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Peristaltic transport of a newtonian fluid with wall properties in an asymmetric channel

Research Article

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Abstract: The present paper investigates the peristaltic motion of a Newtonian fluid with wall properties in a two dimensional flexible channel under long wave length approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and pressure gradient. The effects of elastic parameters and on axial velocity, pressure rise and friction force have been computed numerically. It is noted that pressure rise decreases with increase in elasticity parameters. The friction force has an opposite behavior compared with pressure rise.

MSC: 74F10 • 92C10

Keywords: Peristaltic motion • Newtonian fluid • Pressure drop and Friction force

1. Introduction

Peristalsis is known to be one of the main mechanisms for fluid transport in many biological systems. Peristalsis is a mechanism for fluid transport, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. The word Peristalsis stems from the Greek work 'Peristaltikos' which means clasping and compressing. It consists in narrowing and transverse shortening of a portion of a tube, which then relaxes while the lower portion becomes shortened and narrowed. In physiology, mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferentâĂŹs of the male reproductive organ, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Mechanical devices like finger pumps, roller pumps use peristalsis to pump blood, slurries and corrosive fluids.

The initial mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely long symmetric channel or tube have been investigated by Shapiro et al. [1] and Fung and Yih [2] have been made on peristalsis with reference to mechanical and physiological situations. Several studies (Raju and Devanathan [3], Mitra and Prasad [4], Shubha Verma et al.[5], Venugopal Reddy et al. [6], Raghunath rao et al.[7]) have been made on peristalsis with reference to mechanical and physiological situations.

The study of peristaltic transport in a symmetric channel has been brought out by Eytan and Elad [8] with an application in intra uterine fluid flow in a non-pregnant uterus. Mishra and Rao [9] have investigated the flow in an asymmetric channel generated by peristaltic waves propagating on the walls with different amplitudes and phases. Furthermore, Haroun [10] studied the effect of wall compliance on peristaltic transport of a Newtonian fluid in an asymmetric channel. Ebaid [11] studied the effects of magnetic field and wall slip condition on the peristaltic transport of a Newtonian fluid in an asymmetric channel. Kothandapani and Srinivas [12] discussed the non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel through a porous medium. Wang et al.[13] have studied the magneto hydrodynamic peristaltic flow of a Sisko fluid in a symmetric or asymmetric channel, Sobh [14]

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studied the slip flow in peristaltic transport of a Carreau fluid in an asymmetric channel. Radhakrishnamacharya et al.[15] studied Influence of wall properties on peristaltic transport with heat transfer. Raghunath rao et al. [16] investigated the effect of heat transfer on peristaltic transport of Viscoelastic fluid in a channel with wall properties and Raghunath rao et al.[17] also studied the interaction of peristalsis with heat transfer of Viscoelastic Rivlin Erickson fluid through a porous medium under the magnetic field.

The present research aimed is to investigate the interaction of peristalsis for the motion of a Newtonian fluid with wall properties in an asymmetric flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and pressure drop. The effects of elasticity parameters on axial velocity, pressure rise and friction force at upper and lower wall have been computed numerically.

2. Formulation of the problem

We consider a peristaltic flow of a Newtonian fluid in an asymmetric channel of width $d_1 + d_2$ and the walls of the channel are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.
The geometry of flexible walls are represented by

$$h_1(x,t) = d_1 + a_1 Cos\frac{2\pi}{\lambda}(x-ct), \qquad \text{upper wall} \tag{1}$$

$$h_2(x,t) = -d_2 - a_2 Cos\left[\frac{2\pi}{\lambda}(x-ct)+\theta\right], \qquad \text{lower wall} \tag{2}$$

