Int. J. Adv. Appl. Math. and Mech. 3(1) (2015) 71 – 76 (ISSN: 2347-2529)

Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Local fractional variational iteration transform method to solve partial differential equations arising in mathematical physics

Research Article

Hassan Kamil Jassim *

Department of Mathematics, Faculty of Education for Pure Sciences, University of Thi-Qar, Nasiriyah, Iraq

Received 06 June 2015; accepted (in revised version) 16 August 2015

Abstract: In this paper, we investigate solutions of one-dimensional and two-dimensional diffusion and wave equations on Cantor sets within the local fractional derivatives by using local fractional variational iteration transform method. This method is coupled by the Yang-Laplace transform and variational iteration method. Illustrative examples are included to demonstrate the validity and applicability of the presented method.

MSC: 26A33 • 34A12 • 35R11

Keywords: Diffusion equation • Wave equation • Local fractional variational iteration method • Yang-Laplace transform • Local fractional derivative

1. Introduction

The local fractional calculus was successfully applied to describe the non-differentiable problems arising in mathematical physics [1, 2], such as the diffusion and wave equations on Cantor sets. These problems were studied by several authors by using local fractional decomposition method [3–5], local fractional variational iteration method [5–7], local fractional series expansion method [8], local fractional functional decomposition method [9], local fractional Laplace variational iteration method [10], and local fractional Laplace decomposition method [11]. In this paper, our aims are to present the coupling method of local fractional Laplace transform and variational iteration method, which is called as the local fractional variational iteration transform method, and to use it to solve diffusion and wave equations with local fractional derivative.

2. Local fractional variational iteration Transform Method

To illustrate the basic idea of the LFVITM for the local fractional partial differential equation as:

$$L_\alpha u(x,t) + R_\alpha u(x,t) = g(x,t), \tag{1}$$

where $L_\alpha = \dfrac{\partial^{n\alpha}}{\partial t^{n\alpha}}, n \in N$ is the linear local fractional derivative operator, R_α denotes a lower order local fractional derivative operator and $g(x,t)$ is the nondifferentiable source term.
Applying the Yang-Laplace transform on both sides of (1), we get

$$\widetilde{L}_\alpha \{L_\alpha u(x,t)\} + \widetilde{L}_\alpha \{R_\alpha u(x,t)\} = \widetilde{L}_\alpha \left\{g(x,t)\right\}. \tag{2}$$

Using the property of the Yang-Laplace transform, we have

$$s^{n\alpha}\widetilde{L}_\alpha\{u(x,t)\} - s^{(n-1)\alpha} u(x,0) - s^{(n-2)\alpha} u^{(\alpha)}(x,0) - \cdots - u^{((n-1)\alpha)}(x,0) + \widetilde{L}_\alpha\{R_\alpha u(x,t)\} = \widetilde{L}_\alpha\{g(x,t)\}, \tag{3}$$

* *E-mail address:* hassan.kamil28@yahoo.com

or

$$\widetilde{L}_\alpha\{u(x,t)\} = \frac{1}{s^\alpha}u(x,0) + \frac{1}{s^{2\alpha}}u^{(\alpha)}(x,0) + \cdots + \frac{1}{s^{n\alpha}}u^{((n-1)\alpha)}(x,0) + \frac{1}{s^{n\alpha}}\widetilde{L}_\alpha\{g(x,t)\} - \frac{1}{s^{n\alpha}}\widetilde{L}_\alpha\{R_\alpha u(x,t)\}. \tag{4}$$

Operating with the Yang-Laplace inverse on both sides on Eq. (4) gives

$$u(x,t) = u(x,0) + \cdots + \frac{t^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)}u^{((n-1)\alpha)}(x,0) + \widetilde{L}_\alpha^{-1}\left(\frac{1}{s^{n\alpha}}\widetilde{L}_\alpha\left\{g(x,t) - R_\alpha u(x,t)\right\}\right). \tag{5}$$

Derivative by $\frac{\partial^\alpha}{\partial t^\alpha}$ both side Eq. (5), we have

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = u^{(\alpha)}(x,0) + \cdots + \frac{t^{(n-2)\alpha}}{\Gamma(1+(n-2)\alpha)}u^{((n-1)\alpha)}(x,0) + \frac{\partial^\alpha}{\partial t^\alpha}\widetilde{L}_\alpha^{-1}\left(\frac{1}{s^{n\alpha}}\widetilde{L}_\alpha\left\{g(x,t) - R_\alpha u(x,t)\right\}\right) \tag{6}$$

