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# Well-posedness of neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients

Research Article

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**Abstract:** In this paper, a class of neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients is studied. We establish the well-posedness of mild solutions for neutral impulsive stochastic integro-differential equations with infinite delays driven by Poisson jumps under local non-Lipschitz conditions on the coefficients to the Hilbert space with Lipschitz condition and non-Lipschitz condition being considered as a special case by means of the stopping time technique. Some well-known results are generalized and improved. An example is provided to illustrate the effectiveness of the proposed result.

**MSC:** 93E15 • 60H15 • 35R12

**Keywords:** Resolvent operator • Neutral stochastic integro-differential equation • Impulses • Poisson jumps



## 1. Introduction

The aim of this paper is to establish an well-posedness result of mild solution for a class of neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients described in the form

$$\begin{cases} d[x(t)+\Gamma(t,x_t)] & = A[x(t)+\Gamma(t,x_t)]dt + \left[\int_0^t K(t-s)[x(s)+\Gamma(t,x_s)]ds + F(t,x_t)\right]dt \\ & \quad +\Sigma(t,x_t)dW(t) + \int_{\mathcal{U}} L(t,x(t-),\upsilon)\widetilde{N}(dt,d\upsilon), t\neq t_k, t\in[0,T], \\ \Delta x(t_k) & = x(t_k^+)-x(t_k^-) = I_k(x(t_k)), \quad t=t_k, \quad k=\{1,\cdots,m\}=:\overline{1,m}, \\ x_0(\cdot) & = \varphi\in\mathscr{B}, \end{cases} \tag{1}$$

where the state $x(\cdot)$ takes values in a separable real Hilbert space $\mathbb{H}$; and $A: D(A)\subset\mathbb{H}\to\mathbb{H}$, $K(t): D(K(t))\subset\mathbb{H}\to\mathbb{H}$ are linear, closed, and densely defined operators on $\mathbb{H}$. The history $x_t:(-\infty,0]\to\mathbb{H}$, $x_t(\theta)=x(t+\theta)$ for $t\geq 0$, belong to the phase space $\mathscr{B}$, which will be described axiomatically in Section 2. Assume that the mappings $\Gamma, F:\mathbb{R}^+\times\mathscr{B}\to\mathbb{H}$, $\Sigma:\mathbb{R}^+\times\mathscr{B}\to\mathscr{L}(\mathbb{K};\mathbb{H})$ and $L:\mathbb{R}^+\times\mathbb{H}\times\mathcal{U}\to\mathbb{H}$ are Borel measurable. $I_k:\mathbb{H}\to\mathbb{H}$, $k=\overline{1,m}$ are appropriate functions. Furthermore, let $0=t_0<t_1<\cdots<t_m<t_{m+1}=T$ be prefixed points, where $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t=t_k$, respectively, and $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$, represents the jump of the function $x$ at time $t_k$ with $I_k$ determining the size of the jump. The initial data $\varphi=\{\varphi(t): t\in(-\infty,0]\}$ is an $\mathscr{F}_0$-adapted, $\mathscr{B}$-valued random variable independent of the Wiener process $W$ and the Poisson point process $p(\cdot)$ with $\mathbf{E}\|\varphi\|_{\mathscr{B}}^2<\infty$.

In recent years, the well-posedness of stochastic partial differential equations (SPDEs) have been extensively investigated by many authors (for example, see [1, 2] and the references therein). SPDEs with finite delay have attracted great interest due to their applications in describing many sophisticated dynamical systems in physical, chemistry, biology, economics and social sciences. One can see Refs. ([2–4]) for details. However, in many areas of science and

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engineering there has been an increasing interest in the investigation of functional differential equations incorporating memory or aftereffect, i.e., there is the effect of infinite delay on state equations (see, e.g. [5–8] and the references therein). Therefore, there is real need to discuss functional differential systems with infinite delay. Comparison with finite delay, the problem with infinite delay is clearly more complicated, since the properties of solutions depend on the choice of the phase space $\mathscr{B}$ which is proposed by Hale and Kato in Ref. [9]. On the fundamental theory related to functional differential equations with infinite delay we can see Ref. [10] for details. Moreover, the theory of impulsive differential equations and integro-differential equations with resolvent operators has become an active area of investigation due to their applications in many physical phenomena, see for instance ([11–15] and the references therein). All the above works are established for Wiener process without Poisson jumps processes.

On the other hand, there has not been very much study of SPDEs driven by Poisson jumps, while these have begun to gain attention recently. To be more precise, in [16], Dong discussed the uniqueness of the invariant measure of the Burgers equations with Lévy processes. Albeverio el al. [17] investigated the existence of mild solutions for stochastic differential equations and semilinear equations with non-Gaussian Lévy noise. Taniguchi and Luo [18] considered the existence and behavior of mild solutions to stochastic evolution equations with infinite delays driven by Poisson jumps. For SPDEs with jumps one can see recent monograph [19] as well as papers ([20–22] and the references therein). However, no theory for the well-posedness result of solution to neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients has been established yet. Therefore, motivated by the works [5, 23–27], in this paper we generalize the well-posedness for a class of neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients in the Hilbert space under a class of local non-Lipschitz conditions on the coefficients by means of the stopping time technique. We would like to mention that the non-Lipschitz condition and Lipschitz condition are two special case of the proposed conditions in this paper. The main aim is to close this gap between neutral impulsive stochastic integro-differential equations with infinite delays and Poisson jumps. Our main results concerning (1) rely essentially on techniques using strongly continuous family of operators $\{R(t), t \geq 0\}$, defined on the Hilbert space $\mathbb{H}$ and called their resolvent. The resolvent operator is similar to the semigroup operator for abstract differential equations in Banach spaces. There is a rich theory for analytic semigroups and we wish to develop theories for (1) which yield analytic resolvent. However, the resolvent operator does not satisfy semigroup properties (see, for instance [28, 29]) and our objective in the present paper is to apply the theory developed by Grimmer [30], because it is valid for generators of strongly continuous semigroup, not necessarily analytic.