Where a_1 , a_2 are the amplitudes of the peristaltic waves, 'c' is the wave velocity, $'\lambda'$ is the wave length, t is the time and θ $(0 \leqslant \theta \leqslant \pi)$ is the phase difference. It should be noted that $\theta = 0$ corresponds to symmetric channel with waves out of phase, $\theta = \pi$ with waves in phase, and further a_1 , a_2 , d_1 , d_2 and θ satisfy the following inequality, Mishra and Rao [9]

$$a_1{}^2 + a_2{}^2 + 2a_1 a_2 \cos\theta \leqslant (d_1+d_2)^2$$

The equation of continuity and the equations of motion are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{4}$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{5}$$

where u, v are the velocity components, $'p'$ is the fluid pressure, $'\rho'$ is the density of the fluid, âĂŸvâĂŹ is the coefficient of kinematic viscosity.
The governing equation of motion of the flexible wall may be expressed as

$$L\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix} = p - p_0 \tag{6}$$

where $'L'$ is an operator, which is used to represent the motion of stretched membrane with damping forces such that

$$L \equiv -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t} \tag{7}$$

Here T is the elastic tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to tension in the muscles. This tension may be obtained through the constitutive relation of the muscles when the displacements are known. For simplicity, we assume $p_0 = 0$.

The horizontal displacement will be assumed zero. Hence the boundary conditions for the fluid are

$$u = 0 \quad \text{at} \quad \begin{cases} y = h_1 \\ y = h_2 \end{cases} \tag{8}$$

Continuity of stresses requires that at the interfaces of the walls and the fluid p must be same as that which acts on the fluid at $y = h_1$ and $y = h_2$. The use of x momentum equation leads to

$$\frac{\partial}{\partial x}L\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix} = \frac{\partial p}{\partial x} = \rho v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \tag{9}$$

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function ψsuch that

$$u=\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\psi}{\partial x} \tag{10}$$

and introducing non-dimensional variables

$$x'=\frac{x}{\lambda},\quad y'=\frac{y}{d},\quad u'=\frac{u}{c},\quad v'=\frac{v}{c\delta},\quad \psi'=\frac{\psi}{cd},\quad t'=\frac{ct}{\lambda},\quad h_1{}'=\frac{h_1}{d_1},\quad h_2{}'=\frac{h_2}{d_1},\quad p'=\frac{pd^2}{\mu c\lambda} \tag{11}$$

in equations of motion and the conditions (10)-(2) and (4)-(5), we finally get (after dropping primes)

$$h_1(x,t)=1+aCos2\pi(x-t) \tag{12}$$

$$h_2(x,t)=-d-bCos[2\pi(x-t)+\theta] \tag{13}$$

$$R\delta\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)\right)\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi}{\partial y}\right)+\delta^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial\psi}{\partial y}\right)\right) \tag{14}$$

$$R\delta^3\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial x}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)\right)\right)=\frac{\partial p}{\partial y}+\delta^2\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi}{\partial x}\right)+\delta^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial\psi}{\partial x}\right)\right) \tag{15}$$

$$\frac{\partial\psi}{\partial y}=0\quad\text{on}\quad\begin{cases}y=1+aCos2\pi(x-t)\\ y=-d-b\,Cos[2\pi(x-t)+\theta]\end{cases} \tag{16}$$

$$\begin{aligned}&\left(\frac{\partial^3\psi}{\partial y^3}+\delta^2\frac{\partial^3\psi}{\partial x^2\partial y}\right)-R\delta\left(\frac{\partial^2\psi}{\partial y\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)\\&\qquad=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right)\begin{Bmatrix}h_1\\ h_2\end{Bmatrix}\quad\text{at}\quad y=\begin{cases}1+a\,Cos\,2\pi(x-t)\\ d-b\,Cos[2\pi(x-t)+\theta]\end{cases}\end{aligned} \tag{17}$$

Eliminating p from the Eqs. (14)-(15), we get

$$R\,\delta\left(\left(\frac{\partial}{\partial t}(\nabla^2\psi)\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}(\nabla^2\psi)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}(\nabla^2\psi)\right)\right)=\left(\frac{\partial^2}{\partial y^2}(\nabla^2\psi)+\delta^2\left(\frac{\partial^2}{\partial x^2}(\nabla^2\psi)\right)\right) \tag{18}$$

where

$$\nabla^2=\frac{\partial^2}{\partial y^2}+\delta^2\frac{\partial^2}{\partial x^2}$$

The non-dimensional parameters are:

$R=\dfrac{cd}{v}$ is the Reynolds number.

