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An internal crack problem in an infinite transversely isotropic elastic layer

Research Article

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Abstract: This paper is concerned with an internal crack problem in an infinite transversely isotropic elastic layer. The crack is opened by an internal uniform pressure p_0 along its surface. The layer surfaces are supposed to be acted on by symmetrically applied concentrated forces of magnitude $\frac{P}{2}$ with respect to the centre of the crack. The applied concentrated force may be compressive or tensile in nature. The problem is solved by using integral transform technique. The solution of the problem has been reduced to the solution of a Cauchy type singular integral equation, which requires numerical treatment. The integral equation is solved numerically by using Gauss-Chebychev and Gauss-Laguerre quadratures. The stress-intensity factors and the crack opening displacements are determined and the effects of anisotropy on both cases are shown graphically.

MSC: 74E10 • 74R20

Keywords: Transversely isotropic medium • Fourier integral transform • Singular integral equation • Stress-intensity factor

1. Introduction

The study relating the behaviour of elastic material under applied load needs special attention and care when the elastic body develops a crack in it. It is obvious that the presence of a crack in a structure not only affects the stress distribution but also drastically reduces the life span of the structure.Propagation of elastic disturbance in a solid is also disturbed by the presence of a crack. But cracks are present essentially in all structural materials, either as natural defects or as a result of fabrication processes. Stress distribution in a body which develops a crack in it is entirely different from that in a body without a crack. In literature, considerable effort has been devoted to the study of cracks in solids, due to their applications in industry in general and in fabrication of electronic components in particular.

Presence of a crack in a solid significantly affects its response to the applied load. Stress distributions in the solid with a crack are studied in two regions: the region in the neighbourhood of crack, called the near field region and the region faraway from the crack, called the far field region. Study of stress distribution in the near field region is very important. Stress intensity factor, crack energy etc. are some of the quantities responsible for spreading of a crack. For a solid with a crack in it loaded mechanically or thermally determination of stress intensity factor (SIF) becomes a very important topic in fracture mechanics. The SIF is a parameter that gives a measure of stress concentration around cracks and defects in a solid. SIF needs to be understood if we are to design fracture tolerant materials used in bridges, buildings, aircraft,or even bells. Polishing just won't do if we detect crack. Typically for most materials if a crack can be seen it is very close to the critical stress state predicted by the SIF.

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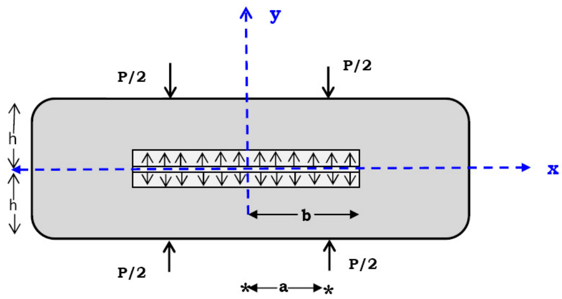
Over the last few decades anisotropic materials have been increasingly used. There are materials which have natural anisotropy such as zinc, magnesium, sapphire, wood, some rocks and crystals, and also there are artificially manufactured materials such as fibre-reinforced composite materials, which exhibit anisotropic character. The advantage of composite materials over the traditional materials lies on their valuable strength, elastic and other properties [1]. A reinforced material may be regarded to some order of approximation, as homogeneous and anisotropic elastic medium having a certain kind of elastic symmetry depending on the symmetry of reinforcement. Some glass fibre reinforced plastics may be regarded as transversely isotropic. Thus solid mechanics problems should not be restricted to the isotropic medium only. Increasing use of anisotropic media demands that the study should be extended to anisotropic medium also. Crack problem in isotropic elastic medium have been extensively studied in literature following classical theory. A comprehensive list of work on crack problems by earlier investigators has been provided in Chaudhuri and Ray [2], Barik et. al.[3], Ozturk and Erdogan [4], Zhou and Hanson [5], Fabrikant [6], Dag and Erdogan [7], Sherief and El- Mahraby [8], Chen et. al. [9], Lee [10], Brock[11]etc.

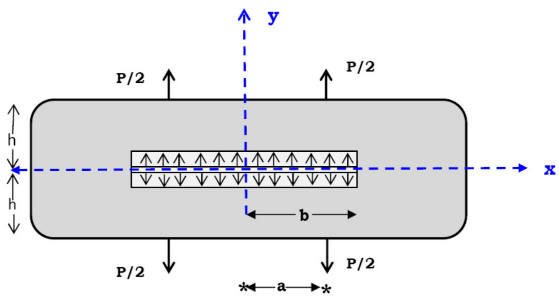
The present investigation aims at studying an internal crack problem in infinite transversely isotropic elastic layer. Following the integral transform technique the problem has been reduced to a problem of Cauchy type singular integral equation, which has been solved numerically. Numerical computations have been done to assess the effects of anisotropy considered in the problem on various subjects of interest and the results have been shown graphically.