The rest of this paper is organized as follows: In Section 2, we recall briefly the notations, concepts and basic results about the Wiener process, Poisson jumps process, deterministic integro-differential equations and the phase space $\mathscr{B}$ which are used throughout this paper. The main results in Section 3 is devoted to prove the well-posedness of mild solutions. An example is given in Section 4 to illustrate the theory. In the last section, concluding remarks are given.

## 2. Preliminaries results

This section is concerned with some basic concepts, notations, definitions, lemmas and preliminary facts which are used through this article. For more details on this section, we refer the reader to [4, 9, 30–32].

Let $(\mathbb{H}, \|\cdot\|_{\mathbb{H}}, \langle\cdot,\cdot\rangle)$ and $(\mathbb{K}, \|\cdot\|_{\mathbb{K}}, \langle\cdot,\cdot\rangle)$ denote two real separable Hilbert spaces, with their vectors norms and their inner products, respectively. We denote by $\mathscr{L}(\mathbb{K};\mathbb{H})$ the set of all linear bounded operators from $\mathbb{K}$ into $\mathbb{H}$, which is equipped with the usual operator norm $\|\cdot\|$. Let $W(t)$ be a $\mathbb{K}$-valued $(\mathscr{F}_t)_{t\geqslant 0}$-Wiener process defined on the probability space $(\Omega, \mathscr{F}, \mathbf{P})$ with covariance operator $Q$, where $Q$ is a positive, self-adjoint, trace class operator on $\mathbb{K}$.

In order to define stochastic integrals $\int_0^t \Phi(s)dW(s)$ [4], we introduce the subspace $\mathbb{K}_0 = Q^{\frac{1}{2}}\mathbb{K}$ of $\mathbb{K}$, which endowed with the inner product, $\langle a,b\rangle_{\mathbb{K}_0} = \langle Q^{-\frac{1}{2}}a, Q^{-\frac{1}{2}}b\rangle_{\mathbb{K}}$ is a Hilbert space. Let $\mathscr{L}_2^0 = \mathscr{L}_2(\mathbb{K}_0;\mathbb{H})$ denote the space of all Hilbert-Schmidt operators from $\mathbb{K}_0$ into $\mathbb{H}$. It turns out to be a separable Hilbert space, equipped with the norm $\|\psi\|^2_{\mathscr{L}_2^0} = tr((\psi Q^{\frac{1}{2}})(\psi Q^{\frac{1}{2}})^*)$, for any $\psi \in \mathscr{L}_2^0$.

Let $p = p(t)$, $t \in D_p$ be a stationary $\mathscr{F}_t$-Poisson point process taking its value in a measurable space $(\mathscr{U}, B(\mathscr{U}))$ with a $\sigma$-finite intensity measure $\lambda(dv)$ by $N(dt,dv)$ the Poisson counting measure associated with $p$, that is, $N(t,\mathscr{U}) = \sum_{s\in D_p, s\leq t} I_{\mathscr{U}}(p(s))$ for any measurable set $\mathscr{U} \in B(K-\{0\})$, which denotes the Borel $\sigma$-field of $(K-\{0\})$. Let

$$\widetilde{N}(dt,dv) := N(dt,dv) - \lambda(dv)dt$$

be the compensated Poisson measure that is independent of $W(t)$. Denote by $\mathscr{P}^2([0,T]\times\mathscr{U};H)$ the space of all predictable mappings $L:[0,T]\times\mathscr{U}\to H$ for which

$$\int_0^t \int_{\mathscr{U}} \mathbf{E}\|L(t,v)\|_H^2 \lambda(dv)dt < \infty.$$

We may then define the $H$-valued stochastic integral $\int_0^t \int_{\mathscr{U}} L(t,v)\widetilde{N}(dt,dv)$, which is a centred square-integrable martingale. For the construction of this kind of integral, we can refer to Protter [32].

Next, to be able to access existence, uniqueness and stability of mild solutions for (1) we need to introduce partial integro-differential equations and resolvent operators.

Let $X, Z$ be two Banach spaces such that $\|z\|_Z := \|Az\|_X + \|z\|_X$ for all $z \in Z$; $A$ and $K(t)$ are closed linear operators on $X$ and satisfy the following assumptions:

(**H**1) The operator $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a strongly continuous semigroup on $X$.

(**H**2) For all $t \geq 0$, $K(t) : D(K(t)) \subseteq X \to X$ is a closed linear operator, $D(A) \subseteq D(K(t))$, and $K(t) \in \mathrm{B}(Z, X)$- the set of all bounded linear operators from $Z$ into $X$. For any $z \in Z$, the map $t \to K(t)z$ is bounded, differentiable and the derivative $t \to \dfrac{dK(t)z}{dt}$ is bounded uniformly continuous on $R^+$.

By Theorem 2.3 in [30], we can see that (**H**1) and (**H**2) imply the following integrodifferential abstract Cauchy problem

$$\frac{dx(t)}{dt} = Ax(t) + \int_0^t K(t-s)x(s)ds, \qquad x(0) = x_0 \in X, \tag{2}$$

has an associated resolvent operator of bounded linear operators $R(t), t \geq 0$ on $X$.

**Definition 2.1** A one-parameter family of bounded linear operator $(R(t))_{t\geq 0}$ on $X$ is called a resolvent operator of (2) if the following conditions are verified.

(a) Function $R(\cdot) : [0, \infty) \to \mathscr{L}(X)$ is strongly continuous and $R(0)x = x$ for all $x \in X$.

