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Local fractional Laplace decomposition method for nonhomogeneous heat equations arising in fractal heat flow with local fractional derivative

Research Article

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Abstract: In this paper, the local fractional Laplace decomposition method is used for solving the nonhomogeneous heat equations arising in the fractal heat flow within local fractional derivative. This method is coupled by the local fractional Adomian decomposition method and Laplace transform. Analytical solutions are obtained by using the local fractional Laplace decomposition method via local fractional calculus theory. The method in general is easy to implement and yields good results. Illustrative examples are included to demonstrate the validity and applicability of the new technique.

MSC: 26A33 • 34A12 • 35R11

Keywords: Fractional heat equation • Local fractional Laplace transform • Local fractional Adomian decomposition method

1. Introduction

Fractals are used in many engineering applications such as porous media modeling, nano fluids, fracture mechanics and many other applications in Nanoscale [1, 2], where various transport phenomena cannot be described by smooth continuum approach and need the fractal nature of the objects to be taken into account. For the transport phenomena performed in fractal objects the local temperature depends on the fractal dimensions where adequate physical results can be achieved by the application of local fractional models and relevant solution approaches.
Fractional heat conduction equation was studied by many researchers [3–17]. For example, Povstenko considered the thermo elasticity based on the fractional heat conduction equation[7]. Youssef suggested the generalized theory of fractional-order thermo elasticity [8]. Ezzat and El-Karamany presented the fractional-order conduction in thermo elastic medium [9]. Ezzat proposed the fractional-order heat transfer in thermoelectric fluid [10]. Sherief et al. reported the fractional-order generalized thermo elasticity with one relaxation time [11]. Vazquez et al. used the second law of thermodynamics to fractional heat conduction equation [12].Hristov considered the inverse Stefan problem and nonlinear heat conduction with Jeffreys fading memory by using the heat balance integral method [13, 14]. Davey and Prosser gave the solutions of the heat transfer on fractal and prefractal domains [15]. Ostoja Starzewski investigated thermo elasticity of fractal media [16]. Qi and Jiang discussed space-time fractional Cattaneo diffusion equation [17]. Bhrawy and Alghamdi applied the Legendre tau-spectral method to find time fractional heat equation with nonlocal conditions [18]. Atangana and Klcman suggested the Sumudu transform solving certain nonlinear fractional heat-like equations [19]. Mohammadi discussed numerical solution of Bagley-Torvik equation using Chebyshev wavelet operational matrix of fractional derivative [33].Salehbhai and Timol found the solution of some fractional differential equations [34]. Aslefallah and Rostamy solved time-fractional differential diffusion equation by theta-method [35].

Recently, the local fractional calculus [20–22] was used to deal with the discontinuous problem for heat transfer in fractal media [23–25]. The nonhomogeneous heat equations arising in fractal heat flow were considered by using

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the local fractional Fourier series method [26]. The local fractional heat conduction equation was investigated by the local fractional variation iteration method [27]. The nondifferentiable solution of one-dimensional heat equations arising in fractal transient conduction was found by the local fractional Adomian decomposition method [28]. Local fractional Laplace variational iteration method [29, 30] was considered to deal with linear partial differential equations.

In this manuscript we use the local fractional Laplace decomposition method to solve the nonhomogeneous heat equations arising in fractal heat flow with local fractional derivative. The structure of the manuscript is suggested as follows. In Section 2 the basic theory of local fractional calculus and local fractional Laplace transform are introduced. Section 3 gives the local fractional Laplace decomposition method. In Section 4, the non-differentiable solutions for nonhomogeneous heat equations arising in fractal heat flow are presented. Finally, the conclusions are considered in Section 5.

2. The heat equations arising in fractal heat flow

In this section, we present heat equations arising in fractal heat flow, the conceptions of local fractional derivative and integral and the local fractional Laplace transform [23, 30, 31].
The heat equations arising in fractal heat flow reads as follows

$$\frac{\partial^{\alpha} T(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} T(x,t)}{\partial x^{2\alpha}} = g(x,t), \tag{1}$$

with the initial conditions

$$T(0,t) = \varphi(t), \frac{\partial^{\alpha} T(0,t)}{\partial x^{\alpha}} = \psi(t). \tag{2}$$

