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On fuzzy strongly g^* - closed sets and g^{**} - closed sets

Research Article

Hamid Reza Moradi[a], **Anahid Kamali**[b, *]

[a] *Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran*

[b] *Department of Mathematics, Khaje Nasir Toosi University of Technology, Tehran, Iran*

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Abstract: In this paper, direct proofs of some properties of new classes of fuzzy sets called fuzzy strongly g^*-closed sets and fuzzy g^{**}-closed sets are introduced. Examples are presented showing that some generalizations can not be obtained.

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Keywords: Fuzzy generalized closed sets • Fuzzy strongly g^* - closed sets • Fuzzy g^{**}-closed sets

1. Introduction

In 1965, Zadeh [1] introduced the concept of fuzzy sets. Subsequently many researchers have been worked in this area and related areas which have applications in different field based on this concept. The concept of generalized closed sets plays a significant role in topology. There are many research papers which deals with different types of generalized closed sets. Levine [2] introduced the concept of generalized closed sets (briefly g-closed) in topological spaces. Chang in [3] introduced the concept of fuzzy topological space. g^*-closed sets were introduced and studied by Veerakumar [4] for general topology. Recently Parimelazhagan and Subramonia pillai introduced strongly g^*-closed sets in topological space [5].

In this paper we investigate the behaviour of fuzzy strongly g^*-closed sets and fuzzy g^{**}-closed sets in fuzzy topological space and its various characterisation are studied.

2. Basic concepts

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection [3]. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets.

Throughout the present paper, (X,τ) or simply X mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote and define the closure and interior for a fuzzy set A by $cl(A) = \wedge\{\mu:\ \mu \geq A,\ 1-\mu \in \tau\}$ and $\operatorname{int}(A) = \vee\{\mu:\ \mu \leq A,\ \mu \in \tau\}$ respectively.
This section contains some basic definition and preliminary results which will be needed in the sequel.

Definition 2.1.
A fuzzy set A of (X,τ) is called,

1. fuzzy semi open (in short, fs-open) if $A \leq cl(\operatorname{int}(A))$ and a fuzzy semi closed (in short, fs-closed) if $\operatorname{int}[cl(A)] \leq A$.

* Corresponding author.
E-mail address: ana.kamali.gh@gmail.com (Anahid Kamali)

2. fuzzy pre open (in short, fp-open) if $A \leq \text{int}[cl(A)]$ and a fuzzy pre-closed (in short, fp-closed) if $cl(\text{int}(A)) \leq A$.
3. fuzzy α-open (in short, $f\alpha$-open) if $A \leq \text{int}[cl(\text{int}(A))]$ and a fuzzy α-closed (in short, $f\alpha$-closed) if $cl(\text{int}[cl(A)]) \leq A$.
4. fuzzy semi pre-open (in short, fsp-open) if $A \leq cl(\text{int}[cl(A)])$and a fuzzy semi pre-closed (in short, fsp-closed) if $\text{int}[cl(\text{int}(A))] \leq A$.
5. fuzzy θ-open (in short, $f\theta$-open) if $A = \text{int}_\theta(A)$ and a fuzzy θ-closed (in short, $f\theta$-closed) if $A = cl_\theta(A)$ where $cl_\theta(A) = \wedge\{cl(\mu) : A \leq \mu, \mu \in \tau\}$.
6. fuzzy generalized closed (in short, fg-closed) if $cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
7. fuzzy generalized semi closed (in short, gfs-closed) if $scl(A) \leq H$, whenever $A \leq H$ and H is fs-open set in X. This set is also called generalized fuzzy weakly semi closed set.
8. fuzzy generalized semi closed (in short, fgs-closed) if $scl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
9. fuzzy generalized pre-closed (in short, fgp-closed) if $pcl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
10. fuzzy α-generalized closed (in short, $f\alpha g$ - closed) if $\alpha cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
11. fuzzy $fg\alpha$-closed if $\alpha cl(A) \leq H$, whenever H is fuzzy open set in X.
12. fuzzy generalized semi pre-closed (in short, fsp-closed) if $spcl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
13. fuzzy semi-pre-generalized closed (in short, $fspg$-closed) if $spcl(A) \leq H$, whenever $A \leq H$ and H is fs-open in X.
14. fuzzy θ-generalized closed (in short, $f\theta g$-closed) if $cl_\theta(A) \leq H$, whenever $A \leq H$ and H is fuzzy open in X.
15. fuzzy g^*- closed (in short, fg^*-closed) if $cl(A) \leq H$, whenever $A \leq H$ and H is fg- open in X.

Definition 2.2.
A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by xqA_p if and only if $P + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB if and only if there exists $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not quasi-coincident. Then we write $A\bar{q}B$.
Note that $A \leq B$ if and only if $A\bar{q}(1 - B)$ [6].

