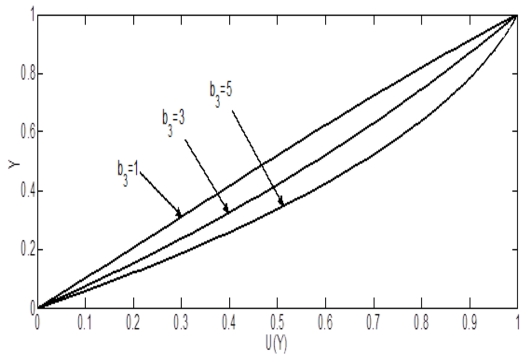

Fig. 2. Variations of the velocity profiles with S=1, $V_0 = 0.1$, $a_6 = 0.01$ and b_3

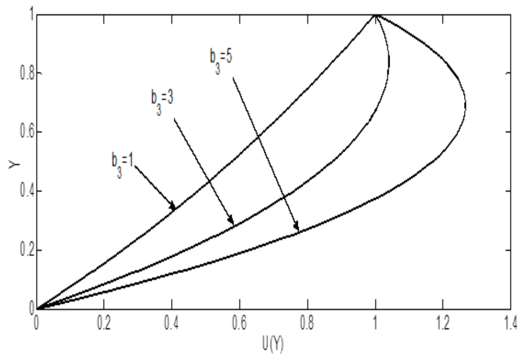

Fig. 3. Variations of the velocity profiles with S=1, $V_0 = 0.1$, $a_6 = 0.03$ and b_3

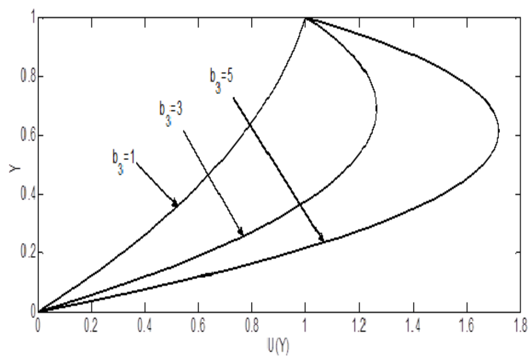

Fig. 4. Variations of the velocity profiles with S=1, $V_0 = 0.1$, $a_6 = 0.05$ and b_3

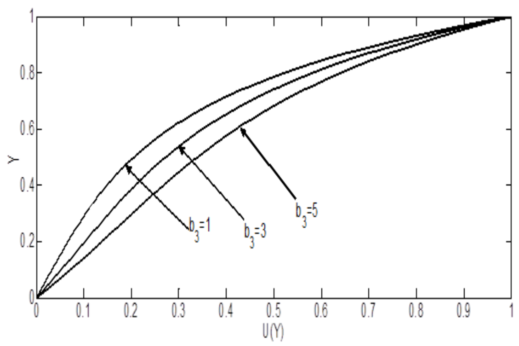

Fig. 5. Variations of the velocity profiles with S=10, $V_0 = 1$, $a_6 = 0.01$ and b_3

It is observed from the Fig. 2, Fig. 3, Fig. 4 that, the velocity profiles raise towards the upper plate as the values of Strain thermal conductivity coefficient(b_3) increases from 1 to 5. Fig. 2 shows that , for small value of Strain thermal conductivity coefficient(i.e. for b_3=1), the straight line profile is realized and the maximum velocity attains at the upper plate. From the Fig. 3, Fig. 4, it is observed that , for large values of Strain thermal conductivity coefficient(i.e. for b_3 =3 ,5) the maximum velocity attains near the centre of the channel. Fig. 2, Fig. 3, Fig. 4 also shows that, as the value of thermo-mechanical stress interaction coefficient(a_6) increases , the velocity profiles raise and the rate of raise of the velocity profiles for small value of (i.e. for b_3 =1) are very slow compared to the rate of raise of those for large values of (i.e. for b_3=3 , 5).
From the Fig. 5, Fig. 6, Fig. 7 , It is noticed that, as the value of thermo-mechanical stress interaction coefficient(a_6) and Strain thermal conductivity coefficient(b_3) both increases, the velocity profiles increase and coincide with the velocity of the upper plate. Fig. 5 indicate that , as the value of Strain thermal conductivity coefficient(b_3) increases , the bending of the velocity profiles increase and attain the maximum velocity at the hotter plate.
It is noticed from the Fig. 8, Fig. 9 and Fig. 10 that , as the value of thermo-mechanical stress interaction coefficient(a_6) and Strain thermal conductivity coefficient(b_3) both increases, the temperature increases and coincide with the temperature of the hotter plate. For small value of the Strain thermal conductivity coefficient(i.e. for b_3=1), the temperature of the fluid sharply raises and attains its maximum temperature at the upper plate. For large values of Strain thermal conductivity coefficient(i.e. for b_3=3 , 5), the maximum temperature occurs near the

