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Application of scaling group transformation for MHD boundary layer flow and heat transfer of nanofluids over moving surface subject to suction/injection in the presence of thermal radiation with chemical reaction

Research Article

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Abstract: In this paper, an investigation is made to study the boundary layer flow of an electrically conducting, viscous incompressible nanofluid over moving surface in the presence of uniform magnetic field with thermal radiation and Chemical reaction. The governing model of partial differential equations for momentum, temperature and concentration are transformed into non linear ordinary coupled differential equations by using a scaling group of transformation.

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1. Introduction

The fluid flow over a moving surface has potential applications and importance in several technical and industrial processes such as in the extrusion of polymer sheet from a die, the lamination and melt-spinning process in the extrusion of polymers or the cooling of a large metallic plate in a bath, glass blowing continuous casting and spinning of fibers. Also magneto hydrodynamic (MHD)has a great importance in many areas of science, engineering, petroleum industries and agriculture field. Several authors [1–4] have investigated the problem under different flow geometry.

The term "nanofluid" represents the fluid in which nano-scale particles are suspended in the base fluid with low thermal conductivity such as water, ethylene glycol mixture and oils. Nanofluids have the potential to reduce thermal resistances,and industrial groups such as electronics, medical, food and manufacturing which gives benefit from such improved heat transfer. Also nanofluids can flow smoothly through micro channels without clogging them as they are very small. Due to this fact many researchers have attracted to investigate the problem of heat transfer characteristics in nanofluids and they found that in the presence of the nano particles in the fluids, the effective thermal conductivity of the fluid and heat transfer characteristics increases appreciably. Firstly the concept of "nanofluid" was introduced by the Choi [5]. The addition of a very small amount of nano particles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to twice was shown by Choi et al. [6]. The characteristic feature of nanofluids is a thermal conductivity enhancement which is suggested by Masuda et al. [7]. This phenomenon suggests the possibility nanofluids in advanced nuclear systems Buongiorno and Hu [8]. A detailed survey of convective transport

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is given by Buongiorno [9]. In recent years, most of the authors [10–14] have contributed to study convective flows of nanofluids which are given in the references.

Radiation effects are very important especially at high operating temperatures in the field like space technology. Hence the study of heat transfer with radiation has becomes very important, particularly in designing pertinent equipment for use in engineering processes with high temperatures. In recent years, most of the authors have studied effects of thermal radiation. Jagdish Prasad and Shateyi [15] investigated a numerical approach for MHD laminar boundary layer flow and heat transfer of nanofluids in the presence of thermal radiation. MHD boundary layer flow of a nanofluid in the presence of thermal stratification due to solar radiation taking into consideration the effects of Brownian motion and thermophoresisis given by Kandasamy et al. [16]. The flow and heat transfer characteristics of a viscous nanofluid over a non linearly stretching sheet in the presence of the thermal radiation investigated by Hadey et al. [17]. Also [18, 19] investigated the effects of thermal radiation and magnetic field on the boundary layer flow of a nanofluid past a stretching sheet.

In the present study, we have considered the problem of boundary layer flow of an electrically conducting, viscous incompressible nanofluid over a stretching surface in the presence of uniform magnetic field with thermal radiation and Chemical reaction. A similarity solution is obtained.

2. Problem formulation

We consider the steady MHD boundary layer flow of a nanofluid over stretching surface in uniform free stream velocity U in the presence of thermal radiation. The flow is takes place at $y \geq 0$ where y is the coordinate measured normal to the moving surface. A uniform transverse magnetic field is applied at y-axis is the coordinate measured normal to the moving surface. Also assumed that, at the surface, temperature T and the nanoparticles fraction C take constant values T_w and C_w respectively where as the values of T and C when y tends to infinity are denoted by T_∞ and C_∞ respectively. Consider the following model of conservation of mass, momentum, thermal energy and nanoparticle respectively.

