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Combined influence of Soret and Dufour effects on unsteady hydromagnetic mixed convective flow in an accelerated vertical wavy plate through a porous medium

Research Article

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Abstract: The combined effects of Soret and Dufour on an unsteady mixed convection magnetohydrodynamics heat and mass transfer flow in an accelerated vertical wavy plate embedded in a porous medium considering thermal radiation, heat generation, and chemical reaction have been investigated. The momentum, energy and mass diffusion equations are coupled non-linear partial differential equations, which are solved by applying finite difference method using symbolic software MATLAB "bvp4c". The features of the fluid flow, heat and mass transfer characteristics are analyzed by plotting graphs and the physical aspects are discussed in detail to interpret the effects of significant parameters of the problem. Comparison of numerical results is made with the previous published results under limiting cases and found to be in good agreement.

MSC: 35Q79 • 65M06

Keywords: Soret • Dufour • Mixed Convection • Finite difference method

1. Introduction

Coupled heat and mass transfer by free convective in porous media has been widely studied in the recent years due to its wide applications in engineering as post accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. The unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate is analyzed by Kishore et al.[1] taking into an account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. Bala and Verma et al.[2] studied the effects of magnetic field and heat and mass transfer on unsteady two dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi infinite moving vertical porous plate under the influence of a uniform transverse magnetic field with temperature dependent heat generation and homogeneous first order chemical reaction. Sivaiah and Srinivasa Raju et al.[3] studied the effects of Hall current and Heat source on unsteady magnetohydrodynamic free convective heat and mass transfer flow in the presence of viscous dissipation by applying finite element technique. Combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed by Chien-Hsin Chen et al.[4], taking into account the effects of Ohmic heating and viscous dissipation. The hydromagnetic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made by Anand Rao

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Nomenclature

A	Positive constant
u	Velocity of the fluid
C'	Dimensionless concentration
C'_w	Concentration near the plate
C'_∞	Concentration in the fluid far
T'	Temperature of the fluid
T'_w	Temperature of the plate
T'_∞	Temperature of the fluid far away from the plate
u'	Velocity component in x' - direction
x'	Spatial co-ordinate along the plate
y'	Spatial co-ordinate normal to the plate
C_p	Specific heat at constant Pressure, $J/kg-K$
D	Chemical molecular diffusivity
U_p	Dimensionless plate translational Velocity
n	Dimensionless frequency of Oscillation
V_0	Suction velocity
M	Hartmann number
D_m	Mass diffusivity
k_T	Thermal diffusion ratio
C_s	Concentration susceptibility
T_m	Mean fluid temperature
Pr	Prandtl number
Sc	Schmidt Number
Sr	Soret Number (Thermal diffusion)
g	Acceleration due to Gravity, 9.81 m/s^2
Gr	Grashof Number
Ec	Eckert number
S	Heat absorption parameter
R	Thermal Radiation absorption parameter
Du	Dufour number (Diffusion thermo)
k'	Permeability of porous medium
Gc	Modified Grashof Number
k_r	Chemical reaction parameter
t'	Non dimensional time in second
K	Permeability parameter
B_0	Magnetic field
u_p	Non dimensional plate translational Velocity
Nu	Nusselt number
Sh	Sherwood number
Re	ReynoldâĂŹs number
y	Spatial coordinate

Greek symbols

ϵ	Porosity of the porous medium
θ	Dimensionless Temperature away from the plate
ν	Kinematics viscosity, m2/s plate
α	Thermal Diffusivity
κ	Thermal conductivity, W/mK
σ	Electrical conductivity, mho/m Henry/meter
ρ	Density $kg/m3$
β	Volumetric co-efficient of thermal expansion, K^{-1}
β^*	Co-efficient of volume expansion with Species concentration
τ	Skin-friction coefficient

et al.[5]. An unsteady two-dimensional MHD free convective and heat transfer flow of a viscous incompressible fluid along a vertical flat plate with internal heat generation/absorption was studied by Alam et al.[6].

Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power and hazards engineering. For some industrial applications such as glass production and furnace design and in space technology applications such as cosmic flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry thermodynamics which operate at higher temperatures, radiation effects can be significant. The influence of thermal radiation on magnetohydrodynamic free convective flow past an exponentially accelerated vertical plate with variable temperature and mass transfer studied by Rajesh and Vijayakumar Verma et al.[7]. Rajesh and Vijayakumar Verma et al.[8] is performed to study the effects of thermal radiation on unsteady free convective flow of an elasto-viscous fluid over a moving vertical plate with variable temperature in the presence of magnetic field through porous medium and a viscous incompressible fluid over a moving vertical plate with variable mass diffusion in the presence of magnetic field through porous medium. The radiation effects on unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation studied by Rao et al.[9]. The method of solution is applied using finite element technique. Anjali Devi and Vasantha kumara et al.[10] was investigated the slip flow effects on unsteady hydromagnetic flow over a stretching surface with thermal radiation heat transfer. The effects of variable viscosity of nano-fluid flow over a permeable wedge embedded in saturated non-Darcy porous medium with chemical reaction and thermal radiation was studied by Makungu James et al.[11]. The effects of chemical reaction and radiation on an unsteady transient MHD free convective and heat transfer of a viscous, incompressible and electrically conducting fluid past a suddenly started infinite vertical porous plate taking into account and the viscous dissipation was presented by Jithender Reddy et al.[12].