We now structure the correctional local fractional function in the form

$$u_{m+1}(x,t) = u_m(x,t) + \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha)}\left\{\begin{array}{l}\frac{\partial^\alpha u_m(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}L_\alpha^{-1}\left(\frac{1}{s^{n\alpha}}L_\alpha\left\{g(x,\xi) - R_\alpha u_m(x,\xi)\right\}\right)\\ u_m^{(\alpha)}(x,0) + \cdots + \frac{\xi^{(n-2)\alpha}}{\Gamma(1+(n-2)\alpha)}u_m^{((n-1)\alpha)}(x,0)\end{array}\right\}(d\xi)^\alpha. \tag{7}$$

Making the local fractional variation, we get

$$\delta^\alpha u_{m+1}(x,t) = \delta^\alpha u_m(x,t) + \delta^\alpha\frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha)}\left\{\begin{array}{l}\frac{\partial^\alpha u_m(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}L_\alpha^{-1}\left(\frac{1}{s^{n\alpha}}L_\alpha\left\{g(x,\xi) - R_\alpha u_m(x,\xi)\right\}\right)\\ -u_m^{(\alpha)}(x,0) + \cdots + \frac{\xi^{(n-2)\alpha}}{\Gamma(1+(n-2)\alpha)}u_m^{((n-1)\alpha)}(x,0)\end{array}\right\}(d\xi)^\alpha. \tag{8}$$

The extremum condition of $u_{m+1}(x,t)$ is given by

$$\delta^\alpha u_{m+1}(x,t) = 0. \tag{9}$$

In view of (9), we have the following stationary conditions:

$$1 + \frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha}|_{\xi=t} = 0, \left[\frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha}\right]^{(\alpha)}|_{\xi=t} = 0. \tag{10}$$

This is turn gives

$$\frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha} = -1 \tag{11}$$

Substituting (11) into (7), we obtained

$$u_{m+1}(x,t) = u_m(x,t) - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left\{\begin{array}{l}\frac{\partial^\alpha u_m(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}L_\alpha^{-1}\left(\frac{1}{s^{n\alpha}}L_\alpha\left\{g(x,\xi) - R_\alpha u_m(x,\xi)\right\}\right)\\ -u_m^{(\alpha)}(x,0) + \cdots + \frac{\xi^{(n-2)\alpha}}{\Gamma(1+(n-2)\alpha)}u_m^{((n-1)\alpha)}(x,0)\end{array}\right\}(d\xi)^\alpha. \tag{12}$$

Finally, the solution $u(x,t)$ is given by

$$u(x,t) = \lim_{m\to\infty} u_m(x,t) \tag{13}$$

3. Illustrative examples

In this section several examples for diffusion and wave equations on Cantor sets is presented in order to demonstrate the simplicity and the efficiency of the above method.

Example 3.1.
Let us consider the following one-dimensional diffusion equation with local fractional derivative in the form

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \tag{14}$$

subject to the initial value

$$u(x,0)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha}). \tag{15}$$

In view of (12) and (14) the local fractional iteration algorithm can be written as follows:

$$u_{m+1}(x,t)=u_m(x,t)-\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\partial^{\alpha}u_m(x,\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{\alpha}}{\partial\xi^{\alpha}}\widetilde{L}_{\alpha}^{-1}\left(\frac{1}{s^{\alpha}}\widetilde{L}_{\alpha}\left\{\frac{\partial^{2\alpha}u_m(x,\xi)}{\partial x^{2\alpha}}\right\}\right)\right](d\xi)^{\alpha}, m\geq 0. \tag{16}$$

We can use the initial condition to select $u_0(x,t)=\dfrac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})$. Using this selection into the correction functional (16) gives the following successive approximations

$$u_0(x,t)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha}), \tag{17}$$

$$\begin{aligned}u_1(x,t)&=u_0(x,t)-\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\partial^{\alpha}u_0(x,\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{\alpha}}{\partial\xi^{\alpha}}\widetilde{L}_{\alpha}^{-1}\left(\frac{1}{s^{\alpha}}\widetilde{L}_{\alpha}\left\{\frac{\partial^{2\alpha}u_0(x,\xi)}{\partial x^{2\alpha}}\right\}\right)\right](d\xi)^{\alpha}\\&=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})\left[1-\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right],\end{aligned} \tag{18}$$

$$\begin{aligned}u_2(x,t)&=u_1(x,t)-\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\partial^{\alpha}u_1(x,\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{\alpha}}{\partial\xi^{\alpha}}\widetilde{L}_{\alpha}^{-1}\left(\frac{1}{s^{\alpha}}\widetilde{L}_{\alpha}\left\{\frac{\partial^{2\alpha}u_1(x,\xi)}{\partial x^{2\alpha}}\right\}\right)\right](d\xi)^{\alpha}\\&=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})\left[1-\frac{t^{\alpha}}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right],\end{aligned} \tag{19}$$

