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The reduced differential transform method for the Black-Scholes pricing model of European option valuation

Research Article

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Abstract: Finance is one of the fastest developing areas in the modern banking and corporate world. In this paper, we apply the Reduced Differential Transform Method (RDTM) for solving Black-Scholese equation for European option valuation. The same algorithm can be used for European put option. The results show that this method is very effective and simple.

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Keywords: Reduced differential transform method • Black-Scholes equation • European option pricing

1. Introduction

Financial securities have become essential tools for corporations and investors over the past few decades. Options can be used, for example, to hedge assets and portfolios in order to control the risk due to the movement in stock prices. In 1973, Fischer Black and Myron Scholes proposed a formula to find the price of financial options, which is called the Black-Scholes equation [2, 4] by Robert Merton. For their contributions, Scholes Merton received the Nobel prize of economy, unfortunately Fisher Black passed away and could not receive it [4]. This together with the sophistication of modern financial products provides a rapidly growing impetus for new mathematical models and modern mathematical methods.

In recent years, some works have been done in order to find the analytical/approximate solution of the Black-Scholes equation such as Adomian decomposition method (ADM), Modified ADM, Variational iteration method (VIM), modified VIM, Homtopy perturbation method (HPM), modified HPM and Homotopy Analysis Method [5–11].

In last three decades, new semiâĂŞanalytical method known as the ADM, HPM , VIM, differential transform method (DTM) has been presented for solving both ordinary differential equations and partial differential equations of various forms. Also it is shown that these methods to be effective, reliable and easier in application when compared to other analytical methods [1, 4, 6, 8–14].

In this paper, we will use the reduced differential transform method (RDTM) to study the Black-Sholes equations for a European option pricing problem.

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2. The Black -Scholes equation

The Black-Scholes model is used for European or American option. We denote by$V$ the values of an option; when the distinction is important we use $C(s,t)$ and $P(s,t)$ to denote a call and put options, respectively. This value is a function of the current value of the underlying asset, S and time t: $V = V(s,t)$. The value of the option also depends on the following parameters:

- δ , the volatility of the underlying asset,
- E, the exercise price,
- T, the expiry,
- r, the risk-free interest rate.

If the option is to buy the asset, it is called option C, if it is to sell asset, it is a put as option P.
The Black-Scholes equation and boundary conditions for a European call with values C(s, t) are, as described in.

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial C^2}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0 \tag{1}$$

with

$$C(0,t)=0, \qquad C(S,t)\sim S, \qquad asS\longrightarrow\infty \tag{2}$$

and

$$C(S,t)=Max(S-E,0) \tag{3}$$

The first thing to do is to get rid of the awkward S and S^2 terms multiplying $\dfrac{\partial C}{\partial S}$ and $\dfrac{\partial^2 C}{\partial S^2}$. At the same time we take the opportunity of making the equation dimensionless as defined in the technical point below, and we turn it in to a forward equation. We set:

$$S=Ee^x, \qquad t=T-\frac{\tau}{\frac{1}{2}C^2}, \qquad C=Ev(x,t) \tag{4}$$

This result in the equation

$$\frac{\partial v}{\partial t}=\frac{\partial v^2}{\partial x^2}+(k-1)\frac{\partial v}{\partial x}-kv \tag{5}$$

Where $k=\dfrac{2r}{\delta^2}$ the initial condition becomes

$$v(x,0)=max(e^x-1,0) \tag{6}$$

Notice that this system of equations contains just two dimensionless parameters $k=\dfrac{2r}{\delta^2}$ which k represents the balance between the rate of interest and the variability of stock returns and the dimensionless time to expiry $\dfrac{1}{2}\delta^2 t$, even though there are four dimensional parameters, E, T , δ^2 and r, in the original statement of the problem.

3. Analysis of the method

In this section we present a breif review of the reduced differential transform method.