But in the above mentioned studies, Dufour and Soret terms have been neglected from the energy and concentration equations respectively. It has been found that energy flux can be generated not only by temperature gradient but also by concentration gradient as well. The energy flux caused by concentration gradient is called Dufour effect and the same by temperature gradient is called the Soret effect. These effects are very significant when the temperature and concentration gradient are very high. We can realize the significance of the Dufour effect from the works of Postelnicu et al.[13], Pal and Mondal et al.[14], and Rushi Kumar and Sivaraj et al.[15]. Previous studies of the flows of heat and mass transfer have focused mainly on a flat wall or a regular channel. It is necessary to study the heat and mass transfer in wavy plates because of its numerous applications. Fluid flow over wavy boundaries may be observed in several natural phenomena, viz., the generation of wind waves on water, the formation of sedimentary ripples in river channels, and dunes in the desert. The analysis of such flows finds applications in different areas, such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces, and film vaporization in combustion chambers. In view of these applications The effects of thermal radiation and Heat source on an unsteady MHD free convection flow past an infinite vertical plate with thermal-diffusion and diffusion-thermo discussed by Srinivasa Raju et al.[16]. A numerical model is developed by Dulal Pal and Sewli Chatterjee et al.[17] to study the MHD mixed convection with the combined action of Soret and Dufour on heat and mass transfer of a power-law fluid over an inclined plate in a porous medium in the presence of variable thermal conductivity, thermal radiation, chemical reaction and Ohmic dissipation and suction/injection. The effect of variable viscosity and thermal conductivity on mixed convection flow and heat transfer along a semi-infinite non-linear stretching sheet embedded in a saturated porous medium in the presence of viscous dissipation, Soret and Dufour, thermal radiation, non-uniform heat source/sink and first order chemical reaction studied by Dulal Pal and Hiranmoy Mondal et al.[18]. Jagdish Prakash et al.[19] studied flow, heat, and mass transfer characteristics of unsteady mixed convective magnetohydrodynamic (MHD) flow of a heat absorbing fluid in an accelerated vertical wavy plate, subject to varying temperature and mass diffusion, with the influence of buoyancy, thermal radiation and Dufour effect. The effect of hall current on an unsteady magnetohydrodynamic flow and mass transfer of an electrically conducting incompressible fluid along an infinite vertical porous plate in presence of thermal-diffusion, diffusion-thermo and chemical reaction has been studied by Sudhakar et al.[20]. The effects of first order homogeneous chemical reaction and thermal diffusion on hydromagnetic free convection heat and mass transfer flow of viscous dissipative fluid past a semi-infinite vertical moving porous plate embedded in a porous medium in the presence of thermal radiation was analyzed by Prabhakar Reddy et al.[21]. G.Jithender Reddy et.al.[24] analyzed the finite element analysis of soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate with heat absorption.

Motivated by the above reference work, the main objective of the present investigation is to study the combined effects of Soret and Dufour on unsteady hydromagnetic mixed convective magnetohydrodynamics heat and mass transfer flow in an accelerated vertical wavy plate embedded in a porous medium considering thermal radiation, heat generation, and chemical reaction have been investigated. The momentum, thermal, and solutal boundary layer equations are transformed into a set of partial differential equations and then solved numerically by the efficient finite difference technique using symbolic using MATLAB “bvp4c”. The analysis of the results obtained in the present work shows that the flow field is appreciably influenced by Dufour and Soret numbers and Thermal radiation parameter on the wall. To reveal the tendency of the solutions, selected results for the velocity components, temperature, and concentration are graphically depicted. The results obtained are good agreement with the results of and the results obtained are good agreement with the results of Jagdish Pakash et al.[19] in the absence of Soret number. The rest of the paper is structured as follows. In Section 2, we formulate the problem, in Section 3, we give the method of solution.

Our results are presented and discussed in Section 5 and in Section 6, we present some brief conclusions.

2. Mathematical Formulation

We consider unsteady two-dimensional flow of an incompressible, viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical permeable wavy plate subject to varying temperature and concentration. The physical model and the coordinate system are shown in Fig. 1.
We made the following assumptions:

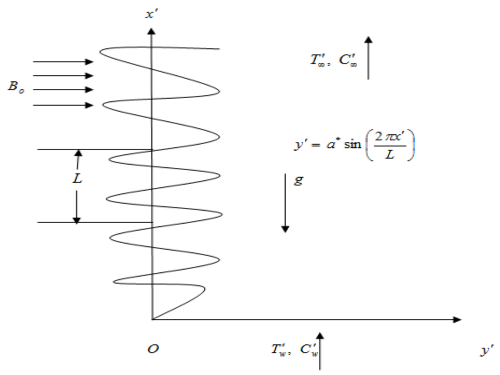

Fig. 1. Flow configuration and coordinate system.

1. In Cartesian coordinate system, let x' axis is taken to be along the plate and the y' axis normal to the plate. Since the plate is considered infinite in x' direction, hence all physical quantities will be independent of x' direction.
2. Let the components of velocity along x' and y' axes be u' and v' which are chosen in the upward direction along the plate and normal to the plate respectively.
   3. Initially, the plate and the fluid are at the same temperature T'_∞ and the concentration C'_∞ . At a time $t' > 0$ the plate temperature and concentration are raised to T'_w and C'_w respectively and are maintained constantly thereafter.
3. A uniform magnetic field of magnitude B_0 is maintained in the y'- direction and the plate moves uniformly along the positive x'- direction with velocity U_0.
4. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field.
5. It is assumed that the external electric field is zero and the electric field due to the polarization of charges is negligible.
6. The homogeneous chemical reaction of first order with rate constant between the diffusing species and the fluid is assumed.
7. The Hall Effect of magnetohydrodynamics and magnetic dissipation (Joule heating of the fluid) are neglected.
8. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (BoussinesqâĂŹs approximation).
9. The governing equations for this investigation are based on the balances of mass, linear momentum, energy and concentration species.

