

## Heat transfer on the MHD flow of couple stress fluid between two concentric rotating cylinders with porous lining

Research Article

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**Abstract:** An attempt has been made to study the couple stress fluid flow in an annulus between two concentric rotating vertical cylinders with porous lining is attached to inside of outer cylinder, under the influence of radial magnetic field. The flow in the annular gap is caused by couple stress fluid and inside the porous sleeve is governed by Darcy's Law. The velocity and temperature distributions, wall shear stress and heat transfer rate on the inner and outer cylinders are calculated numerically by employing finite difference scheme with vanishing couple stresses on the boundary. The effect of various control parameters on inner and outer cylinders are discussed and depicted through graphs.

**MSC:** 35Q79 • 74A10**Keywords:** Porous Lining • MHD Heat transfer • Slip velocity • Darcy law • BJ condition • Rotating cylinders© 2015 The Author(s). This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/3.0/>).

### 1. Introduction

With the growing importance of polar fluids in modern technology and industries, the investigations on such fluids are desirable. During recent years the theory of polar fluids has received much attention and this is because the traditional non-polar fluids cannot precisely describe the characteristics of the fluid flow with suspended particles. The study of such fluids has applications in a number of processes that occur in industry such as extraction of crude oil from petroleum products, aerodynamics heating, electrostatic precipitation, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants, colloidal and suspension solutions. One such fluid is couple stress fluid introduced by Stokes [1] have distinct features, such as polar effects. The main cause of couple stresses is to introduce a length dependent effect which is not observed in non-polar fluids. Dewakar and Iyengar [2] studied Stokes first and second problem for an incompressible couple stress fluid. Habtu Alemayehu and Radhakrishnamacharya [4] investigated the dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium. Punnamchander and Iyengar [5] examined the oscillatory flow of an incompressible couple stress fluid between permeable beds. The fluid is driven by an unsteady oscillatory pressure gradient. Farooq et al. [6] studied the heat transfer flow of couple stress fluids between two heated parallel inclined plates. Tandel and Bhathawala [7] have been derived the mathematical model that conform the hydrological situation of one dimensional vertical ground water recharge by spreading.

The study of flow and temperature fields in an annulus between two concentric rotating cylinders has been widespread interest in many engineering applications. The practical application for such high interest in this subject is journal bearings (Bujurke and Naduvinamani [8], Jaw-Ren Lin [9] and Alessandro et al. [10]) in electrical machineries. A porous journal bearing has been proposed to improve the performance of solid journal bearings. The main

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advantage of porous bearings is used in rotational machinery because of their ability to retain lubricant. One of the great challenges encountered in this problem is the specification of the interface conditions between the fluid and porous layers. Initially an empirical condition is proposed by Beavers and Joseph [11], which matches Darcy's Law for the porous medium with the Navier-stokes equations for the fluid layer through an empirical slip-flow interface condition. The effect of the porous lining thickness on velocity vector and shear stresses at the wall of inner and outer cylinders are examined by Channabasappa et al. [12]. Leong and Lai [13] reported the flow and heat transfer between two concentric, eccentric rotating cylinders with porous sleeve attached to the outer cylinder. Subotic and Lai [14] studied the flow and temperature fields in an annulus between two rotating cylinders with porous lining. Mahmood et al. [15] presented an exact solution for the velocity field and shear stress corresponding to the flow of a generalized Maxwell fluid between to infinite coaxial circular cylinders, by using Laplace and Hankel transforms. Jain et al. [16] study the deductive group method based on general group transformation is applied to derive similarity solutions, these are obtained for more general and systematic along with auxiliary conditions.

In recent years, a number of simple flow problems associated with classical hydrodynamics have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available hydrodynamic solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Interest in rotating hydro-magnetic flow in annular spaces was initiated in the late 1950 with an important analysis by Globe [17], who considered fully developed laminar MHD flow in an annulus. Mahapatra [18] studied the transient motion of an incompressible viscous conducting liquid in the presence of a radial magnetic field. Sengupta and Ghosh [19] have carried out the steady motion of an incompressible viscous fluid rotating in various ways with uniform angular velocities, in presence of a radial magnetic field. Bathaiah and Venugopal [20] investigated the flow in annulus with porous lining under the influence of radial magnetic field. The flow of an electrically conducting fluid in a narrow gap between two concentric rotating vertical cylinders with thin porous lining has been studied by Sai [21]. Makinde et al. [21] studied the MHD flow in the annular regime between two concentric rotating cylinders under the action of a radial magnetic field. Srinivasacharya and Kaladhar [22] presented the hall and ion-slip effects on MHD free convective flow of couple stress fluid between two concentric cylinders. Recently, Ramana Murthy et al. [23] examined the effect of radial magnetic field on micropolar fluid flow due to steady rotation of concentric cylinders with inner porous lining. Such a study has to the authors knowledge not appeared in the scientific communications thus far and is an interesting addition to the existing body of knowledge. Darji and Timol [24] produce similarity analysis via deductive group method based on general group transformation to derive symmetry group and similarity solutions for steady of the laminar free convective boundary layer flow.