(b) For $x \in D(A)$, $R(\cdot) \in \mathrm{C}([0, +\infty); D(A)) \cap \mathrm{C}^1([0, +\infty); X)$, and

$$\frac{dR(t)x}{dt} = AR(t)x + \int_0^t K(t-s)R(s)xds,$$

$$\frac{dR(t)x}{dt} = R(t)Ax + \int_0^t R(t-s)K(s)xds, \quad for \quad t \geq 0.$$

1. There exist constants $M > 0$, $\beta$ such that $\|R(t)\| \leq Me^{\beta t}$ for every $t \geq 0$.

Hence, motivated by Grimmer [30], we can give the mild solution for the integro-differential equation

$$\frac{dx(t)}{dt} = Ax(t) + \int_0^t K(t-s)x(s)ds + \kappa(t), \qquad x(0) = x_0 \in X:$$

$$x(t) = R(t)x_0 + \int_0^t R(t-s)\kappa(s)ds, \qquad \forall t \geq 0,$$

where $\kappa : [0, +\infty) \to X$ is a continuous function.

In the whole of this work, we suppose that the phase space $\mathscr{B}$ is axiomatically defined, we use the approach proposed in [9]. More precisely, we have the following definition.

**Definition 2.1.**

The phase space $\mathscr{B}((-\infty, 0], H)$ (denoted by $\mathscr{B}$ for brevity) is the space of $\mathscr{F}_0$-measurable functions from $(-\infty, 0]$ to $H$ endowed with a seminorm $\|\cdot\|_{\mathscr{B}}$, which satisfies the following axiom:

(**A**$_1$) If $x : (-\infty, T] \to H$, $T > 0$, is such that $x_0 \in \mathscr{B}$, then for every $t \in [0, T]$, the following properties hold:

(i) $x_t \in \mathscr{B}$;

(ii) $\|x(t)\|_H \leq H^* \|x_t\|_{\mathscr{B}}$, which is equivalent to $\|\varphi(0)\|_H \leq H^* \|\varphi\|_{\mathscr{B}}$ for every $\varphi \in \mathscr{B}$;

(iii) $\|x_t\|_{\mathscr{B}} \leq M(t) \sup\limits_{0\leq s\leq t} \|x(s)\|_H + N(t)\|x_0\|_{\mathscr{B}}$,

where $H^* > 0$ is a constant; $M, N : [0, +\infty) \to [1, +\infty)$, $M(\cdot)$ is continuous, $N(\cdot)$ is locally bounded, and $M, N$ are independent of $x(\cdot)$.

(**A**$_2$) The space $\mathscr{B}$ is complete.

**Remark 2.1.**

For convenience, the property (iii) in Definition 2.1 can be replaced by the following condition: $\|x_t\|_{\mathscr{B}} \leq \sup\limits_{0\leq s\leq t} \|x(s)\|_H + N_T\|\varphi\|_{\mathscr{B}}$, where $N_T := \sup\limits_{0\leq s\leq T} N(s)$.

**Definition 2.2.**
Denote by $\mathscr{M} := \mathscr{M}((-\infty, T], H)$ the space of all $H$-valued càdlàg measurable $\mathscr{F}_t$-adapted process $x = x(t), -\infty < t \le T$ such that

(i) $x_0 = \varphi \in \mathscr{B}$ and $x(t)$ is càdlàg on $[0, T]$;

(ii) For all $x \in \mathscr{M}$,

$$\|x\|^2_{\mathscr{M}} := \mathbf{E}\|\varphi\|^2_{\mathscr{B}} + \mathbf{E}\int_0^T \|x\|^2_H dt < \infty.$$

Then $\mathscr{M}$ with the above norm is a Banach space.
Now, we give the definition of mild solution for (1).

**Definition 2.3.**
A càdlàg stochastic process $x : (-\infty, T] \to H, 0 < T < +\infty$ is called a mild solution of (1) on $(-\infty, T]$ if

(i) $x(t)$ is $\mathscr{F}_t$-adapted and $\{x_t : t \in [0, T]\}$ is a $\mathscr{B}$-valued stochastic process;

(ii) For arbitrary $t \in [0, T]$, $\mathbf{P}\{\omega : \int_0^T \|x(s)\|^2_H ds < +\infty\} = 1$, and $x(t)$ satisfies the following integral equation:

$$\begin{aligned} x(t) =& R(t)[\varphi(0) + \Gamma(0,\varphi)] - \Gamma(t, x_t) + \int_0^t R(t-s)F(s, x_s)ds + \sum_{0<t_k<t} R(t-t_k)I_k(x(t_k)) \\ &+ \int_0^t R(t-s)\Sigma(s, x_s)dW(s) + \int_0^t \int_{\mathscr{U}} R(t-s)L(s, x(s-), v)\widetilde{N}(ds, dv); \end{aligned} \tag{3}$$

(iii) $x_0(\cdot) = \varphi \in \mathscr{B}$.

Throughout this paper, for the well-posedness of the mild solution to (1), we shall impose the following assumptions:

(**H**3) (i) For all $\varphi_1, \varphi_2 \in \mathscr{B}$, and $t \in [0, T]$ such that

$$\|F(t,\varphi_1) - F(t,\varphi_2)\|^2_H \vee \|\Sigma(t,\varphi_1) - \Sigma(t,\varphi_2)\|^2_{\mathscr{L}_2^0} \le \tau(\|\varphi_1 - \varphi_2\|^2_{\mathscr{B}}).$$

(ii) For any $x, y \in H$, and $t \in [0, T]$ such that

$$\begin{aligned} &\int_0^t \int_{\mathscr{U}} \|L(t, x(s-), v) - L(t, y(s-), v)\|^2_H \lambda(dv)ds \\ &\vee\Big(\int_0^t \int_{\mathscr{U}} \|L(t, x(s-), v) - L(t, y(s-), v)\|^4_H \lambda(dv)ds\Big)^{\frac{1}{2}} \le \int_0^t \tau(\|x(s) - y(s)\|^2_H)ds; \\ &\Big(\int_0^t \int_{\mathscr{U}} \|L(t, x(s-), v)\|^4_H \lambda(dv)ds\Big)^{\frac{1}{2}} \le \int_0^t \tau(\|x(s)\|^2_H)ds, \end{aligned}$$

where $\tau(\cdot)$ is a concave, nondecreasing, and continuous function from $R^+$ to $R^+$ such that $\tau(0) = 0$, $\tau(u) > 0$ for $u > 0$ and $\int_{0^+} \frac{1}{\tau(u)}du = \infty$.

