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Mixed convection of Non-Newtonian fluid flow and heat transfer over a Non-linearly stretching surface

Research Article

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Abstract: An analysis has been carried out to discuss the non-linear MHD, steady, two dimensional laminar boundary layer flows with mixed convective heat transfer of an incompressible, quiescent viscous, electrically conducting non-newtonian power-law fluids past a non-linearly stretching surface in the presence of a transverse magnetic field is considered. The stretching velocity, the temperature and the transverse magnetic field are assumed to vary in a power-law with the distance from the origin. The flow is induced due to an infinite elastic sheet which is stretched in its own plane. The governing equations are transformed to non-linear ordinary differential equations by means of similarity transformation. These equations are then solved numerically by using Runge-Kutta shooting method along with matlab software technique. The results of this study concerned with the velocity and temperature profiles are displayed and discussed for different values of various parameters. The local skin-friction co-efficient and the local Nusselt number are also analyzed.

MSC: 80A20 • 76D05 • 76D10 • 65L06

Keywords: Mixed convection • MHD flow • Power-law fluid • Stretching sheet • Modified Prandtl number • Heat source/sink parameter

1. Introduction

Mixed (combined) convection flow is combination of forced and free convection flows arise in many transport processes both naturally and in engineering application. These flow patterns are discovered simultaneously by both an external forcing mechanism and internal volumetric forces. They play an important role, for example, in atmospheric boundary-layer flows, heat exchangers, solar collectors, nuclear reactors and in electronic equipment. Such processes occur when the effects of buoyancy force in forced convection or the effects of forced flow in free convection become significant. The interaction of forced and free convection is especially pronounced in situations where forced flow velocity is low and/or the temperature differences are large. This flow is also a relevant type of flow appearing in many industrial processes, such as manufacture and extraction of polymer and rubber sheets, paper production, wire drawing and glass fiber production, melt spinning, continuous casting, etc. This flow has also many industrial applications such as heat treatment of material travelling between a feed roll and wind-up roll or conveyer belts, extrusion of steel, cooling of a large metallic plate in a bath, liquid films in condensation process and in aerodynamics, etc.

It should be mentioned that this type of flow plays a great role in thermal manufacturing applications and is important in establishing the temperature distribution within buildings as well as heat losses or heat loads for heating, ventilating and air conditioning systems. The study of two-dimensional boundary layer flow and heat transfer over a non-linear stretching surface with variable viscosity is very important as its finds many practical applications in geophysics, particularly, geothermal energy extraction and underground storage systems. In the actual manufacturing process, the stretched surface speed and temperature play an important role in the cooling process. Furthermore

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during the manufacture of plastic and rubber sheets, it is often necessary to blow a gaseous medium through the not-yet solidified material. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Mention may be made of wire drawing, annealing, thinning of copper wires, crystal growing, spinning of filaments, continuous casting, glass fiber production and paper production. In all the cases the properties of the final product depend to a great extent on the rate of cooling and the processes of stretching as explained by M.V. Karwe and Y. Jaluria [1, 2]. K. Vajravelu and A. Hadjinicolaou [3] studied the heat transfer characteristic in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Liminar mixed convection boundary layers induced by a linearly stretching permeable surface was studied by Mohamed Ali and Fahd Al-Yousef [4] . MHD Heat transfer Mixed convection flow along a vertical stretching sheet in presence of magnetic field with heat generation was studied by M.Mohebujjaman, Tania S. Khaleque and M.A. Samad [5]. The steady mixed convection boundary layer flow on a vertical surface without the effect of viscous dissapation was studied by Dey and Nath [6]. Hieber [7], Schneider [8], Afzal and Hussain [9] , Ishak [10] and Ishak et al. [11] studied the mixed convection flow above a heated horizontal surface. Molla and Yao [12] have studied on mixed convection of non-newtonian fluids along a heated vertical flat plate. A thorough review on the flow and heat transfer over a permeable stretching surface can be found in Gupta and Gupta [13] . Ishak [14] studied Mixed convection boundary layer flow over permeable vertical surface with prescribed wall heat flux. Non-newtonian power-law fluid flow and heat transfer over a non-linearly stretching surface was studied by Kerehalli vinayakaprasadi et al. [15] . Non-uniform heat generation effect on heat transfer of a non- Newtonian power-law fluid over a non-linearly stretching sheet was studied by Mostafa A.A. Mahmoud [16]. Slip flow effects on unsteady hydromagnetic flow over a stretching surface with thermal radiation was studied by S. P. Anjali Devi et al. [17]. Non-newtonian fluid flow and heat transfer over a non-linearly stretching surface along with porous plate in porous medium was studied by S. Jothimani and T.Vidhya [18]. M. S. Alam and M. Nurul Huda [19] studied a new approach for local similarity solutions of an unsteady hydromagnetic free convective heat transfer flow along a permeable flat surface.