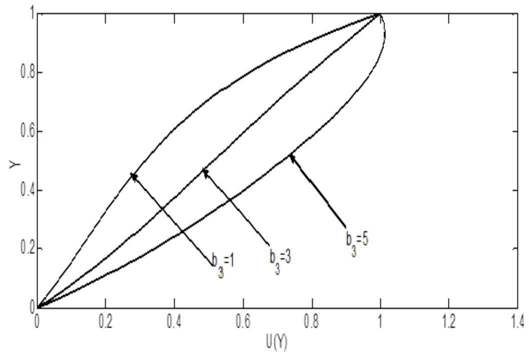

Fig. 6. Variations of the velocity profiles with S=10, $V_0 = 1$, $a_6 = 0.03$ and b_3

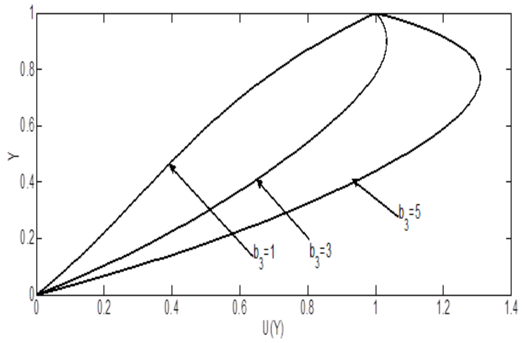

Fig. 7. Variations of the velocity profiles with S=10, $V_0 = 1$, $a_6 = 0.05$ and b_3

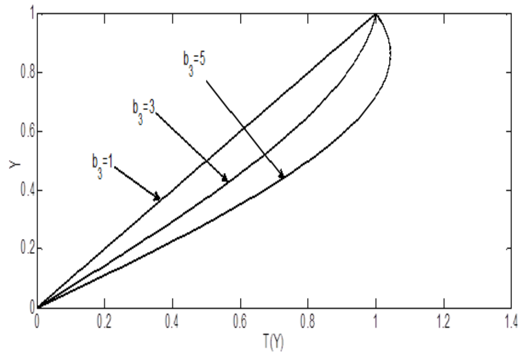

Fig. 8. Variations of the Temperature profiles with S=1, $V_0 = 0.1$, $a_6 = 0.01$ and b_3

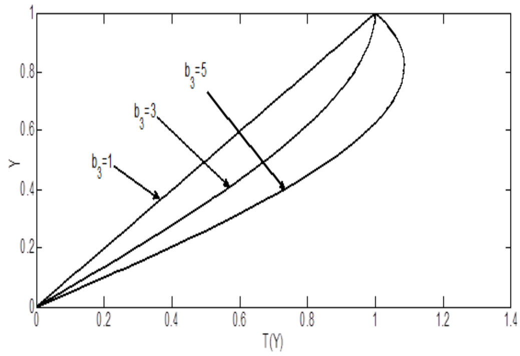

Fig. 9. Variations of the Temperature profiles with S=1, $V_0 = 0.1$, $a_6 = 0.03$ and b_3

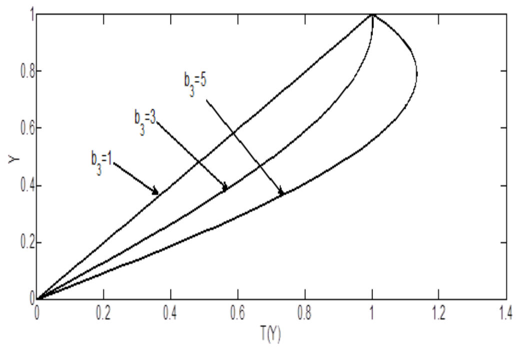

Fig. 10. Variations of the Temperature profiles with S=1, $V_0 = 0.1$, $a_6 = 0.05$ and b_3

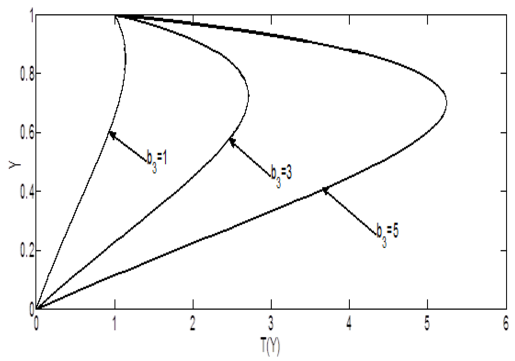

Fig. 11. Variations of the Temperature profiles with S=10, $V_0 = 1$, $a_6 = 0.01$ and b_3

hotter plate.

From the Fig. 11, Fig. 12 and Fig. 13 , It is noticed that, the temperature profiles increase faster as the value of Strain thermal conductivity coefficient(b_3) increases from 1 to 5. For small value of Strain thermal conductivity coefficient(i.e. for b_3=1), the rate of increase of temperature of the fluid is rather very slow while for large values of Strain thermal conductivity coefficient(i.e. for b_3=3 ,5) , the rate of increase of the fluid temperature is very fast.

