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The effect of rigidity on torsional vibrations in a two layered poroelastic cylinder

Research Article

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Abstract: Vibrations in an infinitely long poroelastic composite hollow cylinder are examined by employing Biot's theory of wave propagation in poroelastic media. A two layered poroelastic hollow cylinder consists of two concentric poroelastic cylindrical shells both of which are made of different poroelastic materials. Each of the poroelastic shell is homogeneous and isotropic. The inner and outer boundaries of composite hollow poroelastic cylinder are free from stress. The frequency equation of torsional vibrations of poroelastic composite hollow cylinder is obtained and the effect of rigidity has been discussed. In addition some particular cases such poroelastic composite bore and poroelastic bore are discussed. Non-dimensional phase velocity is computed as a function of non-dimensional wave number. The results are presented graphically for two types of poroelastic composite cylinders and then discussed.

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Keywords: Poro-elasticity • Phase velocity • Core • Casing • Wave number

1. Introduction

Armeniàns [1] studied the torsional waves in composite infinite circular solid rods of two different materials. Bhattacharya [2] investigated torsional wave propagation in a two-layered circular cylinder with imperfect bonding. Following Biot's theory of wave propagation, Tajuddin and Sarma [3, 4] studied torsional vibrations in finite and semi-infinite poroelastic cylinders. Degrande et al. [5] studied the wave propagation in layered dry, saturated and unsaturated poroelastic media. Wisse et al. [6] presented the experimental results of guided wave modes in porous cylinders. Chao et al. [7] studied the shock-induced borehole waves in porous formations. Kang et al. [8] presented torsional vibrations in circular elastic plates with thickness steps. Torsional wave propagation in an initially stressed dissipative cylinder is presented by Selim [9] . Chattopadhyay et. al. [10] investigated propagation of torsional waves in an inhomogeneous layer over an inhomogeneous half space. Tajuddin and Ahmed shah [11] studied torsional vibrations in infinitely long poroelastic cylinders. Akbarov et. al. [12] presented dispersion of torsional waves in a finitely pre-strained hollow sandwich circular cylinder. Ahmed shah and Tajuddin [13] studied vibrations in thick-walled hollow poroelastic spheres. Sumit Gupta et al. [14] solved nonlinear wave-like equations with variable coefficients using homotopy perturbation method. EL-Syed et al. [15] proposed modified Kudryashov method or the rational Exp-function method with the aid of symbolic computation to construct exact solutions of both the coupled equal width wave equation and the (2+1)-dimensional Nizhnik-Novikov-Veselov equations. Barik and Chakraborty [16] studied propagation of magneto elastic surface waves in a two layered infinite plate, under a bias magnetic field. In the present study, the frequency equation of torsional vibrations of a homogeneous, isotropic poroelastic composite hollow circular cylinder of infinite extent is derived in the presence of dissipation. The effect of rigidity has been observed and discussed. Also, the frequency equation for torsional vibrations in poroelastic composite bore has been derived. Let the boundaries of the hollow poroelastic cylinder be free from stress. Non-dimensional phase velocity as a function of non-dimensional wave number is computed in each case i.e., poroelastic composite hollow cylinder, poroelastic composite bore and poroelastic core when it is clamped along its outer surface. The results are presented graphically for two types of poroelastic composite cylinders and then discussed.

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2. Solution of the Problem

Let (r,θ,z) be cylindrical polar co-ordinates. Consider a poroelastic composite hollow cylinder with two poroelastic shells bonded at the interface made of different isotropic poroelastic materials. The inner poroelastic shell is referred as core and the outer poroelastic shell is referred as casing. The prefixes j =1, 2 are used to denote two shells related to poroelastic composite cylinder. The quantities with prefix (1) refer to the core while (2) refer to the casing. The inner radius of core is $'r_1'$, outer radius of casing is $'r_2'$, whereas interface radius is $'a'$.