2. Formulation of the problem

We consider an infinite transversely isotropic layer of thickness 2h weakened by an internal crack of length $2b$, which is opened by an uniform internal pressure p_0 along its surface. The layer is subjected to two different types of loadings on its surfaces in a direction perpendicular to its length (a) a symmetric pair of compressive concentrated normal loads $\frac{P}{2}$, (b) a symmetric pair of tensile concentrated normal loads $\frac{P}{2}$ (Figs. 1(b)-1(b)). It is assumed that the effect of the gravity force is neglected. The problem is formulated in cartesian co-ordinate system (x, y) in which crack lies along x-axis with origin at the centre of the crack. Due to symmetry of the crack location with respect to the layer and also of the applied load with respect to the crack, it is sufficient to consider solution of the problem in the region $0 \le x < \infty$ and $0 \le y \le h$. The strain displacement relations, linear stress-strain relations and equations of equilibrium



(a) Concentrated compressive load condition



(b) Concentrated tensile load condition

Fig. 1. Geometry of the problem

(Ding et. al. 2006) are, respectively, given by

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}; \tag{1}$$

$$\begin{aligned} \sigma_x &= C_{11}\varepsilon_x + C_{12}\varepsilon_y, \\ \sigma_y &= C_{12}\varepsilon_x + C_{11}\varepsilon_y, \\ \tau_{xy} &= \tfrac{1}{2}(C_{11} - C_{12})\gamma_{xy} \end{aligned} \tag{2}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \tag{4}$$

The present problem is mathematically equivalent to the solution of the following equations and boundary conditions:

Equilibrium equations:

$$C_{11}\frac{\partial^2 u}{\partial x^2}+\frac{1}{2}(C_{11}-C_{12})\frac{\partial^2 u}{\partial y^2}+\frac{1}{2}(C_{11}+C_{12})\frac{\partial^2 v}{\partial x\partial y}=0 \tag{5}$$

$$\frac{1}{2}(C_{11}-C_{12})\frac{\partial^2 v}{\partial x^2}+C_{11}\frac{\partial^2 v}{\partial y^2}+\frac{1}{2}(C_{11}+C_{12})\frac{\partial^2 u}{\partial x\partial y}=0 \tag{6}$$

The boundary conditions:

$$\tau_{xy}(x,0)=0, \qquad -\infty<x<\infty \tag{7}$$

$$\tau_{xy}(x,h)=0, \qquad -\infty<x<\infty \tag{8}$$

$$\sigma_y(x,h)=\mp\left[\frac{P}{2}\delta(x-a)+\frac{P}{2}\delta(x+a)\right], \qquad -\infty<x<\infty \tag{9}$$

$$\frac{\partial}{\partial x}[v(x,0)]=\begin{cases} f(x), & -b<x<b;\\ 0, & b<x<\infty.\end{cases} \tag{10}$$

$$\sigma_y(x,0)=-p_0, \qquad -b\le x\le b \tag{11}$$

where u and v are the x and y components of the displacement vectors; $\sigma_x,\sigma_y,\tau_{xy}$ are the normal and sharing stress components; C_{11}, C_{12} are transversely isotropic elastic coefficients; $f(x)$ is an unknown function and $\delta(x)$ is the Dirac delta function. In Eq. (9) positive sign indicates tensile force while negative sign corresponds to compressive force.

3. Method of solution

To solve the partial differential Eqs. (5) and (6), the Fourier transform is applied to the equations with respect to the variable x. The equations in the transformed domain can be written as

$$\frac{1}{2}(C_{11}-C_{12})\frac{d^2\overline{u}}{dy^2}-C_{11}\xi^2\overline{u}-i\xi\frac{1}{2}(C_{11}+C_{12})\frac{d\overline{v}}{dy}=0 \tag{12}$$

$$C_{11}\frac{d^2\overline{v}}{dy^2}-\xi^2\frac{1}{2}(C_{11}-C_{12})\overline{v}-i\xi\frac{1}{2}(C_{11}+C_{12})\frac{d\overline{u}}{dy}=0 \tag{13}$$

where $\overline{u}$ and $\overline{v}$ are the Fourier transforms of u and v, respectively defined by

$$(\overline{u},\overline{v})=\int_{-\infty}^{\infty}(u,v)e^{i\xi x}dx.$$

Now, elimination of $\overline{u}$ from the Eqs. (12) and (13) yields a differential equation in $\overline{v}$;

$$\frac{d^4\overline{v}}{dy^4}-2\xi^2\frac{d^2\overline{v}}{dy^2}+\xi^4\overline{v}=0. \tag{14}$$