Under these assumptions, the governing boundary layer equations of the flow field are:

Continuity equation:

$$\frac{\partial u'}{\partial t'} + \frac{\partial v'}{\partial t'} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + \left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x'}\right) + \upsilon\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) - \frac{\sigma {\beta_0}^2}{\rho}u' - \frac{\upsilon}{k'}u' + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) \tag{2}$$

Energy equation:

$$\frac{\partial T'}{\partial t'}+\left(u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right)=\frac{\kappa}{\rho C_p}\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}\right)-\frac{Q'}{\rho C_p}(T'-T'_\infty)-\frac{1}{\rho C_p}\left(\frac{\partial q'}{\partial x'}+\frac{\partial q'}{\partial y'}\right)+\frac{D_m k_T}{C_s C_p}\left(\frac{\partial^2 C'}{\partial x'^2}+\frac{\partial^2 C'}{\partial y'^2}\right) \tag{3}$$

Species diffusion equation:

$$\frac{\partial C'}{\partial t'}+\left(u'\frac{\partial C'}{\partial x'}+v'\frac{\partial C'}{\partial y'}\right)=D\left(\frac{\partial^2 C'}{\partial x'^2}+\frac{\partial^2 C'}{\partial y'^2}\right)-K'_r(C'-C'_\infty)+\frac{D_m k_T}{T_m}\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}\right) \tag{4}$$

The corresponding boundary conditions are

$$\forall t'\leqslant 0: u'=0,\ T'=T'_\infty,\ C'=C'_\infty \text{ for all } y' \tag{5}$$

$$\forall t'>0: u'=U_p,\ \frac{\partial T'}{\partial y'}=A^* T'_w,\ \frac{\partial C'}{\partial y'}=A^* C'_w \text{ at } y'=a^* sin\left(\frac{2\pi x'}{L}\right) \tag{6}$$

$$\forall t'>0: u'=U_\infty,\ T'\to T'_\infty,\ C'\to C'_\infty \text{ at } y'\to\infty \tag{7}$$

Since the motion is two-dimensional and the length of the plate is large enough, all the physical variables are independent of the coordinate. Therefore:

$$\frac{\partial u'}{\partial x'}=0 \tag{8}$$

So that the suction velocity at the plate surface is a function of time only, and we assume that the suction velocity takes the following exponential form:

$$v=-V_0\left(1+A\epsilon e^{n't'}\right) \tag{9}$$

Where A is a real positive constant, ϵ and $A\epsilon$ are less than unity, and V_0 is a scale of suction velocity which has a non-zero positive constant.
Outside the boundary layer, Eq. (2) gives:

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x'}\right)=\frac{dU_\infty}{dt'}+\left[\frac{\sigma B_0^2}{\rho}+\frac{k'}{\mu}\right]u_\infty \tag{10}$$

So that the pressure p is independent of y .
The fluid is optically thin, with a relatively low density, and the radiative heat flux discussed by Cogley et al. [22] is given by:

$$\frac{\partial q_r}{\partial y'}=4I(T'-T'_\infty) \tag{11}$$

Where I= $\int_0^\infty K_{\lambda w}(\frac{de_{b\lambda}}{dT'})d\lambda$
$K_{\lambda w}$ being the absorption coefficient and $e_{b\lambda}$ is the Plank function.
We now introduce the following non dimensional variables and parameters:
$u=\frac{u'}{V_0},\ v=\frac{v'}{V_0},\ x=\frac{x'}{L},\ y=\frac{V_0 y'}{v},\ U_\infty=\frac{u_\infty}{U_0},\ t=\frac{V_0^2 t'}{U_0},\ U_p=\frac{u_p}{U_0},\ \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ n=\frac{n'v'}{V_0^2},\ M=\frac{\sigma B_0^2 v}{\rho V_0^2},$
$K=\frac{v^2}{V_0^2},\ Gr=v\beta g\frac{T'_w-T'_\infty}{U_0 V_0^2},\ Gc=v\beta^* g\frac{C'_w-C'_\infty}{U_0 V_0^2},\ Pr=\frac{\mu c_p}{\kappa},\ S=\frac{vQ'}{\rho C_p V_0^2},\ Sc=\frac{v}{D},\ R=\frac{4I'}{\rho C_p V_0},\ Sr=\frac{D_m k_T\left(T'_w-T'_\infty\right)}{vV_0 T_m\left(C'_w-C'_\infty\right)},$
$Du=\frac{D_m k_T\left(C'_w-C'_\infty\right)}{vC_s C_p V_0^2\left(T'_w-T'_\infty\right)},\ k_r=\frac{K'_r v}{V_0^2},\ A_1=\frac{V_0}{v},\ h=asin(2\pi x)$
The basic Eqs. (2) to (4) can be expressed in non-dimensional form as:
Momentum equation:

$$\frac{\partial u}{\partial t}-\left(1+\epsilon Ae^{int}\frac{\partial u}{\partial y}\right)=\frac{\partial U_\infty}{\partial t}+\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u+Gr(T)+Gm(C) \tag{12}$$

Energy equation:

$$(Pr)\frac{\partial\theta}{\partial t}-(Pr)\left(1+\epsilon Ae^{int}\frac{\partial\theta}{\partial y}\right)=\left(\frac{\partial^2}{\partial y^2}-(Pr)(R+S)\right)\theta-(Pr)(Du)\left(\frac{\partial^2\phi}{\partial y^2}\right) \tag{13}$$

Species Diffusion Equation:

$$(Sc)\frac{\partial\phi}{\partial t}-(Sc)\left(1+\epsilon Ae^{int}\frac{\partial\phi}{\partial y}\right)=\frac{\partial^2\phi}{\partial y^2}-(K_r)(Sc)\phi+(Sc)(Sr)\left(\frac{\partial^2\theta}{\partial y^2}\right) \tag{14}$$

And the corresponding boundary conditions are:

$$\forall t\le 0: u=0,\theta=0,\phi=0 \text{ for all } y \tag{15}$$

$$\forall t>0: u=U_p,\frac{\partial\theta}{\partial y}=1,\frac{\partial\phi}{\partial y}=1 \text{ at } y=h \tag{16}$$

$$\forall t>0: u'=U_p=1+\epsilon e^{nt},\theta\to 0,\phi\to 0 \text{ at } y\to\infty \tag{17}$$

All the symbols are defined in nomenclature. The mathematical statement of the problem is now complete and embodies the solution of Eqs. (12), (13) and (14) subject to boundary conditions (15)-(17). For practical engineering applications and the design of chemical engineering systems, the skin-friction, Nusselt number (Rate of heat transfer) and Sherwood number (Rate of mass transfer) are important physical parameters for this type of boundary layer flow.