3. Fuzzy strongly g^* - closed sets

Definition 3.1.
Let (X, τ) be a fuzzy topological space. A fuzzy set A of (X, τ) is called fuzzy strongly g^*-closed if $cl(\text{int}(A)) \leq H$, whenever $A \leq H$ and H is fg-open in X.

We are now ready to construct our main results.

Theorem 3.1.
Every fuzzy closed set is fuzzy strongly g^*-closed set in a fuzzy topological space (X, τ).

Proof. Let A be fuzzy closed set in X and H be a fg-open set in X such that $A \leq H$. Since A is fuzzy closed, $cl(A) = A$. Therefore $cl(A) \leq H$. Now $cl(\text{int}(A)) \leq cl(A) \leq H$. Hence A is fuzzy strongly g^*-closed set in X. □

The converse of the above theorem need not be true in general. Which can be seen from the following example.

Example 3.1.
Let $X = \{a, b, c\}$. Fuzzy sets A and B are defined by $A(a) = 0.7$, $A(b) = 0.3$, $A(c) = 0.5$, $B(a) = 0.2$, $B(b) = 0.1$, $B(c) = 0.3$. Let $\tau = \{0, A, 1\}$. Then B is a fuzzy strongly g^*- closed set but it is not a fuzzy closed set in (X, τ).

Theorem 3.2.
Every fuzzy g- closed set is fuzzy strongly g^*-closed sets in X.

Proof. It is obvious. □

Converse of the above theorem need not be true. It can be seen by the following example.

Example 3.2.
Let $X = \{a, b\}$ and the fuzzy sets A and B be defined as follows: $A(a) = 0.3$, $A(b) = 0.3$, $B(a) = 0.5$, $B(b) = 0.4$. Let $\tau = \{0, B, 1\}$. Then A is fuzzy strongly g^*-closed but it is not fg-closed.

Theorem 3.3.
Every fuzzy g^* - closed set is a fuzzy strongly g^* - closed sets in X.

Proof. Suppose that A is fg^*-closed in X. Let H be a fg-open set in X such that $A \le H$ .Then $cl(A) \le H$, since A is fg^*-closed. Now $cl(\text{int}(A)) \le cl(A) \le H$. Hence A is fuzzy strongly g^*-closed set in X. □

However the converse of the above theorem need not be true as seen from the following example.

Example 3.3.
Let $X = \{a, b\}$, $\tau = \{0, A, B, D, 1\}$ and fuzzy sets A, B , D and H are defined as follows: $A(a) = 0.2$, $A(b) = 0.4$, $B(a) = 0.6$, $B(b) = 0.7$, $D(a) = 0.4$, $D(b) = 0.6$, $H(a) = 0.4$, $H(b) = 0.5$. Then H is fuzzy strongly g^*-closed set but it not fg^*-closed in (X, τ).

Theorem 3.4.
A fuzzy set A of fuzzy topological space (X, τ) is fuzzy strongly g^*-closed if and only if $A\bar{q}B$ then $cl(\text{int}(A))\,\bar{q}\,B$ for every fg-closed set B of X.

Proof. Suppose that A is a fuzzy strongly g^*-closed set of X such that $A\bar{q}B$. Then $A \le 1-B$ and $1-B$ is a fuzzy g-open set X. which implies that $cl(\text{int}(A)) \le 1-B$, since A is fuzzy strongly g^*-closed. Hence $cl(\text{int}(A))\,\bar{q}\,B$.

Conversely, let E be a fuzzy g-open let in X such that $A \le E$. Then $A\bar{q}\,1-E$ and $1-E$ is fg-closed set in X. By hypothesis, $cl(int(A))\,\bar{q}\,(1-E)$ which implies $cl(int(A)) \le E$. Hence A is fuzzy strongly g^*-closed in X. □

Theorem 3.5.
If A is a fuzzy strongly g^*-closed set in (X, τ) and $A \le B \le cl(\text{int}(A))$, then B is fuzzy strongly g^* - closed in (X, τ).

Proof. Let A be a fuzzy strongly g^*- closed set in (X, τ). Let $B \le H$ where H is a fuzzy g- open set in X . Then $A \le H$. Since A is fuzzy strongly g^*- closed set, it follows that $cl(\text{int}(A)) \le H$.

Now, $B \le cl(\text{int}(A))$ implies $cl(int(B)) \le cl(\text{int}(cl(\text{int}(A)))) = cl(\text{int}(A))$. We get, $cl(\text{int}(B)) \le H$. Hence, B is fuzzy strongly g^*-closed set in (X, τ). □

Theorem 3.6.
If A is a fuzzy strongly g^* - closed set in $(X, \tau)$and $A \le B \le cl(\text{int}(A))$, then B is fuzzy strongly g^* - closed in (X, τ).