2. Formulation of the problem

Consider a steady two-dimensional boundary layer flow of an incompressible non-newtonian fluid obeying the power-law model in the presence of a transverse magnetic field B_0 . The flow is generated as a consequence of non-linearly stretching of the boundary sheet, caused by simultaneous application of two equal and opposite forces along x - axis, while keeping the origin fixed in the fluid of the ambient temperature T_∞ . The x - axis is measured along the direction of the motion, with the slot at the origin and y - axis is taken normal to it. The continuous stretching sheet is assumed to have power law velocity $U(x) = bx^m, b(>0)$ is a constant and temperature $T_w(x) = T_\infty + Ax^r, x$ is the distance from the slot; A is a constant whose value depends upon the properties of the fluid. Here, m and r are the velocity and temperature exponents, respectively. We neglect the induced magnetic field, which is small in comparison with the applied magnetic field. Further, the external electrical field is assumed to be zero and the electrical field due to polarization of charges is also negligible. Under these assumptions, the basic equations governing the flow and heat-transfer in usual notation are;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{-\partial u}{\partial y} = -\nu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma B_0^2}{\rho}u + g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_s}{\rho C_p}(T - T_\infty). \tag{3}$$

where u and v are the flow velocity components along the x and y -axes respectively, ν is the kinematic viscosity of the fluid, κ -consistency of the fluid, n is the power-law index, ρ is the fluid density , g is the acceleration due to gravity, σ is the electrical conductivity and c_p is the specific heat at constants pressure. If $n < 1$ the fluid is said to be pseudo-plastic (shear thinning fluids), if $n > 1$ it is called dilatant (shear thickening fluids) and when $n = 1$, it is the Newtonian fluid. T is the temperature of the fluid and α is the thermal diffusivity of the fluid. The last term containing Q_s in Eq. (3) represents the temperature dependent volumetric rate of heat source when $Q_s > 0$ and heat sink when $Q_s < 0$. Thus the appropriate boundary conditions are

$$u(x,0) = U(x), T(x,0) = T_w(x) \tag{4}$$

$$u(x, y) \longrightarrow 0, T(x, y) \longrightarrow T_\infty \;\; as \;\; y \longrightarrow \infty \tag{5}$$

where T_w is the temperature of the plate, T_∞ is the temperature of the fluid far away from the plate. In order to obtain the similarity solutions of Eqs. (1)-(5), we assume that the variable magnetic field $B_0(x)$ is of the form $B_0(x) = B_0 x^{\frac{(m-1)}{2}}$. The momentum and energy equations can be transformed to the corresponding ordinary differential equations by the following transformations [15]

$$\eta = \frac{y}{x}(Re_x)^{\frac{1}{n+1}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \psi(x, y) = Ux(Re_x)^{\frac{-1}{n+1}} f(\eta), \tag{6}$$

where η is the similarity variable, $\psi(x, y)$ is the stream function, f and θ are the dimensionless similarity function and temperature, respectively. The velocity components u and v given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