For torsional vibrations, the displacement of the solid ${}_j\mathbf{u}(0,{}_jv,0)$ and liquid ${}_j\mathbf{U}(0,{}_jV,0)$ are

$$ {}_jv = {}_jf(r)\exp[i(kz+\omega t)], \quad {}_jV = {}_jF(r)\exp[i(kz+\omega t)], j=1,2 \tag{1}$$

where ω is the circular frequency of wave, k is the wavenumber and t is the time. Using Eq. (1), the dilatations of solid and liquid media (Biot [17]) vanish and hence the waves considered are essentially shear waves and hence the liquid pressure developed in the solid-liquid aggregate is zero. The equations of motion (Biot [17]) in cylindrical polar coordinates, when v and V are functions of r, z and t, are

$$ {}_jN\left(\nabla^2-\frac{1}{r^2}\right){}_jv = \frac{\partial^2}{\partial t^2}({}_j\rho_{11}\,{}_jv+{}_j\rho_{12}\,{}_jV)+b\frac{\partial}{\partial t}({}_jv-{}_jV), \qquad 0=\frac{\partial^2}{\partial t^2}({}_j\rho_{12}\,{}_jv+{}_j\rho_{22}\,{}_jV)-b\frac{\partial}{\partial t}({}_jv-{}_jV). \tag{2}$$

Substitution of Eq. (1) into the Eq. (2), it reduces to

$$ {}_jN\Delta_jf = -\omega^2\left[{}_jM_jf+{}_jT_jF\right], \quad 0=-\omega^2\left[{}_jT_jf+{}_jX_jF\right], \tag{3}$$

where ${}_jM={}_j\rho_{11}-ib\omega^{-1}$, ${}_jT={}_j\rho_{12}+ib\omega^{-1}$, ${}_jX={}_j\rho_{22}-ib\omega^{-1}$, b is dissipation co-efficient, and

$$ \Delta=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}-k^2. \tag{4}$$

From second equation of (3), we obtain

$$ {}_jF=-\left(\frac{{}_jT}{{}_jX}\right){}_jf. \tag{5}$$

Substituting Eq. (5) into the first equation of (3), one obtains

$$ \left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}+{}_j\xi_3^2\right){}_jf=0. \tag{6}$$

where

$$ {}_j\xi_3^2=\frac{\omega^2}{{}_jV_3^2}-k^2 \quad j=1,2 \tag{7}$$

In Eq. (7), ${}_jV_3$ is shear wave velocity given by ${}_jV_3^2=\dfrac{{}_jN_jX}{{}_jM_jX-{}_jT^2}$. The solution of Eq. (6) is given by

$$ {}_jf={}_jC_1J_1({}_j\xi_3r)+{}_jC_2Y_1({}_j\xi_3r). \tag{8}$$

In Eq. (8), J_1 is Bessel function of first kind of order one while Y_1 is Bessel function of second kind of order one. Thus, the displacement of solid (j =1, 2) from Eq. (1) is

$$ {}_jv=\left({}_jC_1J_1({}_j\xi_3r)+{}_jC_2Y_1({}_j\xi_3r)\right)\exp[i(kz+\omega t)] \tag{9}$$

The relevant stresses pertaining to outer cylinder and inner cylinder are

$$ {}_j\sigma_{r\theta}=-{}_jN\left[{}_jC_1J_2({}_j\xi_3r)+{}_jC_2Y({}_j\xi_3r)\right]\exp[i(kz+\omega t)], \quad j=1,2 \tag{10}$$

3. Boundary conditions and frequency equation

We assume that the outer surface of casing and inner surface of core are free from stress and there is a perfect bonding at the interface, thus the boundary conditions for stress-free vibrations of a poroelastic composite hollow cylinder in case of a pervious surface are

$$\begin{aligned} &at\ r = r_1;\ {}_1(\sigma_{r\theta}) = 0 \\ &at\ r = r_2;\ {}_2(\sigma_{r\theta}) = 0 \\ &at\ r = a;\ {}_1(\sigma_{r\theta}) = {}_2(\sigma_{r\theta}), {}_1v = {}_2v \\ &at\ r = r_1, r_2 \text{ and } a; {}_1s = {}_2s = 0, \end{aligned} \tag{11}$$

while the boundary conditions in case of an impervious surface are

$$\begin{aligned} &at\ r = r_1;\ {}_1(\sigma_{r\theta}) = 0 \\ &at\ r = r_2;\ {}_2(\sigma_{r\theta}) = 0 \\ &at\ r = a;\ {}_1(\sigma_{r\theta}) = {}_2(\sigma_{r\theta}), {}_1v = {}_2v \\ &at\ r = r_1, r_2 \text{ and } a; \frac{\partial_1 s}{\partial r} = \frac{\partial_2 s}{\partial r} = 0, \end{aligned} \tag{12}$$