Proof. Let A be a fuzzy strongly g^*-closed set in (X, τ) and $B \le H$ where H is a fuzzy g-open set in X . Then $A \le H$. Since A is fuzzy strongly g^*-closed set, it follows that $cl(\text{int}(A)) \le H$.

Now $B \le cl(\text{int}(A))$ implies $cl(int(B)) \le cl(\text{int}(cl(\text{int}(A)))) = cl(\text{int}(A))$ We get, $cl(\text{int}(B)) \le H$. Hence, B is fuzzy strongly g^*-closed set in (X, τ). □

Definition 3.2.
A fuzzy set A of (X, τ) is called fuzzy strongly g^* - open set in X if and only if $1-A$ is fuzzy strongly g^*-closed in X. In other words, A is fuzzy strongly g^*-open if and only if $H \le cl(\text{int}(A))$, whenever $H \le A$ of H is fg-closed in X.

Theorem 3.7.
Let A be a fuzzy strongly g^*-open in X and $\text{int}(cl(A)) \le B \le A$ then B is fuzzy strongly g^*-open in X.

Proof. Suppose that A is fuzzy strongly g^*- open in X and $\text{int}(cl(A)) \le B \le A$. Then $1-A$ is fuzzy strongly g^*-closed in X and $1-A \le 1-B \le cl(\text{int}(1-A))$. So $1-B$ is fuzzy strongly g^*-closed. Hence B is fuzzy strongly g^*-open in X. □

Theorem 3.8.
If a fuzzy set A of a fuzzy topological space X is both fuzzy open and fuzzy strongly g^*-closed then it is fuzzy closed.

Proof. Suppose that a fuzzy set A of X is both fuzzy open and fuzzy strongly g^*-closed. Now $A \geq cl(\text{int}(A)) \geq cl(A)$. That is $A \geq cl(A)$. Since $A \leq cl(A)$, we get $A = cl(A)$. Hence A is fuzzy closed in X. □

Theorem 3.9.
If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g^*-closed and fuzzy semi open, then it is fg^*-closed.

Proof. Suppose a fuzzy set A of X is both fuzzy strongly g^*-closed and fuzzy semi open in X. Let H be a fg-open set such that $A \leq H$. Since A is fuzzy strongly g^*-closed, therefore $cl(\text{int}(A)) \leq H$.
Also since A is fs-open, $A \leq cl(\text{int}(A))$. We have $cl(A) \leq cl(\text{int}(A)) \leq H$. Hence A is fg^*-closed in X. □

4. Fuzzy g^{**}- closed sets

In this section, we study properties of fuzzy g^{**}-closed sets. Before proceeding further, we need a definition of g^{**}-closed sets.

Definition 4.1.
A subset A of a topological space (X,τ) is called g^{**}-closed set if $cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy g^*-open in (X,τ). The complement of a fuzzy g^{**}-closed set is called fuzzy g^{**}-open set.

Theorem 4.1.
Every fuzzy closed set is fuzzy g^{**}-closed set, but the converse may not be true in general.

Proof. Let A be a closed set.

TPT: A is fuzzy g^{**}-closed set, (i.e.)

TPT: if $cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy g^*-open Since A is closed, $A = cl(A)$.

Let $A \leq H$, H is fuzzy g^*-open. Then $cl(A) = A \leq H$ therefore A is g^{**}-closed. □

Theorem 4.2.
Every fuzzy g^*-closed set is fuzzy g^{**}-closed set, but the converse may not be true in general.

Proof. Let A be fuzzy g^*-closed set.

TPT: A is fuzzy g^{**}-closed, (i.e.)

TPT: if $cl(A) \leq H$ whenever $A \leq H$ and H is fuzzy g^*-open.

By the definition of fuzzy g^*-closed set, if $cl(A) \leq H$ whenever $A \leq H$ and H is fuzzy g-open let $A \leq H$ where H is fuzzy g^*-open. Since H is fuzzy g^*-open then, H is fuzzy g-open. Thus $cl(A) \leq H$, H is fuzzy g^*-open, A is fuzzy g^{**}- closed. □

Theorem 4.3.
If A is fuzzy strongly g^*-closed and A is fuzzy open then A is fuzzy g^{**}-closed set.

Proof. Let $A \leq H$ and H is fuzzy g^*-open. Since A is fuzzy strongly g^*-closed set $cl(\text{int}(A)) \leq H$, whenever $A \leq H$ and H is fuzzy g-open in X. Since A is open, $\text{int}(A) = A$, also is fuzzy g^*-open then, is fuzzy g-open. Thus $cl(A) \leq H$, U is fuzzy g^*-open, A is fuzzy g^{**}-closed. □

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