$$\nabla .V = 0 \tag{1}$$

$$\rho_f(\frac{\partial v}{\partial t} + v.\nabla v) = -\nabla p + \mu\nabla^2 v - \sigma B_0^2 u + [C\rho_p + (1-C)\{\rho_f[1-\beta(T-T_\infty)]\}]g \tag{2}$$

$$(\rho c)_f(\frac{\partial T}{\partial t} + v.\nabla T) = k\nabla^2 T - \frac{\alpha}{k}\nabla q_r + (\rho c)_p[D_B \nabla C.\nabla T + (\frac{D_T}{T_\infty})\Delta T.\Delta T] \tag{3}$$

$$\frac{\partial C}{\partial t} + v.\nabla C = D_B\nabla^2 C + (\frac{D_T}{T_\infty})\nabla^2 T + K_r(C - C_\infty) \tag{4}$$

here v=(u,v).
where ρ_f the density of base fluid, μ is the dynamic viscosity, σ is the electrical conductivity, ρ is the density, k is the thermal conductivity, c is volumetric volume expansion coefficient of the nanofluid, ρ_p is the density of the particle, $(\rho c)_f$ is the heat capacity of the fluid and $(\rho c)_p$ is the effective heat capacity of the nanoparticle material, $\alpha = \frac{k}{(\rho c)_f}$ is the thermal diffusivity of the fluid, q_r is the heat flux, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, B_0 is the uniform magnetic field of the base fluid, K_r is the rate of chemical reaction, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio of nanoparticles heat capacity and the base fluid heat capacity, g is the gravitational acceleration.
The boundary conditions are:

$$y = 0: u = U(x),\ v = V(x),\ T = T_w,\ C = C_w \qquad y \to \infty: u = 0,\ T = T_\infty,\ C = C_\infty \tag{5}$$

We now write the standard boundary layer approximation, based on a scale analysis and governing equations as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u - \rho_f(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}) + [(1-C_\infty)\rho_{f\infty}\beta g(T-T_\infty) - (\rho_p - \rho_{f\infty})g(C-C_\infty)] = 0 \tag{7}$$

$$\frac{\partial p}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} - \frac{\alpha}{k}\frac{\partial q_r}{\partial y} + \tau[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2] \tag{9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} + K_r(C - C_\infty) \tag{10}$$

Where u and v are the velocity components along x and y axes, also in keeping with the Oberbeck-Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, the momentum equation (2) can be linearized and written in the form of equation (7). One can eliminate p from equations (7) and (8) by cross differentiation. The stream wise velocity and the suction/injection velocity are taken as

$$U(x) = cx^m,\ V(x) = V_0 x^{\frac{m-1}{2}} \tag{11}$$

Here $c > 0$ is constant, T_w is wall temperature and m is constant. Here we consider c=1.
By using the Rosseland diffusion approximation the heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3K_s}\frac{\partial T^4}{\partial y} \tag{12}$$

where σ^* and K_s are the Stefan-Boltzman constant and the Rosseland mean absorption coefficient respectively. It is assume that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 in a Taylor series about and T_∞ and neglecting the higher order terms. Thus we get

$$T^4 \cong T_\infty^4 + 4(T - T_\infty)T_\infty^3 \qquad T^4 \cong 4T_\infty^4 T - 3T_\infty^3 \tag{13}$$

Using (12) and (13) in the following term of equation (9) we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3K_s}\frac{\partial^2 T}{\partial y^2} \tag{14}$$

Introducing the stream function

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

and the non-dimensional quantities

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}$$

so that equation(6) satisfied identically on the other hand we have the following equations.

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \upsilon\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho_f}\frac{\partial \psi}{\partial y} + (1-\phi_\infty)\rho_{f\infty}\beta g\theta\Delta\theta - (\rho_p - \rho_{f\infty})g\phi\Delta\phi \tag{15}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2} + \frac{\alpha}{k}\frac{16\sigma^* T_\infty^3}{3K_s}\frac{\partial^2 \theta}{\partial y^2} + \tau[D_B\frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial \theta}{\partial y})^2] \tag{16}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 \theta}{\partial y^2} + K_r\phi\Delta\phi \tag{17}$$

Along with the boundary conditions

$$\frac{\partial \psi}{\partial y} = x^m,\ \frac{\partial \psi}{\partial x} = -V_0 x^{\frac{m-1}{2}}, \theta = \phi = 1 \quad at \quad y = 0. \qquad \frac{\partial \psi}{\partial y} \to 0, \theta \to 0, \phi \to 0 \quad as \quad y \to 0 \tag{18}$$

