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The stagnation point MHD flow and heat transfer of micropolar fluid over a stretching sheet in the presence of radiation, heat generation and dissipations

Research Article

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Abstract: In this paper, The problem of steady two-dimensional boundary layer MHD stagnation point flow and heat transfer of an incompressible micropolar fluid over a stretching sheet with the presence of radiation, heat generation and dissipations has been investigated. The governing partial differential equations are transformed to ordinary ones using similarity transformation. The resulting nonlinear ordinary differential equations are solved numerically using the finite difference method. The results obtained for velocity, temperature and skin friction are tabulated and shown graphically. Comparing the present results for skin friction with the recent results, it is observed that the results are in excellent agreements.

MSC: 76A02 • 76W05 • 80A20

Keywords: Non-Newtonian fluids • Magnetohydrodynamics • Heat and mass transfer

1. Introduction

Micropolar fluids are fluids with microstructure, and they belong to a class of fluids with nonsymmetric stress tensor that we will call polar fluids. Micropolar fluids may also represent fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium, where the deformation of the particles is ignored. This constitutes a substantial generalization of the Navier-Stokes model and opens a new field of potential applications. The attractiveness and power of the model of micropolar fluids are come from the fact that it is both a significant and a simple generalization of the classical Navier-Stokes model. The theory of micropolar fluids developed by Eringen [1]. Other important applications of micropolar fluids can be seen in Ariman et. al. [2] and [3]. Eringen [4] has also developed the theory of thermomicropolar fluids by the extending theory of micropolar fluids.

The classical two dimensional stagnation point flow impinging on a flat plate, first considered by Hiemenz [5], was extended to the axisymmetric case by Homann [6]. The steady two dimensional flow over a semi infinite flat surface with mass and heat transfer characteristics was considered by Chamkha and Camille [7]. Mahapatra and Gupta [8] presented the numerical solution of the problem of steady two dimensional stagnation point flow of an electrically conducting power law fluid over a stretched surface. Roslinda et al. [9] investigated numerically the unsteady boundary layer flow of an incompressible viscous fluid in the stagnation point region over a stretching sheet. The steady two dimensional laminar MHD mixed convection stagnation point flow with mass and heat transfer over a surface was examined by Abdelkhalek [10] by using perturbation technique. The two dimensional boundary layer stagnation point flow and heat transfer over a stretching sheet with injection/suction effects was considered by Layek et al. [11].

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Ishak et al. [12] investigated numerically the steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature using Keller-box method. Chakraborty and Mazumdar [13] analyzed the approximate solution to the problem of steady laminar flow of a viscous incompressible electrically conducting fluid over a stretching sheet and used least squares method to minimize the residual of a differential equation. Ishak et al. [14] studied the steady two-dimensional stagnation point flow of an incompressible viscous and electrically conducting fluid in the presence of a transverse uniform magnetic field over a vertical stretching sheet. Pop et al. [15] studied the radiation effects on the flow of an incompressible viscous fluid over a flat sheet near the stagnation point and solved system of ordinary differential equations numerically using Runge-Kutta method with a shooting technique. The effect of heat generation and radiation parameters on MHD flow along a uniformly heated vertical flat plate in the presence of a magnetic field has been investigated numerically by Saha et al. [16]. Noor et al. [17] analyzed the magnetohydrodynamic viscous flow due to a shrinking sheet analytically and found that the result obtained by a domian decomposition and homotopy analysis methods are well agreed. Raptis et al. [18] discussed numerically the effect of radiation parameter by solving the problem of steady MHD asymmetric flow of an electrically conducting fluid over a semi infinite stationary plate. Nadeem et al. [19] solved the problem of stagnation point flow of a viscous fluid towards a stretching sheet by homotopy analysis method.