3. Skin-Friction, rate of heat and mass mransfer

The skin friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin friction at the plate, which in the non âĂŞ dimensional form is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{18}$$

The rate of heat transfer coefficient, which in the non âĂŞ dimensional form in terms of the Nusselt number is given by

$$N_u=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \tag{19}$$

The rate of mass transfer coefficient, which in the non âĂŞ dimensional form in terms of the Sherwood number, is given by

$$S_b=-\left(\frac{\partial\phi}{\partial y}\right)_{y=0} \tag{20}$$

4. Method of solution by finite difference method

Eqs. (12)-(14) are coupled non-linear partial differential equations are solved using boundary and initial conditions (15)-(17). All the same, exact or approximate solutions are not possible. Therefore we solve these equations by Crank-Nicolson implicit ïňĄnite difference method for numerical solution. The equivalent ïňĄnite difference scheme of Eqs. (12)-(14) are as follows:

$$\begin{aligned}\left(\frac{u_i^{j+1}-u_i^j}{\Delta t}\right)-B\left(\frac{u_i^{j+1}-u_i^j}{\Delta y}\right)&=(n\epsilon e^{nt})+\left(\frac{u_{i-1}^{j+1}-2u_i^{j+1}+u_{i+1}^{j+1}}{2(\Delta y)^2}+\frac{u_{i-1}^{j}-2u_i^{j}+u_{i+1}^{j}}{2(\Delta y)^2}\right)\\&+(Gr)\left(\frac{\theta_i^{j+1}-\theta_i^j}{2}\right)+(Gc)\left(\frac{\phi_i^{j+1}-\phi_i^j}{2}\right)\\&-\left(M+\frac{1}{K}\right)\left(\frac{u_i^{j+1}-u_i^j}{2}\right)\end{aligned} \tag{21}$$

$$(Pr)\left(\frac{\theta_i^{j+1}-\theta_i^j}{\Delta t}\right)-(Pr)B\left(\frac{\theta_i^{j+1}-\theta_i^j}{\Delta y}\right)=\left(\frac{\theta_{i-1}^{j+1}-2\theta_i^{j+1}+\theta_{i+1}^{j+1}}{2(\Delta y)^2}+\frac{\theta_{i-1}^{j}-2\theta_i^{j}+\theta_{i+1}^{j}}{2(\Delta y)^2}\right)+(Pr)(R+S)\left(\frac{\theta_i^{j+1}-\theta_i^j}{2}\right)+(Pr)(Du)\left(\frac{\phi_{i-1}^{j+1}-2\phi_i^{j+1}+\phi_{i+1}^{j+1}}{2(\Delta y)^2}+\frac{\phi_{i-1}^{j}-2\phi_i^{j}+\phi_{i+1}^{j}}{2(\Delta y)^2}\right) \quad (22)$$

$$(Sc)\left(\frac{\phi_i^{j+1}-\phi_i^j}{\Delta t}\right)-(Pr)B\left(\frac{\phi_i^{j+1}-\phi_i^j}{\Delta y}\right)=\left(\frac{\phi_{i-1}^{j+1}-2\phi_i^{j+1}+\phi_{i+1}^{j+1}}{2(\Delta y)^2}+\frac{\phi_{i-1}^{j}-2\phi_i^{j}+\phi_{i+1}^{j}}{2(\Delta y)^2}\right)+(Sc)(k_r)\left(\frac{\phi_i^{j+1}-\phi_i^j}{2}\right)+(Sc)(Sr)\left(\frac{\theta_{i-1}^{j+1}-2\theta_i^{j+1}+\theta_{i+1}^{j+1}}{2(\Delta y)^2}+\frac{\theta_{i-1}^{j}-2\theta_i^{j}+\theta_{i+1}^{j}}{2(\Delta y)^2}\right) \quad (23)$$

Where $B=1+\epsilon Ae^{nt}$. And the corresponding boundary and initial conditions are:

$$u_{i,0}=0,\ \theta_{i,0}=0,\ \phi_{i,0}=0 \text{ for all } i \quad (24)$$

$$u_{0,j}=U_p,\ \theta_{0,j+1}=\Delta y+\theta_{0,j},\ \phi_{0,j+1}=\Delta y+\phi_{0,j} \quad (25)$$

$$u_{\infty,j}\to U_\infty=1+\epsilon e^{nj\Delta t},\ \theta_{\infty,j}\to 0,\ \phi_{\infty,j}\to 0 \quad (26)$$

Here index i refers to y and j refers to time$t$, $\Delta t=t_{j+1}-t_j$, and $\Delta y=y_{j+1}-y_j$. Knowing the values of u, θ and ϕ at time t, we can compute the values at time $t+\Delta t$ as follows: we substitute$i=1,2,3......,N-1$ where correspond to ∞ , in Eqs. (21)-(23) which make up tridiagonal system of equations, can be figured out by Thomas algorithm as discussed in Carnahan et al.[23]. Subsequently θ and ϕ are known for all values of y at time $t+\Delta t$. Replace these values of and in Eq. (21) and solved by same procedure with initial and boundary condition, we obtain solution for u till desired time t . The implicit Crank-Nicolson ïňĄnite difference method is a second order method $\left(O\left(\Delta t^2\right)\right)$ in time and has no restriction on space and time steps, that is, the method is unconditionally stable. The computation is executed for $\Delta y=0.001$,$\Delta t=0.001$ procedure is repeated till $y=1$.