The local Reynolds number is defined by

$$Re_x = \frac{U^{2-n} x^n}{\nu} \tag{8}$$

The mass conservation Eq. (1) is automatically satisfied by Eq. (7). By assuming the similarity function $f(\eta)$ to depend on the similarity variable η, the momentum Eq. (2) and the heat Eq. (3) transform into the coupled non-linear ordinary differential equation of the form

$$n(-f'')^{n-1} f''' - m f'^2 + \left(\frac{2mn - m + 1}{n+1}\right) f f'' - M_n f' + \lambda\theta = 0 \tag{9}$$

$$\theta'' + Npr\left(\frac{2mn - m + 1}{n+1}\right) f\theta' + Npr(\beta\theta - r f'\theta) = 0 \tag{10}$$

The boundary conditions (4) and (5) now becomes

$$f(\eta) = 0, \;\; f'(\eta) = 1, \;\; \theta(\eta) = 1 \;\; at \;\; \eta = 0 \tag{11}$$

$$f'(\eta) \longrightarrow 0, \theta(\eta) \longrightarrow 0 \;\; as \;\; \eta \longrightarrow \infty \tag{12}$$

where $M_n = \sigma B_0^2 / \rho b$ is the magnetic parameter, $Gr = \dfrac{g\beta x(T_w - T_\infty)}{U^2}(say\lambda)$ is the local Grashof number, $Npr = Npe_x / (Re_x)^{\frac{2}{n+1}}$ is the modified Prandtl number for power-law fluids, $Npe_x = c_p \rho U x / \kappa$ is the convectional Peclet number ([15]), $\beta = Q_s / c_p \rho b$ is the heat source\sink parameter. Here primes denote the differentiation with respective to η. The physical quantities of interest are the skin-friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{2\tau_w}{\rho U^2}, \quad Nu_x = \frac{x q_w}{\kappa(T_w - T_\infty)} \tag{13}$$

respectively, where the wall shear stress τ_w and heat transfer from the sheet q_w are given by

$$\tau_w = \mu_0 \left(\frac{\partial u}{\partial y}\right)_{at y=0}, \quad q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{at\ y=0} \tag{14}$$

with μ_0 and κ being the dynamic viscosity and thermal conductivity, respectively. Using the non-dimensional variable (5), we obtain

$$C_f = \left(\frac{-2\tau_{xy}}{\rho(bx)^2}\right)_{y=0} = 2[-f''(0)]''[Re_x]^{\frac{-1}{n+1}} \tag{15}$$

$Nu_x = -[Re_x]^{\frac{1}{n+1}} \theta'(0)$,where τ_{xy} is the shear stress and Re_x is the local Reynolds number.

3. Results and discussion

The effect of mixed convection of variable thermal conductivity on the MHD boundary layer flow and heat transfer in an electrically conducting power-law fluid over a non-linearly stretching sheet in the presence of heat source/sink parameter is investigated numerically. Numerical computation of the problem is obtained by Runge-Kutta shooting method with help of matlab software. Numerical results are obtained to study the effect of the various values of the power-law index n, the magnetic parameter M_n , the velocity exponent parameter m and the temperature exponent parameter r , the mixed convection parameter λ , the modified Prandtl number β on the flow and heat transfer are shown graphically in the Figs. 1-9. The case $\lambda \gg 1$ corresponds to pure free convection, $\lambda = 1$ corresponds to mixed convection and $\lambda \ll 1$ corresponds to pure forced convection.