(**H**4) There exist a constant $\rho$ such that $\rho > 0$, for any $\varphi_1, \varphi_2 \in \mathscr{B}$ and for all $t \in [0, T]$, we have

$$\|\Gamma(t,\varphi_1) - \Gamma(t,\varphi_2)\|^2_H \le \rho\|\varphi_1 - \varphi_2\|^2_{\mathscr{B}}.$$

(**H**5) The function $I_k \in \mathrm{C}(H, H)$- the space of all continuous functions from $H$ on itself and for any $x, y \in H$ there exists some constant positive $Q_k$ such that

$$\|I_k(x(t_k)) - I_k(y(t_k))\|^2_H \le Q_k\|x - y\|^2_H, \quad k = \overline{1, m}.$$

(**H**6) For all $t \in [0, T]$, there exist a positive constant $C_0$ such that

$$\|\Gamma(t,0)\|^2_H \vee \|F(t,0)\|^2_H \vee \|\Sigma(t,0)\|^2_H \vee \|I_k(0)\|^2_H \vee \int_{\mathscr{U}} \|L(t,0,v)\|^2_H \lambda(dv) \le C_0.$$

**Remark 2.2.**
Let us give some concrete functions $\tau(\cdot)$. Let $\varepsilon \in (0,1)$. Set

$$\tau_1(u) = u, \text{ for } u \geq 0.$$

$$\tau_2(u) = \begin{cases} u\log(\frac{1}{u}) & \text{if } 0 \leq u \leq \varepsilon, \\ \varepsilon\log(\frac{1}{\varepsilon}) + {\tau_2}'(\varepsilon-)(u-\varepsilon) & \text{if } u > \varepsilon. \end{cases}$$

$$\tau_3(u) = \begin{cases} u\log(\frac{1}{u})\log\log(\frac{1}{u}) & \text{if } 0 \leq u \leq \varepsilon, \\ \varepsilon\log(\frac{1}{\varepsilon})\log\log(\frac{1}{\varepsilon}) + {\tau_3}'(\varepsilon-)(u-\varepsilon) & \text{if } u > \varepsilon, \end{cases}$$

where $\varepsilon$ is sufficiently small and ${\tau_i}'$, $i = 2,3$ is the left derivative of $\tau_i$, $i = 2,3$ at the point $\varepsilon$. Then $\tau_i$, $i = 1,2,3$ are concave nondecreasing functions definition on $R^+$ satisfying $\int_{0^+} \frac{1}{\tau_i(x)} dx = \infty$, $i = 1,2,3$. In particular, we see that the Lipschitz conditions and non-Lipschitz conditions are special case of our proposed conditions.

## 3. The main results

In this section, we shall investigate the well-posedness theorem of the mild solution to neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients. In order to prove the well-posedness, we introduce the successive approximations to (3) as follows

$$x^0(t) = \begin{cases} \varphi(t), & for \quad t \in (-\infty, 0], \\ R(t)\varphi(0), & for \quad t \in [0, T], \end{cases} \tag{4}$$

and $x^n$ for $n \geq 1$ is defined by

$$x^n(t) = \begin{cases} \varphi(t), & for \quad t \in (-\infty, 0], \\ R(t)[\varphi(0) + \Gamma(0,\varphi)] - \Gamma(t, x_t^n) + \int_0^t R(t-s)F(s, x_s^{n-1})ds \\ + \sum_{0<t_k<t} R(t-t_k)I_k(x^{n-1}(t_k)) + \int_0^t R(t-s)\Sigma(s, x_s^{n-1})dW(s) \\ + \int_0^t \int_{\mathcal{U}} R(t-s)L(s, x^{n-1}(s-), v)\widetilde{N}(ds, dv), \quad a.s \quad \forall t \in [0, T]. \end{cases} \tag{5}$$

If the coefficients $F, \Sigma, L$ of (1) satisfy the non-Lipchitz conditions, then by using the similar method as in [5, 24], we can be proved that existence and uniqueness of the mild solution to (1).

**Theorem 3.1.**
*Let the assumptions* (**H1**) − (**H6**) *hold and* $0 < \rho < \dfrac{1}{24}$. *Then, there exist a unique mild solution to (1) in* $\mathcal{M}$.

Now, we present the existence and uniqueness of the mild solutions for (1) with the local non-Lipchitz conditions.

**Theorem 3.2.**
*Let the assumptions* (**H1**), (**H2**), (**H3**$^\star$), (**H4**) − (**H6**) *hold. Then, there exist a unique mild solution to (1) in* $\mathcal{M}$, *provided that* $5\big[\rho + m\Lambda \sum_{k=1}^{m} Q_k\big] < 1$.

*Proof.* Let $N$ be a natural integer and let $\overline{T} \in (0, T)$. We define the sequence of the functions $\{F_N\}$, $\{\Sigma_N\}$, and $\{L_N\}$ as follows:

$$F_N(t, x_t) := \begin{cases} F(t, x_t) & \text{if } \|x_t\|_{\mathcal{B}} \leq N, \\ F(t, \frac{N x_t}{\|x_t\|_{\mathcal{B}}}) & \text{if } \|x_t\|_{\mathcal{B}} > N, \end{cases} \quad \Sigma_N(t, x_t) := \begin{cases} \Sigma(t, x_t) & \text{if } \|x_t\|_{\mathcal{B}} \leq N, \\ \Sigma(t, \frac{N x_t}{\|x_t\|_{\mathcal{B}}}) & \text{if } \|x_t\|_{\mathcal{B}} > N, \end{cases}$$

$$L_N\big(t, x(t-), v\big) := \begin{cases} L\big(t, x(t-), v\big) & \text{if } \|x\|_H \leq N, \\ L\Big(t, \frac{N x(t-)}{\|x(t)\|_H}, v\Big) & \text{if } \|x\|_H > N. \end{cases}$$