Liu [20] presented the analytical solutions for the flow and heat transfer in a steady laminar boundary flow of an electrically conducting fluid of second grade subject to a uniform transverse magnetic field past a semi-infinite stretching sheet with power-law surface temperature or power-law heat flux. Prasad et al. [21] discussed the characteristics of heat transfer and momentum in an incompressible electrically conducting non-Newtonian boundary layer flow of a viscoelastic fluid over a stretching sheet by assuming that the fluid viscosity and thermal conductivity vary as an inverse and linear function of temperature and came to know about the significant decrease in wall temperature profile and skin friction of the sheet by increasing magnetic field parameter. Ishak et al. [22] discussed the steady two dimensional stagnation point flow of a micropolar fluid over a shrinking sheet in its own plane and assumed that ambient fluid velocity and shrinking velocity vary linearly. They found that the solutions for a shrinking sheet are non-unique. Attia [23] investigated the steady laminar flow of an incompressible non-Newtonian micropolar fluid impinging on a permeable flat plate with heat transfer and applied a uniform suction or blowing normal to the plate at a constant temperature and discussed the characteristics of the non-Newtonian fluid and uniform suction or blowing on both the flow and heat transfer by obtaining the numerical solution. Ishak [24] studied the thermal boundary layer flow induced by a linearly stretching sheet immersed in an incompressible micropolar fluid with constant surface temperature and found that the heat transfer rate at the surface decreases in the presence of radiation. Lok et al. [25] generalized the classical modified Hiemenz flow for micropolar fluid near an orthogonal stagnation point by discussing the problem of steady two-dimensional flow of a micropolar fluid impinging obliquely on a flat plate. Damesh et al. [26] studied the combined heat and mass transfer by natural convection of a microplar viscous and heat generating or absorbing fluid flow over a uniformly stretched permeable surface in the presence of a first order chemical reaction and solved the governing equations numerically using the fourth order Runge-Kutta method. Ishak and Nazar [27] discussed numerically the steady two-dimensional stagnation point flow of a micropolar fluid over a stretching permeable sheet using finite difference method.

The purpose of the present paper is to study the effect of magnetohydrodynamic and heat transfer on the steady laminar flow of an incompressible non-Newtonian micropolar fluid at a two-dimensional stagnation point with radiation, heat generation and dissipations. The wall temperatures is assumed to be constants. A numerical solution is obtained for the governing (momentum and energy) equations using finite difference approximations, which takes into account the asymptotic boundary conditions. The numerical solution computes the flow and heat profiles for the whole range of the non-Newtonian fluid characteristics, the stretching velocity, magnetic parameter, the heat generation/absorption coefficient, the radiation parameter, the prandtl and brinkman numbers. The algorithm solution is coded using MATLAB to obtain accurate results.

2. Mathematical formulation

Consider a steady ,two-dimensional flow of an incompressible electrically conducting micropolar fluid in the presence of thermal radiation near the stagnation on a horizontal heated plate. It is assumed that the velocity of the flow external to the boundray layer $U(x)$ and the temperature $T_\omega(x)$ of the plate are proportional to the distance x from the stagnation point ,i.e. $U(x)$=cx and $T = T_\omega$ where c is constant. A uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction normal to the plate as shown in figure 1. In addition heat generation and dissipations effects are included.

The governing equations of the flow in two dimensions with heat transfer in the presence of radiation, heat generation and dissipations can be written as:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

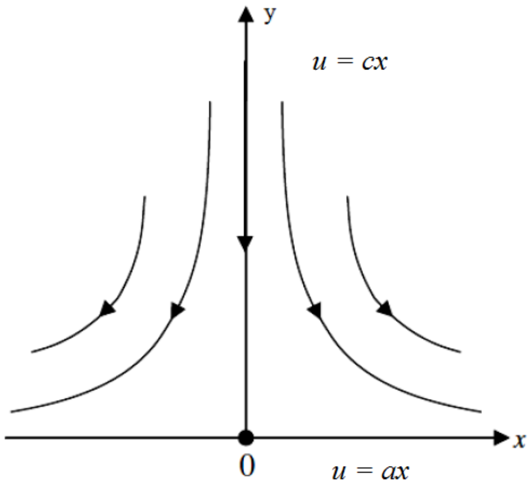

Fig. 1. Physical Configuration and Coordinate System.

Linear momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}(u-U) \tag{2}$$

Angular momentum equation:

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-\kappa\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

Heat equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{c_p\rho}\frac{\partial q_r}{\partial y}+\frac{\sigma {B_O}^2}{\rho c_p}(u-U)^2+\frac{\nu}{c_p}(\frac{\partial u}{\partial y})^2+\frac{Q}{\rho c_p}(T-T_\infty) \tag{4}$$