$a=\dfrac{a_1}{d_1}$, $b=\dfrac{a_2}{d_1}$, $d=\dfrac{d_2}{d_1}$ and $d=\dfrac{d_2}{d_1}$ are geometric parameters.

$E_1=-\dfrac{Td^3}{\lambda^3\rho vc}$, $E_2=\dfrac{mcd^3}{\lambda^3\rho v}$, $E_3=\dfrac{Cd^3}{\lambda^2\rho v}$ are c parameters.

3. Method of solution

We seek perturbation solution in terms of small parameter δ as follows:

$$\psi=\psi_0+\delta\psi_1+\delta^2\psi_2+\ldots.. \tag{19}$$

$$p\;=\;p_0+\delta\,p_1+\delta^2p_2+\ldots\ldots \tag{20}$$

Substituting Eqs. (19)-(20) in Eqs. (14) to (18) and collecting the coefficients of various powers of δ

$$\frac{\partial^4\psi_0}{\partial y^4}=0 \tag{21}$$

$$\frac{\partial p_0}{\partial x}=\frac{\partial^3\psi_0}{\partial y^3} \tag{22}$$

The corresponding boundary conditions are

$$\frac{\partial\psi_0}{\partial y}=0\quad\text{on}\quad\begin{cases}y=1+aCos2\pi(x-t)\\ y=-d-b\,Cos[2\pi(x-t)+\theta]\end{cases} \tag{23}$$

$$\frac{\partial^3\psi_0}{\partial y^3}=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right)\begin{Bmatrix}h_1\\ h_2\end{Bmatrix}\quad\text{at}\quad y=\begin{cases}1+a\,Cos\,2\pi(x-t)\\ d-b\,Cos[2\pi(x-t)+\theta]\end{cases} \tag{24}$$

3.1. Zeroth-order problem

On solving the Eqs. (21) and (22) subject to the conditions (24) and (2), we get

$$\psi_0 = A_1 \frac{y^3}{6} + A_2 \frac{y^2}{2} + A_3 y \tag{25}$$

$$\frac{\partial p_0}{\partial x} = A_1 \tag{26}$$

The first order equations are

$$\left(\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) + \frac{\partial \psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) - \frac{\partial \psi_0}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) \tag{27}$$

$$R\left(\left(\frac{\partial}{\partial t}\left(\frac{\partial \psi_0}{\partial y}\right)\right) + \frac{\partial \psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial \psi_0}{\partial y}\right)\right) - \frac{\partial \psi_0}{\partial x}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) = -\frac{\partial p_1}{\partial x} + \frac{\partial^3 \psi_1}{\partial y^3} \tag{28}$$

The corresponding boundary conditions are

$$\frac{\partial \psi_1}{\partial y} = 0 \quad \text{on} \quad \begin{cases} y = 1 + aCos2\pi(x-t) \\ y = -d - b\, Cos\,[2\pi(x-t)+\theta] \end{cases} \tag{29}$$

$$\frac{\partial^3 \psi_1}{\partial y^3} - R\left(\frac{\partial^2 \psi_0}{\partial y \partial t} + \frac{\partial \psi_0}{\partial y}\frac{\partial^2 \psi_0}{\partial x \partial y} - \frac{\partial \psi_0}{\partial x}\frac{\partial^2 \psi_0}{\partial y^2}\right) = 0 \quad \text{at} \quad y = \begin{cases} 1 + a\, Cos\, 2\pi(x-t) \\ d - b\, Cos\,[2\pi(x-t)+\theta] \end{cases} \tag{30}$$