Fig. 1 depicts the effect of shear thinning ($n < 1$) and shear thickening ($n > 1$) fluids on horizontal velocity profiles f' with η, for different values of velocity exponent parameter m. Horizontal velocity profile $f'(\eta)$ decrease and disclosing the fact that the effect of stretching of the velocity exponent parameter m from negative values to positive values is to decelerate the velocity and hence reduce the momentum boundary layer thickness, which tends to zero as the space variable η increases from the boundary surface. Physically, $m < 0$ implies that the surface is decelerated from the slot, $m = 0$ implies the continuous movement of a flat surface, and $m > 0$ implies the surface is accelerated from the extruded slit.

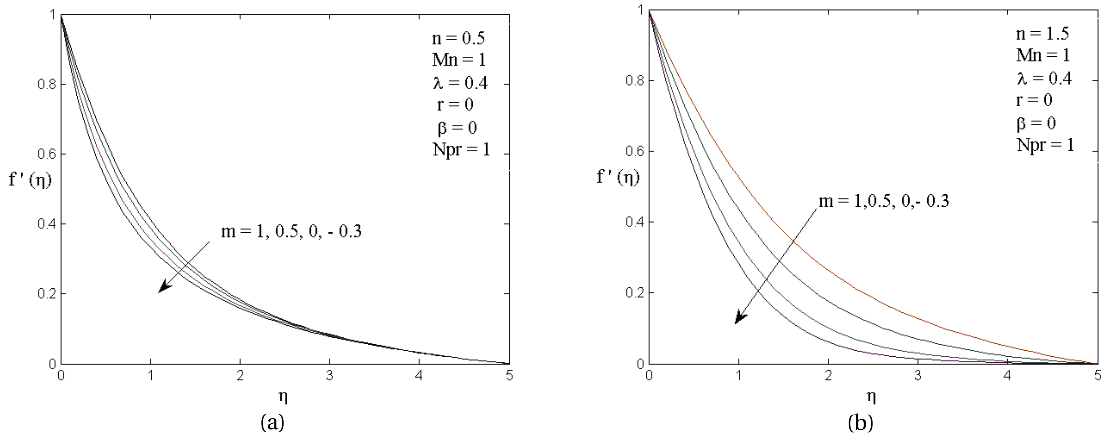

Fig. 1. (a) Velocity profile for various values of m (b) Velocity profile for various values of m

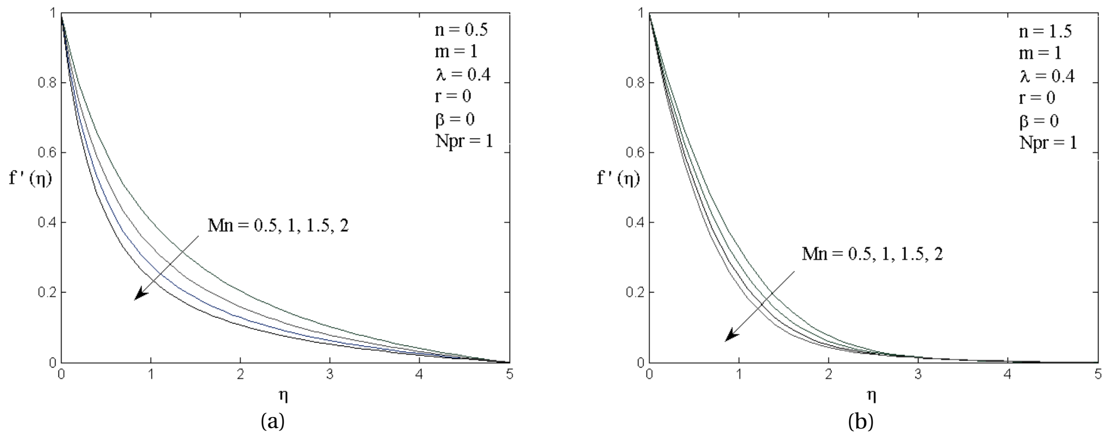

Fig. 2. (a) Velocity profile for various values of M_n (b) Velocity profile for various values of M_n

