Int. J. Adv. Appl. Math. and Mech. 2(4) (2015) 8 - 12 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation

Research Article

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Received 29 March 2015; accepted (in revised version) 05 May 2015

Abstract: In this manuscript, we apply a combined form of the Laplace transform method with the homotopy perturbation method to obtain the solution of Newell-Whitehead Segel equation. This method is called the homotopy perturbation transform method (HPTM). The method can be applied to linear and nonlinear problems. Some examples have been carried out in order to illustrate the efficiency and reliability of the method.

MSC: 41A25 • 41A35

Keywords: Newell-Whitehead-Segel equation • Laplace transform method • Homotopy perturbation method

1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics, we know that most of engineering problems are non-linear, and it is difficult to solve them analytically. The importance of obtaining the exact or approximate solutions of nonlinear partial differential equations in physics and mathematics is still a significant problem that needs new methods to discover exact or approximate solutions. Various powerful mathematical methods have been proposed for obtaining exact and approximate analytic solutions. Some of the classic analytic methods are Lyapunov's artificial small parameter method [1], perturbation techniques [2–5] and Hirota's bilinear method [6, 7] .
In recent years, many research workers have paid attention to study the solutions of nonlinear partial differential equations by using various methods. Among these are the Adomian decomposition method (ADM) [8], HeâĂŹs semi-inverse method [9], the tanh method, the homotopy perturbation method (HPM), the differential transform method and the variational iteration method (VIM) [10–17].
One of the most important of amplitude equations is the Newell-Whitehead-Segel equation which describes the appearance of the stripe pattern in two dimensional systems. Moreover, this equation was applied to a number of problem in a variety systems, e.g., Rayleigh-Benard convection, Faraday instability, nonlinear optics, chemical reactions and biological systems. The approximate solutions of the Newell-Whitehead-Segel equation were presented by Adomian decomposition [18], differential transformation method [19], reduce differential transformation [20]. In this paper a reliable homotopy perturbation transform method is applied for solving Newell-Whitehead-Segel equation. The method can be employed to linear and nonlinear problems. Moreover, some examples are illustrative for demonstrating the advantage of the method.

2. Basic idea of homotopy perturbation transform Method

To illustrate the basic idea of this method, we consider the initial value problem in Newell-Whitehead-Segel equation in the form:

$$\frac{\partial u(x,t)}{\partial t} = k\frac{\partial^2 u(x,t)}{\partial x^2} + k_1 u(x,t) - k_2 u^m(x,t), \tag{1}$$

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with the initial condition

$$u(x,0) = h(x), \tag{2}$$

where k_1 and k_2 are real numbers and k and m are positive integers.
Taking Laplace transform on both sides of the equation (1) and using the linearity of the Laplace transform gives:

$$L\left\{\frac{\partial u(x,t)}{\partial t}\right\} = kL\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} + k_1 L\{u(x,t)\} - k_2 L\left\{u^m(x,t)\right\}. \tag{3}$$

By applying the Laplace transform differentiation property, we have

$$sL\{u(x,t)\} - u(x,0) = kL\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} + k_1 L\{u(x,t)\} - k_2 L\left\{u^m(x,t)\right\}. \tag{4}$$

Thus, we have

$$L\{u(x,t)\} = \frac{h(x)}{s-k_1} + \frac{k}{s-k_1} L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} - \frac{k_2}{s-k_1} L\left\{u^m(x,t)\right\}. \tag{5}$$

Taking the inverse Laplace transform on equation (1), we obtain

$$u(x,t) = L^{-1}\left(\frac{h(x)}{s-k_1}\right) + L^{-1}\left(\frac{k}{s-k_1} L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} - \frac{k_2}{s-k_1} L\left\{u^m(x,t)\right\}\right). \tag{6}$$

In the homotopy perturbation method (HPM), the basic assumption is that the solutions can be written as a power series in p:

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t), \tag{7}$$

and the nonlinear term $N(u) = u^m, m > 1$ can be presented by an infinite series

$$N(u) = \sum_{n=0}^{\infty} p^n H_n(u), \tag{8}$$

where $p \in [0,1]$ is an embedding parameter. $H_n(u)$ is He polynomials [21, 22] that can be generated by formula given below:

$$H_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{n} p^i u_i(x,t)\right)\right], n = 0, 1, \ldots. \tag{9}$$