Definition 3.1.
If the function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_m(x)=\frac{1}{m!}\left[\frac{\partial^m u(x,t)}{\partial t^m}\right] \tag{7}$$

where the t-dimensional spectrum function is the transformed function $U_m(x)$.

In this paper, the lowercase $u(x,t)$ represent the original function while the uppercase $U_m(x)$ stand for the transformed function.

Definition 3.2.
The differential inverse transform of $U_m(x)$ is defined as follows:

$$u(x,t)=\sum_{m=0}^{\infty}U_m(x)t^m \tag{8}$$

Then combining Eqs. (7) and (8) we write

$$u(x,t)=\sum_{m=0}^{\infty}\frac{1}{m!}\Big[\frac{\partial^m u(x,t)}{\partial t^m}\Big]t^m. \tag{9}$$

From the above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion. The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 1.

Table 1. Reduced differential transform

Functional Form	Transformed Form
$w(x,t)=u(x,t)\mp v(x,t)$	$W_m(x)=U_m(x)\mp V_m(x)$
$w(x,t)=\alpha u(x,t)$	$W(x)=\alpha U_m(x)$
$w(x)=\frac{\partial^r}{\partial t^r}u(x,t)$	$W_m(x)=\frac{(m+r)!}{m!}U_{m+r}(x), m\geq 0$
$w(x)=u(x,t)v(x,t)$	$W_m(x)=\sum_{r=0}^{m}V_(r)U(m-r)=\sum_{r=0}^{m}U_(r)V(m-r), m\geq 0$
$w(x,t)=x^s t^n$	$W_m(x)=x^s\delta(m-n)=\begin{cases}1 & m=n,\\ 0 & m\neq n.\end{cases}$
$w(x,t)=x^s t^n u(x,t)$	$W_m(x)=x^s U(m-n)$
$w(x,t)=\frac{\partial}{\partial x}u(x,t)$	$W_m(x)=\frac{\partial}{\partial x}U_m(x)$

4. Solving the Black-Scholes equation with RDTM

We considered the following Black-Scholes equation [4]

$$\frac{\partial v}{\partial t}=\frac{\partial v^2}{\partial x^2}+(k-1)\frac{\partial v}{\partial x}-kv, \tag{10}$$

with initial condition

$$V(x,0)=max(e^x-1,0) \tag{11}$$

To solve Eq. (10) by RDTM, in view of Table 1, we obtain the following recurrence formula

$$(m+1)V_{m+1}(x)=\frac{\partial^2}{\partial x^2}V_m(x)+(k-1)\frac{\partial}{\partial x}V_m(x)-kV_m(x), \qquad m=0,1,2,\cdots,$$

also from (11) we have

$$V_0=max(e^x-1,0). \tag{12}$$

Now for $m=0,1,2,\cdots$ we calculate V_m :

$$m=0, \qquad V_1=\frac{\partial^2}{x^2}V_0(x)+(k-1)\frac{\partial}{\partial x}V_0(x)-kV_0(x) \tag{13}$$

$$=\frac{\partial^2}{\partial x^2}\max(e^x-1,0)+(k-1)\frac{\partial}{\partial x}\max(e^x-1,0)-k\max(e^x-1,0)=k\begin{cases}0, & x<1,\\ 1, & \text{otherwise.}\end{cases} \tag{14}$$

$$m=1, \qquad 2V_2=\frac{\partial^2}{x^2}V_1(x)+(k-1)\frac{\partial}{\partial x}V_1(x)-kV_n(x)=-\frac{k^2}{2!}\begin{cases}0, & x<1,\\ 1, & \text{otherwise.}\end{cases} \tag{15}$$

$$m = 2, \qquad 3V_3 = \frac{\partial^2}{x^2} V_2(x) + (k-1)\frac{\partial}{\partial x} V_2(x) - kV_n(x) = \frac{k^3}{3!} \begin{cases} 0, & x < 1, \\ 1, & \text{otherwise.} \end{cases} \tag{16}$$

$$m = 3, \qquad 4V_4 = \frac{\partial^2}{x^4} V_n(x) + (k-1)\frac{\partial}{\partial x} V_4(x) - kV_4(x) \qquad = -\frac{k^4}{4!} \begin{cases} 0, & x < 1, \\ 1, & \text{otherwise.} \end{cases} \tag{17}$$

and so on.

