Int. J. Adv. Appl. Math. and Mech. 3(1) (2015) 1 - 9 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

A novel computable extension of fractional kinetic equations arising in astrophysics

Research Article

V.B.L. Chaurasia[a], **Yudhveer Singh**[b, *]

[a] *Department of Mathematics, University of Rajasthan, -302055, Rajasthan, India*

[b] *Department of Mathematics, Jaipur National University, Jaipur-302025, Rajasthan, India*

Received 24 Septamber 2014; accepted (in revised version) 13 July 2015

Abstract: The objective of the present paper is to develop the solutions of generalized fractional kinetic equations involving the generalized Mittag-Leffler function and I-function. The use of mathematical physics in distinguished astrophysical problems has attracted astronomers and physicists to special attention on available mathematical tools that can be used in solving several problems of astrophysics. The manifold generality of the generalized Mittag-Leffler and I-function is discussed in terms of the solution of the above fractional kinetic equations. Special cases involving the generalized Mittag-Leffler function, $\bar{H}$-function, Fox H-function and generalized M-series are considered. The obtained results imply the known results more precisely.

MSC: 26A33 • 33E12 • 33C60 • 44A10

Keywords: Fractional kinetic equation • Generalized Mittag-Leffler function • I-function • Riemann-Liouville operator • Generalized M-series • Laplace transform

1. Introduction

In last few decades fractional kinetic-equations have been extensively used in describing and solving various problems of applied sciences. Fractional calculus and special functions have also contributed a lot to mathematical physics and its various branches. Nuclear reactions are highly energetic process involving subatomic particles found within the nuclei of atoms, are in two general forms. One in which reactions involve the splitting of atomic nuclei in to smaller subatomic particles, called Fission and another one is the process which create larger atoms from the nuclei of smaller atoms, called the Fusion. In the fusion the nuclei of two or more atoms collide with sufficient force, they fuse to form a single, larger nucleus. During this process part of mass of the fused nucleus is converted in to energy. This produced energy is released as heat, light and various forms of radiation. Such nuclear reactions require extremely high temperature to induce the necessary immense collisions. Because such immense amount of heat energy are required, fusion reactions are also known as thermonuclear reactions. The stars themselves are formed and fueled by thermonuclear reactions. Thus thermonuclear fusion play an important role in the formation of stars and to keep them shining for billions of years. A star can be taken as a symmetric gas sphere in thermal and hydrostatic equilibrium with negligible rotation and magnetic fields. The star is characterized by its mass, luminosity, effective surface temperature, radius, central density and central temperature. The Steller structures and their mathematical models are investigated on the basis of the above characteristics in addition to some additional information related to the equation of state, nuclear energy generation rate, and opacity. The assumption of thermal equilibrium and hydrostatic equilibrium indicate that there is no time dependence in the mathematical model, which involve mathematical equations describing the internal structure of the star ([1–3]). Energy in such stellar structures in being produced by the process of chemical reactions. Computation of such chemical reactions is of the prime importance as it plays the central role in the evolution of such stellar structures. The two important nuclear reactions in stars, during their

* Corresponding author.
E-mail addresses: drvblc@yahoo.com (V.B.L. Chaurasia), yudhvir.chahal@gmail.com (Yudhveer Singh)

evolution, are pp chain (proton-proton chain) and CNO cycle (involves nuclei of carbon, nitrogen and oxygen). The total energy production and luminosity of the star is based on the pp chain and the composition of stellar plasma described by CNO cycle. The production and destruction of nuclei in such chemical reactions can be described by the reaction- equations. Solution of such reaction- (Linear/nonlinear) equations determine distribution function of the dynamical states of single particle. The linear reaction- equation $\frac{dy}{dx} = y^q$, lead to new insights into generalized Boltzmann-Gibbs statistical mechanics which is called nonextensive statistical mechanics. Recently, Ferro et al. [4] studied that a very small deviation from the Maxwell-Boltzman particle distribution and the use of nonextensive statistical mechanics can be applied to describe the modified nuclear reaction rate of stellar plasmas which is consistent with need of the modification of the nuclear reaction rates of stellar plasma and their chemical composition. If an arbitrary reaction is characterized by a time dependent quantity N = N(t) then it is possible to calculate the rate of change of $\frac{dN}{dt}$ by mathematical equation

$$\frac{dN}{dt} = -d + p, \tag{1}$$

where d is the distruction rate and p is the production rate of N.