Eq. (14) is an ordinary differential equation in independent variable y, its general solution can be obtained as

$$\overline{v}(\xi,y)=(A+By)e^{-\xi y}+(C+Dy)e^{\xi y} \tag{15}$$

where A,B,C and D are the unknown constants. Eliminating derivatives of $\overline{u}$ from Eqs. (12)-(13) and using Eq. (15) we have

$$\overline{u}(\xi,y)=\frac{1}{i\xi}\left[\{-\xi(A+By)+\frac{3C_{11}-C_{12}}{C_{11}+C_{12}}B\}e^{-\xi y}+\{\xi(C+Dy)+\frac{3C_{11}-C_{12}}{C_{11}+C_{12}}D\}e^{\xi y}\right] \tag{16}$$

where $i=\sqrt{-1}$. Application of Fourier inversion formula on the Eq. (15) and (16) we get

$$v(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\{(A+By)e^{-\xi y}+(C+Dy)e^{\xi y}\}e^{-i\xi x}d\xi \tag{17}$$

$$u(x,y)=\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\xi}\left[\{\xi(A+By)-\frac{3C_{11}-C_{12}}{C_{11}+C_{12}}B\}e^{-\xi y}-\{\xi(C+Dy)+\frac{3C_{11}-C_{12}}{C_{11}+C_{12}}D\}e^{\xi y}\right]e^{-i\xi x}d\xi \tag{18}$$

Combination of Eqs. (1), (2), (17) and (18) yields

$$\begin{aligned}\sigma_x(x,y)=&\frac{1}{2\pi}\int_{-\infty}^{\infty}[\{\xi(C_{11}-C_{12})e^{-\xi y}\}A+\{\xi y(C_{11}-C_{12})+\frac{2C_{11}C_{12}+C_{12}^2-3C_{11}^2}{C_{11}+C_{12}}\}e^{-\xi y}B\\&+\{-\xi(C_{11}-C_{12})e^{\xi y}\}C+\{-\xi y(C_{11}-C_{12})+\frac{2C_{11}C_{12}+C_{12}^2-3C_{11}^2}{C_{11}+C_{12}}\}e^{\xi y}D]e^{-i\xi x}d\xi\end{aligned} \tag{19}$$

$$\begin{aligned}\sigma_y(x,y)=&\frac{1}{2\pi}\int_{-\infty}^{\infty}[\{-\xi(C_{11}-C_{12})e^{-\xi y}\}A+\{-\xi y(C_{11}-C_{12})+\frac{-2C_{11}C_{12}+C_{12}^2+C_{11}^2}{C_{11}+C_{12}}\}e^{-\xi y}B\\&+\{\xi(C_{11}-C_{12})e^{\xi y}\}C+\{\xi y(C_{11}-C_{12})+\frac{-2C_{11}C_{12}+C_{12}^2+C_{11}^2}{C_{11}+C_{12}}\}e^{\xi y}D]e^{-i\xi x}d\xi\end{aligned} \tag{20}$$

$$\begin{aligned}\tau_{xy}(x,y)=&\frac{C_{11}-C_{12}}{2}\frac{i}{2\pi}\int_{-\infty}^{\infty}[\{-2\xi e^{-\xi y}\}A+\{-2\xi y+\frac{4C_{11}}{C_{11}+C_{12}}\}e^{-\xi y}B+\{-2\xi e^{\xi y}\}C\\&+\{-2\xi y-\frac{4C_{11}}{C_{11}+C_{12}}\}e^{\xi y}D]e^{-i\xi x}d\xi.\end{aligned} \tag{21}$$

Utilizing of the boundary conditions (7)-(10), the unknown constants A, B, C and D can be found out and substitution of these values into the equation (11) will lead to the following singular integral equation:

$$\frac{1}{\pi}\int_{-b}^{b}f(t)\left[\frac{1}{t-x}+k_1(x,t)]dt=\frac{2C_{11}}{C_{11}^2-C_{12}^2}[-p_0\pm\frac{P}{2\pi}k_2(x)\right],\qquad(-b<x<b) \tag{22}$$

where

$$k_1(x,t)=-2\int_0^{\infty}\frac{(1+2\xi h+2\xi^2h^2)e^{-2\xi h}-e^{-4\xi h}}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}sin\xi(t-x)d\xi \tag{23}$$

and

$$k_2(x)=\int_0^{\infty}\frac{2e^{-\xi h}[1+\xi h+(-1+\xi h)e^{-2\xi h}]}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}[cos\xi(x-a)+cos\xi(x+a)]d\xi. \tag{24}$$