subject to the boundary conditions

$$u=ax,\ v=0,\ N=-n\frac{\partial u}{\partial y},\ T=T_w,\ at\ y=0 \tag{5}$$

$$u\to U(x)=cx,\ N\to 0,\ T\to T_\infty,\ as\ y\to\infty \tag{6}$$

where u and v are the velocity components along the x and y axes, respectively and T is the fluid temperature in the boundary layer. Furthermore, $\mu,\ k,\ \rho,\ j,\ N,\ \gamma,\sigma,B_o,\ \alpha=\frac{K}{\rho c_p}$, c_p and q_r are respectively the dynamics viscosity, vortex viscosity(or the microinvasion viscosity), fluid density, micro-inertia per unit mass,microrotation vector (or angular velocity), spin gradient viscosity, thermal diffusivity, the electric conductivity, the strength of magnetic field, specific heat at constant pressure and the radiative heat flux. we notice that n is a constant such that when $n=\frac{1}{2}$ indicates that, the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration [28] of micro-elements which will be considered here. We follow the usual procedure by assuming that $\gamma=\left(\mu+\frac{k}{2}\right)j=\mu\left(1+\frac{K}{2}\right)j$, where $K=\left(\frac{k}{\mu}\right)$ is the material parameter (or the Eringen micropolar parameter) and $j=\frac{\nu}{a}$ as a reference length. This assumption is involved to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity (see Ishak et al. [29]). By using the Rosseland approximation (Brewster [30]) the radiative heat flux q_r is given by:

$$q_r=-4\frac{\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

$$\frac{\partial q_r}{\partial y}=-\frac{4\sigma^*}{3k^*}\frac{\partial^2 T^4}{\partial y^2} \tag{8}$$

where σ^* the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting the second and higher order terms lead to

$$T^4 \cong 4T_\infty^3(T - 3T_\infty^4) \tag{9}$$

$$\frac{\partial^2 T^4}{\partial y^2} \cong 4T_\infty^3 \frac{\partial^2 T}{\partial y^2} \tag{10}$$

by substituting from Eqs.(10) and (8) in Eq. (4) we obtain the following equation.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma {B_O}^2}{\rho c_p}(u-U)^2 + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho c_p}(T - T_\infty) \tag{11}$$

3. Similarity transformation

The partial differential Eqs. (1–3) and Eq. (11) are transformed into non-dimensional form by using the following dimensionless variables:
$\eta = (\frac{a}{\nu})^{\frac{1}{2}} y$, $u = axf'(\eta)$, $v = -(a\nu)^{\frac{1}{2}} f(\eta)$, $N = ax(\frac{a}{\upsilon})^{\frac{1}{2}} h(\eta)$ and $T = \Delta T\theta(\eta) + T_\infty$ or $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ $U(x) = cx$, from which we obtain
1.$u = axf'(\eta)$.

$$\frac{\partial u}{\partial x} = af'(\eta) \tag{12}$$

$$\frac{\partial u}{\partial y} = axf''(\eta)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \tag{13}$$

$$\frac{\partial^2 u}{\partial y^2} = axf'''(\eta)\left(\frac{a}{\nu}\right) \tag{14}$$

$$u\frac{\partial u}{\partial x} = a^2 x f'^2(\eta) \tag{15}$$

$$v\frac{\partial u}{\partial y} = -a^2 x f(\eta) f''(\eta) \tag{16}$$

2.$v = -(a\nu)^{\frac{1}{2}} f(\eta)$

$$\frac{\partial v}{\partial y} = -(a\nu)^{\frac{1}{2}} f'(\eta)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} = -af'(\eta) \tag{17}$$

3.$U(x) = cx$

$$\frac{dU}{dx} = c \tag{18}$$

$$U\frac{dU}{dx} = c^2 x \tag{19}$$

4.$N = ax\left(\frac{a}{\nu}\right)^{\frac{1}{2}} h(\eta)$

$$\frac{\partial N}{\partial x} = a\left(\frac{a}{\nu}\right)^{\frac{1}{2}} h(\eta) \tag{20}$$

$$u\frac{\partial N}{\partial x} = axf'(\eta)a\left(\frac{a}{\nu}\right)^{\frac{1}{2}} h(\eta) = a^2 x\left(\frac{a}{\nu}\right)^{\frac{1}{2}} f'(\eta)h(\eta) \tag{21}$$

$$\frac{\partial N}{\partial y} = ax\left(\frac{a}{\nu}\right)^{\frac{1}{2}} h'(\eta)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} = \frac{a^2 x}{\nu} h'(\eta) \tag{22}$$

$$v\frac{\partial N}{\partial y} = -(a\nu)^{\frac{1}{2}} f(\eta)\left(\frac{a^2 x}{\nu}\right) h'(\eta) \tag{23}$$

$$\frac{\partial^2 N}{\partial y^2} = \frac{a^2 x}{\nu} h''(\eta)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} = \frac{a^2 x}{\nu}\left(\frac{a}{\nu}\right)^{\frac{1}{2}} h''(\eta) \tag{24}$$