5. Results and discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The system of Eqs. (12) to (14) with the boundary conditions (15)-(17) was solved numerically by the finite difference method. Graphical representations of the numerical results are illustrated in Fig. 2 through Fig. 20 to show the influences of different numbers on the boundary layer flow. In this study, we investigate the influence of the effects of material parameters such as thermal Grashof number (Gr), solutal Grashof number (Gc), Prandtl number (Pr), Schmidt number (Sc), Hartmann number (M), Permeability parameter (K), Soret number (Sr), Dufour number (Du), Heat absorption parameter (S), Thermal Radiation absorption parameter (R) and Chemical reaction parameter (k_r) separately in order to clearly observe their respective effects on the velocity, temperature and concentration profiles of the flow. Also Skin-friction coefficient , Rate of heat and mass transfer coefficients in terms of Nusselt number (Nu) and Sherwood number (Sh) respectively have been observed through tabular forms. For the physical significance, the numerical discussions in the problem and at $t=1.0$, stable values for velocity, temperature and concentration fields are obtained. To examine the effect of parameters related to the problem on the velocity field and Skin-friction numerical computations are carried out at $Pr=0.71$ and $Sc=0.22$. To find solution of this problem, we have placed an infinite vertical plate in a finite length in the flow. Hence, we solve the entire problem in a finite boundary. However, in the graphs, the y values vary from 0 to 1, and the velocity, temperature, and concentration tend to zero as y tend to 1. This is true for any value of y . Thus, we have considered finite length. Throughout the computations we employ $Gr=4.0$,$Gc=2.0$, $Pr=0.71$, $R=1.0$, $M=1.0$,$t=1.0$,$Sc=0.22$, $a=0.5$, $X=1.0$, $n=0.2$, $U_p=0.5$, $S=1.0$, $k_r=1.0$, $\varepsilon=0.02$, $Du=1.0$ and $Sr=1.0$.

The influence of thermal Grashof number Gr on the velocity is shown in Fig. 2. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated

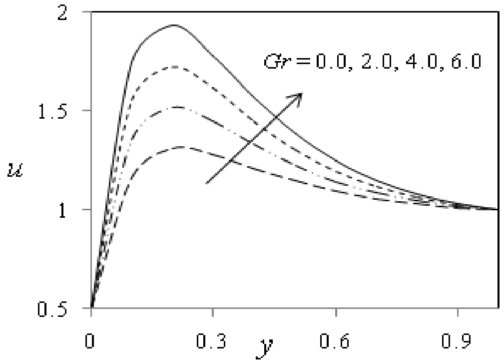

Fig. 2. Effect of *Gr* on Velocity profiles

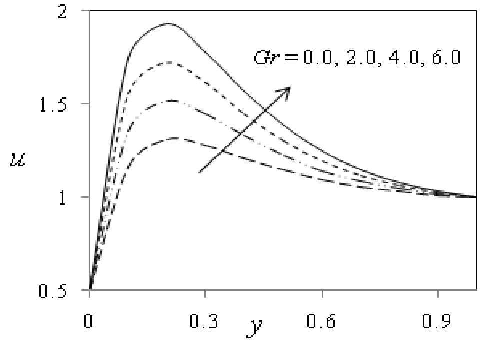

Fig. 3. Effect of *Gc* on Velocity profiles.

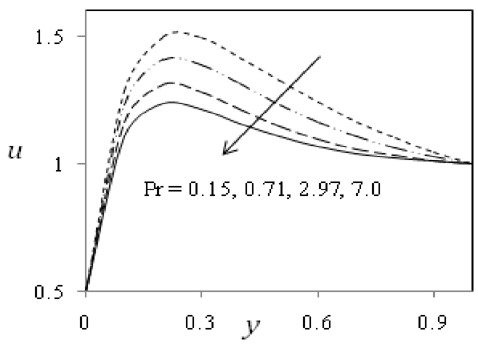

Fig. 4. Effect of *Pr* on Velocity profiles .

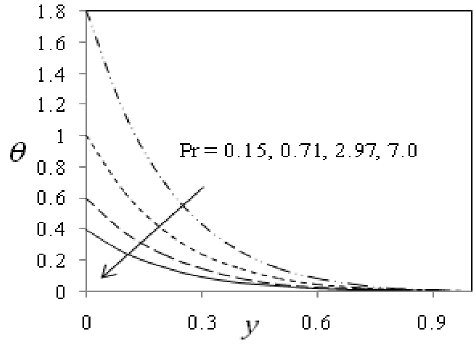

Fig. 5. Effect of *Pr* on Temperature profiles.

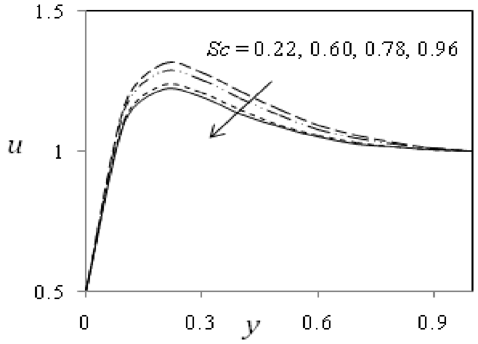

Fig. 6. Effect of *Sc* on Velocity profiles.

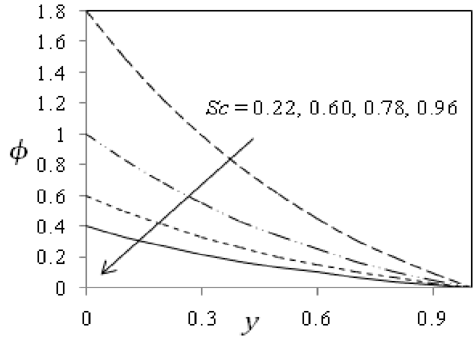

Fig. 7. Effect of *Sc* on Concentration profiles.

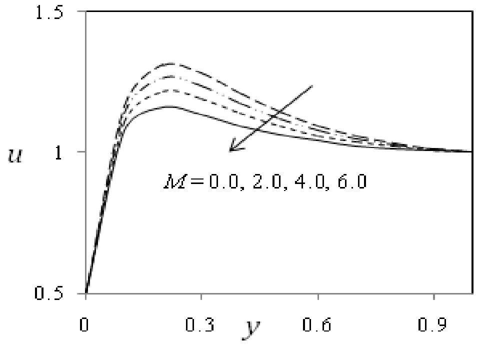

Fig. 8. Effect of *M* on Velocity profiles.