Haubold and Mathai [5] have established a functional differential equation between rate of change of reaction, the destruction rate and the production rate as follows

$$\frac{dN}{dt} = -d\,(N_t) + p\,(N_t), \tag{2}$$

where N = N(t) the rate of reaction, d = d(N) the rate of destruction, p = p(N) the rate of production and N_t denotes the function defined by $N_t(t^*) = N(t - t^*)$, $t^* > 0$.

They have studied a special case of (2), for spatial fluctuations or inhomogeneities in the quantity N(t) are neglected, given by the equation

$$\frac{dN_i}{dt} = -c_i N_i(t). \tag{3}$$

with the initial condition that $N_i(t = 0) = N_0$ is the number density of species i at time t = 0, $c_i > 0$, known as standard kinetic equation.

The solution of equation (3) is given by

$$N_i(t) = N_0\, e^{-c_i t}. \tag{4}$$

An alternative form of the same equation can be obtained on integration

$$N(t) - N_0 = c\,{}_0D_t^{-1} N(t), \tag{5}$$

where ${}_0D_t^{-1}$ is the standard integral operator. Haubold and Mathai [5] have given the fractional generalization of the standard kinetic equation (5) as

$$N(t) - N_0 = c^{\nu}\,{}_0D_t^{-\nu} N(t), \tag{6}$$

where ${}_0D_t^{-\nu}$ is the well-known standard Riemann-Liouville fractional integral operator (Oldham and Spanier [6] ; Samko et. al [7] ; Miller and Ross [8]) defined by

$${}_0D_t^{-\nu} f(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-u)^{\nu-1} f(u)\, du, \;\; R(\nu) > 0, \tag{7}$$

the solution of the fractional kinetic equation (6) in the computable series representation is given by (Haubold and Mathai [5])

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k + 1)} (ct)^{\nu k}, \tag{8}$$

due to the importance of kinetic equation in mathematical physics many authors have generalized the standard kinetic equation time to time.

In the recent paper of Haubold and Mathai [5] have derived the fractional kinetic equation and thermonuclear function in terms of well known Mittag-Leffler function. As an extension of the work of Haubold and Mathai [5] , Saxena et al. [9] have generalized the standard kinetic equation with generalized Mittag-Leffler functions and R-function. Further, Chaurasia and Pandey [4] established a computable fractional generalization of the fractional kinetic equation, which can be used to compute the change of chemical composition in stars like the sun. Further, Chaurasia and Kumar [11] generalized and studied the kinetic equation with generalized M-series of Sharma [12] .

The F-function (Hartley and Lorenzo [13]), the Mittag-Leffler function (Mittag-Leffler [4]), the generalized Mittag-Leffler function (Prabhakar [15]), the R-function, the Lorenzo-Hartley function (Lorenzo and Hartley [16]) and the new generalized Mittag-Leffler function (Salim and Faraj [17]) are some generalized function for fractional calculus. Such function provide direct solutions and understanding for fundamental linear fractional order differential equations and related initial value problems. In this paper, we introduce and investigate the further computable generalization of the generalized fractional kinetic equation. The fractional kinetic equation and its solution, discussed in term of the new generalized Mittag-Leffler function and I-function [18] , are written in compact and easily computable form.

2. Definitions

Definition 2.1.

The Swedish mathematician Mittag-Leffler [14] introduced the function $E_\nu(z)$ defined as

$$E_\nu(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\nu k+1)} z \in C \tag{9}$$

A generalization of $E_\nu(z)$ was studied by Wiman [19] where he defined the function $E_{\nu,\mu}(z)$as

$$E_{\nu,\mu}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\nu k+\mu)}, (z,\nu,\mu,\in C; R(\nu)>0, R(\mu)>0) \tag{10}$$

Prabhakar [15] introduced the function $E^{\gamma}_{\nu,\mu}(z)$ in the form

$$E^{\gamma}_{\nu,\mu}(z) = \sum_{k=0}^{\infty} \frac{\gamma_k z^k}{\Gamma(\nu k+\mu) k!}, \quad (z,\nu,\mu,\gamma \in C; R(\nu)>0, Re(\mu)>0, Re(\gamma)>0) \tag{11}$$