Suppose that there is the relation

$$|f(x) - f(x_0)| < \epsilon^{\alpha}, \quad 0 < \alpha \leq 1, \tag{3}$$

with $|x - x_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in R$, then the function $f(x)$ is called local fractional continuous at $x = x_0$ and it is denoted by $\lim_{x \to x_0} f(x) = f(x_0)$.
The local fractional derivative of $f(x)$ of order α at $x = x_0$ is given by

$$f^{(\alpha)}(x_0) = \frac{d^{\alpha}}{dx^{\alpha}} f(x)|_{x=x_0} = \lim_{x \to x_0} \frac{\triangle^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}} \tag{4}$$

where $\triangle^{\alpha}(f(x) - f(x_0) \cong \Gamma(\alpha+1)(f(x) - f(x_0))$.
The local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is given by

$${}_aI_b^{(\alpha)} f(x) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^{\alpha} = \frac{1}{\Gamma(1+\alpha)} \lim_{\triangle t \longrightarrow 0} \sum_{j=0}^{N-1} f(t_j)(\triangle t_j)^{\alpha}. \tag{5}$$

where the partition of the interval $[a,b]$ is denoted as (t_j, t_{j+1}), $j = 0,...,N-1$, $t_0 = a$ and $t_N = b$ with $\triangle t_j = t_{j+1} - t_j$ and $\triangle t = \max\{\triangle t_0, \triangle t_1, \ldots\}$. The Yang-Laplace transform of $f(x)$ is given by

$$L_{\alpha}\{f(x)\} = f_s^{L,\alpha}(s) = \frac{1}{\Gamma(1+\alpha)} \int_0^{\infty} E_{\alpha}(-s^{\alpha}x^{\alpha}) f(x)(dx)^{\alpha}, \quad 0 < \alpha \leq 1, \tag{6}$$

where the latter integral converges and $s^{\alpha} \in R^{\alpha}$. The inverse formula of the Yang-Laplace transform of $f(x)$ is given by

$$L_{\alpha}^{-1}\left(f_s^{L,\alpha}(s)\right) = f(x) = \frac{1}{(2\pi)^{\alpha}} \int_{\beta-i\omega}^{\beta+i\omega} E_{\alpha}(s^{\alpha}x^{\alpha}) f_s^{L,\alpha}(s)(ds)^{\alpha}, \quad 0 < \alpha \leq 1 \tag{7}$$

where $s^{\alpha} = \beta^{\alpha} + i^{\alpha}\omega^{\alpha}$; fractal imaginary unit i^{α} and $Re(s) = \beta > 0$.
The properties for local fractional Laplace transform used in the paper are given as [23]

$$L_{\alpha}\{af(x) + bg(x)\} = af_s^{L,\alpha}(s) + bg_s^{L,\alpha}(s) \tag{8}$$

$$L_{\alpha}\{f^{(2\alpha)}(x)\} = s^{2\alpha} f_s^{L,\alpha}(s) - s^{\alpha} f(0) - f^{(\alpha)}(0) \tag{9}$$

$$L_{\alpha}\{\cos_{\alpha}(cx^{\alpha})\} = \frac{c}{s^{2\alpha} + c^2} \tag{10}$$

$$L_{\alpha}\{\cos_{\alpha}(cx^{\alpha})\} = \frac{s^{\alpha}}{s^{2\alpha} + c^2} \tag{11}$$

$$L_{\alpha}\{x^{k\alpha}\} = \frac{\Gamma(1+k\alpha)}{s^{(k+1)\alpha}} \tag{12}$$

3. Analysis of the method

Let us consider the following linear operator with local fractional derivative:

$$Ł_\alpha u(x,t) + R_\alpha u(x,t) = h(x,t), \tag{13}$$

where $Ł_\alpha = \dfrac{\partial^{2\alpha}}{\partial x^{2\alpha}}$ denotes the linear local fractional differential operator, R_α is the remaining linear operator, and $h(x,t)$ is a source term. Taking Yang-Laplace transform on Eq. (13), we obtain

$$L_\alpha\{Ł_\alpha u(x,t)\} + L_\alpha\{R_\alpha u(x,t)\} = L_\alpha\{h(x,t)\}. \tag{14}$$