Since the considered vibrations are shear vibrations, the dilatations of solid and liquid media each is zero, thereby liquid pressure developed in solid-liquid aggregate will be identically zero and no distinction between pervious and impervious surface is made. Hence, fourth equation in each of the Eqs. (11) and |eqref12 is satisfied identically. Eqs.(9), (10) and (11) results in a system of four homogeneous equations in constants ${}_jC_1$ and ${}_jC_2$ $(j = 1, 2)$ such a homogeneous system has non-trivial solution only if the determinant of the coefficients of the unknowns vanishes identically. Thus by eliminating the constants, the frequency equation of torsional vibrations in poroelastic composite hollow cylinder is obtained as

$$\begin{vmatrix} J_2({}_1\xi_3 r_1) & Y_2({}_1\xi_3 r_1) & 0 & 0 \\ {}_1N_1\xi_3 J_2({}_1\xi_3 a) & {}_1N_1\xi_3 Y_2({}_1\xi_3 a) & {}_2N_2\xi_3 J_2({}_2\xi_3 a) & {}_2N_2\xi_3 Y_2({}_2\xi_3 a) \\ J_1({}_1\xi_3 a) & Y_1({}_1\xi_3 a) & J_1({}_2\xi_3 a) & Y_1({}_2\xi_3 a) \\ 0 & 0 & J_2({}_2\xi_3 r_2) & Y_2({}_2\xi_3 r_2) \end{vmatrix} = 0. \tag{13}$$

When shear modulus of the casing is larger than that of core, we can assume that casing is perfectly rigid. Letting the shear modulus of the casing approaches to infinity i.e., ${}_2N \to \infty$ then the shear wave velocity of casing approaches to infinity. Under this limiting condition, the frequency Eq. (13) of vibrations of poroelastic composite hollow cylinder reduces to

$$C_1 C_2 = 0, \tag{14}$$

$$C_1 = \begin{vmatrix} J_2({}_1\xi_3 r_1) & Y_2({}_1\xi_3 r_1) \\ J_1({}_1\xi_3 a) & Y_1({}_1\xi_3 a) \end{vmatrix} \text{ and } C_2 = \begin{vmatrix} J_2({}_2\xi_3 a) & Y_2({}_2\xi_3 a) \\ J_2({}_2\xi_3 r_2) & Y_2({}_2\xi_3 r_2) \end{vmatrix}. \tag{15}$$

From Eq. (14) it is clear that vibrations of poroelastic composite hollow cylinder related to core and casing are uncoupled when the solid in casing is rigid, also we obtain $C_1 = 0$ or $C_2 = 0$. The equation

$$\text{or } J_2({}_1\xi_3 r_1) Y_1({}_1\xi_3 a) - J_1({}_1\xi_3 a) Y_2({}_1\xi_3 r_1) = 0 \tag{16}$$

represents the frequency equation of vibrations of poroelastic core when it is clamped along its outer surface, whereas the equation

$$\text{or } J_2({}_2\xi_3 a) Y_2({}_2\xi_3 r_2) - J_2({}_2\xi_3 r_2) Y_2({}_2\xi_3 a) = 0 \tag{17}$$

represents the frequency equation of vibrations of hollow rigid casing when the boundaries are free from stress. When the outer radius r_2 of casing tends to ∞ , the frequency Eq. (13) of poroelastic composite hollow cylinder reduces to

$$\begin{vmatrix} J_2({}_1\xi_3 r_1) & Y_2({}_1\xi_3 r_1) & 0 \\ {}_1N_1\xi_3 J_2({}_1\xi_3 a) & {}_1N_1\xi_3 Y_2({}_1\xi_3 a) & {}_2N_2\xi_3 Y_2({}_2\xi_3 a) \\ J_1({}_1\xi_3 a) & Y_1({}_1\xi_3 a) & Y_1({}_2\xi_3 a) \end{vmatrix} = 0, \tag{18}$$

which is the frequency equation of torsional vibrations in poroelastic composite bore. If the poroelastic constants of core vanish, the above frequency Eq. (18) of poroelastic composite bore reduces to

$$Y_2({}_2\xi_3 a) = 0, \tag{19}$$

which is the frequency equation of torsional vibrations in poroelastic bore of radius a.