The kernels $k_1(x,t)$ and $k_2(x)$ are bounded and continuous in the closed interval $-b\le x\le b$. The integral equation must be solved under the following single-valuedness condition

$$\int_{-b}^{b}f(t)dt=0. \tag{25}$$

Before further proceeding it will be convenient to introduce non-dimensional variables r and s by rescaling all lengths in the problems by length scale b:

$$x=br,\,t=bs \tag{26}$$

$$f(t)=f(bs)=\frac{p_0}{C_{11}-C_{12}}\phi(s),\omega=\xi b. \tag{27}$$

In terms of non-dimensional variables the integral Eq. (22) and single valuedness condition (25) become

$$\frac{1}{\pi}\int_{-1}^{1}[\frac{1}{s-r}+k_1^*(r,s)]\phi(s)ds=\frac{2C_{11}}{C_{11}+C_{12}}[-1\pm\frac{Q}{\pi}k_2^*(r)],\qquad(-1<r<1) \tag{28}$$

and

$$\int_{-1}^{1}\phi(s)ds=0 \tag{29}$$

where

$$k_1^*(r,s)=\int_0^{\infty}\frac{-2[(1+2\xi h+2\xi^2h^2)e^{-2\xi h}-e^{-4\xi h}]}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}sin\omega(s-r)\,d\omega, \tag{30}$$

$$k_2^*(r)=\int_0^{\infty}\frac{2e^{-\xi h}[1+\xi h+(-1+\xi h)e^{-2\xi h}]}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}\{cos\omega(r-a^*)+cos\omega(r+a^*)\}\,d\omega, \tag{31}$$

$a^*=\dfrac{a}{b}$ and Q is the load ratio defined as:

$$Q=\frac{P}{2bp_0}. \tag{32}$$

4. Solution of integral equations

The singular integral Eq. (28) is a Cauchy-type singular integral equation for an unknown function $\phi(s)$. For the evaluation of displacement and stress components it is necessary to solve the integral Eq. (28) for the unknown function $\phi(s)$. Expressing now the solution of equation (28) in the form

$$\phi(s) = \frac{\psi(s)}{\sqrt{1-s^2}}, \qquad (-1 < s < 1) \tag{33}$$

where $\psi(s)$ is a regular and bounded unknown function and using the Gauss-Chebyshev formula [12] to evaluate the integral Eq. (28), we obtain

$$\frac{1}{N}\left[\sum_{k=1}^{N}\left[\frac{1}{s_k - r_j} + k_1^*(r_j, s_k)\right]\psi(s_k)\right] = \frac{2C_{11}}{C_{11}+C_{12}}[-1 \pm \frac{Q}{\pi}k_2^*(r_j)], \quad j = 1,2,....,N-1 \tag{34}$$

and

$$\frac{1}{N}\sum_{k=1}^{N}\psi(s_k) = 0 \tag{35}$$

where s_k and r_j are given by

$$s_k = cos\left(\frac{2k-1}{2N}\pi\right), (k = 1,2,3,.....,N) \tag{36}$$

$$r_j = cos\left(\frac{j\pi}{N}\right), (j = 1,2,3,.....,N-1). \tag{37}$$

We observe that corresponding to $(N-1)$ collocation points $x_j = cos(\frac{j\pi}{2(N+1)})$, $j = 1,2,..,(N-1)$ the Eqs. (34) and (35) represent a set of N linear equations in N unknowns $\psi(s_1), \psi(s_2),....,\psi(s_N)$. This linear algebraic system of equations are solved numerically by utilizing Gaussian elimination method.

5. Determination of stress intensity factor

Presence of a crack in a solid significantly affects the stress distribution compared to that when there is no crack. While the stress distribution in a solid with a crack in the region far away from the crack is not much disturbed, the stresses in the neighbourhood of the crack tip assumes a very high magnitude. In order to predict whether the crack has a tendency to expand further, the stress intensity factor (SIF), a quantity of physical interest, has been defined in fracture mechanics. The load at which failure occurs is referred to as the fracture strength. The stress intensity factor is defined as

$$k(b) = \lim_{r\to 1}\sqrt{2b(r-1)}\sigma_y^*(r,0). \tag{38}$$

Use of the Eqs. (27) and (22) and after some manipulation, the expression for $k(b)$ is obtained as

$$\frac{k(b)}{p_0\sqrt{b}} = -\frac{C_{11}+C_{12}}{2C_{11}}\psi(1) \tag{39}$$

where $\psi(1)$ can be found out from $\psi(s_k)$, $(k = 1,2,3,....,N)$ using the interpolation formulas given by [13]. Following the method as in [14] we obtain the crack surface displacement in the form

$$v(x,0) = \int_{-b}^{x} f(t)dt, \qquad (-b < x < b). \tag{40}$$

Using the Eqs. (26) and (27) into the Eq. (40), we can express the dimensionless normal displacement as

$$v'(r,0) = \frac{v(x,0)(C_{11}-C_{12})}{p_0 b} = \int_{-1}^{r}\phi(s)ds, \qquad (-1 < r < 1) \tag{41}$$