In this paper, the numerical solutions have been analyzed for the flow and temperature fields in an annulus with a porous lining between two rotating cylinders in the presence of radial magnetic field. The solutions obtained with the effects of various control parameters on the velocity profiles and temperature distributions are carefully examined.

## 2. Formulation of the problem

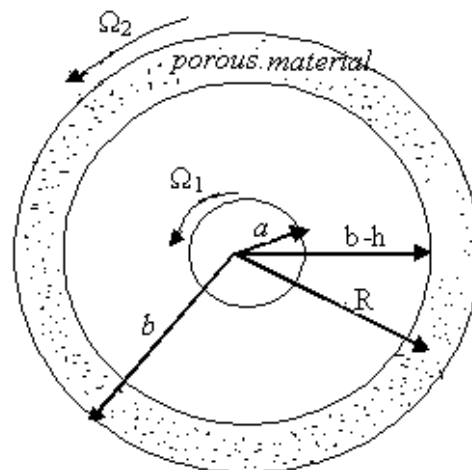


Fig. 1. Physical model and coordinate system.

Consider a steady, laminar, incompressible, and electrically conducting couple stress fluid between 2 coaxial concentric circular cylinders of radii  $a$  and  $b$  ( $a < b$ ). Choose the cylindrical polar coordinate system  $(r, \theta, z)$  with  $z$ -axis as the common axis for both cylinders. The inner and outer cylinders are rotating with constant angular velocities  $\Omega_1$  and  $\Omega_2$  respectively. There is a non-erodible porous lining of thickness  $h$  on the inside of outer cylinder. A uniform

magnetic field ( $B_0/r$ ) is applied in the radial direction. The motion is rotationally symmetric and two dimensional. Hence, the axial velocity  $w$  is zero and also the derivatives of transverse velocity with respect to  $\theta$  and  $z$  vanishes. Assume that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. Assuming that the flow caused by rotation of these cylinders. The flow configuration and the coordinates system are shown in Fig. 1.

With the above assumptions, the equations of governing the couple stress fluid under the usual MHD approximations are:

$$\frac{du}{dr} + \frac{u}{r} = 0 \quad (1)$$

$$\rho \left( u \frac{du}{dr} - \frac{v^2}{r} \right) = -\frac{dp}{dr} \quad (2)$$

$$\rho \left( u \frac{du}{dr} + \frac{uv}{r} \right) = \mu D^2 v - \eta D^4 v - \frac{\sigma_e b^2 B_0^2 v}{r^2} \quad (3)$$

$$c_p u \frac{dT}{dr} = \frac{\mu}{\rho} \left[ 2 \left( \frac{du}{dr} \right)^2 + 2 \frac{u^2}{r^2} + \left( \frac{dv}{dr} \right)^2 + \frac{v^2}{r^2} - 2 \frac{v}{r} \frac{dv}{dr} \right] + \frac{\eta}{\rho} (D^2 v)^2 + \frac{K_T}{\rho} \left[ \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right] \quad (4)$$

where  $u$  and  $v$  are radial and transverse components of the velocity,  $r$  is the radial distance,  $p$  is the fluid pressure,  $\rho$  is the fluid density,  $\mu$  is the coefficient of viscosity,  $\sigma_e$  is the electrical conductivity,  $\eta$  is the couple stress viscosity,  $c_p$  is the specific heat of the fluid at constant pressure,  $T$  is the temperature of the fluid,  $K_T$  is the thermal conductivity and where

$$D^2 = \frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2}$$

The boundary conditions are defined as

$$u = 0 \text{ at } r = a \text{ and } r = b - h \quad (5)$$

$$v = a\Omega_1 \text{ at } r = a \quad (6)$$

$$D^2 v = 0 \text{ at } r = a \text{ and } r = b - h \quad (7)$$

$$v = v_B \text{ at } r = b - h \quad (8)$$

$$T = T_1 \text{ at } r = a \quad (9)$$

$$\left[ \frac{dT}{dr} \right]_{r=b-h} = \frac{\beta}{\sqrt{K}} (T_B - T_0) \quad (10)$$

where  $v_B$  is the slip velocity,  $K$  is the porosity of the lining material,  $\beta$  is the Biot number,  $T_B$  is the slip temperature at the interface and  $T_0$  is the ambient temperature. The boundary conditions (5) and (6) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (7) implies that the couple stresses are vanishes on the surfaces. The slip velocity  $v_B$  is obtained by using Beavers and Joseph condition

$$\frac{dv}{dr} = \frac{\alpha}{\sqrt{K}} (v_B - Q_D) \text{ at } r = b - h \quad (11)$$

here  $\alpha$  is the slip parameter and  $Q_D$  is the Darcy's velocity in the permeable lining. It is given by

$$Q_D = r\Omega_2 + \Phi \quad (12)$$

where  $r\Omega_2$  is the velocity in the porous medium due to the rotation of the porous medium with angular velocity  $\Omega_2$  and  $\Phi$  is given by

$$\Phi = \frac{K \int_0^{2\pi} \int_{b-h}^b \rho r \Omega_2^2 r d\theta dr}{\mu \int_0^{2\pi} \int_{b-h}^b r d\theta dr} = \frac{2K\Omega_2^2 \rho}{3\mu} \left( \frac{3b^2 - 3bh + h^2}{2b - h} \right) \quad (13)$$

From the continuity equation (1) and boundary condition (5), we observe that  $u = 0$  everywhere.