4. Results and discussion

Non-dimensional phase velocity is calculated for two types of composite cylinders, namely composite cylinder-I and composite cylinder-II. Composite cylinder-I consists of core made up of sandstone saturated with water (Yew and Jogi [18]) and casing is made up of sandstone saturated with kerosene (Fatt [19]), where as in composite cylinder-II, the core is sandstone saturated with kerosene and casing is sandstone saturated with water. For given poroelastic parameters, frequency equations when non-dimensionalized, constitute a relation between non-dimensional phase velocity and non-dimensional wave number. Different values of a/r and r/a, viz., 1.1 and 3 are taken for numerical computation. These values, respectively, represent thin and thick poroelastic shells.

Figs. 1-4 depict phase velocity of torsional vibrations of poroelastic composite hollow cylinders I and II for different combinations of thin and thick shells. In Fig. 1, phase velocity for thin core and thin casing has been plotted. The phase velocity of composite cylinder II is more than that of cylinder I when wave number is between 0 and 5.8 and it is same for both the cylinders I and II when wave number is more than 7. Fig. 2 shows the phase velocity for thin core and thick casings. The variation in phase velocity is more in composite cylinder II than in cylinder II. The variation of phase velocity for thin casing and thick core is shown in Fig. 3. The phase velocity is nearly same for both the cylinders when the wave number is less than 2 and phase velocity is maximum when wave number is 3 in case of cylinder II. Fig. 4 shows the phase velocity for thick core and thick casings. The phase velocity is more for cylinder II than that of cylinder I when the wave number is between 1.5 and 5.7.

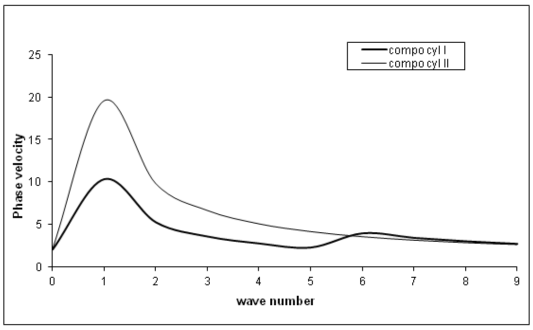

Fig. 1. Variation of phase velocity with the wave number – poroelastic composite hollow cylinder – Thin core and thin casing

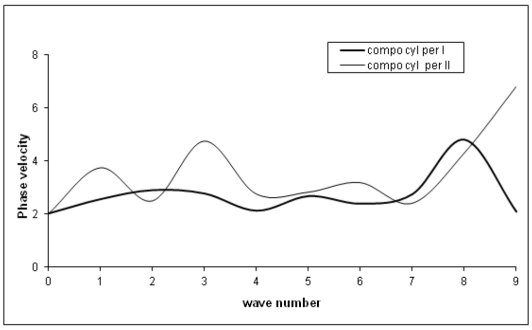

Fig. 2. Variation of phase velocity with the wave number – poroelastic composite hollow cylinder – Thin core and thick casing

The variation in phase velocity for poroelastic core when it is clamped along its outer surface is shown in Figs. 5-6. In particular, thin core is considered in Fig. 5, whereas thick core is considered in Fig. 6. From Fig. 5, it is clear that the phase velocity is almost same for both cylinders. There is gradual decrease in phase velocity when wave number is more than 1. In case of thick core, the variation in phase velocity is more than the case of thin core.

The variation in phase velocity for poroelastic composite bore is shown in Figs. 7-8. In particular, composite bore with thin core is considered in Fig. 7, whereas composite bore with thick core is considered in Fig. 8. From Fig. 7, it is clear that the phase velocity for composite bore I higher than that of cylinder II when wave number is less than 4.5 for an impervious surface. Also, the maximum phase velocity is observed when wave number is 8 for an impervious surface for composite bore I.