By applying the local fractional Laplace transform differentiation property, we have

$$s^{2\alpha}L_\alpha\{u(x,t)\} - s^\alpha u(0,t) - u^{(\alpha)}(0,t) + L_\alpha\{R_\alpha u(x,t)\} = L_\alpha\{h(x,t)\}. \tag{15}$$

or

$$L_\alpha\{u(x,t)\} = \frac{1}{s^\alpha}u(0,t) + \frac{1}{s^{2\alpha}}u^{(\alpha)}(0,t) + \frac{1}{s^{2\alpha}}L_\alpha\{h(x,t)\} - \frac{1}{s^{2\alpha}}L_\alpha\{R_\alpha u(x,t)\}. \tag{16}$$

Taking the inverse of local fractional Laplace transform on Eq. (16), we have

$$u(x,t) = u(0,t) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t) + L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{h(x,t)\}\right) - L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{R_\alpha u(x,t)\}\right). \tag{17}$$

We are going to represent the solution in an infinite series given below:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{18}$$

Substituting Eq. (18) into Eq. (17), which give us this result

$$\sum_{n=0}^{\infty} u_n(x,t) = u(0,t) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t) + L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{h(x,t)\}\right) - L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\left\{R_\alpha \sum_{n=0}^{\infty} u_n(x,t)\right\}\right). \tag{19}$$

When we compare the left and right hand sides of Eq. (19) we obtain

$$\begin{aligned} u_0(x,t) &= u(0,t) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t) + L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{h(x,t)\}\right), \\ u_1(x,t) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{R_\alpha u_0(x,t)\}\right), \\ u_2(x,t) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{R_\alpha u_1(x,t)\}\right) \end{aligned} \tag{20}$$

The recursive relation, in general form is

$$\begin{aligned} u_0(x,t) &= u(0,t) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t) + L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{h(x,t)\}\right) \\ u_{n+1}(x,t) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\{R_\alpha u_n(x,t)\}\right), \end{aligned} \tag{21}$$

4. Illustrative examples

In this section, we given some illustrative examples for solving the nonhomogeneous heat equation arising in fractal heat flow within local fractional operator by using local fractional Laplace decomposition method.

Example 4.1.
The nonhomogeneous local fractional heat equation with the nondifferentiable sink term is presented as follows:

$$\frac{\partial^\alpha T(x,t)}{\partial t^\alpha} - \frac{\partial^{2\alpha} T(x,t)}{\partial x^{2\alpha}} = -\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-t^\alpha), \tag{22}$$

with the initial condition

$$T(0,t) = 0, \frac{\partial^\alpha T(0,t)}{\partial x^\alpha} = E_\alpha(-t^\alpha). \tag{23}$$

In view of Eqs. (21) and (22) the local fractional iteration algorithm can be written as follows:

$$T_0(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} E_{\alpha}(-t^{\alpha}) + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_{\alpha}(-t^{\alpha}),$$
$$T_{n+1}(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}} L_{\alpha}\left\{\frac{\partial^{\alpha} T_n(x,t)}{\partial t^{\alpha}}\right\}\right), n \geq 0. \quad (24)$$

Therefore, from (24) we give the components as follows:

$$T_0(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} E_{\alpha}(-t^{\alpha}) + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_{\alpha}(-t^{\alpha}), \quad (25)$$
$$T_1(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}} L_{\alpha}\left\{\frac{\partial^{\alpha} T_0(x,t)}{\partial t^{\alpha}}\right\}\right)$$
$$= -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_{\alpha}(-t^{\alpha}) - \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_{\alpha}(-t^{\alpha}), \quad (26)$$
$$T_2(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}} L_{\alpha}\left\{\frac{\partial^{\alpha} T_1(x,t)}{\partial t^{\alpha}}\right\}\right)$$
$$= \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_{\alpha}(-t^{\alpha}) + \frac{x^{7\alpha}}{\Gamma(1+7\alpha)} E_{\alpha}(-t^{\alpha}), \quad (27)$$
$$T_3(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}} L_{\alpha}\left\{\frac{\partial^{\alpha} T_2(x,t)}{\partial t^{\alpha}}\right\}\right)$$
$$= -\frac{x^{7\alpha}}{\Gamma(1+7\alpha)} E_{\alpha}(-t^{\alpha}) - \frac{x^{9\alpha}}{\Gamma(1+9\alpha)} E_{\alpha}(-t^{\alpha}), \quad (28)$$
$$\vdots$$

Hence, we finally have

$$T(x,t) = E_{\alpha}(-t^{\alpha})\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} - \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} + \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} + \cdots\right)$$
$$= \frac{x^{\alpha}}{\Gamma(1+\alpha)} E_{\alpha}(-t^{\alpha}). \quad (29)$$

The result is the same as the one which is obtained by the local fractional Laplace variational iteration method [30].