5. $T = \triangle T\theta(\eta) + T_\infty$

$$\frac{\partial T}{\partial x} = 0 \ and \ u\frac{\partial T}{\partial x} = 0 \tag{25}$$

$$\frac{\partial T}{\partial y} = \triangle T\theta'(\eta)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} \triangle T\theta'(\eta) \tag{26}$$

$$v\frac{\partial T}{\partial y} = -a\triangle T f(\eta)\theta'(\eta) \tag{27}$$

$$\frac{\partial^2 T}{\partial^2 y} = \frac{a}{\nu}\triangle T\theta''(\eta) \tag{28}$$

Substituting these transformations from Eqs. (12–28) into Eqs. (1–6) gives the following ordinary differential equations as follows

$$(1+K)f''' + ff'' + C^2 - f'^2 + Kh' + H_a^2(C - f') = 0, \tag{29}$$

$$\left(1+\frac{K}{2}\right)h'' + fh' - f'h - K(2h + f'') = 0, \tag{30}$$

Therefore, according to ref. [28], we have $h(\eta) = -\frac{1}{2}f''(\eta)$, and hence Eqs.(29) and (30) reduce to the single equation as

$$\left(1+\frac{K}{2}\right)f''' + ff'' + C^2 - f'^2 + H_a^2(C - f') = 0. \tag{31}$$

By Substituting these transformation in Eq.(11) give the following equation

$$\left(1+\frac{4}{3R_d}\right)\theta'' + P_r f\theta' + H_a^2 B_r (C - f')^2 + B_r f''^2 + BP_r\theta = 0. \tag{32}$$

with boundary conditions

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = C, \tag{33}$$

$$\theta(0) = 1, \theta(\infty) = 0. \tag{34}$$

where $C = \frac{c}{a}$ is a stretching parameter where c and a are constant, $H_a^2 = \frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter, $P_r = \frac{c_p\mu}{k}$ is the Prandtl number, $R_d = \frac{kk^*}{4\sigma^* T_\infty^3}$ is the radiation parameter, $B_r = \frac{\mu U^2}{k\Delta T}$ is Brinkman number and $B = \frac{Q}{a\rho c_p}$ is the dimensionless heat generation/absorption parameter.

For micropolar boundary layer flow, the wall skin friction τ_w is given by:

$$\tau_w = \left[(\mu + k)\frac{\partial u}{\partial y} + kN\right]_{y=0}, \tag{35}$$

Using $U(x) = cx$ as characteristic velocity, the skin friction coefficient C_f can be defined as

$$C_f = \frac{\tau_w}{\rho u^2}, \tag{36}$$

Substituting by similarity transformations and (36) into (35), we get

$$C_f Re_x^{\frac{1}{2}} = \left(1+\frac{K}{2}\right) f''(0). \tag{37}$$

where $Re_x = xu/\nu$ is local Reynolds number

4. Numerical method for solution

The governing ordinary differential Eqs. (31) and (32) are nonlinear. Most of the physical systems are inherently nonlinear in nature and are of great interest to physicists, engineers and mathematicians. Problems involving nonlinear ordinary differential equations are difficult to solve and give rise to interesting phenomena such as chaos. We use a finite difference of second based numerical algorithm to solve the system of differential Eqs. (31) and (32). Following the procedures of Chamkha and Camille [7], Ashraf and Ashraf [31], Ashraf and Sumra [32], Ashraf et al. [33] and Ashraf et al. [34], we reduce the order of this equations by one with the help of the substitution $p = f'$ become as follows:

$$\left(1+\frac{K}{2}\right)p''+fp'+C-p^2+H_a(C-p)=0 \tag{38}$$

$$\left(1+\frac{4}{3R_d}\right)\theta''+P_r f\theta'+H_a^2B_r(C-f')^2+B_r p'^2+BP_r\theta=0 \tag{39}$$

subject to the boundary conditions (33) and (34) which become

$$f(0)=0,\ p(0)=1,\ p(\infty)=C \tag{40}$$

$$\theta(0)=1,\theta(\infty)=0 \tag{41}$$

The system of nonlinear differential Equations (38) and (39) is solved under the boundary conditions (40) and (41) by applying centeral finite difference method. Hence

$$p_i'=\frac{p_{i+1}-p_{i-1}}{2\Delta}+O(\Delta)^2. \tag{42}$$

$$p_i''=\frac{p_{i+1}-2p_i+p_{i-1}}{\Delta^2}+O(\Delta)^2. \tag{43}$$

$$\theta_i'=\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta}+O(\Delta)^2. \tag{44}$$

$$\theta_i''=\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{\Delta^2}+O(\Delta)^2. \tag{45}$$