Introducing the following non-dimensional transformations

$$\begin{aligned} v' &= \frac{v}{b\Omega_2}, & r' &= \frac{r}{b}, & r_0 &= \frac{a}{b}, & \sigma &= \frac{b}{\sqrt{K}}, & \lambda &= \frac{\Omega_1}{\Omega_2}, & e &= \frac{h}{b}, & p &= \frac{P}{\rho b^2 \Omega_2^2}, & v_B &= \frac{V_B}{b\Omega_2}, & Re &= \frac{b^2 \Omega_2}{\nu} \\ S &= \frac{\eta}{\mu b^2}, & M^2 &= \frac{\sigma_e b^2 B_0^2}{\mu}, & T' &= \frac{T - T_0}{T_1 - T_0} \end{aligned} \quad (14)$$

In view of (14), Eqs. (3) and (4) reduce to (after dropping the primes),

$$v^i v + \frac{2}{r} v''' + \left( -\frac{3}{r^2} + a_1 \right) v'' + \left( \frac{3}{r^3} + \frac{a_1}{r} \right) v' + \left( -\frac{3}{r^4} + \frac{a_2}{r^2} \right) v = 0 \quad (15)$$

Where  $a_1 = \frac{-1}{S}$  and  $a_2 = \frac{M^2 + 1}{S}$

$$r^2 \frac{d^2 T}{dr^2} + r \frac{dT}{dr} = -Br \left( r \frac{dv}{dr} - v \right)^2 - Sr^2 Br D^2 v \quad (16)$$

$Br = \frac{\mu b^2 \Omega_2^2}{K_T (T - T_0)}$  Brinkman Number The boundary conditions in the non-dimensional forms are

$$v = \lambda r_0 \text{ at } r = r_0 \quad (17)$$

$$v = v_B \text{ at } r = 1 - e \quad (18)$$

where the non-dimensional slip velocity  $v_B$  is given by

$$\left( \frac{dv}{dr} \right)_{r=1-e} = \alpha \sigma v_B - \alpha \sigma (1 - e) - \frac{2\alpha Re (e^2 - 3e + 3)}{3\sigma(2 - e)} \quad (19)$$

$$T = 1 \text{ at } r = r_0 \quad (20)$$

$$\left( \frac{dT}{dr} \right)_{r=1-e} = \beta \sigma T_B \quad (21)$$

where  $\sigma$  is the porosity parameter and  $T_B$  is the non-dimensional slip temperature.

### 3. Method of solution

The set of non-linear ordinary differential equations (15) and (16) with the boundary conditions (17) - (21) have been solved numerically using finite difference method. We discretise the interval  $[r_0, 1 - e]$  into  $n$  subintervals with  $n + 1$  nodes, starting from first node  $r_0$  to the last node  $r_n = 1 - e$ . A step size of  $h = 0.005$  was selected to be satisfactory for convergence criteria of  $10^{-5}$  in this case.

**Skin friction:** The coefficient of skin friction at the inner and outer cylinder in the non-dimensional form is given by:

$$C_f = \frac{2\bar{\tau}_r \theta}{Re} \text{ at } r = r_0 \text{ and } r = 1 - e \quad (22)$$

where  $\bar{\tau}_r \theta = \frac{S}{2h^3} v_{i-2} - \left( \frac{1}{2h} + \frac{S}{h^3} + \frac{2S}{r_i h^2} + \frac{S}{2hr_i^2} \right) v_{i-1} + \left( -\frac{1}{r_i} + \frac{4S}{r_i h^2} - \frac{S}{r_i^3} \right) v_i + \left( \frac{1}{2h} + \frac{S}{h^3} + \frac{2S}{r_i h^2} + \frac{S}{2hr_i^2} \right) v_{i+1} - \frac{S}{2h^3} v_{i+2}$

**Rate of Heat Transfer:** The rate of heat transfer at the wall of the inner cylinder is given by

$$q = \left( \frac{dT}{dr} \right)_{r=r_0} = \frac{T_1 - T_{-1}}{2h\beta\sigma} \quad (23)$$