3.2. First-order problem

On solving the Eqs. (27) and (28) subject to the conditions (29) and (30), we obtain

$$\psi_1 = R\left[\frac{y^7}{2520} A_1 A_6 + \frac{y^6}{360} A_2 A_6 + \frac{y^5}{120}(A_4 + A_2 A_7 + A_3 A_6 + A_1 A_8) + \frac{y^4}{24}(A_5 + A_3 A_7)\right] + \frac{y^3}{6} B_1 + \frac{y^2}{2} B_2 + B_3 y \tag{31}$$

$$\left(y - y^2\right) A_2 A_7 + \left(y^2 - \frac{y^3}{2}\right) A_1 A_7 + \left(1 - y\right) A_2 A_8 - A_3 A_8 - A_9 \tag{32}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

$$\Delta\, p = \int_0^1 \frac{dp}{dx} dx \tag{33}$$

where $\frac{dp}{dx} = \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + ...$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F_1 = \int_0^1 h_1{}^2\left(-\frac{dp}{dx}\right) dx \qquad \text{(Upper wall)} \tag{34}$$

$$F_2 = \int_0^1 h_2{}^2\left(-\frac{dp}{dx}\right) dx \qquad \text{(Lower wall)} \tag{35}$$

where

$$A_1 = 4E_1\pi^3\,[a\, Sin\, 2\pi\,(x-t) - b\, Sin\, 2\pi\,(x-t)+\theta] + 4E_2\pi^3\,[a\, Sin\, 2\pi\,(x-t) - b\, Sin\, 2\pi\,(x-t)+\theta]$$
$$+2E_3\pi^2\,[a\, Cos\, 2\pi\,(x-t) - b\, Cos\, 2\pi\,(x-t)+\theta],$$

$$A_2 = -\frac{A_1}{2}(h_1 + h_2), \quad A_3 = A_1 h_1 h_2, \quad A_4 = A_{1t}, \quad A_5 = A_{2t}, \quad A_6 = A_{1x},$$

$$A_7 = A_{2x}, \quad A_8 = A_{3x}, \quad A_9 = A_{3t}, \quad B_1 = R(A_9 + A_3 A_8),$$

$$B_2 = \frac{R}{h_1 - h_2}[\frac{1}{24}(A_4 + A_2 A_7 + A_3 A_6 - A_1 A_8)(h_1{}^4 - h_2{}^4) - \frac{1}{6}(A_5 + A_3 A_7)(h_1{}^3 + h_2{}^3)$$
$$-\frac{1}{60}(A_2 A_6)(h_1{}^5 + h_2{}^5) + \frac{1}{360}(A_1 A_6)(h_1{}^6 - h_2{}^6)] + \frac{1}{2}(h_1 + h_2) B_1$$

$$B_3 = R[-\frac{1}{12}(A_5 + A_3 A_7)(h_1{}^3 + h_2{}^3) + \frac{1}{48}(A_4 + A_2 A_7 + A_3 A_6 - A_1 A_8)(h_1{}^4 + h_2{}^4)$$
$$-\frac{1}{120}(A_2 A_6)(h_2{}^5 - h_1{}^5) + \frac{1}{720}(A_1 A_6)(h_1{}^6 + h_2{}^6)] - \frac{1}{4} B_1 (h_1{}^2 + h_2{}^2) - \frac{1}{2} B_2 (h_1 + h_2)$$

It is observed that, if we put $b = d = 1$ and $\theta = 1$ then the results of the problem agree with the work of Mitra and Prasad [4].