By substituting from Eqs. (42–45) into Eqs. (38) and (39), we obtain

$$\begin{aligned}&\left(1+\frac{K}{2}\right)\left[\frac{p_{i-1}-2p_i+p_{i+1}}{\Delta^2}\right]+f_i\left[\frac{p_{i+1}-p_{i-1}}{2\Delta}\right]\\&+C^2-p_i^2+H_a^2(C-p_i)=0\end{aligned} \tag{46}$$

$$\begin{aligned}&\left[(2+K)-\Delta f_i\right]p_{i-1}+\left[-2(2+K)-2\Delta^2p_i-2\Delta^2H_a^2\right]p_i\\&+\left[(2+K)+\Delta f_i\right]p_{i+1}=-2\Delta^2C(C+H_a^2)\end{aligned} \tag{47}$$

$$\begin{aligned}&\left(1+\frac{4}{3R_d}\right)\left[\frac{\theta_{i-1}-2\theta_i+\theta_{i+1}}{\Delta^2}\right]+P_rf\left[\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta}\right]\\&+H_a^2B_r(C-p_i)^2+B_r\left[\frac{p_{i+1}-p_{i-1}}{2\Delta}\right]^2+BP_r\theta_i=0\end{aligned} \tag{48}$$

To compute the velocity $f(\eta)$ from $f'(\eta)$ for this problem by applying Trapezium rule as the following form:

$$f(\eta)=\int_0^\eta f'(\eta)d\eta=\int_0^\eta p(\eta)d\eta=\frac{\Delta\eta}{2}[p(0)+2\{p(\Delta\eta)+p(2\Delta\eta)+.......+p(\eta-\Delta\eta)\}+p(\eta)] \tag{49}$$

5. Results and discussion

Table 1 and Fig. 2–4 present the velocity profiles f, f', and the profile of temperature θ respectively, for various values of C. They show that increasing the parameter C increases both f and f' and decreases θ when the others parameters are constants. They show also that f increases while θ decreases by increasing η for any value of C, and the value of f' decreases or increases and then fixed depend on the value of C.

Table 2 and Fig. 5 and Fig. 6 present the velocity profiles of f and f', respectively, for various values of K. The figures show that increasing the parameter K decreases both f and f' when the others parameters are constants. They show also that f increases by increasing η for any value of K, while the value of f' increases and then fixed depend for all values of K.

Table 3 and Fig. 7 presents the profile of temperature θ for various values of K. It is observed that θ decreases with increasing the parameters K and η when the others parameters are constants.

Table 4 and Fig. 8 and Fig. 9 present the profile of temperature θ for various values of R_d. It is observed that θ decreases with increasing the parameters R_d and η when the others parameters are constants.

Table 5 and Fig. 10 and Fig. 11 present the profile of temperature θ for various values of P_r. It is observed that θ and η decreases with increasing P_r $E_c = B = 0$ and the others parameters are constants, whileθ increases and then decreases with increasing P_r after a critical value of η for other values of E_c and B.

Table 6 and Fig. 12 and Fig. 13 present the profile of temperature θ for various values of B. It is observed that θ increases with increasing the parameter B when others parameters are constants, while η decreases at any balue of B.

Table 7 and Fig. 14 and Fig. 15 present the profile of temperature θ for various values of E_c. It is observed that θ increases with increasing E_c when the others parameters are constants, while η increases and then decreases at any value of E_c.