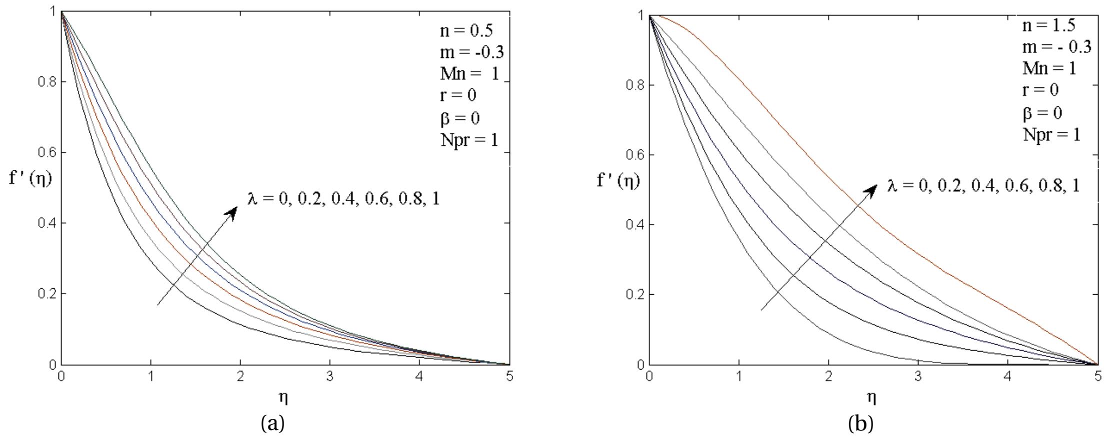

Fig. 3. (a) Velocity profile for various values of λ (b) Velocity profile for various values of λ

Fig. 2 shows the shear thinning ($n < 1$) and shear thickening ($n > 1$) fluids on the velocity profiles $f'(\eta)$ with η for different values of magnetic parameter M_n. It is observed that an increase in magnetic parameter M_n gives rise to decrease in the velocity. This is due to the fact that by the application of transverse magnetic field in an electrically conducting fluid produce to a resistive force known as Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer.

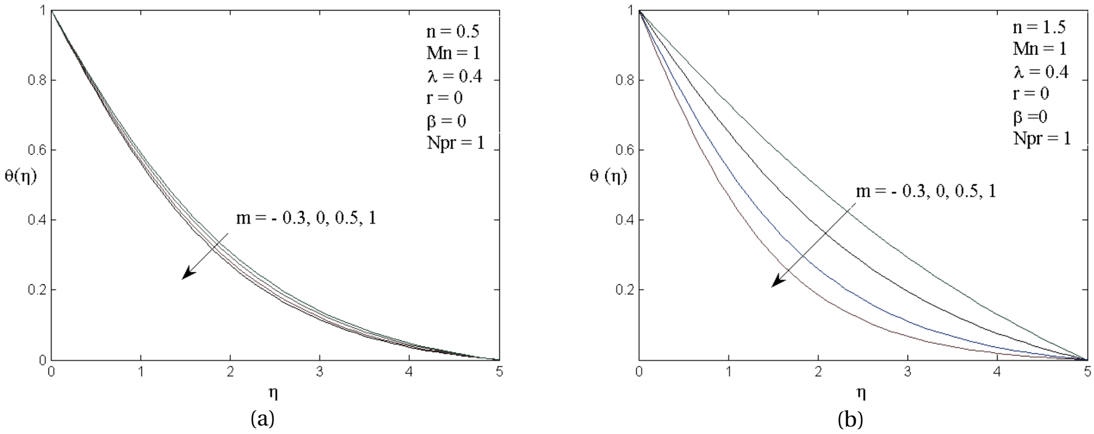

Fig. 4. (a) Temperature profile for various values m (b) Temperature profile for various values of m