Hence we have

$$v(x,t) = max(e^x - 1, 0) + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(tk)^n}{n!} \begin{cases} 0, & x < 1, \\ 1, & \text{otherwise.} \end{cases} \tag{18}$$

Eq. (18) is the exact solution of (10)

5. Conclusion

In this paper, we use the RDTM to solve the Black-Scholes equation for a simple European Option. Using this method, we obtained a new efficient recurrent relation to solve Black-Scholes equation. The RDTM is more efficient and better than DTM since less computational is involved. This method could also be useful for solving other Partial differential equations of financial mathematics.

References

[1] B. A.Taha, The Use of Reduced Differential Transform Method for Solving Partial Differential Equations with Variable Coefficients, Journal of Basrah Researches (Sciences) Volume 37. Number 4. C (2011) 226-233.

[2] F. Black, M. Scholes, The pricing of option and corporate liabilities, The Journal of Political Economy, 81, (1973), 637-654. 637-654.

[3] J.F. Price, Optional mathematics is not optional, Notices of the American Mathematical Society, 43, (1996), 964–971.

[4] T. Allahviranloo , Sh. S. Behzadi, The use of iterative methods for solving Black-Scholes equation, Int. J. Industrial Mathematics, Vol. 5, No. 1, (2013) 1–11.

[5] L. Blanco-Cocom, A. G. Estrella, E. Avila-Vales, Solution of the Black-Scholes equation via the Adomian decomposition method, International Journal of Applied Mathematical Research, 2 (4) (2013) 486–494.

[6] V. Gulkac, The homotopy perturbation method for the Black-Scholes equation, Journal of Statistical Computation and Simulation, 80(12), (2010) 1349-1354.

[7] Y. Qiu and J. Lorenz, A nonlinear Black-Scholes Equation, Int. J. Business Performance and Supply Chain Modelling,1(1), (2009) 33-40.

[8] A.A Elbeleze, A. Kilicman and B.M. Taib, Homotopy Perturbation Method for Fractional Black- Scholes European Option Pricing Equations Using Sumudu Transform, Mathematical Problems in Engineering,(2013),1-7.

[9] S. Kumar, D. Kumar and J. Singh, Numerical Computation of Fractional Black-Scholes Equation arising in Financial Market, Egyptian Journal of Basic and Applied Sciences, 1(2014) 177-183.

[10] J. Ahmad, M. Shakeel, Q.M. UI Hassan, S.T. Mohyud-Din, Analytical Solution of Black-Scholes Model Using Fractional Variational Iteration Method, International Journal of Modern Mathematical Sciences, 5(3),(2013) 133-142.

[11] S. Kumar, A. Yildirin, Y. Khan, H. Jafari, K. Sayevand, L. Wei, Analytical Solution of Fractional Black-Scholes European Option Pricing Equations Using Laplace Transform, Journal of Fractional Calculus and Applications, 2(8), (2012) 1-9.

[12] A. Taghavi, A. Babaei, A. Mohammadpour, Analytical approximations of the porous medium equations by reduced differential transform method, Int. J. Adv. Appl. Math. and Mech. 2(3) (2015) 95 - 100

[13] H.K. Jassim, Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation, Int. J. Adv. Appl. Math. and Mech. 2(4) (2015) 8 - 12 .

[14] H. Jafari, E. Tayyebi, S. Sadeghi, C.M. Khalique, A new modification of the Adomian decomposition method for nonlinear integral equations, Int. J. Adv. Appl. Math. and Mech. 1(4) (2014) 33 - 39.