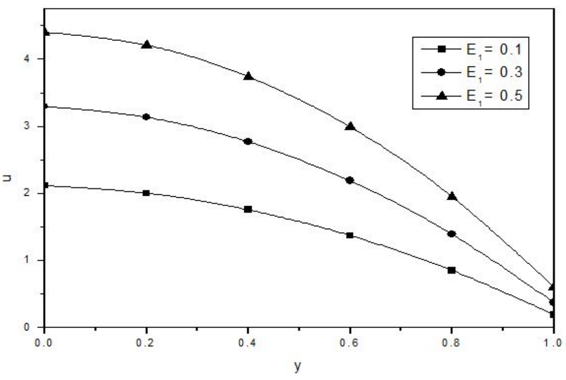

Fig. 1. Effect of the rigidity of the wall E_1 on variation of axial velocity u for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_2 = 0.2, E_3 = 0.3$

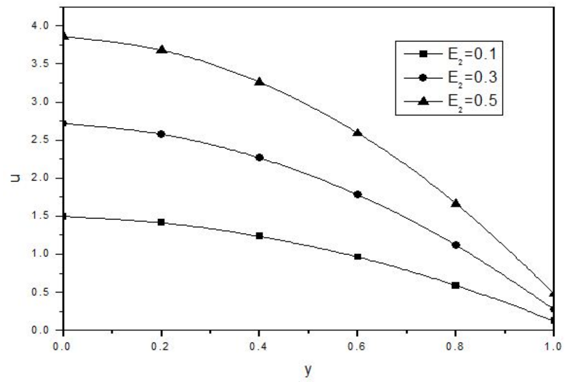

Fig. 2. Effect of the stiffness of the wall E2 on variation of axial velocity u for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_3 = 0.3$

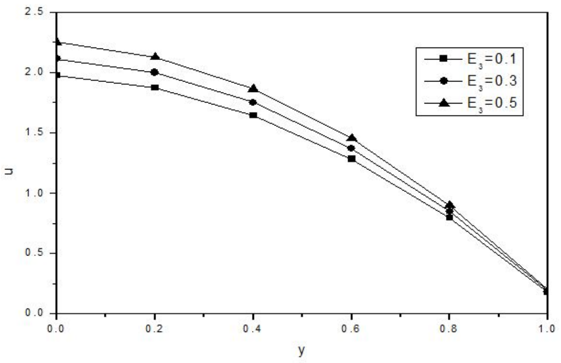

Fig. 3. Effect of the damping nature of the wall E_3 on variation of axial velocity u for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2$

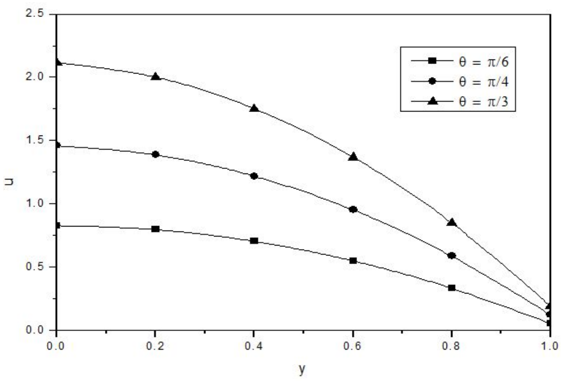

Fig. 4. Effect of the phase difference θ on variation of axial velocity u for $d = 0.1, b = 0.1, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2, E_3 = 0.3$

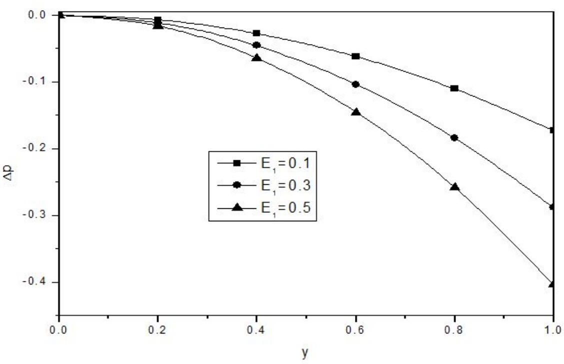

Fig. 5. Effect of the rigidity of the wall E_1 on variation of Pressure drop Δp for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_2 = 0.2, E_3 = 0.3$