$$\begin{aligned}u_3(x,t)&=u_2(x,t)-\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\partial^{\alpha}u_2(x,\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{\alpha}}{\partial\xi^{\alpha}}\widetilde{L}_{\alpha}^{-1}\left(\frac{1}{s^{\alpha}}\widetilde{L}_{\alpha}\left\{\frac{\partial^{2\alpha}u_2(x,\xi)}{\partial x^{2\alpha}}\right\}\right)\right](d\xi)^{\alpha}\\&=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})\left[1-\frac{t^{\alpha}}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right],\end{aligned} \tag{20}$$

$$\vdots$$

$$u_m(x,t)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})\left[\sum_{k=0}^{m}(-1)^k\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right] \tag{21}$$

Finally, the solution is

$$\begin{aligned}u(x,t)&=\lim_{m\to\infty}u_m(x,t)\\&=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\cos_{\alpha}(x^{\alpha})\left[\sum_{k=0}^{\infty}(-1)^k\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right]\\&=\frac{x^{\alpha}}{\Gamma(1+\alpha)}+E_{\alpha}(-t^{\alpha)}\cos_{\alpha}(x^{\alpha}).\end{aligned} \tag{22}$$

Example 3.2.
Consider the following two-dimensional diffusion equation with local fractional derivative in the form

$$\frac{\partial^{\alpha}u(x,,y,t)}{\partial t^{\alpha}}-\frac{1}{2}\left(\frac{\partial^{2\alpha}u(x,y,t)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y,t)}{\partial y^{2\alpha}}\right)=0, \tag{23}$$

and the initial condition

$$u(x,y,0)=E_{\alpha}(x^{\alpha}+y^{\alpha}). \tag{24}$$

Applying Eqs. (12) and (23), we obtain the correction function can be written as follows:

$$\begin{aligned}u_{m+1}(x,y,t)=&u_m(x,y,t)\\&-\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\partial^{\alpha}u_m(x,y,\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{\alpha}}{\partial\xi^{\alpha}}\widetilde{L}_{\alpha}^{-1}\left(\frac{1}{s^{\alpha}}\widetilde{L}_{\alpha}\left\{\frac{1}{2}\left(\frac{\partial^{2\alpha}u_m(x,y,\xi)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u_m(x,y,\xi)}{\partial y^{2\alpha}}\right)\right\}\right)\right](d\xi)^{\alpha}.\end{aligned} \tag{25}$$

We can use the initial condition to select $u_0(x,y,t) = E_\alpha(x^\alpha + y^\alpha)$. Using this selection into the correction functional (25) gives the following successive approximations

$$\begin{aligned}
u_0(x,y,t) &= E_\alpha(x^\alpha + y^\alpha),\\
u_1(x,y,t) &= u_0(x,y,t) - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\partial^\alpha u_0(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}\widetilde{L}_\alpha^{-1}\left(\frac{1}{s^\alpha}\widetilde{L}_\alpha\left\{\frac{1}{2}\left(\frac{\partial^{2\alpha} u_0(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_0(x,y,\xi)}{\partial y^{2\alpha}}\right)\right\}\right)\right](d\xi)^\alpha\\
&= E_\alpha(x^\alpha + y^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)}\right],\\
u_2(x,y,t) &= u_1(x,y,t) - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\partial^\alpha u_1(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}\widetilde{L}_\alpha^{-1}\left(\frac{1}{s^\alpha}\widetilde{L}_\alpha\left\{\frac{1}{2}\left(\frac{\partial^{2\alpha} u_1(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_1(x,y,\xi)}{\partial y^{2\alpha}}\right)\right\}\right)\right](d\xi)^\alpha\\
&= E_\alpha(x^\alpha + y^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right],\\
u_3(x,y,t) &= u_2(x,y,t) - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\partial^\alpha u_2(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}\widetilde{L}_\alpha^{-1}\left(\frac{1}{s^\alpha}\widetilde{L}_\alpha\left\{\frac{1}{2}\left(\frac{\partial^{2\alpha} u_2(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_2(x,y,\xi)}{\partial y^{2\alpha}}\right)\right\}\right)\right](d\xi)^\alpha\\
&= E_\alpha(x^\alpha + y^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right],\\
&\vdots\\
u_m(x,y,t) &= E_\alpha(x^\alpha + y^\alpha)\left[\sum_{k=0}^{m}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right].
\end{aligned}$$

Finally, the solution is

$$\begin{aligned}
u(x,y,t) &= \lim_{m\to\infty} u_m(x,y,t)\\
&= E_\alpha(x^\alpha + y^\alpha)\left[\sum_{k=0}^{\infty}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right]\\
&= E_\alpha(x^\alpha + y^\alpha) + t^{\alpha)}.
\end{aligned} \tag{26}$$

Example 3.3.

