Int. J. Adv. Appl. Math. and Mech. 2(3) (2015) 252 - 257 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

International Journal of Advances in Applied Mathematics and Mechanics

On the properties of generalized multiplicative coupled fibonacci sequence of r^{th} order

Research Article

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Received 02 February 2015; accepted (in revised version) 28 February 2015

Abstract: Coupled Fibonacci sequences of lower order have been generalized in number of ways.In this paper the Multiplicative Coupled Fibonacci Sequence has been generalized for r^{th} order with some new interesting properties.

MSC: 11B39 • 11B37 • 11B99

Keywords: Fibonacci sequence • Coupled Fibonacci sequence • Recurrence relation

1. Introduction

Definition 1.1.

The Multiplicative Coupled Fibonacci Sequence of 2^{nd} order is defined as, Let $\{X_i\}_{i=0}^{i=\infty}$ and $\{Y_i\}_{i=0}^{i=\infty}$ be two infinite sequences and four arbitary real numbers a, b, c, d are given.The Multiplicative Coupled Fibonacci Sequence of 2^{nd} order is generated by the following four different ways:

First scheme:

$$X_{n+2} = X_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+2} = Y_{n+1} \cdot Y_n, n \geq 0$$

Second scheme:

$$X_{n+2} = Y_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+2} = X_{n+1} \cdot Y_n, n \geq 0$$

Third scheme:

$$X_{n+2} X_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+2} = Y_{n+1} \cdot X_n, n \geq 0$$

Fourth scheme:

$$X_{n+2} = Y_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+2} = X_{n+1} \cdot X_n, n \geq 0$$

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Definition 1.2.

The Multiplicative Coupled Fibonacci Sequence of 3^{rd} order is defined as, Let $\{X_i\}_{i=0}^{i=\infty}$ and $\{Y_i\}_{i=0}^{i=\infty}$ be two infinite sequences and six arbitary real numbers a, b, c, d, e, f are given.The Multiplicative Coupled Fibonacci Sequence of 3^{rd} order is generated by the following eight different ways:

First scheme:

$$X_{n+3} = Y_{n+2} \cdot Y_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+3} = X_{n+2} \cdot X_{n+1} \cdot X_n, n \geq 0$$

Second scheme:

$$X_{n+3} = X_{n+2} \cdot X_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+3} = Y_{n+2} \cdot Y_{n+1} \cdot Y_n, n \geq 0$$

Third scheme:

$$X_{n+3} = Y_{n+2} \cdot Y_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+3} = X_{n+2} \cdot X_{n+1} \cdot Y_n, n \geq 0$$

Fourth scheme:

$$X_{n+3} = Y_{n+2} \cdot X_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+3} = X_{n+2} \cdot Y_{n+1} \cdot X_n, n \geq 0$$

Fifth scheme:

$$X_{n+3} = Y_{n+2} \cdot X_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+3} = X_{n+2} \cdot Y_{n+1} \cdot Y_n, n \geq 0$$

Sixth scheme:

$$X_{n+3} = X_{n+2} \cdot X_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+3} = Y_{n+2} \cdot Y_{n+1} \cdot X_n, n \geq 0$$

Seventh scheme:

$$X_{n+3} = X_{n+2} \cdot Y_{n+1} \cdot Y_n, n \geq 0$$
$$Y_{n+3} = Y_{n+2} \cdot X_{n+1} \cdot X_n, n \geq 0$$

Eigth scheme:

$$X_{n+3} = X_{n+2} \cdot Y_{n+1} \cdot X_n, n \geq 0$$
$$Y_{n+3} = Y_{n+2} \cdot X_{n+1} \cdot Y_n, n \geq 0$$

In recent years many authors have been generalized Coupled Fibonacci sequences of lower order in number of ways.In this paper the Multiplicative Coupled Fibonacci Sequence has been generalized for r^{th} order with some new interesting properties.