3. Group theoretic treatment

Similarity analysis by the group-theoretic method is based on concepts derived from the theory of continuous group transformation group. This method was first introduced by Birkhoff [20] and Morgan [21] and later on a number of authors like Na and Hansen [22], Timol and Kalthia [23], Pakdemirli [24] and most recently Patil et al. [25] have contributed much to the development of the theory.Recently, this theory is found to give more adequate treatment of boundary layer equations (Refer Seshadri and Na [26]). We now introduce the simplified form of Lie-group transformation, namely scaling group of transformation ((Mukhopadhyay et al. [27]),

$$\Gamma: x^* = xe^{\epsilon\alpha_1}, y^* = ye^{\epsilon\alpha_2}, \psi^* = \psi e^{\epsilon\alpha_3} \qquad u^* = ue^{\epsilon\alpha_4}, v^* = ve^{\epsilon\alpha_5}, \theta^* = \theta e^{\epsilon\alpha_6}, \phi^* = \phi e^{\epsilon\alpha_7} \tag{19}$$

where

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7$

are transformation parameters and ϵ is a small parameter. The transformation (19) may be considered as a point transformation, which transformed the coordinates (x,y,ψ,u,v,θ,ϕ)to coordinates ($x^*, y^*, \psi^*, u^*, v^*, \theta^*, \phi^*$). Using relation (19) into equations (15-17)we get

$$e^{\epsilon\alpha_1+2\epsilon\alpha_2-2\epsilon\alpha_3}(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial y^*\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}) = \nu e^{3\epsilon\alpha_2-\epsilon\alpha_3}\frac{\partial^3\psi^*}{\partial y^{*3}} - e^{\epsilon\alpha_2-\epsilon\alpha_3}\frac{\sigma B_0^2}{\rho_f}\frac{\partial\psi^*}{\partial y^*} + e^{-\epsilon\alpha_6}(1-\phi_\infty)\rho_{f\infty}\beta g\theta\Delta\theta - e^{-\epsilon\alpha_7}(\rho_p-\rho_{f\infty})g\phi\Delta\phi \quad (20)$$

$$e^{\epsilon\alpha_1+\epsilon\alpha_2-\epsilon\alpha_3-\epsilon\alpha_6}(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}) = \alpha e^{2\epsilon\alpha_2-\epsilon\alpha_6}\frac{\partial^2\theta^*}{\partial y^{*2}} + e^{2\epsilon\alpha_2-\epsilon\alpha_6}\frac{\alpha}{k}\frac{16\sigma^* T_\infty^3}{3K_s}\frac{\partial^2\theta^*}{\partial y^{*2}} + \tau[D_B e^{2\epsilon\alpha_2-\epsilon\alpha_6-\epsilon\alpha_7}\frac{\partial\phi^*}{\partial y^*}\frac{\partial\theta^*}{\partial y^*} + e^{2\epsilon\alpha_2-\epsilon\alpha_6}\frac{D_T}{T_\infty}(\frac{\partial\theta^*}{\partial y^*})^2] \quad (21)$$

$$e^{\epsilon\alpha_1+\epsilon\alpha_2-\epsilon\alpha_3-\epsilon\alpha_7}(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\phi^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\phi^*}{\partial y^*}) = D_B e^{2\epsilon\alpha_2-\epsilon\alpha_7}\frac{\partial^2\phi^*}{\partial y^{*2}} + e^{2\epsilon\alpha_2-\epsilon\alpha_6}\frac{D_T}{T_\infty}\frac{\partial^2\theta^*}{\partial y^{*2}} + e^{-\epsilon\alpha_7}K_r\phi^*\Delta\phi \quad (22)$$

The above system will remains invariant under the group of transformation Γ will have the following relations between the parameters

$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_6 = -\alpha_7$

$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_6 = 2\alpha_2 - \alpha_6 - \alpha_7 = 2\alpha_2 - 2\alpha_6$

$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_7 = 2\alpha_2 - \alpha_6 = -\alpha_7$

These relations gives

$\alpha_2 = \frac{1}{4}\alpha_1, \alpha_3 = \frac{3}{4}\alpha_1, \alpha_6 = \alpha_7 = 0$