Let us consider the one-dimensional wave equation involving local fractional operator

$$\frac{\partial^\alpha u(x,t)}{\partial t^{2\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \tag{27}$$

subject to the initial value

$$u(x,0) = 0, \frac{\partial^\alpha u(x,0)}{\partial t^\alpha} = E_\alpha(x^\alpha). \tag{28}$$

In view of (12) and (27) the local fractional iteration algorithm can be written as follows:

$$u_{m+1}(x,t) = u_m(x,t) - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\partial^\alpha u_m(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha}\widetilde{L}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}\widetilde{L}_\alpha\left\{\frac{\partial^{2\alpha} u_m(x,\xi)}{\partial x^{2\alpha}}\right\}\right) - E_\alpha(x^\alpha)\right](d\xi)^\alpha. \tag{29}$$

We can use the initial condition to select $u_0(x,t) = \frac{t^\alpha}{\Gamma(1+\alpha)}E_\alpha(x^\alpha)$. Using this selection into the correction functional

(29) gives the following successive approximations

$$
\begin{aligned}
u_0(x,t) &= \frac{t^\alpha}{\Gamma(1+\alpha)} E_\alpha(x^\alpha),\\
u_1(x,t) &= u_0(x,t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_0(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ \frac{\partial^{2\alpha} u_0(x,\xi)}{\partial x^{2\alpha}} \right\} \right) - E_\alpha(x^\alpha) \right] (d\xi)^\alpha\\
&= E_\alpha(x^\alpha) \left[\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} \right],\\
u_2(x,t) &= u_1(x,t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_1(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ \frac{\partial^{2\alpha} u_1(x,\xi)}{\partial x^{2\alpha}} \right\} \right) - E_\alpha(x^\alpha) \right] (d\xi)^\alpha\\
&= E_\alpha(x^\alpha) \left[\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)} \right],\\
u_3(x,t) &= u_2(x,t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_1(x,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ \frac{\partial^{2\alpha} u_2(x,\xi)}{\partial x^{2\alpha}} \right\} \right) - E_\alpha(x^\alpha) \right] (d\xi)^\alpha\\
&= E_\alpha(x^\alpha) \left[\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)} + \frac{t^{7\alpha}}{\Gamma(1+7\alpha)} \right],\\
&\vdots\\
u_m(x,t) &= E_\alpha(x^\alpha) \left[\sum_{k=0}^{m} \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \right].
\end{aligned}
$$

Finally, the solution is

$$
\begin{aligned}
u(x,t) &= \lim_{m\to\infty} u_m(x,t)\\
&= E_\alpha(x^\alpha) \left[\sum_{k=0}^{\infty} \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \right]\\
&= E_\alpha(x^\alpha) \sinh_\alpha(t^\alpha).
\end{aligned} \tag{30}
$$

Example 3.4.
Consider the following two-dimensional wave equation with local fractional derivative in the form

$$
\frac{\partial^\alpha u(x,,y,t)}{\partial t^{2\alpha}} - 2\left(\frac{\partial^{2\alpha} u(x,y,t)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y,t)}{\partial y^{2\alpha}} \right) = 0, \tag{31}
$$

and the initial condition

$$
u(x,y,0) = \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha), \frac{\partial^\alpha u(x,y,0)}{\partial t^\alpha} = 0. \tag{32}
$$

Applying Eqs. (12) and (31), we obtain the correction function can be written as follows:

$$
\begin{aligned}
u_{m+1}(x,y,t) =& u_m(x,y,t)\\
&- \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_m(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ 2\left(\frac{\partial^{2\alpha} u_m(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_m(x,y,\xi)}{\partial y^{2\alpha}} \right) \right\} \right) \right] (d\xi)^\alpha.
\end{aligned} \tag{33}
$$