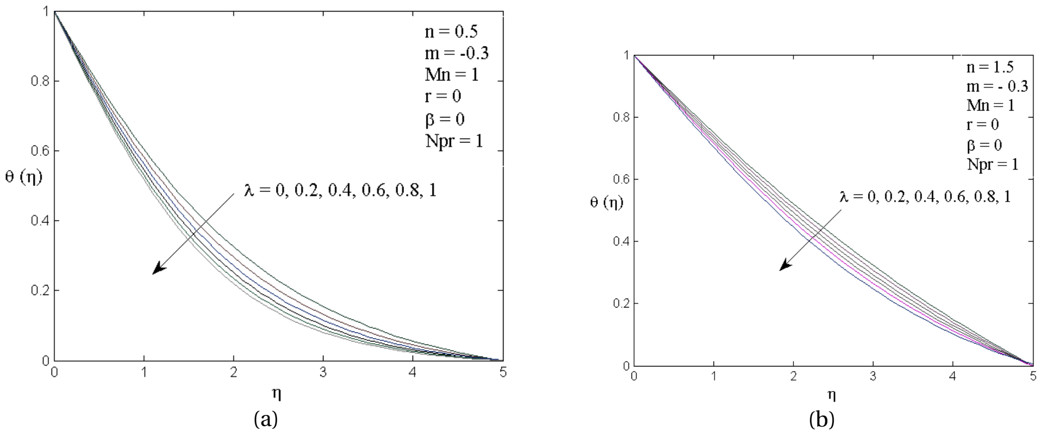

Fig. 5. (a) Temperature profile for various values of λ (b) Temperature profile for various values of λ

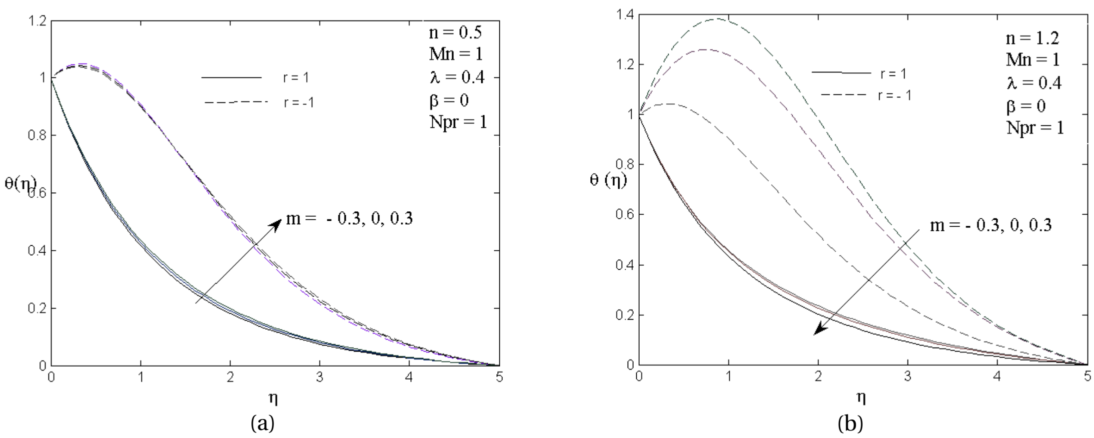

Fig. 6. (a) Temperature profile for various values of r (b) Temperature profile for various values of r

Fig. 3 represents the velocity profiles $f'(\eta)$ with η for several sets of values of the mixed convection parameter λ for the shear thinning ($n < 1$) and shear thickening ($n >$) fluids. The mixed convection parameter λ serves to sustain the strength of the fluid flow and ultimately the rate of heat transfer increases within the boundary layer.