Shukla and Prajapati [20] (see also Srivastava and Tomovski [21]]) defined and investigated the function $E^{\gamma,\delta}_{\nu,\mu}(z)$as

$$E^{\gamma,\delta}_{\nu,\mu}(z) = \sum_{k=0}^{\infty} \frac{\gamma_{\delta k}}{\Gamma(\nu k+\mu)} \frac{z^k}{k!}, \quad (z,\nu,\mu,\gamma \in C; R(\nu)>0, R(\mu)>0, R(\gamma)>0, \delta \in (0,1)\cup N) \tag{12}$$

A new generalization of Mittag-Leffler function was defined by Salim [22] as

$$E^{\gamma,\delta}_{\nu,\mu}(z) = \sum_{u=0}^{\infty} \frac{\gamma_k}{\Gamma(\nu k+\mu)} \frac{z^k}{(\delta)_k}, \quad (z,\nu,\mu,\delta \in C, R(\nu)>0, R(\mu)>0, R(\gamma)>0, R(\delta)>0) \tag{13}$$

Further he introduced a new generalization of Mittag-Leffler function defined as

$$E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(z) = \sum_{k=0}^{\infty} \frac{\gamma_{qk}}{\Gamma(\nu k+\mu)} \frac{z^k}{(\delta)_{\alpha k}}, \tag{14}$$

where $z,\nu,\mu,\gamma,\delta \in C$; $min\{R(\nu), R(\mu), R(\gamma), R(\delta)\} > 0; \alpha, q > 0 \; and \; q \leq R(\nu)+\alpha$.
Equation (14) is a generalization of equations (9) – (13).
$E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(z)$ in terms of other functions:
Wright generalized function [23] as

$$E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} {}_2\psi_2 \left[{}^{(\gamma,q),(1,1)}_{(\delta,\alpha),(\mu,\nu)} ; z \right]. \tag{15}$$

Fox's H-function [24] as

$$E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} H^{1,2}_{2,3} \left[-z \left| {}^{(0,1),(1-\gamma,q)}_{(0,1),(1-\mu,\nu),(1-\delta,\alpha)} \right. \right]. \tag{16}$$

2.1. The I-function and its relationship with other functions

Definition 2.2.
Rathie [18] introduced the I-function is defined as Mellin-Barnes contour integral:

$$I^{M,N}_{P,Q}[t] = I^{M,N}_{P,Q}\left[t\left|^{(a_1,\gamma_1,A_1),\ldots,(a_P,\gamma_P,A_P)}_{(b_1,\delta_1,B_1),\ldots,(b_Q,\delta_Q,B_Q)}\right.\right] = \frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\theta(\xi)\,t^{\xi}\,dt, \tag{17}$$

where

$$\theta(\xi) = \frac{\prod_{j=1}^{M}\{\Gamma(b_j-\delta_j\xi)\}^{B_j}\prod_{j=1}^{N}\{\Gamma(1-a_j+\gamma_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\delta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\{\Gamma(a_j-\gamma_j\xi)\}^{A_j}}, \tag{18}$$

where $A_j, j = 1,\ldots,$ P, j = 1,... ,Q are not, in general, positive integers. Clearly for non-integral values A_j or B_j, ((17)) is not expressible as an H-function. Here t may be real or complex but is not equal to zero and an empty product is interpreted as unity; P,Q,M and N are integers such that $0 \leq M \leq Q$; $0 \leq N \leq P$; $\gamma_j > 0\ (j = 1,...,P)$, $\delta_j > 0\ (j = 1,...,Q)$; a_j (j = 1,... ,P) and b_j (j = 1,... ,Q) are complex parameters.