which can be obtained numerically, using Simpson's $\frac{1}{3}$ integration formula and the appropriate interpolation formula. It has been verified that the results of [15] follow directly [with the exception of one / two printing errors / omissions] from our results by using the transformations:

$$C_{11} = C_{22} = \lambda + 2\mu, C_{12} = \lambda, C_{33} = \mu.$$

6. Numerical results and discussions:

The present study is related to the study of an internal crack problem in an infinite transversely isotropic elastic layer. In our present discussion we have considered the transversely isotropic material as Sapphire, Magnesium and Graphite to illustrate theoretical results. The numerical values of the elastic coefficients for the materials are listed in Table 1 [16].

Table 1. Basic data for three transversely isotropic materials.

Quantity	Unit	Sapphire	Magnesium	Graphite
c_{11}	Nm^{-2}	4.96×10^{11}	5.974×10^{10}	1.628×10^{11}
c_{12}	Nm^{-2}	1.64×10^{11}	2.624×10^{10}	0.362×10^{11}

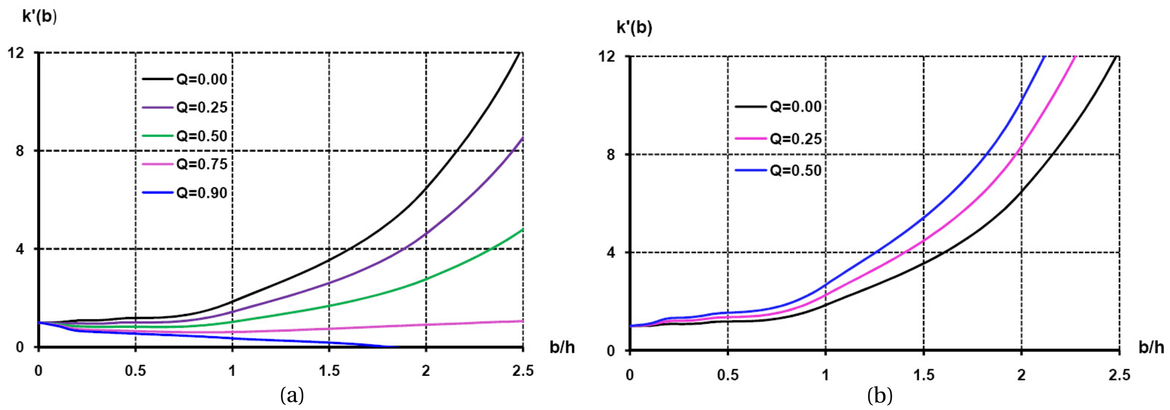


Fig. 2. (a) Variation of normalized stress-intensity factor $k'(b)$ for various values of Q in the case of compressive concentrated forces ($a/b = 0.50$). (b) Variation of normalized stress-intensity factor $k'(b)$ for various values of Q in the case of tensile concentrated forces ($a/b = 0.50$).

Utilizing the input parameters and following the standard numerical method described in section 4 we compute the normal displacement component and the stress intensity factor and are shown graphically. Considering the elastic material of the layer as Sapphire, the variation of normalized stress intensity factor with $\frac{b}{h}$ are shown in Figs. 2(a)-2(b) for both the cases of two symmetric transverse pair of compressive and tensile concentrated forces. It is observed from Fig. 2(a) that for compressive concentrated forces the normalized stress-intensity factor $k'(b)$ decreases with the increase of the load ratio Q, and the increase of $k'(b)$ is quite significant for smaller values of Q. It is also observed from Fig. 2(a) that the load ratio Q has not much effect on the stress intensity factor $k'(b)$ when the crack length is sufficiently small. Fig. 2(b) represents the variations of $k'(b)$ with crack length under tensile nature of the concentrated force. Contrary to the previous case it is observed that $k'(b)$ increases with Q. For small crack length, the behaviour of $k'(b)$ is similar to the case of compressive concentrated load.

Figs. 3(a)-3(b) display the variation of normalized stress-intensity factor $k'(b)$ for different position of loading.