Substituting (7) and (8) in (6), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = L^{-1}\left(\frac{h(x)}{s-k_1}\right) + p\left[L^{-1}\left(\frac{k}{s-k_1} L\left\{\sum_{n=0}^{\infty} p^n \frac{\partial^2 u_n(x,t)}{\partial x^2}\right\} - \frac{k_2}{s-k_1} L\left\{\sum_{n=0}^{\infty} p^n H_n(u)\right\}\right)\right], \tag{10}$$

which is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of p, the following approximations are obtained.

$$\begin{aligned} p^0 &: u_0(x,t) = L^{-1}\left(\frac{h(x)}{s-k_1}\right), \\ p^1 &: u_1(x,t) = L^{-1}\left(\frac{k}{s-k_1} L\left\{\frac{\partial^2 u_0(x,t)}{\partial x^2}\right\} - \frac{k_2}{s-k_1} LH_0(u)\right), \\ p^2 &: u_2(x,t) = L^{-1}\left(\frac{k}{s-k_1} L\left\{\frac{\partial^2 u_1(x,t)}{\partial x^2}\right\} - \frac{k_2}{s-k_1} LH_1(u)\right), \\ &\vdots \end{aligned} \tag{11}$$

Proceeding in this same manner, the rest of the components $u_n(x,t)$ can be completely obtained and the series solution is thus entirely determined. Finally, we approximate the analytical solution $u(x,t)$ by truncated series:

$$u(x,t) = \lim_{N\to\infty}\left(\sum_{n=0}^{N} u_n(x,t)\right). \tag{12}$$

The above series solutions generally converge very rapidly.

3. Illustrative examples

In this section, some initial value problems are presented to show the advantages of the proposed method which can be applied to linear and nonlinear problem.

Example 3.1.
Consider linear Newell-Whitehead-Segel equation

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} - 3u(x,t), \tag{13}$$

subject to initial condition

$$u(x,0) = \exp 2x. \tag{14}$$

Taking the Laplace transform on both sides of equation (13) subject to the initial condition (14), we have

$$L\{u(x,t)\} = \frac{\exp 2x}{s+3} + \frac{1}{s+3} L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\}. \tag{15}$$

Taking the inverse Laplace transform on equation (15), we obtain

$$u(x,t) = L^{-1}\left(\frac{\exp 2x}{s+3}\right) + L^{-1}\left(\frac{1}{s+3} L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\}\right). \tag{16}$$

Now, applying the homotopy perturbation method, we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = L^{-1}\left(\frac{\exp 2x}{s+3}\right) + +p\left[L^{-1}\left(\frac{k}{s+3} L\left\{\sum_{n=0}^{\infty} p^n \frac{\partial^2 u_n(x,t)}{\partial x^2}\right\}\right)\right]. \tag{17}$$

Comparing the coefficients of like powers of p , we have

$$\begin{aligned}
p^0 &: u_0(x,t) = L^{-1}\left(\frac{\exp 2x}{s+3}\right) = \exp 2x - 3t,\\
p^1 &: u_1(x,t) = L^{-1}\left(\frac{1}{s+3} L\left\{\frac{\partial^2 u_0(x,t)}{\partial x^2}\right\}\right) = 4t \exp 2x - 3t,\\
p^2 &: u_2(x,t) = L^{-1}\left(\frac{1}{s+3} L\left\{\frac{\partial^2 u_1(x,t)}{\partial x^2}\right\}\right) = 8t^2 \exp 2x - 3t,\\
p^3 &: u_3(x,t) = L^{-1}\left(\frac{1}{s+3} L\left\{\frac{\partial^2 u_2(x,t)}{\partial x^2}\right\}\right) = \frac{32}{3} t^3 \exp 2x - 3t,\\
&\vdots
\end{aligned} \tag{18}$$