Table 1. The variation of f, f' and θ at $C = 0.5, 1$ and 1.5 where others parameters are constant

η	C=0.5			$C = 1.0$			$C = 1.5$		
	f	f'	θ	f	f'	θ	f	f'	θ
0	0	1	1	0	1	1	0	1	1
1	0.737806064	0.616445516	0.652366105	1.002000003	1.000000003	0.582706369	1.306722576	1.431051587	0.525474278
2	1.2704499	0.522246586	0.375563306	1.998000007	1.000000002	0.273063168	2.788607163	1.494924242	0.200372543
3	1.775175955	0.50334772	0.187212204	3.000000008	1.000000001	0.099433619	4.291322993	1.499824737	0.05241771
4	2.276452749	0.500393337	0.077970263	4.002000008	1	0.027393036	5.794320905	1.499997325	0.009165031
5	2.774466777	0.500034877	0.023616594	4.998000008	1	0.00518064	7.288320901	1.499999982	0.001010123
6	3.275467232	0.5	0	6.000000008	1	0	8.791320901	1.5	0

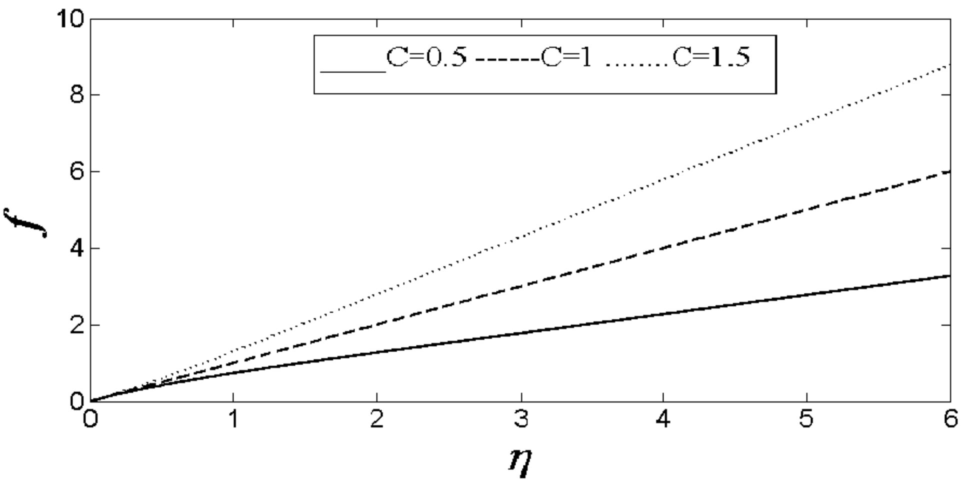

Fig. 2. Effect of C on the profile of f at H_a=1 and K=0.

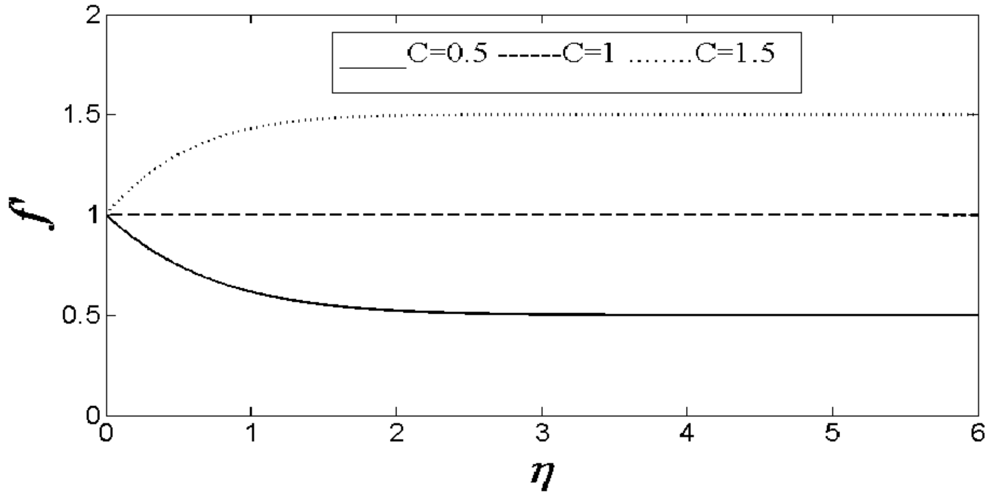

Fig. 3. Effect of C on the profile of f' at H_a=1 and K=1.