Example 4.2.
We now consider the nonhomogeneous local fractional heat equation with the nondifferentiable source term:

$$\frac{\partial^{\alpha} T(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} T(x,t)}{\partial x^{2\alpha}} = \frac{x^{\alpha}}{\Gamma(1+\alpha)} \cos_{\alpha}(t^{\alpha}), \quad (30)$$

with the initial condition

$$T(0,t) = 0, \frac{\partial^{\alpha} T(0,t)}{\partial x^{\alpha}} = \sin_{\alpha}(t^{\alpha}). \quad (31)$$

Making use of Eqs. (21) and (30) the local fractional iteration algorithm can be written as follows:

$$T_0(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} \sin_{\alpha}(t^{\alpha}) - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} \cos_{\alpha}(t^{\alpha}),$$
$$T_{n+1}(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}} L_{\alpha}\left\{\frac{\partial^{\alpha} T_n(x,t)}{\partial t^{\alpha}}\right\}\right), n \geq 0. \quad (32)$$

Therefore, from Eq. (32) we give the components as follows:

$$T_0(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_{\alpha}(t^{\alpha}) - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\cos_{\alpha}(t^{\alpha}), \tag{33}$$

$$\begin{aligned} T_1(x,y) &= L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_0(x,t)}{\partial t^{\alpha}}\right\}\right) \\ &= \frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\cos_{\alpha}(t^{\alpha}) + \frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\sin_{\alpha}(t^{\alpha}), \end{aligned} \tag{34}$$

$$\begin{aligned} T_2(x,y) &= L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_1(x,t)}{\partial t^{\alpha}}\right\}\right) \\ &= -\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\sin_{\alpha}(t^{\alpha}) + \frac{x^{7\alpha}}{\Gamma(1+7\alpha)}\cos_{\alpha}(t^{\alpha}), \end{aligned} \tag{35}$$

$$\begin{aligned} T_3(x,y) &= L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_2(x,t)}{\partial t^{\alpha}}\right\}\right) \\ &= -\frac{x^{7\alpha}}{\Gamma(1+7\alpha)}\cos_{\alpha}(t^{\alpha}) + \frac{x^{9\alpha}}{\Gamma(1+9\alpha)}\sin_{\alpha}(t^{\alpha}), \end{aligned} \tag{36}$$

$$\vdots$$

Consequently, we obtain

$$T(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha}\sin_{\alpha}(t^{\alpha}). \tag{37}$$

The result is the same as the one which is obtained by the local fractional Laplace variational iteration method [30].

Example 4.3.
Let us consider the nonhomogeneous local fractional heat equation

$$\frac{\partial^{\alpha}T(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha}T(x,t)}{\partial x^{2\alpha}} = 1 \tag{38}$$

with the initial condition

$$T(0,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)}, \frac{\partial^{\alpha}T(0,t)}{\partial x^{\alpha}} = 0. \tag{39}$$

In view of Eqs. (21) and (38) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} T_0(x,t) &= \frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \\ T_{n+1}(x,t) &= L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_n(x,t)}{\partial t^{\alpha}}\right\}\right), n \geq 0. \end{aligned} \tag{40}$$

Therefore, from Eq. (40) we give the components as follows:

$$T_0(x,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{41}$$

$$\begin{aligned} T_1(x,t) &= L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_0(x,t)}{\partial t^{\alpha}}\right\}\right) \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \end{aligned} \tag{42}$$

$$T_2(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_0(x,t)}{\partial t^{\alpha}}\right\}\right) = 0, \tag{43}$$

$$T_2(x,t) = L_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}L_{\alpha}\left\{\frac{\partial^{\alpha}T_0(x,t)}{\partial t^{\alpha}}\right\}\right) = 0, \tag{44}$$

$$\vdots$$

Consequently, we obtain

$$T(x,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{45}$$

5. Conclusions

In this work we derived the nonhomogeneous heat equations arising in fractal heat flow based upon the local fractional calculus. The obtained solutions are nondifferentiable functions, which are Cantor functions and they discontinuously depend on the local fractional derivative. It is shown that the local fractional Laplace decomposition method is an efficient and simple tool for solving local fractional differential equations.

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