The rate of heat transfer at the interface is given by

$$q^* = \left( \frac{dT}{dr} \right)_{r=1-e} = \beta \sigma T_B \quad (24)$$

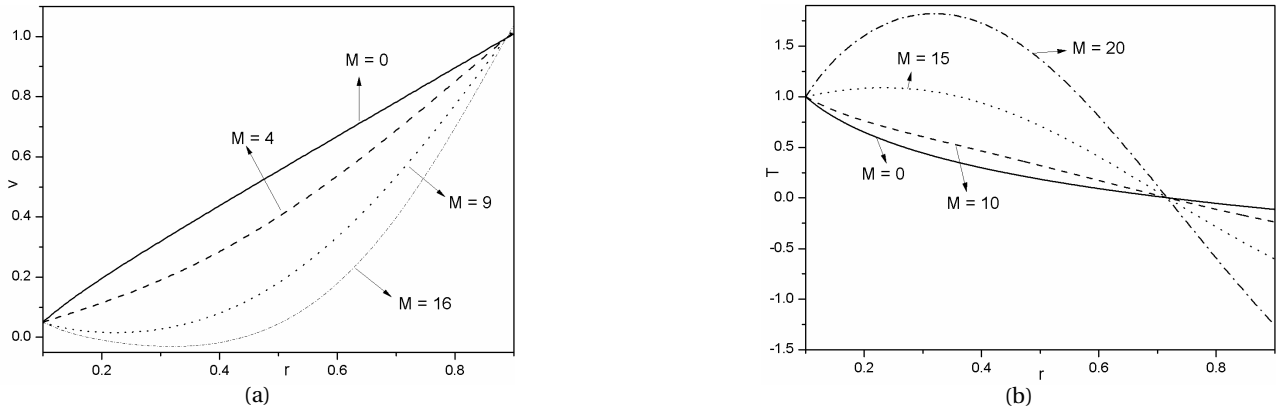


Fig. 2. Velocity and temperature profiles for various values of  $M$

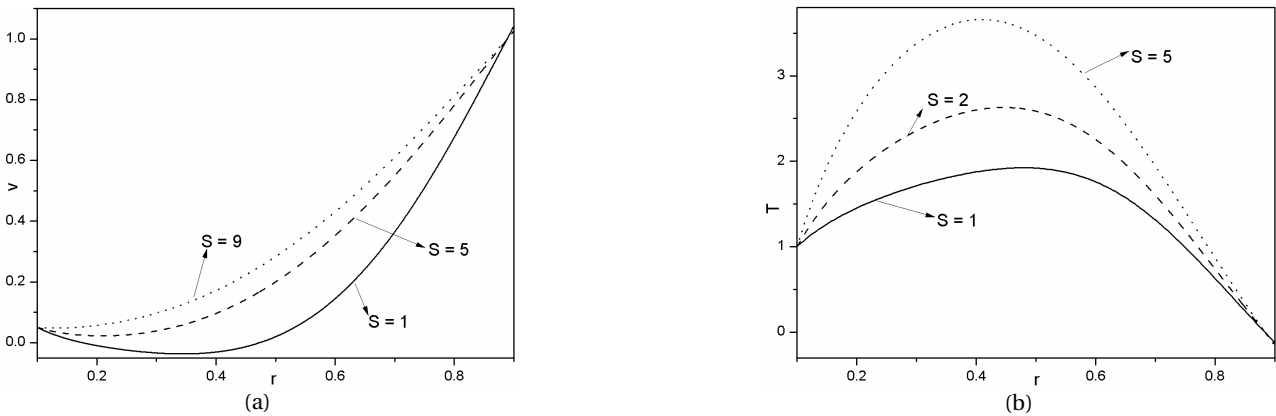


Fig. 3. Velocity and temperature profiles for various values of  $S$

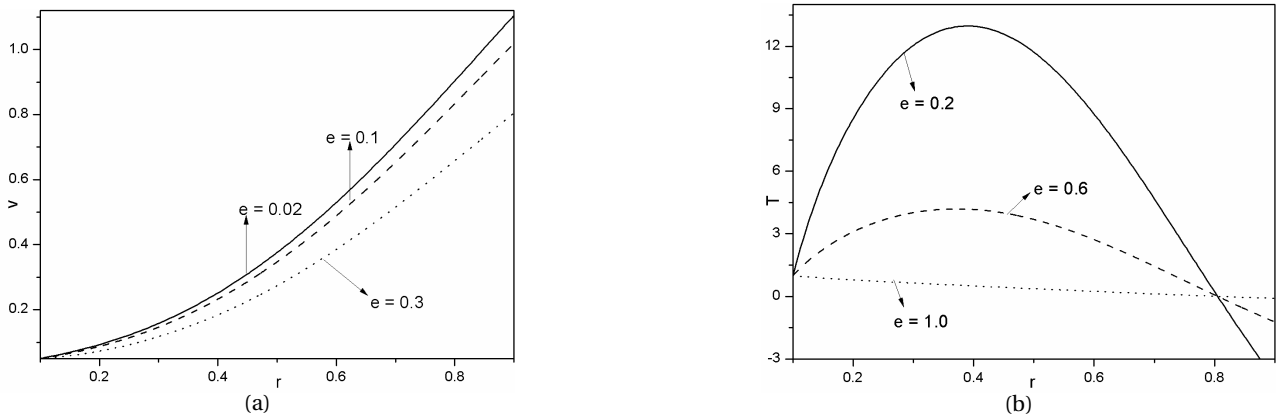


Fig. 4. Velocity and temperature profiles for various values of  $e$ .

#### 4. Results and Discussions

The solutions for velocity ( $v$ ), temperature ( $T$ ), skin friction coefficient ( $C_f$ ) and rate of heat transfer ( $q$ ) have been computed and are shown graphically in Figs. Figs. 2 - 12. The effects of Magnetic parameter ( $M$ ), couple stress parameter ( $S$ ), porous lining thickness parameter ( $e$ ), slip parameter ( $\alpha$ ), Porosity parameter ( $\sigma$ ), ratio of angular velocities parameter ( $\lambda$ ), biot number ( $\beta$ ) and brinkman number ( $Br$ ) have been discussed.