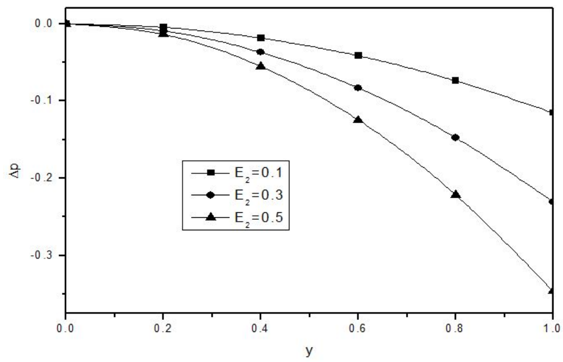

Fig. 6. Effect of the rigidity of the wall E_2 on variation of Pressure drop Δp for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_3 = 0.3$

4. Results and discussions

In this section, we have presented the graphical results of the solutions. The expression for axial velocity u, pressure rise Δp, Friction force at upper wall F_1 and Friction force at lower wall F_2 are calculated numerically using mathematics software for the different values of the rigidity of the wall (E_1), the stiffness of the wall (E_2), damping nature of the wall (E_3) and phase difference (θ). The axial velocity u is shown in Figs. 1 to 4, We noticed that the variation u with respect to E_1, E_2 and θ is appreciably large in the central region $y = 0$ and approaches the prescribed value at $y = 1$, while the variation with E_3 is much less than that u with respect to E_3. The axial velocity enhances with E_1, E_2, E_3, θ. The pressure rise is shown in Figs. 5 to 8. The pressure rise decreases with increase in E_1, E_2, E_3, θ. The Frictional force F is shown in Figs. 9 to 16 for the different values of E_1, E_2, E_3, θ. Figs. 9-12 represent the variation of Frictional force F_1 at upper wall against y for a different parametric values. These figures show that frictional force at

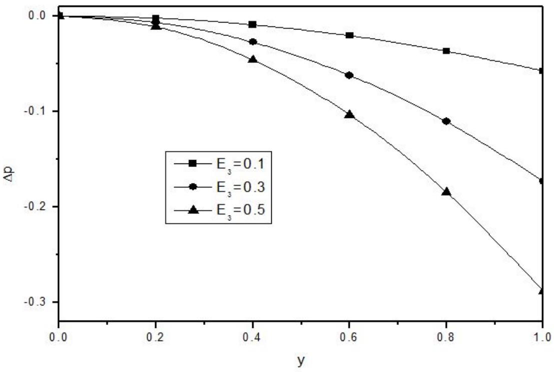

Fig. 7. Effect of the damping nature of the wall E_3 on variation of Pressure drop Δp for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2$

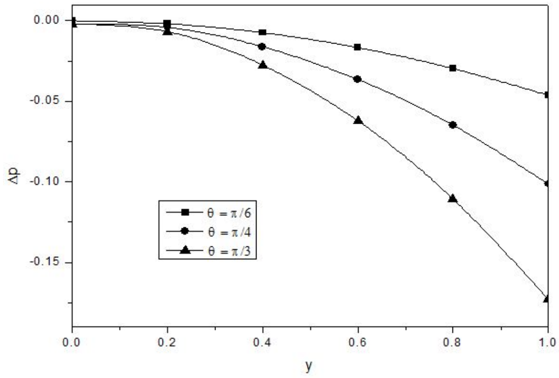

Fig. 8. Effect of the phase difference θ on variation of Pressure drop Δp for $d = 0.1, b = 0.1, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2, E_3 = 0.3$