2. Multiplicative Coupled Fibonacci Sequence of r^{th} order

Definition 2.1.

The Multiplicative Coupled Fibonacci Sequence of r^{rd} order is defined as, Let $\{X_i\}_{i=0}^{i=\infty}$ and $\{Y_i\}_{i=0}^{i=\infty}$ be two infinite sequences and $2r$ arbitary real numbers $x_0, x_1, x_2, x_3......, x_{r-1}$ and $y_0, y_1, y_2, y_3......, y_{r-1}$ are given.The Multiplicative Coupled Fibonacci Sequence of r^{th} order is generated by the following 2^r different ways:

First scheme:

$$X_{n+r} = Y_{n+r-1} \cdot Y_{n+r-2} \cdot Y_{n+r-3} \cdots Y_n, n \geq 0$$
$$Y_{n+r} = X_{n+r-1} \cdot X_{n+r-2} \cdot X_{n+r-3} \cdots X_n, n \geq 0$$

$$\vdots$$

$(2^r)^{th}$ **scheme:**

$$X_{n+r} = X_{n+r-1} \cdot X_{n+r-2} \cdot X_{n+r-3} \cdots X_n, n \geq 0$$
$$Y_{n+r} = Y_{n+r-1} \cdot Y_{n+r-2} \cdot Y_{n+r-3} \cdots Y_n, n \geq 0$$

are given below:

Table 1. First few terms of the sequences under 2^{rth} scheme

n	X_{n+r}	Y_{n+r}
0	$y_0 \cdot y_1 \cdot y_2 \cdot y_3 \ldots\ldots \cdot y_{r-1}$	$x_0 \cdot x_1 \cdot x_2 \cdot x_3 \ldots\ldots \cdot x_{r-1}$
1	$x_0 \cdot x_1 \cdot x_2 \cdot x_3 \ldots\ldots \cdot x_{r-1} \cdot y_1 \cdot y_2 \cdot y_3 \ldots\ldots \cdot y_{r-1}$	$x_1 \cdot x_2 \cdot x_3 \ldots\ldots \cdot x_{r-1} \cdot y_0 \cdot y_1 \cdot y_2 \cdot y_3 \ldots\ldots \cdot y_{r-1}$
2	$x_0 \cdot x_1^2 \cdot x_2^2 \cdot x_3^2 \ldots\ldots \cdot x_{r-1}^2 \cdot y_0 \cdot y_1 \cdot y_2^2 \cdot y_3^2 \ldots\ldots \cdot y_{r-1}^2$	$x_0 \cdot x_1 \cdot x_2^2 \cdot x_3^2 \ldots\ldots \cdot x_{r-1}^2 \cdot y_0 \cdot y_1^2 \cdot y_2^2 \cdot y_3^2 \ldots\ldots \cdot y_{r-1}^2$
3	$x_0^2 \cdot x_1^3 \cdot x_2^4 \cdot x_3^4 \ldots\ldots \cdot x_{r-1}^4 \cdot y_0^2 \cdot y_1^3 \cdot y_2^3 \cdot y_3^4 \ldots\ldots \cdot y_{r-1}^4$	$x_0^2 \cdot x_1^3 \cdot x_2^3 \cdot x_3^4 \ldots\ldots \cdot x_{r-1}^4 \cdot y_0^2 \cdot y_1^3 \cdot y_2^4 \cdot y_3^4 \ldots\ldots \cdot y_{r-1}^4$
4	$x_0^4 \cdot x_1^5 \cdot x_2^7 \cdot x_3^8 \ldots\ldots \cdot x_{r-1}^8 \cdot y_0^4 \cdot y_1^6 \cdot y_2^7 \cdot y_3^7 \cdot y_4^8 \ldots\ldots \cdot y_{r-1}^8$	$x_0^4 \cdot x_1^5 \cdot x_2^7 \cdot x_3^7 \cdot x_4^8 \ldots\ldots \cdot x_{r-1}^8 \cdot y_0^4 \cdot y_1^6 \cdot y_2^7 \cdot y_3^8 \ldots\ldots \cdot y_{r-1}^8$

3. Main Results

In this section many of the fabulous properties of generalized multiplicative coupled Fibonacci sequence of r^{th} order under 2^{rth} scheme are established.