In Figs. 2(a)-2(b) the velocity and temperature profiles are presented for different values of Magnetic parameter  $M$ . The non-dimensional velocity  $v$  decreases with the increase in  $M$ , as indicated in Fig. 2(a). This is due to the fact that the imposed magnetic field normal to the flow direction. This magnetic field gives rise to a resistance force

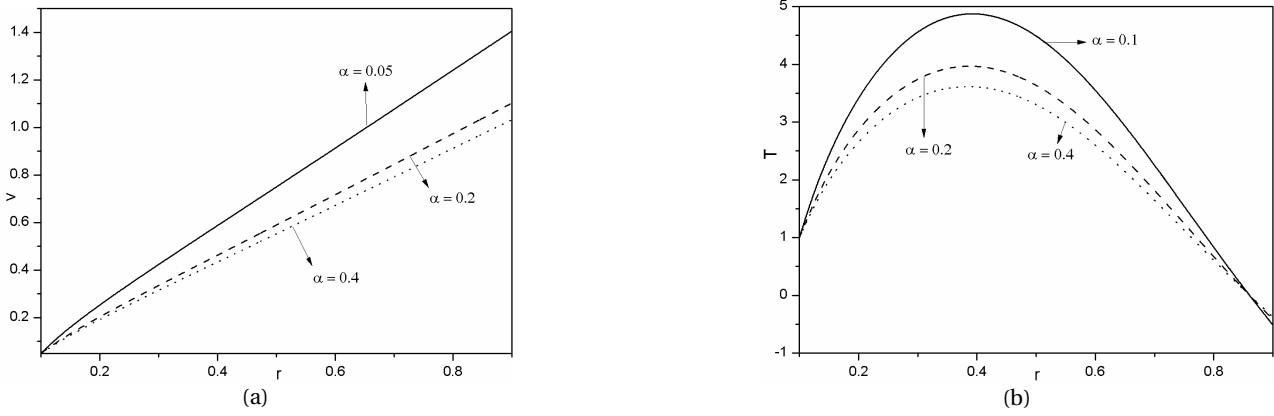


Fig. 5. Velocity and temperature profiles for various values of  $\alpha$ .

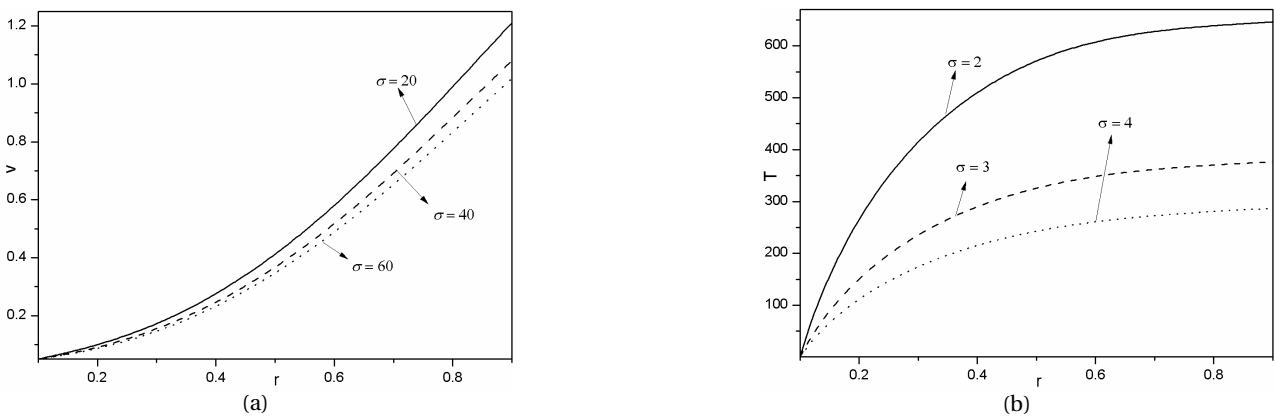


Fig. 6. Velocity and temperature profiles for various values of  $\sigma$ .