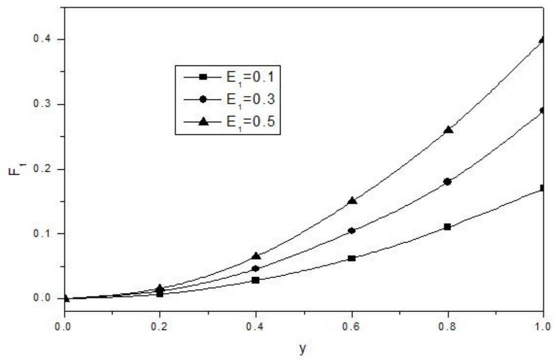

Fig. 9. Effect of the rigidity of the wall E_1 on variation of Friction force F_1 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_2 = 0.2, E_3 = 0.3$

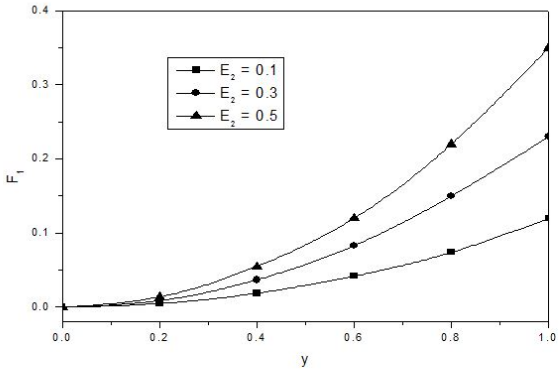

Fig. 10. Effect of the stiffness of the wall E_2 on variation of Friction force F_1 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_3 = 0.3$

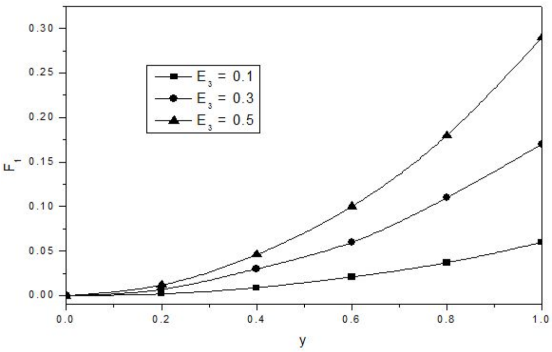

Fig. 11. Effect of the damping nature of the wall E_3 on variation of Friction force F_1 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2$

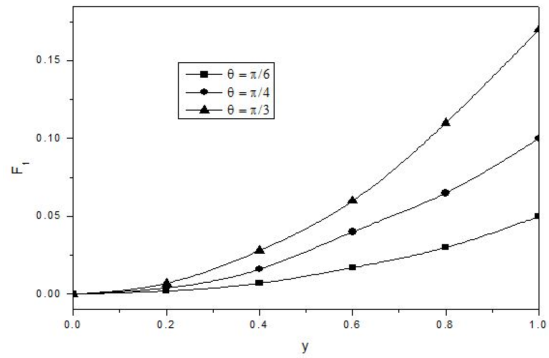

Fig. 12. Effect of the phase difference θ on variation of Friction force F_1 for $d = 0.1, b = 0.1, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2, E_3 = 0.3$

upper wall increases with increase in E_1, E_2, E_3, θ. Frictional force at upper wall opposite behavior in comparison with pressure rise (Sobh) [13]. Figs. 13-16 represents the Frictional force F2 at lower wall. From Figs. 13-14, F_2 increases with E_1, E_2 and θ in the region $y = 0$ to $y = 0.8$ and F_2 graphically enhances in magnitude in the remaining region. F_2 graphically enhances in magnitude with increase in E_3.