Then, the functions $\{F_N\}$, $\{\Sigma_N\}$, and $\{L_N\}$ satisfies assumption ($\mathbf{H}_3$). Thus, by Theorem 3.1, there exists a unique solution $x^\ell(t) \in \mathcal{M}$, with $\ell = \{N, N+1\}$ such that

$$\begin{aligned} x^\ell(t) = {} & R(t)[\varphi(0) + \Gamma(0,\varphi)] - \Gamma(t, x_t^\ell) + \int_0^t R(t-s)F_\ell(s, x_s^\ell)ds + \int_0^t R(t-s)\Sigma_\ell(s, x_s^\ell)dW(s) \\ & + \sum_{0<t_k<t} R(t-t_k)I_k(x^\ell(t_k)) + \int_0^t \int_{\mathcal{U}} R(t-s)L_\ell\big(s, x^\ell(s-), v\big)\widetilde{N}(ds, dv). \end{aligned} \tag{6}$$

For sufficiently large integer $N$, define the stopping times

$$\gamma_N := \overline{T} \wedge \inf\{t \in [0,T] \mid \quad \|x_t^N\|_{\mathscr{B}} \geq N\}, \quad \gamma_{N+1} := \overline{T} \wedge \inf\{t \in [0,T] \mid \quad \|x_t^{N+1}\|_{\mathscr{B}} \geq N+1\},$$
$$\sigma_N := \overline{T} \wedge \inf\{t \in [0,T] \mid \quad \|x^N\|_H \geq N\}, \quad \sigma_{N+1} := \overline{T} \wedge \inf\{t \in [0,T] \mid \quad \|x^{N+1}\|_H \geq N+1\},$$

$$\theta_N := \gamma_N \wedge \gamma_{N+1} \wedge \sigma_N \wedge \sigma_{N+1}.$$

We claim that $x^{N+1}(t) = x^N(t)$, for all $t \in [0, \overline{T} \wedge \theta_N]$, a.s. $\omega$.
By (6), we infer that

$$\begin{aligned}
&\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|x^{N+1}(s) - x^N(s)\|_H^2 \\
\leq & 5\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|\Gamma(s, x_s^{N+1}) - \Gamma(s, x_s^N)\|_H^2 \\
&+ 5\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \Big\|\int_0^s R(s-r)\big[F_{N+1}(r, x_r^{N+1}) - F_N(r, x_r^N)\big]dr\Big\|_H^2 \\
&+ 5\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \Big\|\int_0^s R(s-r)\big[\Sigma_{N+1}(r, x_r^{N+1}) - \Sigma_N(r, x_r^N)\big]dW(r)\Big\|_H^2 \\
&+ 5\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \Big\|\int_0^s \int_{\mathscr{U}} R(s-r)\big[L_{N+1}(r, x^{N+1}(r-), v) - L_N(r, x^N(r-), v)\big]\widetilde{N}(dr, dv)\Big\|_H^2 \\
&+ 5\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \Big\|\sum_{0<t_k<t} R(t-t_k)\big[I_k(x^{N+1}(t_k)) - I_k(x^N(t_k))\big]\Big\|_H^2 \\
\leq & 5\rho\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|x_s^{N+1} - x_s^N\|_{\mathscr{B}}^2 + 5\Lambda T\mathbf{E}\int_0^{t\wedge\theta_N} \|F_{N+1}(s, x_s^{N+1}) - F_N(s, x_s^N)\|_H^2 ds \\
&+ C_1\mathbf{E}\int_0^{t\wedge\theta_N} \|\Sigma_{N+1}(s, x_s^{N+1}) - \Sigma_N(s, x_s^N)\|_{\mathscr{L}_2^0}^2 ds \\
&+ C_2\mathbf{E}\int_0^{t\wedge\theta_N}\int_{\mathscr{U}} \|L_{N+1}(s, x^{N+1}(s-), v) - L_N(s, x^N(s-), v)\|_H^2 \lambda(dv)ds \\
&+ C_2\mathbf{E}\Big(\int_0^{t\wedge\theta_N}\int_{\mathscr{U}} \|L_{N+1}(s, x^{N+1}(s-), v) - L_N(s, x^N(s-), v)\|_H^4 \lambda(dv)ds\Big)^{\frac{1}{2}} \\
&+ 5m\Lambda\sum_{k=1}^m Q_k \mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|x^{N+1}(s) - x^N(s)\|_H^2,
\end{aligned}$$

where $C_1$, $C_2$ are positive constants. Since for $s \in [0, \theta_N]$, we know that

$$F_{N+1}(s, x_s^N) = F_N(s, x_s^N), \quad \Sigma_{N+1}(s, x_s^N) = \Sigma_N(s, x_s^N), \quad L_{N+1}(s, x^N(s-), v) = L_N(s, x^N(s-), v).$$

Thus, we have

$$\begin{aligned}
&\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|x^{N+1}(s) - x^N(s)\|_H^2 \\
\leq & 5\Big(\rho + m\Lambda\sum_{k=1}^m Q_k\Big)\mathbf{E} \sup_{s\in[0,t\wedge\theta_N]} \|x^{N+1}(s) - x^N(s)\|_H^2 \\
&+ 5\Lambda T\mathbf{E}\int_0^{t\wedge\theta_N} \|F_{N+1}(s, x_s^{N+1}) - F_{N+1}(s, x_s^N)\|_H^2 ds \\
&+ C_1\mathbf{E}\int_0^{t\wedge\theta_N} \|\Sigma_{N+1}(s, x_s^{N+1}) - \Sigma_{N+1}(s, x_s^N)\|_{\mathscr{L}_2^0}^2 ds \\
&+ C_2\mathbf{E}\int_0^{t\wedge\theta_N}\int_{\mathscr{U}} \|L_{N+1}(s, x^{N+1}(s-), v) - L_{N+1}(s, x^N(s-), v)\|_H^2 \lambda(dv)ds \\
&+ C_2\mathbf{E}\Big(\int_0^{t\wedge\theta_N}\int_{\mathscr{U}} \|L_{N+1}(s, x^{N+1}(s-), v) - L_{N+1}(s, x^N(s-), v)\|_H^4 \lambda(dv)ds\Big)^{\frac{1}{2}}.
\end{aligned}$$