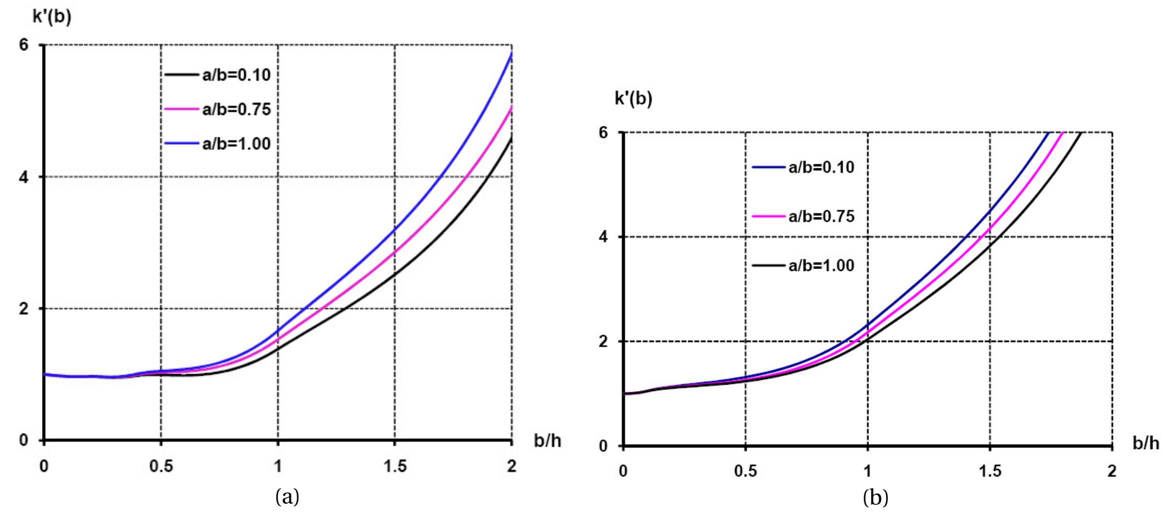


Fig. 3. (a) Variation of normalized stress intensity factor $k'(b)$ for different values a/b in the case of compressive concentrated forces ($Q = 0.25$). (b) Variation of normalized stress intensity factor $k'(b)$ for different values of a/b in the case of tensile concentrated forces ($Q = 0.25$).

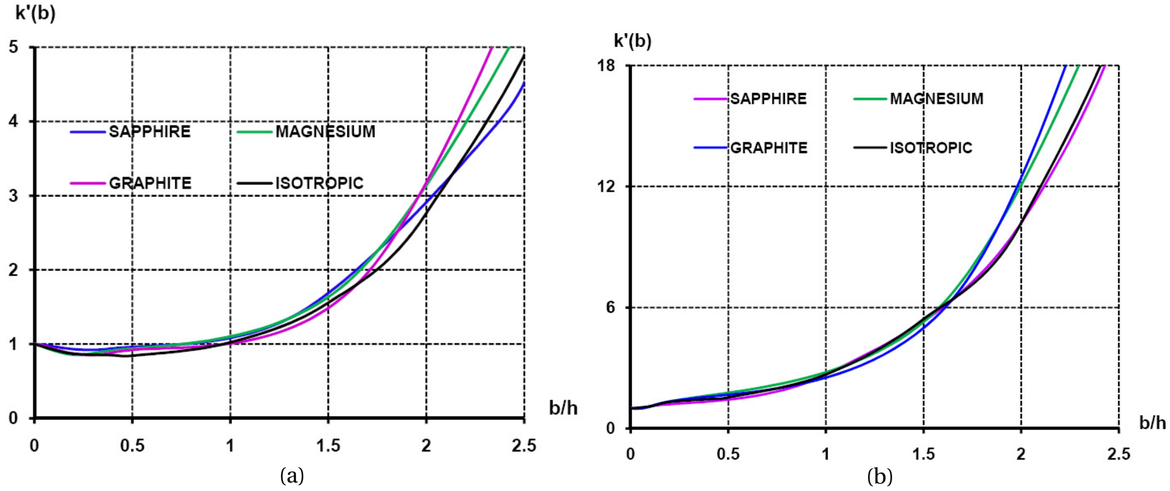


Fig. 4. (a) Effect of anisotropy on normalized stress intensity factor $k'(b)$ in the case of compressive concentrated forces $(a/b = 0.50, Q = 0.50)$. (b) Effect of anisotropy on normalized stress intensity factor $k'(b)$ in the case of tensile concentrated forces $(a/b = 0.50, Q = 0.50)$.