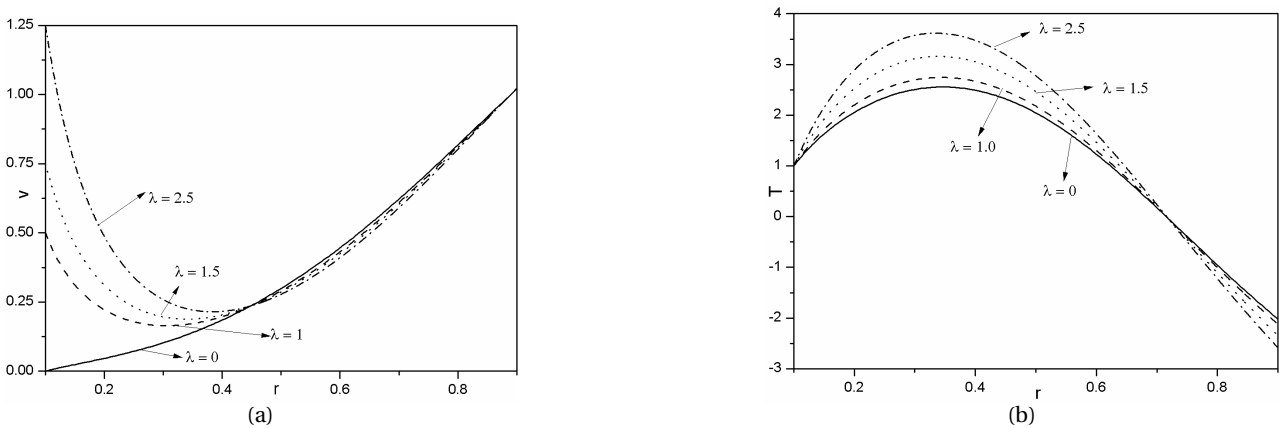


Fig. 7. Velocity and temperature profiles for various values of  $\lambda$ .

and slows down the movement of fluid. From Fig. 2(b), it is observed that the temperature profiles first increases and then decreases with the increase in  $M$  at points vicinity to the inner cylinder while it decreases with the increase in  $M$  at points sufficiently close to the interface. We have also seen that  $T$  first increases in the neighborhood of the inner cylinder and then decreases as  $r$  increases. It is also noticed that when Couple stress parameter  $S$  increases, the velocity  $v$  and temperature  $T$  increases as shown in Figs. 3(a) and 3(b). It is observed from Fig. 4(a), the velocity  $v$  decreases as the parameter  $e$  increase. Fig. 4(b) plotted against  $r$  for different values of  $e$ . From this it is clear that the temperature  $T$  first decreases and then increases with the increase in  $e$  at points sufficiently close to the inner cylinder while it increases with the increase in  $e$  at points sufficiently close to the interface. Figs. 5 and 6 depict the

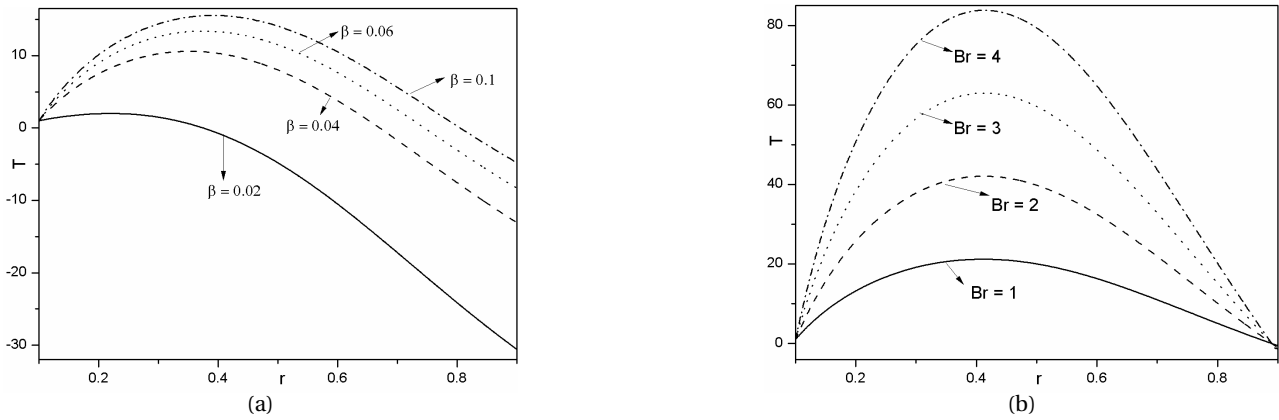


Fig. 8. Temperature profiles for various values of  $\beta$  and  $Br$

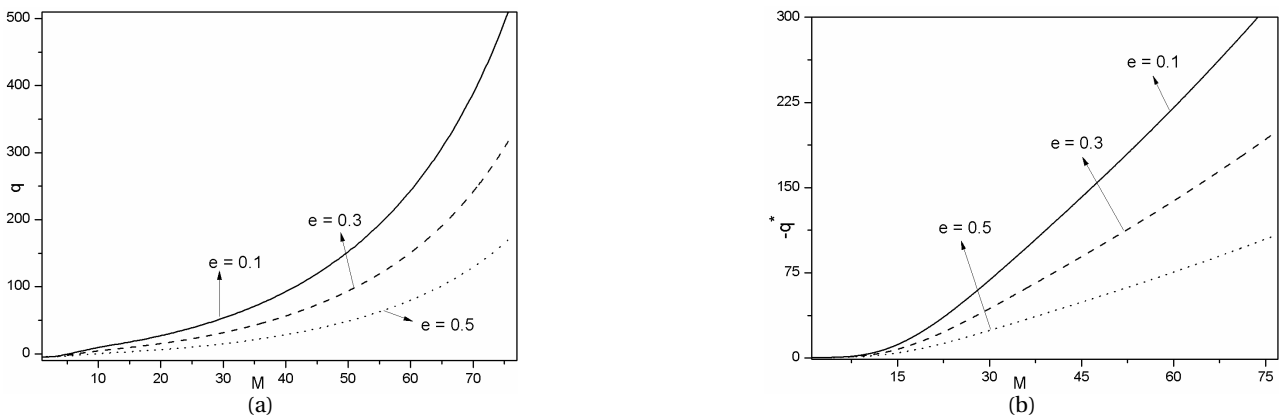


Fig. 9. Rate of heat transfer at the inner and outer cylinders for various values of  $e$ .