The contour in (17) is presumed to be the imaginary axis Re(ξ) = 0 which is suitable indented in order to avoid the singularities of the gamma functions and to keep these singularities at appropriate sides. For A_j not an integer, the pole of the gamma functions of the numerator of (18) is converted to branch points. The branch cuts can be chosen in order that the path of integration can be distorted for the contour Re(ξ) = 0 as long as there is no coincidence of poles from $\Gamma(b_j-\delta_j\xi)\, and\, \Gamma(1-a_j+\gamma_j\xi)$. The sufficient conditions for the absolute convergence of the contour integral (17) is given by

$$\nabla = \sum_{j=1}^{M}|B_j\delta_j| + \sum_{j=1}^{N}|A_j\gamma_j| - \sum_{j=M+1}^{Q}|B_j\delta_j| - \sum_{j=N+1}^{P}|A_j\gamma_j| > 0, \tag{19}$$

this condition provides exponential decay of the integrand in (17), and region of absolute convergence of (17) is

$$|arg\ t| \leq \frac{\nabla\pi}{2} \tag{20}$$

If we take B_j (j = 1,... ,M) and A_j (j = 1,... ,N) unity in (17), I-function reduces to $\bar{H}$-function (Gupta and Soni [25]). When the exponents $A_j = 1$ (j = 1,... ,P) and $B_j = 1$ (j = 1,... ,Q) in (17) the I-function reduces to the familiar Fox's H-function defined by Fox [24] .

3. Extensions of generalized fractional kinetic equations

Theorem 3.1.
Let $t,\nu,\mu,\gamma,\delta \in C$, $min\{R(\nu),R(\mu),R(\gamma),R(\delta)\} > 0$; $\alpha > 0$, $q > 0$, $c > 0$, $d > 0$, *then for the solution of the fractional kinetic equation*

$$N(t) - N_0\,t^{\mu-1}\,E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(-d^{\nu}t^{\nu}) = -c^{\nu}\,{}_0D_t^{-\nu}\,N(t), \tag{21}$$

has a solution of the form

$$N(t) = N_0\,t^{\mu-1}\sum_{r=0}^{\infty}(-c^{\nu}t^{\nu})^r\,E^{\gamma,\delta,q}_{\nu,\nu r+\mu,\alpha}(-d^{\nu}\,t^{\nu}). \tag{22}$$

Proof. We know that (Erdélyi et al. [26]) the Laplace transform of the Riemann-Liouville fractional integration is given by

$$L\{{}_0D_t^{-\nu}\,f(t)\,;\,p\} = p^{-\nu}\,F(p), \tag{23}$$

where

$$F(p) = \int_0^{\infty} e^{-pu}\,f(u)\,du. \tag{24}$$

Now taking Laplace transform of both sides of (21), we have

$$L\{N(t);p\} - N_0\, L\{t^{\mu-1} E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(-d^\nu t^\nu);p\} = L\{-c^\nu\, {}_0D_t^{-\nu} N(t)\,;\,p\}$$

$$N(p) - N_0 \int_0^\infty e^{-pt}\, t^{\mu-1} \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-d^\nu t^\nu)^k}{\Gamma(\nu k+\mu)\,(\delta)_{\alpha k}} dt = -\,c^\nu p^{-\nu} N(p), \tag{25}$$

$$N(p) - N_0 \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-d^\nu)^k}{\Gamma(\nu k+\mu)\,(\delta)_{\alpha k}} \int_0^\infty e^{-pt}\, t^{\nu k+\mu-1}\, dt = -\,c^\nu p^{-\nu} N(p)$$

$$N(p)\,[1 + c^\nu p^{-\nu}] = N_0 \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-d^\nu)^k}{(\delta)_{\alpha k}\, p^{\nu k+\mu}}$$

$$N(p) = N_0 \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-d^\nu)^k}{(\delta)_{\alpha k}} \left\{ p^{-(\nu k+\mu)} \sum_{r=0}^{\infty} (-1)^r\,(c^\nu p^{-\nu})^r \right\}. \tag{26}$$

Taking Inverse Laplace transform of both side of equation (26), we have

$$L^{-1}\{N(p)\} = N_0 \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-d^\nu)^k}{(\delta)_{\alpha k}} \left\{ L^{-1}\left[\sum_{r=0}^{\infty} (c^\nu)^r\, p^{-(\nu k+\nu r+\mu)} \right] \right\}, \tag{27}$$

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-d^\nu)^k}{(\delta)_{\alpha k}} \left\{ \sum_{r=0}^{\infty} (-1)^r\,(c^\nu)^r \frac{t^{\nu k+\nu r+\mu-1}}{\Gamma(\nu k+\nu r+\mu)} \right\}, \tag{28}$$