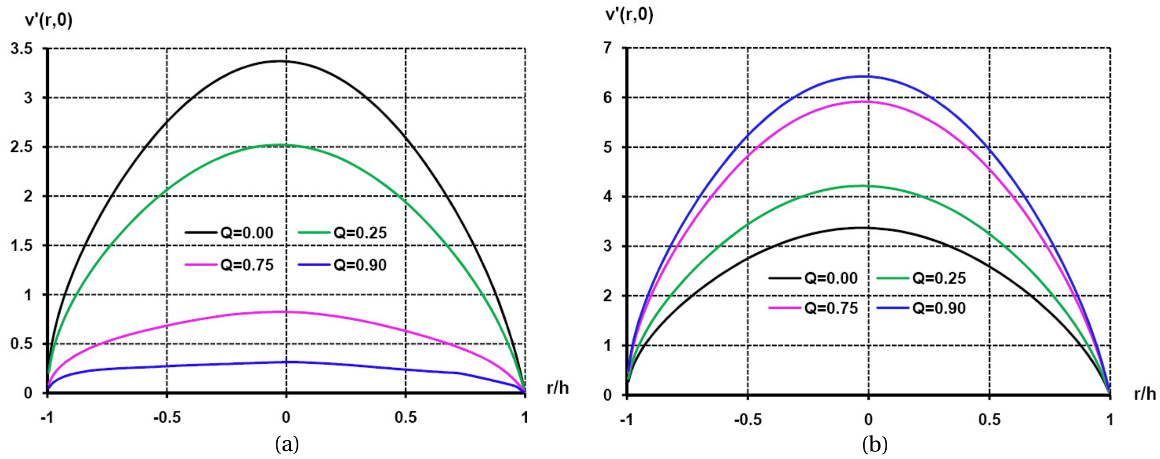


Fig. 5. (a) Variation of normalized crack surface displacement $v'(r,0)$ for various values of Q in the case of compressive concentrated forces $(a/b = 0.5, b/h = 1)$. (b) Variation of normalized crack surface displacement $v'(r,0)$ for various values of Q in the case of tensile concentrated forces $(a/b = 0.5, b/h = 1)$.

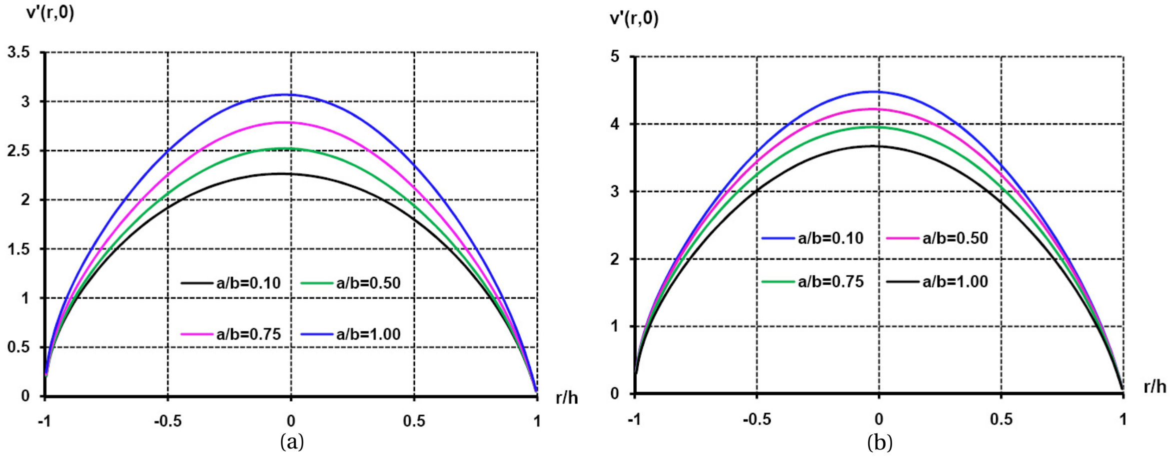


Fig. 6. (a) Variation of normalized crack surface displacement $v'(r,0)$ for various values of a/b in the case of compressive concentrated forces $(Q = 0.25, b/h = 1)$. (b) Variation of normalized crack surface displacement $v'(r,0)$ for various values of a/b in the case of tensile concentrated forces $(Q = 0.25, b/h = 1)$.