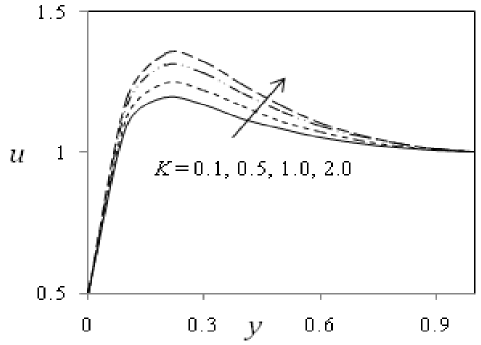

Fig. 9. Effect of *K* on Velocity profiles.

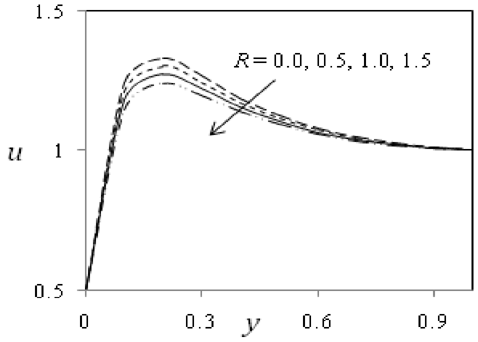

Fig. 10. Effect of R on Velocity profiles .

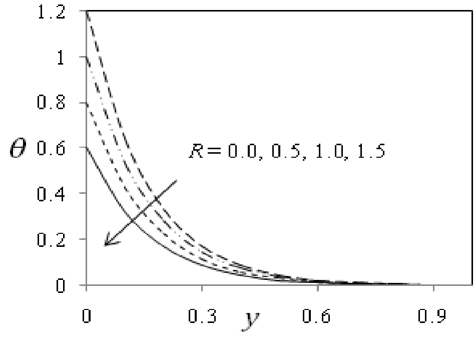

Fig. 11. Effect of R on Velocity profiles.

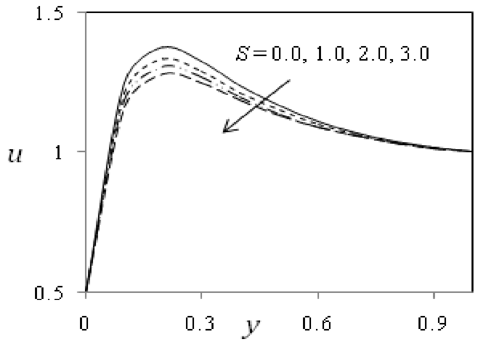

Fig. 12. Effect of S on Velocity profiles .

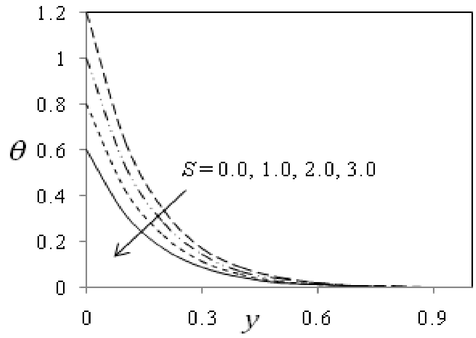

Fig. 13. Effect of S on Temperature profiles.

due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number i.e., free convection effects. The positive values of Gr correspond to cooling of the plate by natural convection. Heat is therefore conducted away from the vertical plate into the fluid which increases the temperature and thereby enhances the buoyancy force. In addition, it is seen that the peak values of the velocity increases rapidly near the plate as thermal Grashof number increases and then decays smoothly to the free stream velocity.

Fig. 3 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gc. The solutal Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the solutal Grashof number. Fig. 4 and Fig. 5 illustrate the velocity and temperature profiles for different values of Prandtl number Pr. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig. 5, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

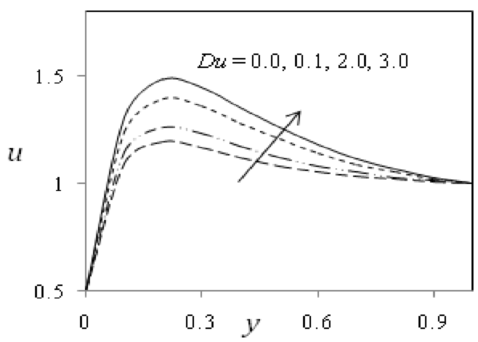

Fig. 14. Effect of Du on Velocity profiles.

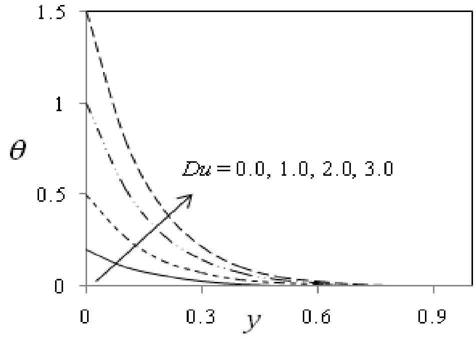

Fig. 15. Effect of Du on Temperature profiles.

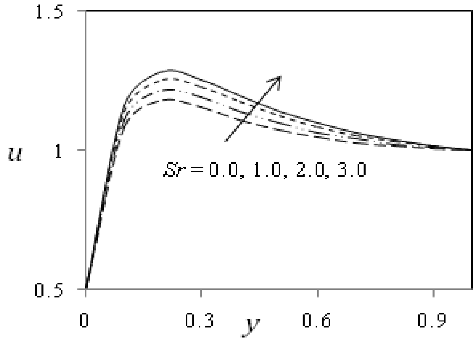

Fig. 16. Effect of Sr on Velocity profiles.

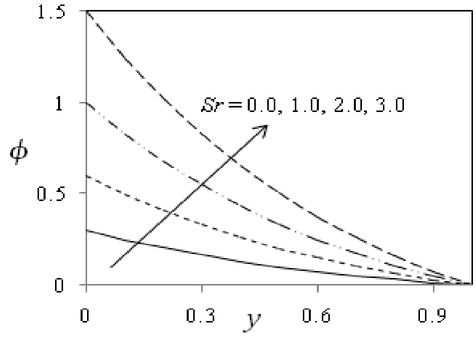

Fig. 17. Effect of Sr on Concentration profiles.