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^\nu t^\nu)^r\, E^{\gamma,\delta,q}_{\nu,\nu r+\mu,\alpha}(-d^\nu t^\nu), \tag{29}$$

which is equation (22), this completes the proof of Theorem. □

Theorem 3.2.
If $\nu > 0, \mu > 0, c > 0, d > 0, \gamma_j > 0\,(j = 1,...,P)$ and $\delta_j > 0\,(j = 1,...,Q)$, then for the solution of fractional kinetic equation

$$N(t) - N_0\, t^{\mu-1}\, I^{M,N}_{P,Q}\left[-d^\nu t^\nu \,\Big|^{(a_1,\gamma_1,A_1),...,(a_P,\gamma_P,A_P)}_{(b_1,\delta_1,B_1),...,(b_Q,\delta_Q,B_Q)} \right] = -c^\nu\, {}_0D_t^{-\nu} N(t), \tag{30}$$

has a solution of the form

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^\nu t^\nu)^r\, I^{M,N+1}_{P+1,Q+1}\left[-d^\nu t^\nu \,\Big|^{(1-\mu,\nu;1),(a_1,\gamma_1,A_1),...,(a_P,\gamma_P,A_P)}_{(b_1,\delta_1,B_1),...,(b_Q,\delta_Q,B_Q),(1-\mu-\nu r,\nu;1)} \right] \tag{31}$$

Proof. Applying the Laplace transform both side of equation (30), we get

$$N(p) - N_0 \frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \theta(\xi)\,(-d^\nu)^\xi \frac{\Gamma(\nu\xi+\mu)}{p^{\nu\xi+\mu}}\, d\xi = -\,c^\nu p^{-\nu} N(p), \tag{32}$$

$$N(p) = \frac{N_0}{(1+c^\nu p^{-\nu})} \frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \theta(\xi)\,(-d^\nu)^\xi \frac{\Gamma(\nu\xi+\mu)}{p^{\nu\xi+\mu}}\, d\xi,$$

$$N(p) = N_0 \sum_{r=0}^{\infty} (-1)^r\,(c^\nu p^{-\nu}) \frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \theta(\xi)\,(-d^\nu) \frac{\Gamma(\nu\xi+\mu)}{p^{\nu\xi+\mu}}\, d\xi, \tag{33}$$

Now taking Inverse Laplace transform both side of (33), we obtain the desired result(31). □

4. special cases

Corollary 4.1.
If $min\{R(\nu), R(\nu-\mu), R(\gamma), R(\delta)\} > 0$, $\alpha, q, c, d > 0$, *then for the solution of equation*

$$N(t) - N_0\, t^{\nu-\mu-1}\, E^{\gamma,\delta,q}_{\nu,\nu-\mu,\alpha}(-d^\nu\, t^\nu) = -c^\nu\, {}_0D_t^{-\nu}\, N(t), \tag{34}$$

the following result holds If $min\{R(\nu), R(\nu-\mu), R(\gamma), R(\delta)\} > 0$, $\alpha, q, c, d > 0$, *then for the solution of equation*

$$N(t) - N_0\, t^{\nu-\mu-1}\, E^{\gamma,\delta,q}_{\nu,\nu-\mu,\alpha}(-d^\nu\, t^\nu) = -c^\nu\, {}_0D_t^{-\nu}\, N(t), \tag{35}$$

the following result holds.

$$N(t) = N_0\, t^{\nu+\mu-1} \sum_{r=0}^{\infty} (-c^\nu\, t^\nu)^r\, E^{\gamma,\delta,q}_{\nu,\nu r+\nu-\mu,\alpha}(-d^\nu\, t^\nu). \tag{36}$$

If we put $\alpha = q = 1$ *in (21), then we arrive at the following result.*

Corollary 4.2.
If $R(\nu) > 0$, $R(\mu) > 0$, $R(\gamma) > 0$, $c > 0$ *and* $d > 0$, *then for the solution of equation*

$$N(t) - N_0\, t^{\mu-1}\, E^{\gamma,\delta}_{\nu,\mu}(-d^\nu\, t^\nu) = -c^\nu\, {}_0D_t^{-\nu}\, N(t), \tag{37}$$

the following result hold

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^\nu\, t^\nu)^r\, E^{\gamma,\delta}_{\nu,\nu r+\mu}(-d^\nu\, t^\nu). \tag{38}$$

If we take $\delta = \alpha = q = 1$ and $c = d$ in (21), then we arrive at the following result obtained by Saxena et al. [9].