Hence, by assumption (**H3**$^\star$), Proposition 7.3 in [4] and Lemma 2.2 in [20], we get the following inequalities

$$
\begin{aligned}
&\mathbf{E}\sup_{s\in[0,t]}\|x^{N+1}(s\wedge\theta_N)-x^N(s\wedge\theta_N)\|_H^2\\
\leq&\frac{5\Lambda T}{C_3}\mathbf{E}\int_0^t\|F_{N+1}(s\wedge\theta_N,x^{N+1}_{s\wedge\theta_N})-F_{N+1}(s\wedge\theta_N,x^N_{s\wedge\theta_N})\|_H^2ds\\
&+\frac{C_1}{C_3}\mathbf{E}\int_0^t\|\Sigma_{N+1}(s\wedge\theta_N,x^{N+1}_{s\wedge\theta_N})-\Sigma_{N+1}(s\wedge\theta_N,x^N_{s\wedge\theta_N})\|^2_{\mathcal{L}_2^0}ds\\
&+\frac{C_2}{C_3}\mathbf{E}\int_0^t\int_{\mathcal{U}}\|L_{N+1}\big(s\wedge\theta_N,x^{N+1}((s\wedge\theta_N)-),v\big)\\
&\qquad\qquad-L_{N+1}\big(s\wedge\theta_N,x^N((s\wedge\theta_N)-),v\big)\|_H^2\lambda(dv)ds\\
&+\frac{C_2}{C_3}\mathbf{E}\Big(\int_0^t\int_{\mathcal{U}}\|L_{N+1}\big(s\wedge\theta_N,x^{N+1}((s\wedge\theta_N)-),v\big)\\
&\qquad\qquad-L_{N+1}\big(s\wedge\theta_N,x^N((s\wedge\theta_N)-),v\big)\|_H^4\lambda(dv)ds\Big)^{\frac{1}{2}}\\
\leq&\frac{5\Lambda T+C_1+C_2}{C_3}\int_0^t\tau_{N+1}\Big(\mathbf{E}\big(\sup_{r\in[0,s]}\|x^{N+1}(r\wedge\theta_N)-x^N(r\wedge\theta_N)\|_H^2\big)\Big)ds,
\end{aligned}
$$

where $C_3:=\big(1-5\rho-5m\Lambda\sum_{k=1}^m Q_k\big)$.

For all $t\in[0,\overline{T}]$, by Bihari's inequality [33], we obtain that

$$\mathbf{E}\sup_{s\in[0,t]}\|x^{N+1}(s\wedge\theta_N)-x^N(s\wedge\theta_N)\|_H^2=0.$$

This means that, for all $t\in[0,\overline{T}\wedge\theta_N]$, we always have $x^{N+1}(t)=x^N(t),\quad a.s.\quad\omega.$
For each $\omega\in\Omega$, there exists an $N_0(\omega)>0$, such that $\overline{T}\in(0,\theta_{N_0}]$. For all $t\in[0,\overline{T}]$, define $x(t)$ by $x(t)=x^{N_0}(t)$. Since $x(t\wedge\theta_N)=x^N(t\wedge\theta_N)$, it holds that

$$
\begin{aligned}
x(t\wedge\theta_N)=&R(t)[\varphi(0)+\Gamma(0,\varphi)]-\Gamma(t,x_t^N)+\int_0^{t\wedge\theta_N}R(t-s)F_N(s,x_s^N)ds\\
&+\sum_{0<t_k<t\wedge\theta_N}R(t-t_k)I_k(x^N(t_k))+\int_0^{t\wedge\theta_N}R(t-s)\Sigma_N(s,x_s^N)dW(s)\\
&+\int_0^{t\wedge\theta_N}\int_{\mathcal{U}}R(t-s)L_N(s,x^N(s-),v)\widetilde{N}(ds,dv)\\
=&R(t)[\varphi(0)+\Gamma(0,\varphi)]-\Gamma(t,x_t)+\int_0^{t\wedge\theta_N}R(t-s)F(s,x_s)ds\\
&+\sum_{0<t_k<t\wedge\theta_N}R(t-t_k)I_k(x(t_k))+\int_0^{t\wedge\theta_N}R(t-s)\Sigma(s,x_s)dW(s)\\
&+\int_0^{t\wedge\theta_N}\int_{\mathcal{U}}R(t-s)L(s,x(s-),v)\widetilde{N}(ds,dv).
\end{aligned}
$$

Letting $N\to\infty$, for all $t\in[0,T]$, we infer that

$$
\begin{aligned}
x(t)=&R(t)[\varphi(0)+\Gamma(0,\varphi)]-\Gamma(t,x_t)+\int_0^tR(t-s)F(s,x_s)ds+\sum_{0<t_k<t}R(t-t_k)I_k(x(t_k))\\
&+\int_0^tR(t-s)\Sigma(s,x_s)dW(s)+\int_0^t\int_{\mathcal{U}}R(t-s)L(s,x(s-),v)\widetilde{N}(ds,dv).
\end{aligned}
$$

The uniqueness is obtained by stopping our process. The proof is thus complete. □