Therefore the solution $u(x,t)$ is given by

$$u(x,t) = \exp 2x - 3t\left(1 + 4t + 8t^2 + \frac{32}{3}t^3 + \cdots\right) \tag{19}$$

$$= \exp 2x + t \tag{20}$$

Example 3.2.
Consider nonlinear Newell-Whitehead-Segel equation

$$\frac{\partial u(x,t)}{\partial t} = 5\frac{\partial^2 u(x,t)}{\partial x^2} + 2u(x,t) + u^2(x,t), \tag{21}$$

subject to initial condition

$$u(x,0) = \beta, \tag{22}$$

where β is arbitrary constant. Taking the Laplace transform on both sides of equation (21) subject to the initial condition (22), we have

$$L\{u(x,t)\} = \frac{\beta}{s-2} + \frac{5}{s-2} L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} + \frac{1}{s-2} L\{u^2(x,t)\}. \tag{23}$$

Taking the inverse Laplace transform on equation (23), we obtain

$$u(x,t) = L^{-1}\left(\frac{\beta}{s-2}\right) + L^{-1}\left(\frac{5}{s-2}L\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} + \frac{1}{s-2}L\left\{u^2(x,t)\right\}\right). \tag{24}$$

Now, applying the homotopy perturbation method, we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = L^{-1}\left(\frac{\beta}{s-2}\right) + p\left[L^{-1}\left(\frac{5}{s-2}L\left\{\sum_{n=0}^{\infty} p^n \frac{\partial^2 u_n(x,t)}{\partial x^2}\right\}\right) + \frac{1}{s-2}L\left\{\sum_{n=0}^{\infty} p^n H_n(u)\right\}\right]. \tag{25}$$

Comparing the coefficients of like powers of p , we have

$$\begin{aligned}
p^0 &: u_0(x,t) = L^{-1}\left(\frac{\beta}{s-2}\right) = \beta \exp 2t,\\
p^1 &: u_1(x,t) = L^{-1}\left(\frac{5}{s-2}L\left\{\frac{\partial^2 u_0(x,t)}{\partial x^2}\right\} + \frac{1}{s-2}L\{H_0(u)\}\right) = \frac{\beta^2}{2}\exp 2t\left(\exp 2t - 1\right),\\
p^2 &: u_2(x,t) = L^{-1}\left(\frac{5}{s-2}L\left\{\frac{\partial^2 u_1(x,t)}{\partial x^2}\right\} + \frac{1}{s-2}L\{H_1(u)\}\right) = \frac{\beta^3}{4}\exp 2t\left(\exp 2t - 1\right)^2,\\
p^3 &: u_3(x,t) = L^{-1}\left(\frac{5}{s-2}L\left\{\frac{\partial^2 u_2(x,t)}{\partial x^2}\right\} + \frac{1}{s-2}L\{H_2(u)\}\right) = \frac{\beta^4}{8}\exp 2t\left(\exp 2t - 1\right)^3,\\
&\vdots
\end{aligned} \tag{26}$$

Therefore the solution $u(x,t)$ is given by

$$\begin{aligned}
u(x,t) &= \exp 2t\left(\beta + \frac{\beta^2}{2}\exp 2t\left(\exp 2t - 1\right) + \frac{\beta^3}{4}\exp 2t\left(\exp 2t - 1\right)^2 + \frac{\beta^4}{8}\exp 2t\left(\exp 2t - 1\right)^3 + \cdots\right)\\
&= \frac{2\beta \exp 2t}{2 + \beta\left(1 - \exp 2t\right)}.
\end{aligned} \tag{27}$$

4. Conclusion

In this work, the homotopy perturbation transform method (HPTM) was successfully used for solving Newell-Whitehead-Segel equation with initial condition. The results show that the homotopy perturbation transform method (HPTM) is powerful and efficient technique in finding exact and approximate solutions for nonlinear differential equations. It is worth mentioning that HPTM is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result; the size reduction amounts to an improvement of the performance of the approach. The fact that the HPTM solves nonlinear problems without using AdomianâĂŹs polynomials is a clear advantage of this technique over the decomposition method. In conclusion, the HPTM may be considered as a nice refinement in existing numerical techniques and might find the wide applications.

Acknowledgments

The author acknowledges Ministry of Higher Education and Scientific Research in Iraq for its support this work.

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