Fig. 4 depicts temperature profiles for different values of velocity exponent parameter m. The effect of increasing values of velocity exponent parameter m is to increase the temperature profiles for shear thinning fluids($n < 1$) and

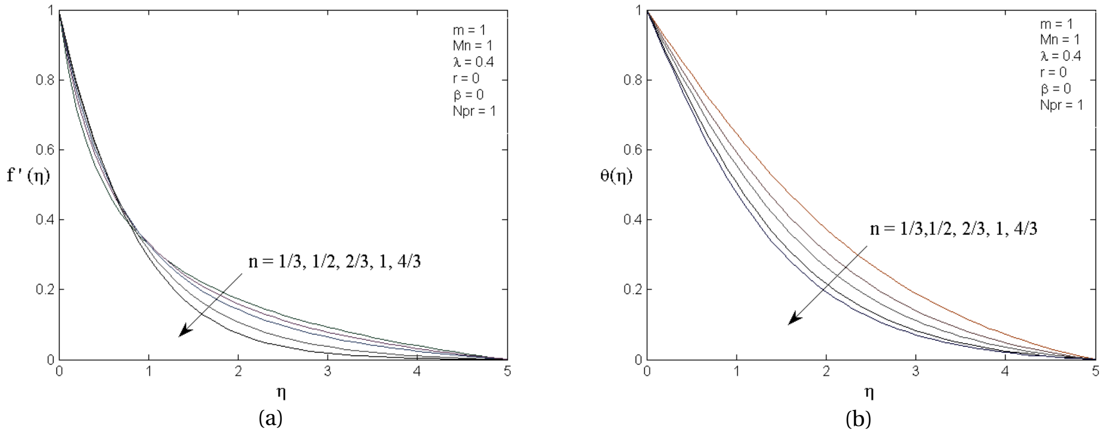

Fig. 7. (a) Velocity profile for various values of n (b) Temperature profile for various values of n

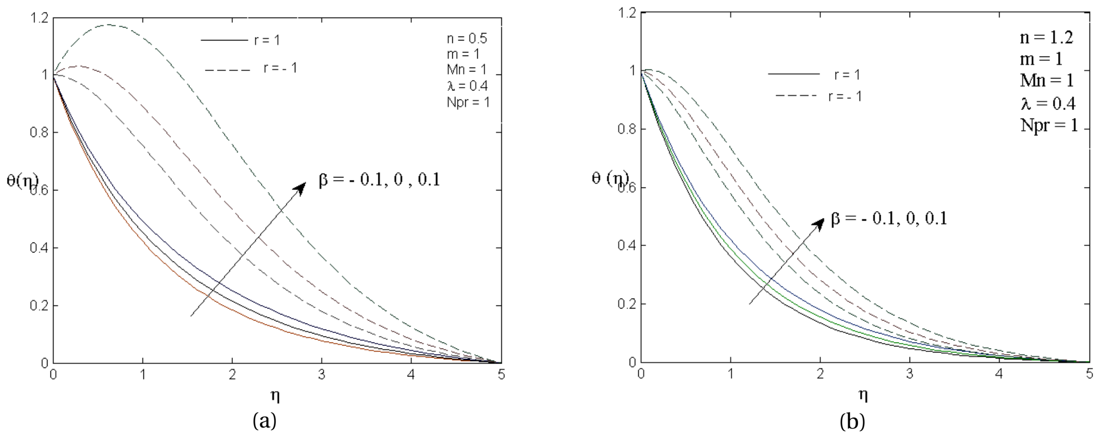

Fig. 8. (a) Temperature profile for various values of β (b) Temperature profile for various values of β