Theorem 3.1.
For every integer $n \geq 0, r \geq 0$

$$X_{n(r+1)} \cdot Y_0 = Y_{n(r+1)} \cdot X_0 \tag{1}$$

Proof. If $n = 0$,then result is true because

$$X_o \cdot Y_0 = Y_0 \cdot X_0$$

Assume that the result is true for some integer $n \geq 1$
Now,

$$X_{n+r} = Y_{n+r-1} \cdot Y_{n+r-2} \cdot Y_{n+r-3} \cdots Y_n$$
$$Y_{n+r} = X_{n+r-1} \cdot X_{n+r-2} \cdot X_{n+r-3} \cdots X_n,$$

Using Induction Method,
For, $n+1$

$$\begin{aligned} X_{(n+1)(r+1)} \cdot Y_0 &= \left[Y_{n(r+1)+r} \cdot Y_{n(r+1)+r-1} \cdot Y_{n(r+1)+r-2} \cdots Y_{n(r+1)+1}\right] \cdot Y_0 \\ &= \left[X_{n(r+1)+(r-1)} \cdot X_{n(r+1)+(r-2)} \cdot X_{n(r+1)+(r-3)} \cdots X_{n(r+1)}\right] \cdot \left[Y_{n(r+1)+r-1} \cdot Y_{n(r+1)+r-2} \cdots Y_{n(r+1)+1}\right] \cdot Y_0 \\ &= \left[X_{n(r+1)+(r-1)} \cdot X_{n(r+1)+(r-2)} \cdot X_{n(r+1)+(r-3)} \cdots X_{n(r+1)+1}\right] \cdot \left[Y_{n(r+1)+r-1} \cdot Y_{n(r+1)+r-2} \cdots Y_{n(r+1)+1}\right]\left[X_{n(r+1)} \cdot Y_0\right] \end{aligned}$$

Using induction hypothesis,

$$\begin{aligned} &= \left[X_{n(r+1)+(r-1)} \cdot X_{n(r+1)+(r-2)} \cdot X_{n(r+1)+(r-3)} \cdots X_{n(r+1)+1}\right] \cdot \left[Y_{n(r+1)+r-1} \cdot Y_{n(r+1)+r-2} \cdots Y_{n(r+1)+1}\right]\left[Y_{n(r+1)} \cdot X_0\right] \\ &= \left[X_{n(r+1)+(r-1)} \cdot X_{n(r+1)+(r-2)} \cdot X_{n(r+1)+(r-3)} \cdots X_{n(r+1)+1}\right] \cdot \left[X_{n(r+1)+r} \cdot X_0\right] \\ &= \left[Y_{n(r+1)+r+1} \cdot X_0\right] \\ &= \left[Y_{(n+1)(r+1)} \cdot X_0\right] \end{aligned}$$

Hence result is true for $n+1$. □

Theorem 3.2.
:For every integer $n \geq 0, r \geq 0$

$$X_{n(r+1)+1} \cdot Y_1 = Y_{n(r+1)+1} \cdot X_1 \tag{2}$$

Proof. : If $n = 0$,then result is true because

$$X_1 \cdot Y_1 = Y_1 \cdot X_1$$

Assume that the result is true for some integer $n \geq 1$
Now,

$$X_{n+r} = Y_{n+r-1} \cdot Y_{n+r-2} \cdot Y_{n+r-3} \cdots Y_n$$
$$Y_{n+r} = X_{n+r-1} \cdot X_{n+r-2} \cdot X_{n+r-3} \cdots X_n,$$