The boundary equation yield

$\alpha_4 = m\alpha_1 = \frac{1}{2}\alpha_1, \alpha_5 = \frac{m-1}{2}\alpha_1 = \frac{-1}{4}\alpha_1 (as\, m = \frac{1}{2})$

In view of these, the boundary conditions are transformed to

$$\frac{\partial\psi^*}{\partial y^*} = x^{*\frac{1}{2}}, \frac{\partial\psi^*}{\partial x^*} = -V_0 x^{*\frac{-1}{4}}, \theta^* = \theta\, \phi^* = \phi \quad at \quad , y^* = 0 \quad \frac{\partial\psi^*}{\partial y^*} = 0, \theta^* \to 0, \phi^* \to 0 \quad , as \quad y^* \to \infty \quad (23)$$

Therefore we get

$x^* = xe^{\epsilon\alpha_1}, y^* = ye^{\epsilon\frac{\alpha_1}{4}}, \psi^* = \psi e^{\epsilon\frac{3\alpha_1}{4}} \quad u^* = ue^{\epsilon\frac{\alpha_1}{2}}, v^* = ve^{\epsilon\frac{-\alpha_1}{4}}\theta^* = \theta, \phi^* = \phi$

The "absolute invariants" under the above group of transformation are

$y^* x^{*\frac{-1}{4}} = \eta, \psi = x^{*\frac{3}{4}} f(\eta)\ \theta^* = \theta, \phi^* = \phi$

Substituting these invariants in equations (15-18), we get the

$$f''' + \frac{1}{P_r}[\frac{3}{4}ff'' - \frac{1}{2}f'^2] + Ra_x[\theta - N\phi] + Mf' = 0 \quad (24)$$

$$\frac{1}{P_r}(\frac{3+4R}{3})\theta'' + N_b\theta'\phi' + N_t\theta'^2 + \frac{3}{4}f\theta' = 0 \quad (25)$$

$$\phi'' + \frac{3}{4}\mathrm{Le\,f}\phi' + \frac{N_t}{N_b}\theta'' + \gamma\, Le\, Re_x\,\phi = 0 \quad (26)$$

With the transformed boundary value conditions,

$$\left.\begin{array}{l} f'=1, \quad f=-\frac{4V_0}{3}, \quad \theta=\phi=1 \quad at \quad \eta=0 \\ f'\to 0, \quad \theta\to 0, \quad \phi\to 0 \quad as \quad \eta\to\infty \end{array}\right\} \tag{27}$$

where -$\frac{4V_0}{3}$ > 0 corresponds to suction and -$\frac{4V_0}{3}$ < 0 corresponds to injection whereas V_0 is the velocity of suction if and injection if where P_r=$\frac{\upsilon}{\alpha}$ is the Prandtl number,R=$\frac{4\sigma^* T_\infty^3}{K K_s}$ is the radiation parameter, Le=$\frac{\upsilon}{D_B}$ is the Lewis number,Ra_x=$\frac{(1-\phi_\infty)\beta g\Delta\theta}{\upsilon\alpha}$ is the local Rayleigh number, N=$\frac{(\rho_p-\rho_{f\infty})\Delta\phi}{(1-\phi_\infty)\beta\Delta\theta\rho_{f\infty}}$ is the Buoyancy ratio, N_b=$\frac{(\rho c)_p\Delta\phi D_B}{\alpha(\rho c)_f}$ is the Brownian motion parameter, N_t=$\frac{(\rho c)_p\Delta\theta D_T}{\alpha(\rho c)_f T_\infty}$ s the thermophoresis parameter,M=$\frac{\sigma B_0^2 U}{\rho_f}$ is the magnetic parameter,γ=$\frac{\upsilon K_r}{U^2}$ is the chemical reaction parameter,Re_x=$\frac{U^2}{\upsilon}$ is the local Reynolds number.

4. Conclusion

Similarity solution of MHD boundary layer flow of an electrically conducting, viscous incompressible nanofluid over a moving surface in the presence of uniform magnetic field with thermal radiation is studied. The governing partial differential equations are transformed into nonlinear ordinary coupled differential equations by using scaling group of transformation. This solution depends on the Lewis number Le, the Buoyancy ratio N, the Brownian motion parameter Nb and the thermophoresis parameter Nt. Present similarity equations are in well agreement to those available in literature.

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