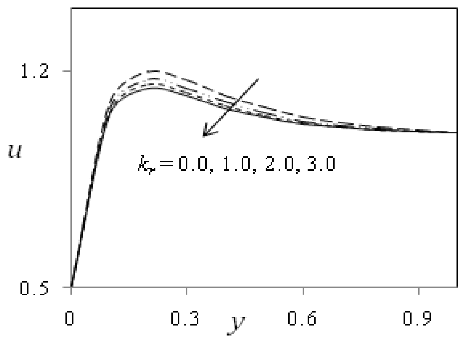

Fig. 18. Effect of k_r on Velocity profiles.

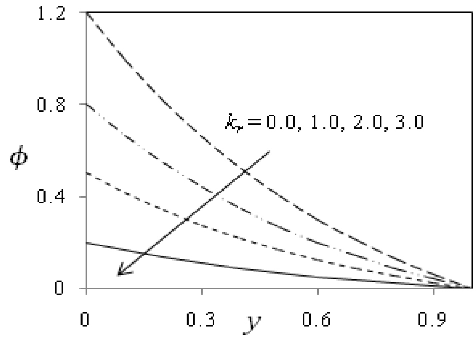

Fig. 19. Effect of k_r on Concentration profiles.

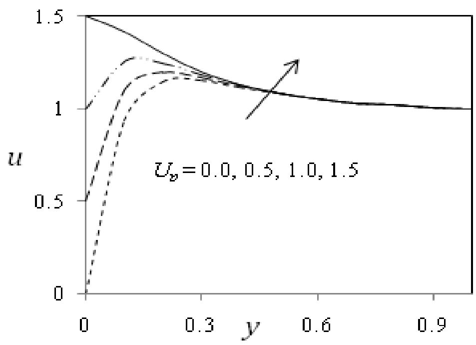

Fig. 20. Effect of U_p on Velocity profiles .

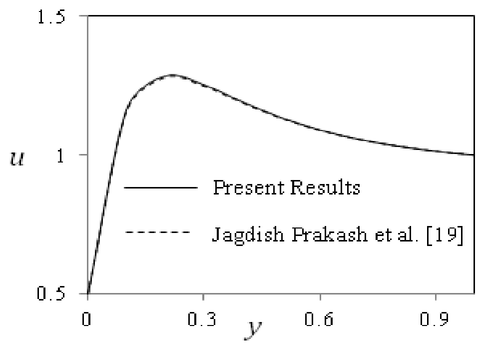

Fig. 21. Comparison of Numerical results (Present results) with Analytical results (Jagdish Prakash et al. [19]) in absence of Soret effect.

For different values of the Schmidt number Sc, the velocity and concentration profiles are plotted in Fig. 6 and Fig. 7 respectively. The Schmidt number Sc embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass - transfer (concentration) boundary layer. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers, which is evident from Fig. 6 and Fig. 7. The effect of magnetic field parameter M on the velocity is shown in Fig. 8. The velocity decreases with an increase in the magnetic parameter. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. The effect of the permeability parameter K on the velocity field is shown in Fig. 9. The parameter K as defined in Eq. (2) is inversely proportional to the actual permeability K of the porous medium. An increase in K will therefore increase the resistance of the porous medium (as the permeability physically becomes less with increasing K) which will tend to decelerate the flow and reduce the velocity. The influence of the thermal radiation parameter R on the velocity and

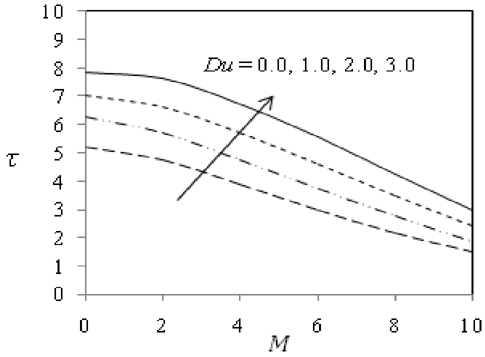

Fig. 22. Effect of Du on Skin-friction profiles .

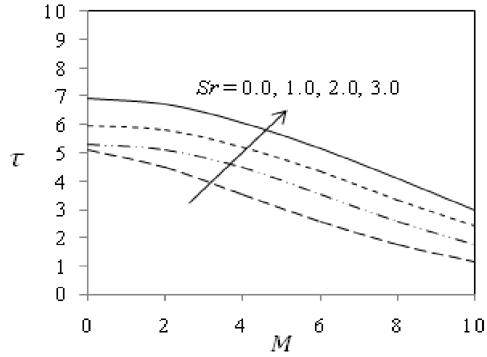

Fig. 23. Effect of Sr on Skin-friction profiles.

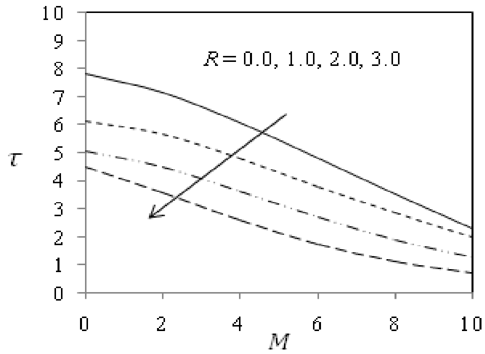

Fig. 24. Effect of R on Skin-friction profiles.

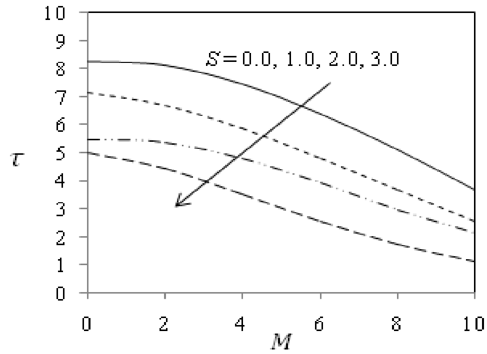

Fig. 25. Effect of S on Skin-friction profiles .