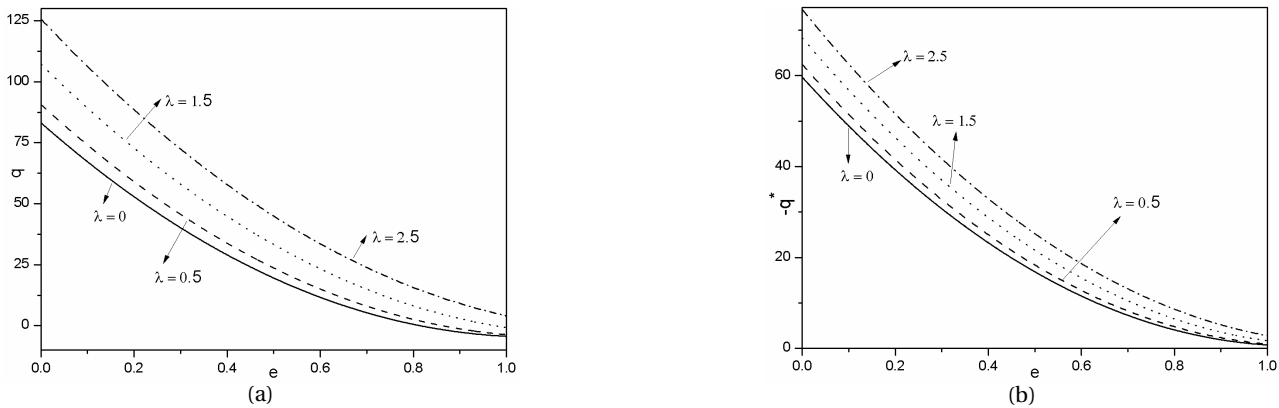


Fig. 10. Rate of heat transfer at the inner and outer cylinders for various values of  $\lambda$ .

variation of the slip parameter, porosity parameter on  $v$  and  $T$ . It can be noticed that from these figures the velocity and the temperature decrease with an increase in the parameters  $\alpha$  and  $\sigma$ . Figs. 7(a) and 7(b) represent the effect of parameter  $\lambda$  on  $v$  and  $T$ . Fig. 7(a) shows that the velocity increases with  $\lambda$  throughout the flow domain except in a close neighborhood of the porous lining in which velocity decreases with increase in  $\lambda$ . From Fig. 7(b), it is observed that the temperature decreases with the increase in  $\lambda$  at points sufficiently close to the inner cylinder. While the reverse trend is observed towards the porous lining. It can be noticed that from Figs. 8(a)-8(b), the temperature profiles increase as the parameters  $\beta$  and  $Br$  increases.

Table 1 shows that the effects of the porous lining thickness parameter ( $e$ ), slip parameter ( $\alpha$ ) and ratio of angular velocity parameter ( $\lambda$ ) on the skin friction ( $Cf$ ) at the inner and outer cylinders respectively. From this table it is seen

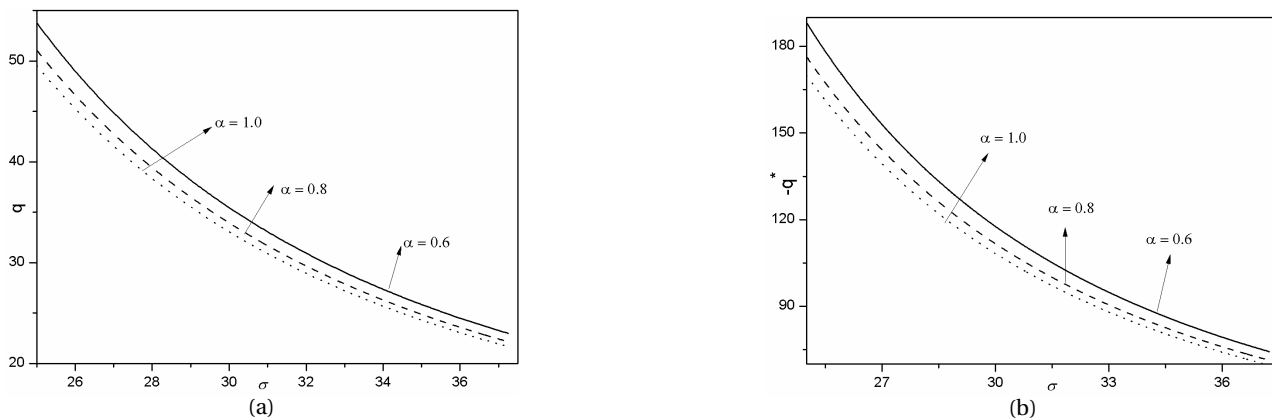


Fig. 11. Rate of heat transfer at the inner and outer cylinders for various values of  $\alpha$ .