5. Conclusions

In the present paper we have discussed the peristaltic transport of a Newtonian fluid with wall properties in an asymmetric channel. The governing equations of motion are solved analytically using long wave length approxima-

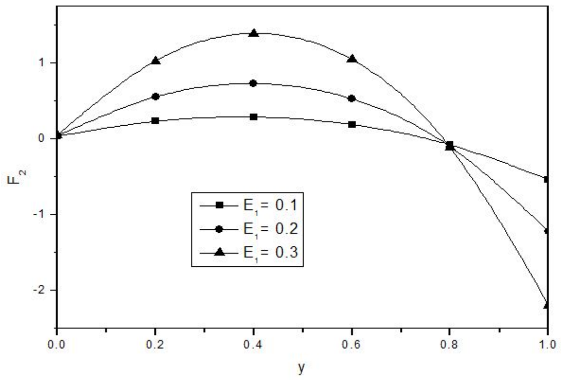

Fig. 13. Effect of the rigidity of the wall E_1 on variation of Friction force F_2 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_2 = 0.2, E_3 = 0.3$

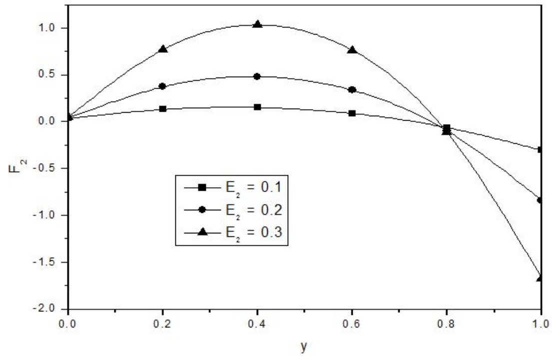

Fig. 14. Effect of the stiffness of the wall E_2 on variation of Friction force F_2 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_3 = 0.3$

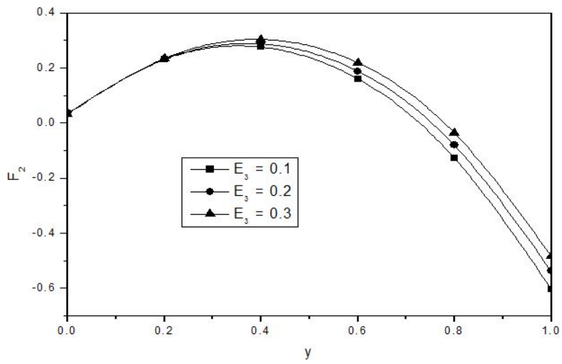

Fig. 15. Effect of the damping nature of the wall E_3 on variation of Friction force F_2 for $d = 0.1, b = 0.1, \theta = \pi/3, a = 0.1, \delta = 0.01, R = 1, E_1 = 0.1, E_2 = 0.2$

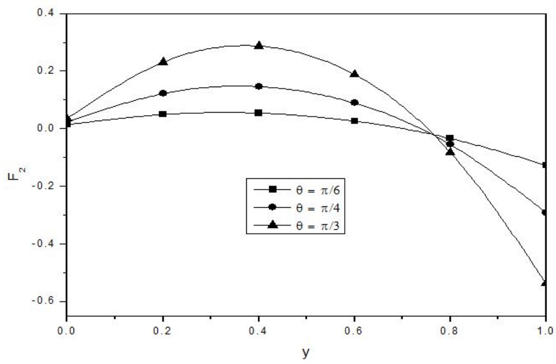

Fig. 16. Effect of the phase difference θ on variation of Friction force F_2 for $d = 0.1, b = 0.1, a = 0.1, \delta = 0.01, R = 1, E1 = 0.1, E_2 = 0.2, E_3 = 0.3$

tion. Furthermore, the effect of elastic parameters and phase difference on pressure rise and friction force have been computed numerically and explained graphically. We conclude the following observations:

1. The axial velocity enhances with E_1, E_2, E_3 and θ.
2. Pressure rise Δp experiences depreciation with E_1, E_2, E_3 and θ.
3. The frictional force at upper wall increases with increase in E_1, E_2, E_3 and θ.
4. The frictional force at lower wall increases with E_1, E_2, and θ.
5. F_2 graphically enhances in magnitude with increase in E_3.
6. We observe that the Friction force F has an opposite behavior compared with pressure rise.

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