Using Induction Method,
For, $n+1$

$$\begin{aligned}
X_{(n+1)(r+1)+1}\cdot Y_1 &= X_{n(r+1)+(r+2)}\cdot Y_1\\
&=\left[Y_{n(r+1)+r+1}\cdot Y_{n(r+1)+r}\cdot Y_{n(r+1)+r-1}\cdots Y_{n(r+1)+2}\right]\cdot Y_1\\
&=\left[X_{n(r+1)+r}\cdot X_{n(r+1)+(r-1)}\cdot X_{n(r+1)+(r-2)}\cdots X_{n(r+1)+1}\right]\cdot\left[Y_{n(r+1)+r}\cdot Y_{n(r+1)+r-1}\cdots Y_{n(r+1)+2}\right]\cdot Y_1\\
&=\left[X_{n(r+1)+r}\cdot X_{n(r+1)+(r-1)}\cdot X_{n(r+1)+(r-2)}\cdots X_{n(r+1)+2}\right]\cdot\left[Y_{n(r+1)+r}\cdot Y_{n(r+1)+r-1}\cdots Y_{n(r+1)+2}\right]\left[X_{n(r+1)+1}\cdot Y_1\right]
\end{aligned}$$

Using induction hypothesis,

$$\begin{aligned}
&=\left[X_{n(r+1)+r}\cdot X_{n(r+1)+(r-1)}\cdot X_{n(r+1)+(r-2)}\cdots X_{n(r+1)+2}\right]\cdot\left[Y_{n(r+1)+r}\cdot Y_{n(r+1)+r-1}\cdots Y_{n(r+1)+2}\right]\left[Y_{n(r+1)+1}\cdot X_1\right]\\
&=\left[X_{n(r+1)+r}\cdot X_{n(r+1)+(r-1)}\cdot X_{n(r+1)+(r-2)}\cdots X_{n(r+1)+2}\right]\cdot\left[X_{n(r+1)+(r+1)}\cdot X_1\right]\\
&=\left[Y_{n(r+1)+(r+1)+1}\cdot X_1\right]\\
&=\left[Y_{(n+1)(r+1)+1}\cdot X_1\right]
\end{aligned}$$

Hence result is true for $n+1$. □

In the similar way, it could be proved for the following results by induction method.

Theorem 3.3.
:For every integer $n\geq 0, r\geq 0$

$$X_{n(r+1)+2}\cdot Y_2 = Y_{n(r+1)+2}\cdot X_2 \tag{3}$$

Theorem 3.4.
:For every integer $n\geq 0, r\geq 0$

$$X_{n(r+1)+3}\cdot Y_3 = Y_{n(r+1)+3}\cdot X_3 \tag{4}$$

Theorem 3.5.
:For every integer $n\geq 0, r\geq 0$ and $m\geq 0$

$$X_{n(r+1)+m}\cdot Y_m = Y_{n(r+1)+m}\cdot X_m \tag{5}$$

Proof. : If $n=0$,then result is true because

$$X_m\cdot Y_m = Y_m\cdot X_m$$

Assume that the result is true for some integer $n\geq 1$
Now,

$$\begin{aligned}
X_{n+r} &= Y_{n+r-1}\cdot Y_{n+r-2}\cdot Y_{n+r-3}\cdots Y_n\\
Y_{n+r} &= X_{n+r-1}\cdot X_{n+r-2}\cdot X_{n+r-3}\cdots X_n,
\end{aligned}$$