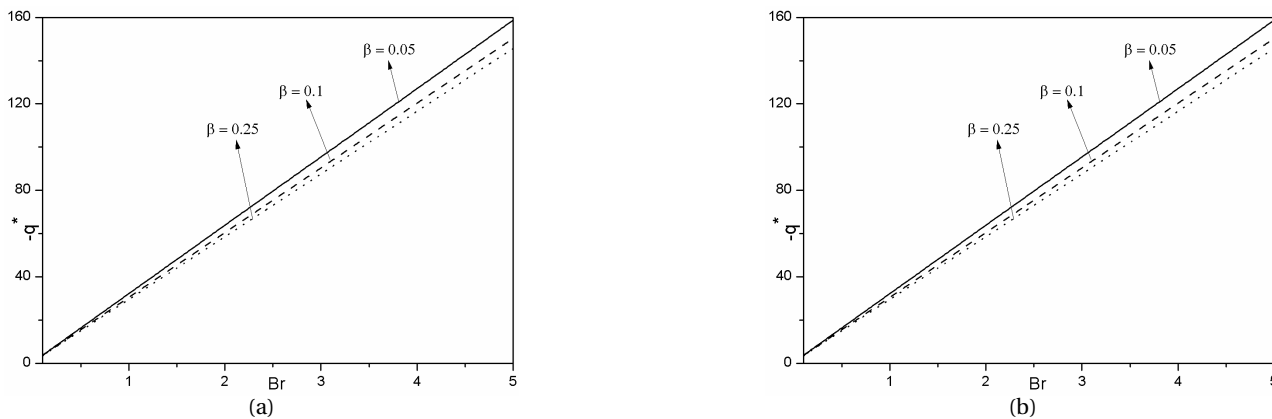


Fig. 12. Rate of heat transfer at the inner and outer cylinders for various values of  $\beta$ .

that both  $Cf_{in}$  and  $Cf_{out}$  increases with  $e$  or  $M$ . The effect of magnetic parameter is to decrease the skin friction coefficients. It is noticed that increase and decrease with increasing values of  $a$  or  $\lambda$ .

Fig. 9 represents the effects of the porous lining thickness parameter  $e$  on  $q$  and  $-q^*$ . It is observed that the rate of heat transfer decreases with an increase in the parameter  $e$ . From Figs. 10-12, it is also noticed that both  $q$  and  $-q^*$  increases with the increase in  $\lambda$ ,  $\alpha$  and  $Br$ .

### 5. Conclusions

In this paper, the effect of temperature profiles on fully developed electrically conducting couple stress fluid flow between two concentric cylinders with inner porous lining has been studied. Using similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations. The similarity solutions are obtained numerically by applying finite difference method. From the present study it is noticed that as the magnetic field strength increases velocity decreases. While the opposite trend is observed for skin friction coefficient at the inner and outer cylinders, respectively. Also, it is noticed that the presence of couple stresses in the fluid increases the velocity and temperature.



**Table 1.** Effect of Skin friction coefficient ( $C_f$ ) on inner and outer cylinders for various values of  $e$ ,  $\alpha$  and  $\lambda$ .

M	e = 0.1		e = 0.2		e = 0.3	
	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$
0	0.572391	0.000888003	0.750951	0.00116502	0.929119	0.00144143
4	0.57172	0.00517775	0.749919	0.00501975	0.927728	0.0048621
8	0.571043	0.00943847	0.748882	0.00884827	0.926332	0.00825936
12	0.57036	0.0136706	0.74784	0.012651	0.924932	0.0116336
16	0.569671	0.0178746	0.746793	0.0164283	0.923527	0.0149851
20	0.568976	0.0220508	0.74574	0.0201805	0.922117	0.0183143
24	0.568275	0.0261998	0.744682	0.0239082	0.920703	0.0216215
$\sigma$	$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$	
	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$
25	-0.770684	0.0692592	-0.586943	0.0666229	-0.500854	0.0653877
27.5	-0.252024	0.0618177	-0.106328	0.0597273	-0.0376248	0.0587415
30	0.137096	0.0562347	0.256095	0.0545274	0.312506	0.053718
32.5	0.436671	0.0519366	0.53624	0.050508	0.583648	0.0498278
35	0.67234	0.0485553	0.75733	0.0473359	0.797948	0.0467531
37.5	0.861162	0.0458462	0.934927	0.0447878	0.970293	0.0442804
40	1.01485	0.0436411	1.07978	0.0427096	1.11099	0.0422617
e	$\lambda = 1$		$\lambda = 2$		$\lambda = 3$	
	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$	$C_{fin}$	$C_{fout}$
0.05	-0.19407	0.0270275	0.682494	0.0291042	1.55906	0.0311808
0.1	-0.14048	0.0257786	0.736084	0.0278553	1.61265	0.0299319
0.15	-0.0869388	0.0245309	0.789625	0.0266075	1.66619	0.0286841
0.2	-0.0334518	0.0232844	0.843112	0.025361	1.71968	0.0274376
0.25	0.0199767	0.0220392	0.896541	0.0241159	1.7731	0.0261925
0.3	0.0733417	0.0207956	0.949906	0.0228722	1.82647	0.0249488

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