We can use the initial condition to select $u_0(x,y,t) = \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha)$. Using this selection into the correction functional (33) gives the following successive approximations

$$
\begin{aligned}
u_0(x,y,t) &= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha),\\
u_1(x,y,t) &= u_0(x,y,t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_0(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ 2\left(\frac{\partial^{2\alpha} u_0(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_0(x,y,\xi)}{\partial y^{2\alpha}} \right) \right\} \right) \right] (d\xi)^\alpha\\
&= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha) \left[1 - \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)} \right],\\
u_2(x,y,t) &= u_1(x,y,t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left[\frac{\partial^\alpha u_1(x,y,\xi)}{\partial \xi^\alpha} - \frac{\partial^\alpha}{\partial \xi^\alpha} \widetilde{L}_\alpha^{-1} \left(\frac{1}{s^{2\alpha}} \widetilde{L}_\alpha \left\{ 2\left(\frac{\partial^{2\alpha} u_1(x,y,\xi)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_1(x,y,\xi)}{\partial y^{2\alpha}} \right) \right\} \right) \right] (d\xi)^\alpha\\
&= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha) \left[1 - \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{8t^{4\alpha}}{\Gamma(1+4\alpha)} \right],\\
&\vdots\\
u_m(x,y,t) &= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha) \left[\sum_{k=0}^{m} (-1)^k \frac{2^{2k} t^{2k\alpha}}{\Gamma(1+2k\alpha)} \right].
\end{aligned}
$$

Finally, the solution is

$$\begin{aligned} u(x,y,t) &= \lim_{m\to\infty} u_m(x,y,t) \\ &= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha)\left[\sum_{k=0}^{\infty}(-1)^k\frac{2^{2k}t^{2k\alpha}}{\Gamma(1+2k\alpha)}\right] \\ &= \sin_\alpha(x^\alpha)\sin_\alpha(y^\alpha)\cos_\alpha(2t^\alpha). \end{aligned} \tag{34}$$

4. Conclusions

In this work the diffusion and wave equations on Cantor sets within the local fractional differential operators had been analyzed using the local fractional variational iteration transform method. The non-differentiable solutions are obtained. The present method is a powerful tool for solving many differential equations on Cantor sets within the local fractional derivative.

Acknowledgments

The author acknowledges Ministry of Higher Education and Scientific Research in Iraq for its support this work.

References

[1] X. J. Yang, Local Fractional Functional Analysis and Its Applications, Asian Academic Publisher Limited, Hong Kong, 2011.
[2] X. J. Yang, Advanced Local Fractional Calculus and Its Applications, World Science Publisher, New York, 2012.
[3] X. J. Yang, D. Baleanu, W. P. Zhong, Approximation solutions for diffusion equation on Cantor time-space, Proceeding of the Romanian Academy A, vol.14, no.2 (2013) 127–133.
[4] Z. P. Fan, H. K. Jassim, R. K. Rainna, and X. J. Yang, Adomian Decomposition Method for Three-Dimensional Diffusion Model in Fractal Heat Transfer Involving Local Fractional Derivatives, Thermal Science 19(1) (2015) 137–141.
[5] D. Baleanu, J. A. Machado, C. Cattani, M. C. Baleanu, X. J. Yang, Local fractional variational iteration and decomposition methods for wave equation on Cantor sets within local fractional operators, Abstract and Applied Analysis, vol. 2014, Article ID 535048 (2014) 1–6
[6] X. J. Yang, et al., Local Fractional Variational Iteration Method for Diffusion and Wave Equations on Cantor Sets, Romanian Journal of Physics 59(1-2) (2014)36–48.
[7] S. Xu, X. Ling, Y. Zhao, H. K. Jassim, A Novel Schedule for Solving the Two-Dimensional Diffusion in Fractal Heat Transfer, Thermal Science 19(1) (2015) 99–103.
[8] A. M. Yang, X. J. Yang, Z. B. Li, Local fractional series expansion method for solving wave and diffusion equations on Cantor sets. Abstract and Applied Analysis, vol. 2013, Article ID 351057 (2013) 1–5.
[9] Y. Cao, W. G. Ma, L. C. Ma, Local fractional functional method for solving diffusion equations on Cantor sets, Abstract and Applied Analysis, vol. 2014, Article ID 803693 (2014) 1–6.
[10] H. K. Jassim, C. Unlu, S. P. Moshokoa, C. M. Khalique, Local Fractional Laplace Variational Iteration Method for Solving Diffusion and Wave Equations on Cantor Sets within Local Fractional Operators, Mathematical Problems in Engineering, vol. 2015, Article ID 309870 (2015) 1–9.
[11] H. K. Jassim, Local Fractional Laplace Decomposition Method for Nonhomogeneous Heat Equations Arising in Fractal Heat Flow with Local Fractional Derivative, International Journal of Advances in Applied Mathematics and Mechanics 2(4) (2015) 1–7.