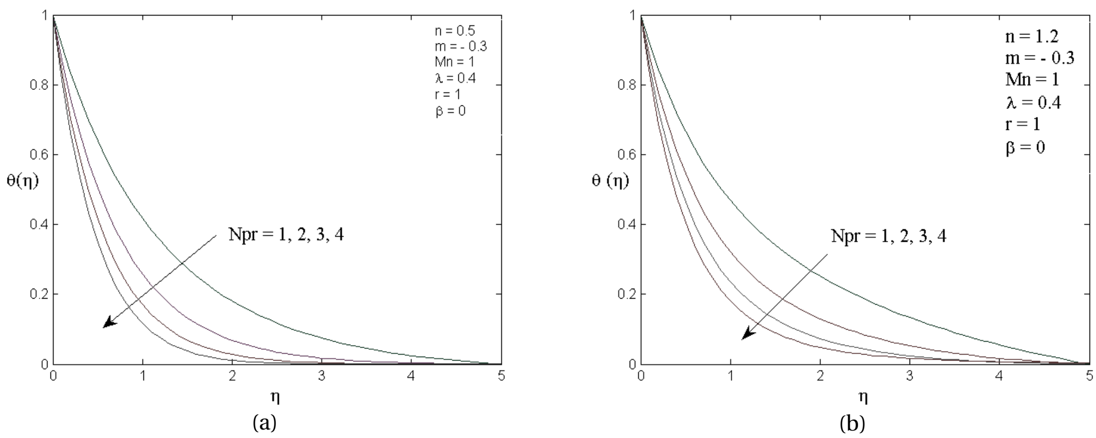

Fig. 9. (a) Temperature profile for various values of Npr (b) Temperature profile for various values of Npr

decrease the temperature profiles for shear thickening ($n > 1$) fluids. It elucidates that the thermal boundary layer thickness increases as m increases for shear thinning fluids whereas thermal boundary layer thickness decreases as m increases for shear thickening fluids. Also it is seen that boundary layer becomes closer to each other for shear thinning fluids ($n < 1$).

Fig. 5 demonstrate the temperature profiles for several sets of values of the mixed convection parameter λ for shear thinning and shear thickening fluids. It is clear that the temperature of the fluid decreases as the mixed convection parameter λ increases.

The graphs for the temperature profiles for different values of temperature exponent parameter r and velocity exponent parameter m are plotted in **??** for shear thinning and shear thickening fluids respectively. From these figures we examine that the increase in the temperature exponent parameter r leads to decrease the temperature profiles for shear thickening fluids. For shear thinning fluids temperature profiles increases and suddenly this trend is changes then temperature profiles decreases. Physically, when $r > 0$ temperature flows from the stretching sheet into the ambient medium, when $r < 0$ the wall temperature gradient positive and the temperature flows into the stretching sheet from the ambient medium and when $r = 0$ the thermal boundary conditions becomes isothermal.

Fig. 7 depict the effects of fluid power law index parameter n on the dimensionless velocity and temperature profiles. We examine that the velocity increases when an increase in n. It is found that a cross flow occurs at $\eta = 0.4(approx.)$. Also the temperature decreases when an increase in n.

Fig. 8 illustrate the effects of heat source/ sink parameter β on the profiles of temperature for shear thinning and shear thickening fluids, It is clear that the increase in the temperature exponent parameter r and heat source\sink parameter β leads to increases the temperature profiles.

Fig. 9 demonstrates the temperature profiles for several sets of values of the modified Prandtl number Npr for shear thinning and shear thickening fluids. It is observed that the temperature of the fluid decreases as Prandtl number increases.

4. Conclusions

- The power-law index n is to decrease the momentum boundary layer thickness and is to decrease the thermal boundary layer thickness for increasing values of power-law index namely, shear thinning, Newtonian and shear thickening fluids.
- The effect of velocity exponent parameter is to reduce the momentum boundary layer thickness. Also increasing values of m is to reduce the temperature for shear thickening fluids and increase the temperature for shear thinning fluids.
- The increasing value of magnetic parameter M_n results in flattering the horizontal velocity profiles and increases the temperature profile.
- We see that velocity increases but the temperature decreases as we increase the buoyancy parameter both the case of n.
- Increasing value of temperature exponent parameter r is to decrease the temperature for shear thickening and not much significant difference in shear thinning fluids.
- The effect of modified Prandtl number Npr is to decrease the thermal boundary layer thickness and the wall temperature gradient.
- The internal heat source \ sink parameter β increases the temperature profile.

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