Corollary 4.3.
If $R(\nu) > 0$, $R(\mu) > 0$, $R(\gamma) > 0$ *and* $c > 0$, *then for the solution of the equation*

$$N(t) - N_0\, t^{\mu-1}\, E^{\gamma}_{\nu,\mu}(-c^\nu\, t^\nu) = -c^\nu\, {}_0D_t^{-\nu}\, N(t), \tag{39}$$

the following result hold

$$N(t) = N_0\, t^{\mu-1}\, E^{\gamma+1}_{\nu,\mu}(-c^\nu\, t^\nu). \tag{40}$$

If we set $\delta = \alpha = q = \gamma = 1$ in equation (21), then we arrive the results obtained by the Saxena et al.[27] .

Corollary 4.4.
If $R(\nu) > 0$, $R(\mu) > 0$, $c > 0$ *and* $d > 0$, *then for the solution of the equation*

$$N(t) - N_0\, t^{\mu-1}\, E_{\nu,\mu}(-d^\nu t^\nu) = -c^\nu\, {}_0D_t^{-\nu}\, N(t), \tag{41}$$

the following result holds

$$N(t) = N_0\, \frac{t^{\mu-\nu-1}}{c^\nu - d^\nu}\, [E_{\nu,\mu-\nu}(-d^\nu t^\nu) - E_{\nu,\mu-\nu}(-c^\nu t^\nu)]. \tag{42}$$

when $\delta = \alpha = q = \gamma = 1$ and $c = d$ in equation (21), then we arrive at the following result given by Saxena et al. [27].

Corollary 4.5.
If $R(\nu) > 0$, $R(\mu) > 0$ and $c > 0$, then for the solution of the equation

$$N(t) - N_0\, t^{\mu-1} E_{\nu,\mu}(-c^{\nu} t^{\nu}) = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{43}$$

the following result holds.

$$N(t) = \frac{N_0\, t^{\mu-1}}{\nu} [E_{\nu,\mu-1}(-c^{\nu} t^{\nu}) + (1+\nu-\mu) E_{\nu,\mu}(-c^{\nu} t^{\nu})]. \tag{44}$$

If we take $A_j (j = N+1,\ldots,P)$ and $B_j (j = 1,\ldots,M)$ unity in equation (30), then we arrive at the result.

Corollary 4.6.
If $\nu > 0$, $\mu > 0$, $c > 0$, $d > 0$, $\gamma_j > 0 (j = 1,...,P)$ and $\delta_j > 0 (j = 1,...,Q)$, then for the solution of equation

$$N(t) - N_0\, t^{\mu-1} \bar{H}^{M,N}_{P,Q} \left[-d^{\nu} t^{\nu} \left|^{(a_j,\gamma_j;A_j)_{1,N},(a_j,\gamma_j)_{N+1,P}}_{(b_j,B_j)_{1,M},(b_j,\delta_j;B_j)_{M+1,Q}} \right. \right] = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{45}$$

the following result holds

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^{\nu} t^{\nu})^r \bar{H}^{M,N+1}_{P+1,Q+1} \left[-d^{\nu} t^{\nu} \left|^{(1-\mu,\nu;1),(a_j,\gamma_j;A_j)_{1,N},(a_j,\gamma_j)_{N+1,P}}_{(b_j,\delta_j)_{1,M},(b_j,\delta_j;B_j)_{M+1,Q},(1-\mu-\nu r,\nu;1)} \right. \right]. \tag{46}$$

For existence condition of $\bar{H}$-function (Gupta and Soni [25]).