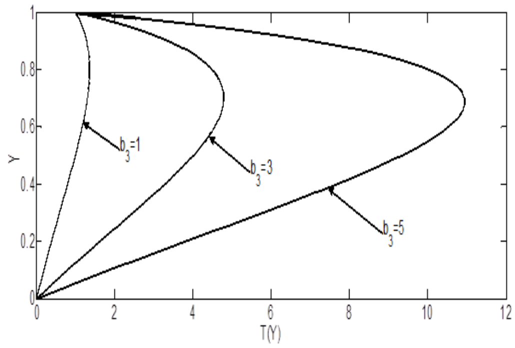

Fig. 12. Variations of the Temperature profiles with S=10, $V_0 = 1$, $a_6 = 0.03$ and b_3

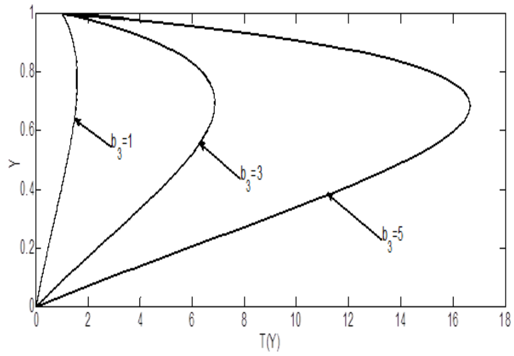

Fig. 13. Variations of the Temperature profiles with S=10, $V_0 = 1$, $a_6 = 0.05$ and b_3

Acknowledgements

The authors would like to thank different software companies (MatLab and LATEX) for developing the codes that help in the computation, verification of graphical results and editing the paper .

References

[1] S. L. Koh, A. C. Eringin, On the foundations of non-linear thermo-elastic fluids. Int.J. Engg. Scil. 1 (1963) 199 -229.
[2] B. D. Coleman, V. J. Mizel, On the existence of caloric equations of state, Journal of Chem.Phys. 40(1964) 1116-1125.
[3] A. E. Green, P. M. Naghdi, A dynamical theory of interacting Continua , Int.J. Engg. Sci. 3(1965) 231-241.
[4] P. D. Kelly, Some viscometric flows of incompressible thermo-viscous fluids, Int. J. Engg. sci. 2(1965) 519-537.
[5] P. Nageswara Rao, Some problems in thermo- Viscous fluid Dynamics, Ph. D thesis, K. U Warangal, 1979.
[6] E. Nagaratnam, Some steady and unsteady flows of thermo- viscous fluids, Ph. D thesis, J.N.T.U Hyderabad, 2006.
[7] P. Nageswara Rao, N. Ch. Pattabhi Ramacharyulu , Steady flow of a second order thermo- viscous fluid over an infinite plate, Proc.Ind.Acad.Sci., 88A 2(1979) 157-162.
[8] W. E. Langlois, Steady flow of slightly visco-elastic fluid between rotating cylinders, Q.APP.Math. 1(1963) 61-71.
[9] R. S. Rivlin, The solution of problems in second-order elasticity theory, J.Rat.Mech.Anal. 2(1954) 53-59.
[10] N. Pandya, A. K. Shukla, Soret-Dufour and Radiation effect on unsteady MHD flow over an inclined porous plate embedded in porous medium with viscous dissipation, Int. J. Adv. Appl. Math. and Mech. 2(1) (2014)107âĂŞ119.
[11] M. S. Alam, M. Nurul Huda , A new approach for local similarity solutions of an unsteady hydromagnetic free convective heat transfer flow along a permeable flat surface, Int. J. Adv. Appl. Math. and Mech. 1(2) (2013) 39-52.
[12] Ramesh Chand, Arvind Kumar, Thermal instability of rotating Maxwell visco-elastic fluid with variable gravity in porous medium, Int. J. Adv. Appl. Math. and Mech. 1(2) (2013) 30-38.
[13] L. Preziosi, A. Farina, On Darcy's Law for growing porous media, Int.J.Non-linear mechanics 37(2002) 485-491.
[14] K. V. Yamamoto, Z. Yoshida, Flow through a porous wall with convective acceleration, Journal of Physical Society 3(1974) 774-779.
[15] G. S. Beaver, D. D. Joseph, Boundary conditions at a naturally permeable wall, J. Fluid Mechanism 30(1967) 197-207.
[16] Srinivas Joshi, P.Nageswar Rao and N.Ch. Pattabhi Ramacharyulu, Steady flow of thermo-viscous fluid between two parallel porous plates in relative motion, IEEMS 9(2010) 58-65.
[17] J. Bear, Dynamic of fluids in porous media, American Elsevier Pub.Co.inc., Newyork, 1970.
[18] R. E. Collis, Flow of fluids through porous materials, Reinhold publishing corporation, Newyork, 1961.
[19] S. K. Lakshmana Rao, P. L. Bhatnagar , Steady motion of Non-newtonian fluids in tubes, Proc. 3rd congress on ISTAM (1957) 225-234.