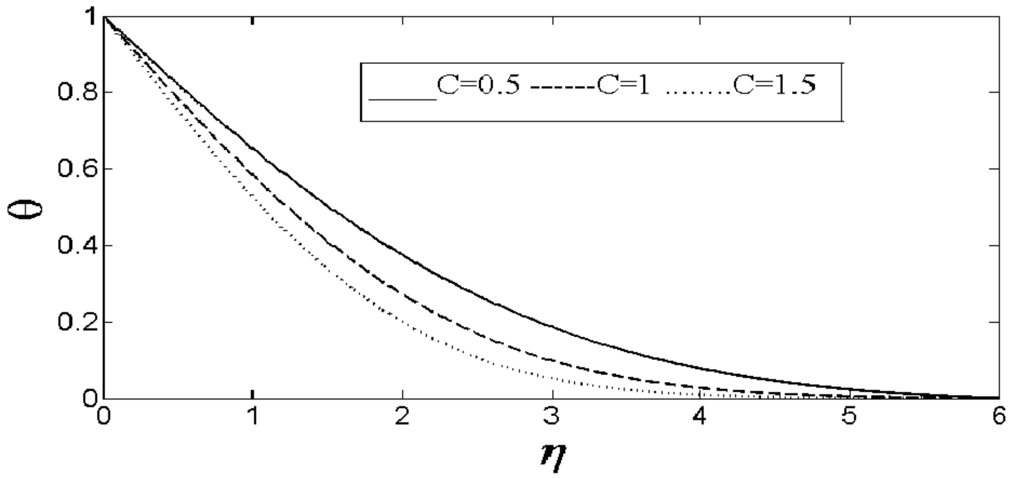

Fig. 4. Effect of C on the profile of θ at H_a=1, K=1, P_r=0.7, E_c=0, $R_d = 1$ and B=0.

Table 2. The variation of f and f' at $K = 0, 1$ and 2 where others parameters are constant

η	K=0		$K = 1.0$		$K = 2.0$	
	f	f'	f	f'	f	f'
0	0	1	0	1	0	1
1	2.213594755	1.459426393	2.048342229	1.431051584	1.940156627	1.406886351
2	5.158526905	1.498762741	4.921229728	1.49492424	4.737983757	1.489040291
3	8.163893974	1.499988466	7.920742778	1.499824737	7.72398066	1.49925679
4	11.16989207	1.49999997	10.92659557	1.499997325	10.72886454	1.499972147
5	14.15789207	1.5	13.91459432	1.499999982	13.71683296	1.499999434
6	17.16389207	1.5	16.92059432	1.5	16.72283252	1.5

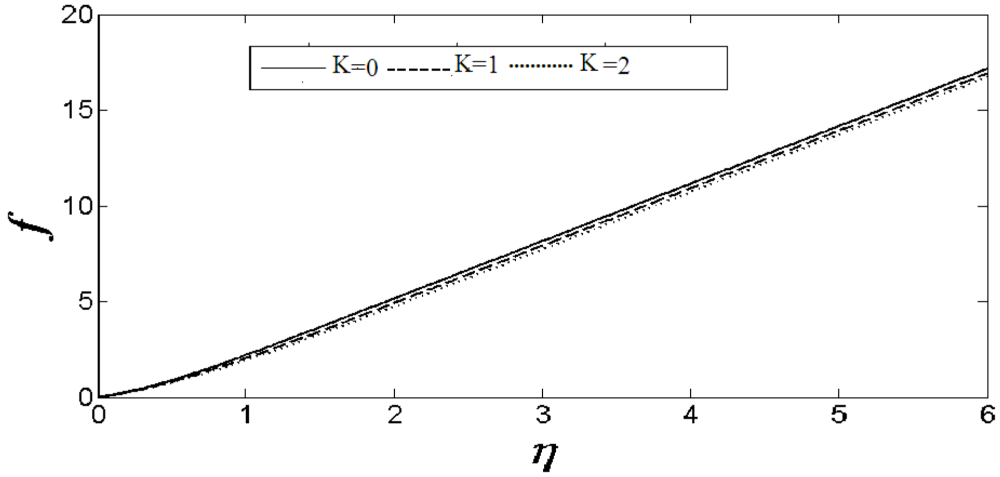

Fig. 5. Effect of K on the profile of f at H_a=0.5 and C=3.