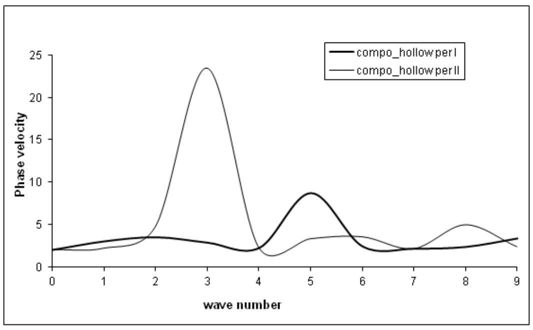

Fig. 3. Variation of phase velocity with the wave number – poroelastic composite hollow cylinder – Thick core and thin casing

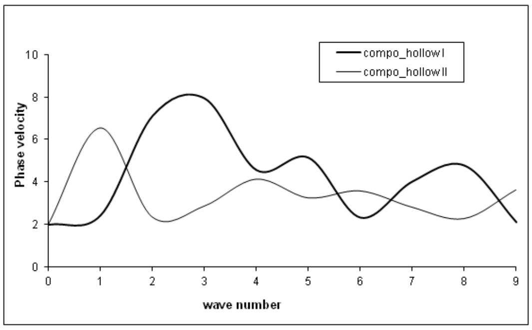

Fig. 4. Variation of phase velocity with the wave number – poroelastic composite hollow cylinder – Thick core and thick casing

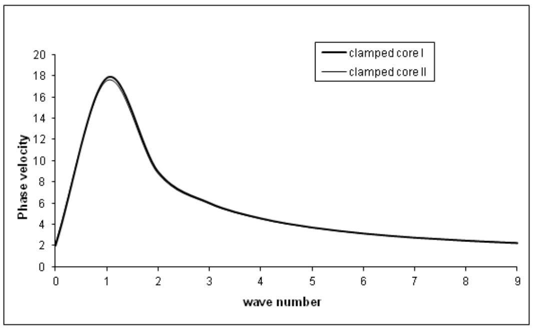

Fig. 5. Variation of phase velocity with the wave number – poroelastic shell clamped along its outer surface – Thin shell

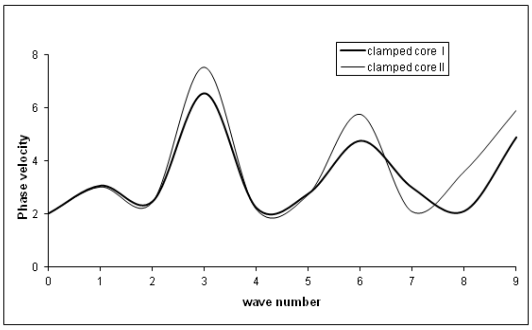

Fig. 6. Variation of phase velocity with the wave number – poroelastic shell clamped along its outer surface – Thick shell

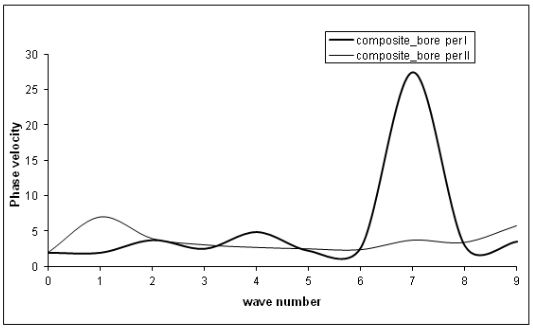

Fig. 7. Variation of phase velocity with the wave number – poroelastic composite bore – Thin core

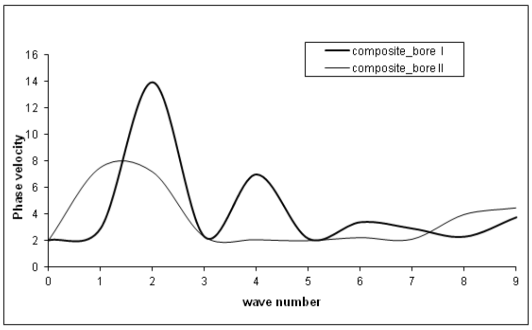

Fig. 8. Variation of phase velocity with the wave number – poroelastic composite bore – Thick core

5. Concluding remarks

- Vibrations of poroelastic composite hollow cylinder related to core and casing are uncoupled when the solid in casing is rigid.
- Phase velocity in poroelastic composite hollow cylinder II is higher than that of composite hollow cylinder I in case of thin core and thin casing.
- Phase velocity of poroelastic thin shells I and II is same when they are clamped along each of their outer surface, whereas the phase velocity in nearly same in case of thick shells.

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