Corollary 4.7.
If $\nu > 0$, $\mu > 0$, $c > 0$, $d > 0$, $\gamma_j > 0 (j = 1,...,P)$ and $\delta_j > 0 (j = 1,...,Q)$, then for the solution of equation

$$N(t) - N_0\, t^{\mu-1} \bar{H}^{M,N}_{P,Q+1} \left[-d^{\nu} t^{\nu} \left|^{(a_j,\gamma_j;A_j)_{1,N},(a_j,\gamma_j)_{N+1,P}}_{(b_j,B_j)_{1,M},(b_j,\delta_j;B_j)_{M+1,Q},(1-\mu,\nu;1)} \right. \right] = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{47}$$

the following result holds

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^{\nu} t^{\nu})^r \bar{H}^{M,N}_{P,Q+1} \left[-d^{\nu} t^{\nu} \left|^{(a_j,\gamma_j;A_j)_{1,N},(a_j,\gamma_j)_{N+1,P}}_{(b_j,\delta_j)_{1,M},(b_j,\delta_j;B_j)_{M+1,Q},(1-\mu-\nu r,\nu;1)} \right. \right]. \tag{48}$$

For $A_j = Bj = 1$ in equation (47), we arrive at the result.

Corollary 4.8.
If $\nu > 0$, $\mu > 0$, $c > 0$, $d > 0$, $\gamma_j > 0$ and $\delta_j > 0$, then for the solution of equation

$$N(t) - N_0\, t^{\mu-1} H^{M,N}_{P,Q+1} \left[-d^{\nu} t^{\nu} \left|^{(a_j,\gamma_j)_{N+1,P}}_{(b_j,\delta_j)_{1,Q},(1-\mu,\nu)} \right. \right] = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{49}$$

the following result holds

$$N(t) - N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^{\nu} t^{\nu})^r H^{M,N}_{P,Q+1} \left[-d^{\nu} t^{\nu} \left|^{(a_j,\gamma_j)_{1,P}}_{(b_j,\delta_j)_{1,Q},(1-\mu-\nu r,\nu)} \right. \right] \tag{50}$$

Finally, for existence condition of H-function, we refer to the book Srivastava et al. [28].

On specializing the parameters in above Corollary 4.8 suitable and then making some obvious changes there in, we arrive at the result recently obtained by the Chaurasia and Kumar [11] .

Corollary 4.9.
If $\nu > 0$, $\mu > 0$, $c > 0$ and $d > 0$, then for the solution of equation

$$N(t) - N_0\, t^{\mu-1}\, {}_P \overset{\nu,\mu}{M}{}_Q(a_1,...,a_P; b_1,...,b_Q; -d^{\nu} t^{\nu}) = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{51}$$

the following result holds

$$N(t) = N_0\, t^{\mu-1} \sum_{r=0}^{\infty} (-c^{\nu} t^{\nu})^r\, {}_P \overset{\nu,\mu+r\nu}{M}{}_Q(a_1,...,a_P; b_1,...,b_Q; -d^{\nu} t^{\nu}). \tag{52}$$

5. Concluding remarks

Kinetic equations are used in computation of chemical changes in star (like the sun). Haubold and Mathai [5] investigated the standard kinetic equation which is suitable for incorporating changes in the Maxwell-Boltzmann distribution function. Further Saxena, Mathai and Haubold [9, 27] investigated some other fractional generalizations of the standard kinetic equation which contain the thermonuclear function as a time constant.

In this paper, we have introduced an extended fractional generalization of the standard kinetic equation and established solution for the same. Fractional kinetic equations can be used to compute the particle reaction rate and describe the statistical mechanics associated with the particles distribution function. The generalized fractional kinetic equation discussed in this paper, involves the generalized Mittag-Leffler function $E^{\gamma,\delta,q}_{\nu,\mu,\alpha}(-d^{\nu}t^{\nu})$and I-function. It can be seen that the fractional kinetic equation, discussed in this paper, contain a number of the known (and also new) fractional kinetic equations involving various other special functions (the generalized Mittag-Leffler function, Mittag-Leffler functions, $\bar{H}$-function, Fox H-function and generalized M-series etc.). The results obtained in the present paper provide an extension of the results given by Haubold and Mathai [5] , Saxena, Mathai and Haubold [9, 27] and Chaurasia and Kumar [11] .

References

[1] V. Kourganoff , Introduction to the physics of stellars interiors, Dordrecht: D. Reidel publishing Company,1973.

[2] J. Predang , Lecture notes in stellar stability, Parts I and II, Instituto di Astronomia, padova, 1976.