Using Induction Method,
For, $n+1$

$$\begin{aligned}
X_{(n+1)(r+1)+m}\cdot Y_m &= X_{n(r+1)+(r+m+1)}\cdot Y_m\\
&=\left[Y_{n(r+1)+r+m}\cdot Y_{n(r+1)+r+m-1}\cdot Y_{n(r+1)+r+m-2}\cdots Y_{n(r+1)+m+1}\right]\cdot Y_m\\
&=\left[X_{n(r+1)+r+m-1}\cdot X_{n(r+1)+(r+m-2)}\cdot X_{n(r+1)+(r+m-3)}\cdots X_{n(r+1)+m}\right]\\
&\cdot\left[Y_{n(r+1)+r+m-1}\cdot Y_{n(r+1)+r+m-2}\cdots Y_{n(r+1)+m+1}\right]\cdot Y_m\\
&=\left[X_{n(r+1)+r+m-1}\cdot X_{n(r+1)+(r+m-2)}\cdot X_{n(r+1)+(r+m-3)}\cdots X_{n(r+1)+m+1}\right]\\
&\cdot\left[Y_{n(r+1)+r+m-1}\cdot Y_{n(r+1)+r+m-2}\cdots Y_{n(r+1)+m+1}\right]\left[X_{n(r+1)+m}\cdot Y_m\right]
\end{aligned}$$

Using induction hypothesis,

$$
\begin{aligned}
&= \left[X_{n(r+1)+r+m-1} \cdot X_{n(r+1)+(r+m-2)} \cdot X_{n(r+1)+(r+m-3)} \cdots X_{n(r+1)+m+1}\right] \\
&\cdot \left[Y_{n(r+1)+r+m-1} \cdot Y_{n(r+1)+r+m-2} \cdots Y_{n(r+1)+m+1}\right]\left[Y_{n(r+1)+m} \cdot X_m\right] \\
&= \left[X_{n(r+1)+r+m-1} \cdot X_{n(r+1)+(r+m-2)} \cdot X_{n(r+1)+(r+m-3)} \cdots X_{n(r+1)+m+1}\right]\left[X_{n(r+1)+(r+m)} \cdot X_m\right] \\
&= \left[Y_{n(r+1)+(r+m)+1} \cdot X_m\right] \\
&= \left[Y_{(n+1)(r+1)+m} \cdot X_m\right]
\end{aligned}
$$

Hence result is true for $n+1$. □

Theorem 3.6.

For every integer $n \geq 0, r \geq 0$

1. $\prod_{i=1}^{i=n} X_{ri+1} = \prod_{i=1}^{i=rn} Y_i$
2. $\prod_{i=1}^{i=n} Y_{ri+1} = \prod_{i=1}^{i=rn} X_i$

Proof. : If $n = 1$,then result is true because

$$X_{r+1} = Y_r \cdot Y_{r-1} \cdot Y_{r-2} \cdots Y_1$$

Assume that the result is true for some integer $n \geq 1$
Now,

$$
\begin{aligned}
X_{n+r} &= Y_{n+r-1} \cdot Y_{n+r-2} \cdot Y_{n+r-3} \cdots Y_n \\
Y_{n+r} &= X_{n+r-1} \cdot X_{n+r-2} \cdot X_{n+r-3} \cdots X_n,
\end{aligned}
$$

Using Induction Method,
For, $n+1$

$$\prod_{i=1}^{i=n+1} X_{ri+1} = \prod_{i=1}^{i=n} X_{ri+1} \cdot X_{r(n+1)+1}$$

Using induction hypothesis,

$$
\begin{aligned}
&= \prod_{i=1}^{i=rn} Y_i \cdot X_{rn+r+1} \\
&= \prod_{i=1}^{i=rn} Y_i \cdot Y_{rn+r} \cdot Y_{rn+r-1} \cdot Y_{rn+r-2} \cdots Y_{rn+1} \\
&= \prod_{i=1}^{i=rn+r} Y_i
\end{aligned}
$$

Hence result is true for $n+1$. □

In the similar way (2) can be proved using induction method.

4. Conclusions

The identities of generalized multiplicative coupled Fibonacci sequence of r^{th} order are described here, the ideas can be extended for generalized multiplicative coupled Fibonacci sequence of r^{th} order with negative integers.

5. Acknowledgments

Authors are thankful to reviewers for their valuable comments in improvement of the paper.

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