## 4. Application

In this section, an example is provided to illustrate the obtained theory. We consider the following neutral impulsive stochastic integro-differential equations with infinite delays driven by Poisson jumps of the form:

$$\begin{cases} \frac{\partial}{\partial t}\Big[u(t,\xi)+\int_{-\infty}^{0}\gamma(\theta,u(t+\theta,\xi))d\theta\Big]=\frac{\partial^2}{\partial\xi^2}\Big[u(t,\xi)+\int_{-\infty}^{0}\gamma(\theta,u(t+\theta,\xi))d\theta\Big] \\ +\int_0^t\widehat{k}(t-s)\frac{\partial^2}{\partial\xi^2}\Big[u(s,\xi)+\int_{-\infty}^{0}\gamma(\theta,u(s+\theta,\xi))d\theta\Big]ds+\int_{-\infty}^{0}f(\theta,u(t+\theta,\xi))d\theta \\ +\sigma(t,u(t+\theta,\xi))dW(t)+\int_{\mathscr{U}}u(t-,\xi)v\widetilde{N}(dt,dv),\quad \xi\in[0,\pi],t\neq t_k,\quad t\geq 0, \\ u(t_k^+)-u(t_k^-)=(1+c_k)u(\xi(t_k)),\qquad for\quad t=t_k,\quad k=\overline{1,m}, \\ u(t,0)+\int_{-\infty}^{0}\gamma(\theta,u(t+\theta,0))d\theta=0\quad for\quad t\geq 0, \\ u(t,\pi)+\int_{-\infty}^{0}\gamma(\theta,u(t+\theta,\pi))d\theta=0\quad for\quad t\geq 0, \\ u(\theta,\xi)=u_0(\theta,\xi)\qquad for\quad \theta\in(-\infty,0],\quad \xi\in[0,\pi], \end{cases} \tag{7}$$

where $W(t)$ is a standard one-dimensional Wiener process in $H$ defined on a stochastic space $(\Omega,\mathscr{F},\mathbf{P})$; $\mathscr{U}=\{v\in R: 0<\|v\|_R\leq\bar{a},\bar{a}>0\}$; $\gamma,f:R^-\times R\to R$ and $\sigma:R^+\times R\to\mathscr{L}(R)$ are continuous functions, $\widehat{k}:R^+\to R$ is continuous; $c_k\geq 0$ for $k=\overline{1,m}$ and $\sum_{k=1}^{m}c_k<\infty$; $u_0:(-\infty,0]\times[0,\pi]\to R$ is given càdlàg function such that $u_0(\cdot)\in L^2([0,\pi])$ is $\mathscr{F}_0$-measurable and satisfies $\mathbf{E}\|u_0\|^2_{\mathscr{B}}<\infty$.

Let $p=p(t)$, $t\in D_p$ be a $K$-valued $\sigma$-finite stationary Poisson point process (independent of $W(t)$) on a complete probability space with the usual condition $(\Omega,\mathscr{F},(\mathscr{F}_t)_{t\geq0},\mathbf{P})$. Let $\widetilde{N}(ds,dv):=N(ds,dv)-\lambda(dv)ds$, with the characteristic measure $\lambda(dv)$ on $\mathscr{U}\in B(K-\{0\})$. Assume that $\int_{\mathscr{U}}v^2\lambda(dv)<\infty\quad and\quad\int_{\mathscr{U}}v^4\lambda(dv)<\infty$.

To rewrite (7) into the abstract from of (1) we consider the space $H=L^2([0,\pi])$ with the norm $\|\cdot\|$ and $K=R^1$. Let $e_n(x):=\sqrt{\frac{2}{\pi}}\sin nx$, $n=1,2,3,\ldots$ denote the completed orthonormal basics in $H$ and $W(t)=\sum_{n=1}^{\infty}\sqrt{\lambda_n}\beta_n(t)e_n,\quad t\geq0$, $\lambda_n>0$, where $\{\beta_n(t)\}_{n\geq0}$ are one-dimensional standard Brownian motions mutually independent on a usual complete probability space $(\Omega,\mathscr{F},(\mathscr{F}_t)_{t\geq0},\mathbf{P})$.

Defined $A:H\to H$ by $A=\frac{\partial^2}{\partial x^2}$, with domain $D(A)=H^2([0,\pi])\cap H_0^1([0,\pi])$, here $H_0^1([0,\pi])=\{w\in L^2([0,\pi]): \frac{\partial w}{\partial z}\in L^2([0,\pi]), w(0)=w(\pi)=0\}$ and $H^2([0,\pi])=\{w\in L^2([0,\pi]):\frac{\partial w}{\partial z},\frac{\partial^2 w}{\partial z^2}\in L^2([0,\pi])\}$. Then $Ax=-\sum_{n=1}^{\infty}n^2\langle x,e_n\rangle e_n$, $x\in D(A)$, where $n=1,2,3,\ldots$ is also the orthonormal set of eigenvector of $A$. It is wellknown that $A$ is the infinitesimal generator of a strongly continuous semigroup $\{S(t)\}_{t\geq0}$ on $H$ and is given (see Pazy [31], page 70) by $S(t)x=\sum_{n=1}^{\infty}e^{-n^2t}\langle x,e_n\rangle e_n$, $x\in H$. Thus, (**H**1) is true.

Let $K(t):D(A)\subset H\to H$ be the operator defined by $K(t)(z)=\widehat{k}(t)Az$ for $t\geq0$ and $z\in D(A)$. Let $\mathscr{B}=BC(R^-;H)$ denote the Banach space of all bounded continuous functions from $R^-$ to $H$, equipped with the following norm

$$\|\phi\|_{\mathscr{B}}=\sup_{\theta\leq0}\|\phi(\theta)\|_H=\sup_{\theta\leq0,\xi\in[0,\pi]}\|\phi(\theta)(\xi)\|_H,\quad \phi\in\mathscr{B}.$$

Then, the space $\mathscr{B}$ satisfies all conditions of axioms ($\mathbf{A}_1$) and ($\mathbf{A}_2$).