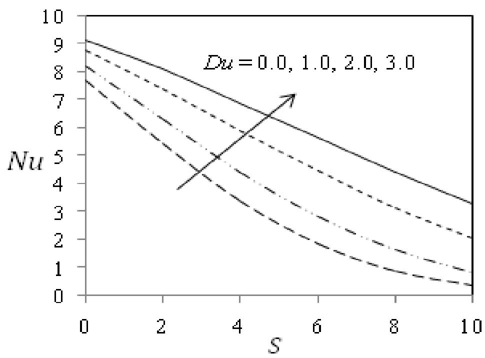

Fig. 26. Effect of R on Nusselt number Nu.

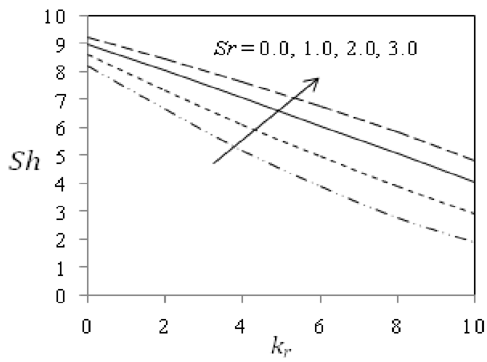

Fig. 27. Effect of Sr on Sherwood number Sh.

temperature are shown in Fig. 10 and Fig. 11 respectively. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in decreasing velocity and temperature within the boundary layer. The effect of increasing the value of the Heat absorption parameter S is to decrease the boundary layer as shown in Fig. 12, which is as expected due to the fact that when heat is absorbed the buoyancy force decreases which retards the flow rate and thereby giving rise to decrease in the velocity profiles.

Fig. 13 has been plotted to depict the variation of temperature profiles against for different values of Heat absorption parameter S by fixing other physical parameters. From this graph we observe that temperature decrease with increase in the Heat absorption parameter S because when heat is absorbed, the buoyancy force decreases the temperature profiles. For different values of the Dufour number Du, the velocity and temperature profiles are plotted in Fig. 14 and Fig. 15 respectively. The Dufour number Du signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in the Dufour number causes a rise in the velocity and temperature throughout the boundary layer. For $Du \leq 1$, the temperature profiles decay smoothly from the plate to the free stream value. However for $Du > 1$, a distinct velocity overshoot exists near the plate, and thereafter the profile falls to zero at the edge of the boundary layer. Fig. 16 and Fig. 17 depict the velocity and concentration profiles for different values of the Soret number (Sr). The Soret number Sr defines the effect of the temperature gradients inducing significant mass diffusion effects. It is noticed that an increase in the Soret number Sr results in an increase in the velocity and concentration within the boundary layer.

Fig. 18 and Fig. 19 display the effects of the chemical reaction parameter (k_r) on the velocity and concentration profiles, respectively. As expected, the presence of the chemical reaction significantly affects the concentration profiles as well as the velocity profiles. It should be mentioned that the studied case is for a destructive chemical reaction (k_r). In fact, as chemical reaction (k_r) increases, the considerable reduction in the velocity profiles is predicted, and the presence of the peak indicates that the maximum value of the velocity occurs in the body of the fluid close to the surface but not at the surface. Also, with an increase in the chemical reaction parameter, the concentration decreases. It is evident that the increase in the chemical reaction (k_r) significantly alters the concentration boundary layer thickness but does not alter the momentum boundary layers.

Fig. 20 illustrates the variation of velocity distribution across the boundary layer for several values of plate (U_p) moving velocity in the direction of the fluid flow. Although we have different initial plate moving velocities, the velocity decrease to the constant values for given material parameters. In order to ascertain the accuracy of the numerical results, the present study is compared with the previous study in absence of Soret number (Sr).

The velocity profiles in absence of Soret number for $Gr = 4.0, Gc = 2.0, Pr = 0.71, R = 1.0, M = 1.0, t = 1.0, Sc = 0.22, a = 0.5, X = 1.0, n = 0.2, U_p = 0.5, S = 1.0, k_r = 1.0, \varepsilon = 0.02, Du = 1.0$ are compared with the analytical solutions of Jagdish Prakash et al.[19] in Fig. 21. It is observed that the present results are in good agreement with that of Jagdish Prakash et al.[19]. Figs. 22-25 illustrate the effects of Du, Sr, R and S on skin-friction distribution (τ) against the Hartmann number (M). Fig. 22 and Fig. 23 show that increases in Dufour and Soret numbers increases the skin-friction. It is observed from Fig. 24 and Fig. 25 that the skin-friction diminishes when increasing the thermal radiation parameter and Heat absorption parameter. The significance of Dufour number in the Nusselt number distribution (Nu) with respect to the heat absorption parameter (S) is shown in Fig. 26. We observe that an increase in Dufour number gradually increases the magnitude of Nusselt number. Fig. 27 is plotted to display the effect of Soret number on Sherwood number (Sh) with respect to the chemical reaction parameter (k_r). The Sherwood number strictly increases for the higher values of Soret number.

6. Conclusions

We have analyzed the combined effects of Soret and Dufour on an unsteady mixed convective flow of a heat absorbing fluid in an accelerated vertical wavy plate, subject to varying temperature and thermal radiation. We noticed the following key observations from this study.

- Velocity decreases with an increase in Hartmann number, whereas it increases with an increase in Soret number, Dufour number, Porous permeability parameter, thermal Grashof number and solutal Grashof number.
- An increase in Prandtl number or heat absorption parameter or thermal radiation parameter decreases the heat transfer, but the Dufour number reverses the trend.
- The fluid concentration decreases for increasing Schmidt number and chemical reaction parameter, but the Soret number reverse trend.
- The effects of various significant physical parameters on skin-friction, Nusselt number and Sherwood number are illustrated graphically.
- Furthermore, it is clear from this study that the Soret and Dufour effects should be considered for fluids with light molecular weight.

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