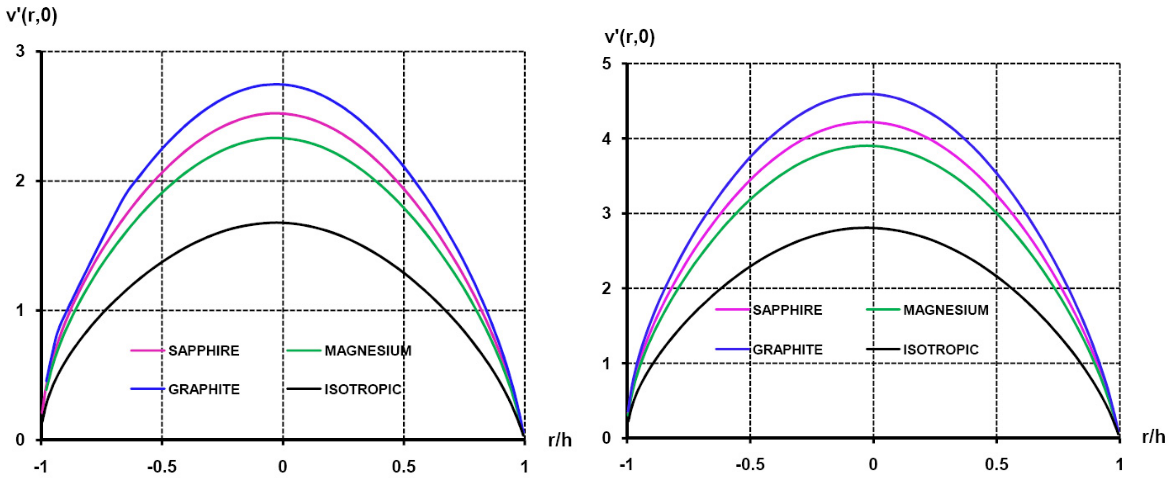


Fig. 7. (a) Effect of anisotropy on normalized crack surface displacement $v'(r,0)$ in the case of compressive concentrated forces $(a/b = 0.50, Q = 0.25, b/h = 1)$. (b) Effect of anisotropy on normalized crack surface displacement $v'(r,0)$ in the case of tensile concentrated forces $(a/b = 0.50, Q = 0.25, b/h = 1)$.

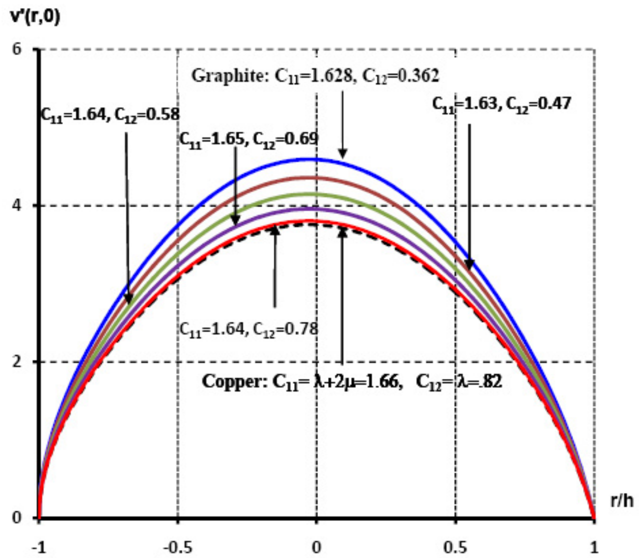


Fig. 8. Transition of normalized crack surface displacement $v'(r,0)$ from anisotropic materials to isotropic one (as $C_{12} \to \lambda$ and $C_{11} \to \lambda + 2\mu$) in the case tensile forces

It is noted that in the case of compressive concentrated forces, $k'(b)$ increases with increasing $\frac{a}{b}$, but it decreases in the case of tensile concentrated forces. In Figs. 4(a)-4(b) normalized stress intensity factor experiences the effect of anisotropy in the cases of two symmetric transverse pair of compressive and tensile concentrated forces for load ratio $Q = 0.5$.

Figs. 5(a)-5(b) depict the variation of normalized crack surface displacement $v'(r,0)$ with $\frac{r}{h}$ for different values of load ratio Q. It is clear from Fig. 5(a) that for two symmetric transverse pair of compressive concentrated forces the normalized crack surface displacement $v'(r,0)$ decreases as load ratio Q increases, but for tensile concentrated forces the normalized crack surface displacement decreases as load ratio Q also decreases. For both the cases of compressive and tensile concentrated forces the graphs show that the normalized crack surface displacement is symmetrical with respect to origin.

Figs. 6(a)-6(b) illustrate the role of the point of application of loading on the normalized crack surface displacement for a particular load ratio $Q = 0.25$ and $\frac{b}{h} = 1.00$. It is observed in Fig. 6(a) that for compressive concentrated loading the normalized crack surface displacement increases with the increased values of $\frac{a}{b}$ but behaviour is just opposite (Fig. 6(b)) for tensile concentrated loading. The effect of anisotropy on normalized crack surface displacement $v'(r,0)$ is observed in Figs. 7(a)-7(b) for both the cases of compressive and tensile concentrated forces as $\frac{a}{b} \to 0$, $\frac{b}{h} = 1.00$ and load ratio $Q = 0.25$. As expected it is observed from Figs. 5-7,that the normalized crack surface displacement $v'(r,0)$ assumes its maximum magnitude near the origin.

It is to be noted that all physical quantities like stress intensity factor, crack surface displacement etc. in isotropic medium are the degenerate case of corresponding results obtained for transversely isotropic media. For instance, Fig. 8 exhibits that variation of crack surface displacement in our discussion approaches the corresponding variation of crack surface displacement in Copper (with λ= 82 GPa; μ= 42 GPa), an isotropic medium.

An important observation may be available from Fig. 2(a). Under the compressive load condition we find that with the increase of Q, there are critical crack lengths for which the stress intensity factor has zero value i,e. for which the stress magnitude σ_y is finite at the crack tips. For example, from Fig. 2(a), if $Q = 0.9$, the critical crack length is approximately 3.5.

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