For $\xi\in[0,\pi]$ and $\phi\in\mathscr{B}$, let us introduce the operators $\Gamma,F:R^+\times\mathscr{B}\to H$, $\quad\Sigma:R^+\times\mathscr{B}\to\mathscr{L}(K;H)$, $\quad L:R^+\times\mathscr{B}\times\mathscr{U}\to H$ and $I_k:H\to H$, $\quad k=\overline{1,m}$ by

$$\Gamma(t,\phi)(\xi)=\int_{-\infty}^{0}\gamma(\theta,\phi(\theta)(\xi))d\theta, F(t,\phi)(\xi)=\int_{-\infty}^{0}f(\theta,\phi(\theta)(\xi))d\theta,\quad L(t,\phi(\xi),v)=\phi(\xi)v,$$
$$\Sigma(t,\phi)(\xi)=\sigma(t,\phi(\theta)(\xi)),\quad I_k(\phi(\xi))(t_k)=(1+c_k)\phi(\xi(t_k)),\quad k=\overline{1,m}.$$

If we put,

$$\begin{cases} x(t) &=u(t,\xi),\quad \text{for } t\geq0 \text{ and } \xi\in[0,\pi], \\ \varphi(\theta)(\xi) &=u_0(\theta,\xi),\quad \text{for } \theta\in(-\infty,0] \text{ and } \xi\in[0,\pi]. \end{cases}$$

Then (7) takes the following abstract form:

$$\begin{cases} d[x(t)+\Gamma(t,x_t)] &=A[x(t)+\Gamma(t,x_t)]dt+\Big[\int_0^tK(t-s)[x(s)+\Gamma(s,x_s)]ds+F(t,x_t)\Big]dt \\ &\quad+\Sigma(t,x_t)dW(t)+\int_{\mathscr{U}}L(t,x(t-),v)\widetilde{N}(dt,dv),t\neq t_k,t\in[0,T], \\ \Delta x(t_k) &=x(t_k^+)-x(t_k^-)=I_k(x(t_k)),\quad t=t_k,\quad k=\overline{1,m}, \\ x_0(\cdot) &=\varphi\in\mathscr{B}. \end{cases}$$

Moreover, if $\widehat{k}$ is bounded and $C^1$, where C stand for the space of all continuous functions such that $\widehat{k}'$ is bounded and uniformly continuous, then (**H**1) and (**H**2) are satisfied and hence (2) has a resolvent operator $(R(t))_{t\geq 0}$ on $H$. As a consequence of the continuity of $\gamma$, $f$, it follows that $\Gamma$, $F$ are continuous on $R^+ \times \mathscr{B}$ with values in $H$, and from the continuity of $\sigma$ it follows that $\Sigma$ is continuous on $R^+ \times \mathscr{B}$ with values in $\mathscr{L}(K,H)$. Thus, (7) can be expressed as (1) with $A, \Gamma, K, F, \Sigma, L, I_k, k = \overline{1,m}$ as defined above.

Now we suppose the following assumptions

(i) For $\theta \leq 0$, $\gamma(\theta, 0) = 0$.

(ii) There exists a function $\gamma_1 \in L^1(R^-, R)$ such that for $\theta \leq 0$ and $\xi_1, \xi_2 \in R$

$$\sup_{\xi_1 \neq \xi_2} \frac{|\gamma(\theta,\xi_1) - \gamma(\theta,\xi_2)|}{|\xi_1 - \xi_2|} \leq \gamma_1(\theta).$$

(iii) There exists a function $\gamma_2$ is measurable nonnegative function on $(-\infty, 0]$ such that for $\theta \leq 0$ and $\xi_1, \xi_2 \in R$ : $|f(\theta,\xi_1) - f(\theta,\xi_2)|^2 \leq \gamma_2(\theta)\tau(|\xi_1 - \xi_2|^2)$, where $\tau(\cdot)$ is define as (**H**3).

With the above assumptions $\Gamma$ is well defined from $R^+ \times \mathscr{B} \to H$. In fact, given $\xi \in [0,\pi]$, $\phi \in \mathscr{B}$ and a sequence $(\xi_n)_{n\geq 0} \subset [0,\pi]$ such that $\xi_n \to \xi$, we have

$$\|\Gamma(t,\phi)(\xi_n) - \Gamma(t,\phi)(\xi)\| \leq \sqrt{\pi}\int_{-\infty}^{0} \gamma_1(\theta)\|\phi(\theta)(\xi_n) - \phi(\theta)(\xi)\| d\theta.$$

By continuity of $\phi$, we have $\lim_n \phi(\theta)(\xi_n) = \phi(\theta)(\xi)$.

Thus, by Lebesgue convergence theorem we deduce that $\Gamma(\phi) \in H$ for all $\phi \in \mathscr{B}$. Moreover, for all $\phi_1, \phi_2 \in \mathscr{B}$, we have

$$\|\Gamma(t,\phi_1) - \Gamma(t,\phi_2)\| = \sup_{\xi\in[0,\pi]} \|\Gamma(t,\phi_1)(\xi) - \Gamma(t,\phi_2)(\xi)\| \leq \sqrt{\pi}\int_{-\infty}^{0} \gamma_1(\theta) d\theta \|\phi_1 - \phi_2\|_{\mathscr{B}},$$

and

$$\|F(t,\phi_1) - F(t,\phi_2)\|^2 \leq \sqrt{\pi}\int_{-\infty}^{0} \gamma_2(\theta) d\theta \tau(\|\phi_1 - \phi_2\|^2_{\mathscr{B}}).$$

We also suppose that

(a) $0 < \sqrt{\pi}\int_{-\infty}^{0} \gamma_1(\theta) d\theta < \frac{1}{\sqrt{24}}$ and $0 < \sqrt{\pi}\int_{-\infty}^{0} \gamma_2(\theta) d\theta < 1$;

(b) $\sigma$ satisfies (**H**3), that is $\|\sigma(t,\zeta) - \sigma(t,\mu)\|^2 \leq \tau(\|\zeta - \mu\|^2_{\mathscr{B}})$.

Thus, all the assumptions of Theorem 3.1 and Theorem 3.2 are fulfilled. Therefore, the system (7) has a unique mild solution.

## 5. Conclusion

In this paper, we have studied a class of neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients in real separable Hilbert spaces. Sufficient conditions for the well-posedness of mild solutions for neutral impulsive stochastic integro-differential equations with local non-Lipschitz coefficients are derived by means of the stopping time technique combined with theories of resolvent operators for integro-differential equations. In addition, an example illustrating the applicability of the general theory are also provided. The results presented in this paper extend and improve the results in [5, 23–27].

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