[3] D.D. Clayton, Principles of Stellar Evolution and Nucleosynthesis, 2nd edn, The University of Chicago and London, 1983.

[4] F. Ferro, A. Lavago, P. Quarati, Temperature dependence of modified CNO nuclear reaction rate in dense stellar plasmas, arXiv:nuclth/0312106V1, 2003.

[5] H.J. Haubold, A.M. Mathai, The fractional kinetic equation and thermonuclear functions, Ap and SS 273 (2000) 53-63.

[6] K.B. Oldham, J. Spanier, the Fractional Calculus. Theory and Applications of Differentiation and Integration of arbitrary order, New York, Academic Press, 1974.

[7] S.G. Samko, A.A. Kilbas, O.I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, New York, Gordon and Breach, 1993.

[8] K.S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, New York, John Wiley and Sons, 1993.

[9] R.K. Saxena, A.M. Mathai, H.J.Haubold, On generalized fractional kinetic equations, Physica A 344 (2004) 653-664.

[10] V.B.L. Chaurasia, S.C.Pandey, On the new computable solution of the generalized fractional kinetic equations involving the generalized function for the fractional calculus and related functions, Ap and SS 317 (2008) 213-219.

[11] V.B.L. Chaurasia, D. Kumar, On the solutions of Generalized fractional kinetic equations, Adv. Stud. Theor. Phys. 16 (2010) 773-780.

[12] M. Sharma, M., R. Jain, A note on a generalized M-series as a special function of fraction calculus, Fract. Clac. Appl. Anal. 12(4) (2009) 499-452.

[13] T.T. Hartley, C.F. Lorenzo, A solution to the fundamental linear fractional order differential equation NASA/TP-(1998) 208963.

[14] G.M. Mittag-Leffler, Sur la nouvelle function, C.R. Acad. Sci. Paris 137 (1903) 554-558.

[15] T.R. Prabhakar, A singular integral equation with the generalized Mittag-Leffler function in the kernel. Yokohama Math. J. 19 (1971) 7-15.

[16] C.F. Lorenzo,T.T. Hartley, Generalized function for the fractional Calculus. NASA/TP -(1999) 209429.

[17] T.O. Salim, A.W. Faraj, A Generalization of Mittag-Leffler Function and Integral Operator associated with fractional Calculus, J. Fract. Calc. Appl. 3 (2012) 1-13.

[18] A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Mathematiche 52 (1997) 297-310.

[19] A. Wiman, Uber den fundamental staz in der theori der functionen , Acta Math. 29 (1905) 191-201.

[20] A.K. Shukla, J.C. Prajapati, On a generalization of Mittag-Leffler function and its properties, J. Math. Anal. Appl. 336 (2007) 797-811.

[21] H.M. Srivastava, Z. Tomovski, Fractional Calculus with an integral operator containing a generalized Mittag-Leffler function in the kernel, Appl. Math. Comput. 211 (2009) 198-210.

[22] T.O. Salim, Some properties relating to the generalized Mittag-Leffler function, Adv. Appl. Math. Anal. 4 (2009) 21-30.

[23] A. Erd*é*lyi., W.Magnus, F. Oberhettinger, ,F.G. Tricomi, Higher Transcendental Functions, Vol.1, New York, Toronto and London, McGraw-Hill, 1953.

[24] C. Fox, The asymptotic expansion of generalized hypergeometric functions. Proc. Lond. Math. Soc. Ser. 27 (1928)

389-400.

[25] K.C. Gupta, R.C. Soni, On a basic formula involving the product of $\bar{H}$ -function and Fox H-function, J. Raj. Acad. Phy. Sci. 4(3) (2005) 157-164.

[26] A. Erdályi, W. Magnus, F. Oberhettinger, F.G. Tricomi, Tables of integrals transform, Vol.2, New York, Toronto and London, McGraw-Hill, 1954.

[27] R.K. Saxena. A.M., Mathai, H.J. Haubold, On fractional kinetic equataions, Ap and SS 282 (2002) 281-287.

[28] H.M. Srivastava, K.C. Gupta, S.P. Goyal, The H-function of One and Two Variables with Applications, NewDelhi and Madras, South Asian Publishers, 1982.