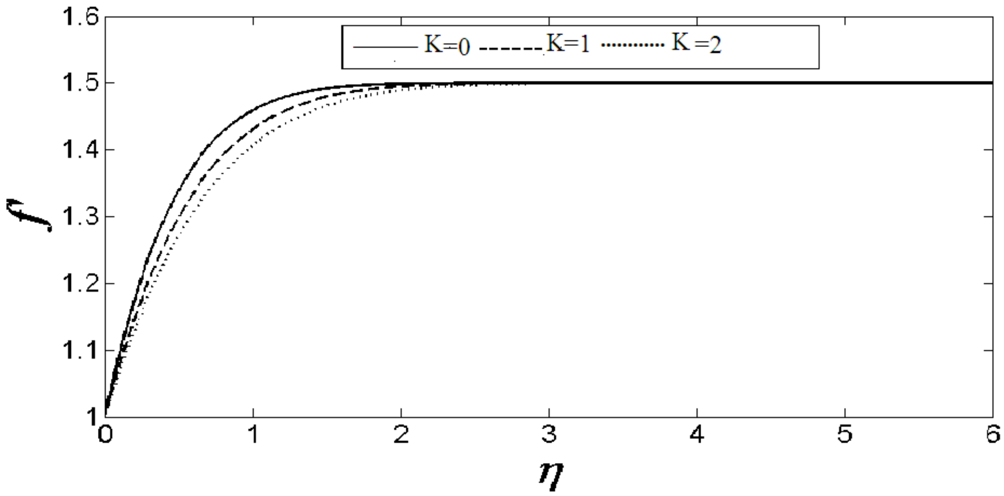

Fig. 6. Effect of K on the profile of f' at H_a=1 and C=1.5.

Table 3. The variation of θ at $K = 0, 5$ and $K = 10$ where others parameters are constant

η	θ		
	$K = 0$	$K = 5$	$K = 10$
0	1	1	1
1	0.740342252	0.71040458	0.700040953
2	0.515197029	0.465624402	0.448465896
3	0.331429337	0.278286672	0.260081769
4	0.188306192	0.146058634	0.131918789
5	0.080434407	0.057716189	0.050365647
6	0	0	0

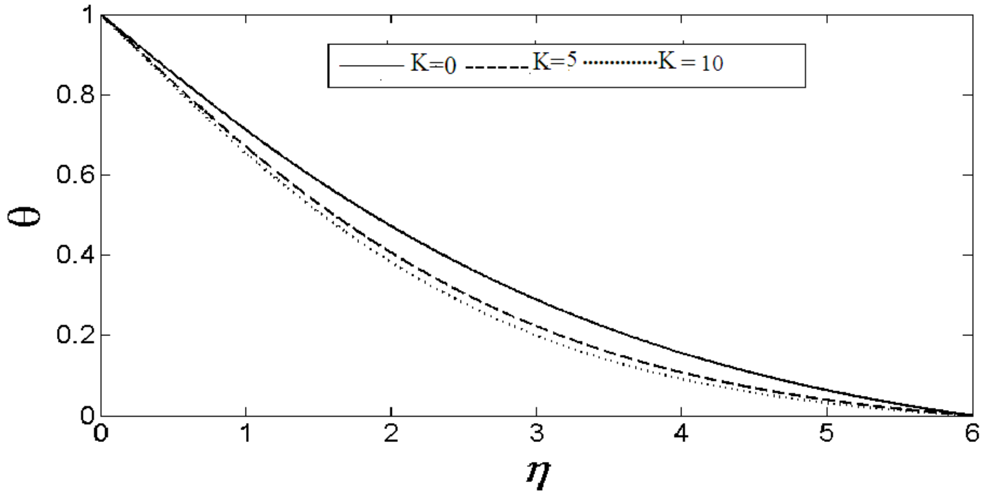

Fig. 7. Effect of K on the profile of θ at C=0.1, H_a=0, P_r=0.7, E_c=0, $R_d = 1$ and B=0.

Table 4. The variation of θ at $R_d = 1,2$ and 3 where others parameters are constant

η	θ					
	K=0.5, H_a=0.5, C=2, P_r=0.7, E_c=2 and B=2.			K=1, H_a=1, C=1.5, P_r=0.7, E_c=1 and B=2.		
	$R_d = 1$	$R_d = 2$	$R_d = 3$	$R_d = 1$	$R_d = 2$	$R_d = 3$
0	1	1	1	1	1	1
1	0.850583559	0.796084507	0.767886137	0.528298825	0.45840664	0.42696949
2	0.396687298	0.273685303	0.223979334	0.203177105	0.133691045	0.10774482
3	0.100991427	0.04035213	0.024599636	0.053686632	0.022810003	0.014562355
4	0.014024739	0.002558613	0.001020825	0.009486074	0.002205045	0.001016917
5	0.001037979	7.05268E-05	1.63416E-05	0.00105623	0.000117528	3.60707E-05
6	0	0	0	0	0	0

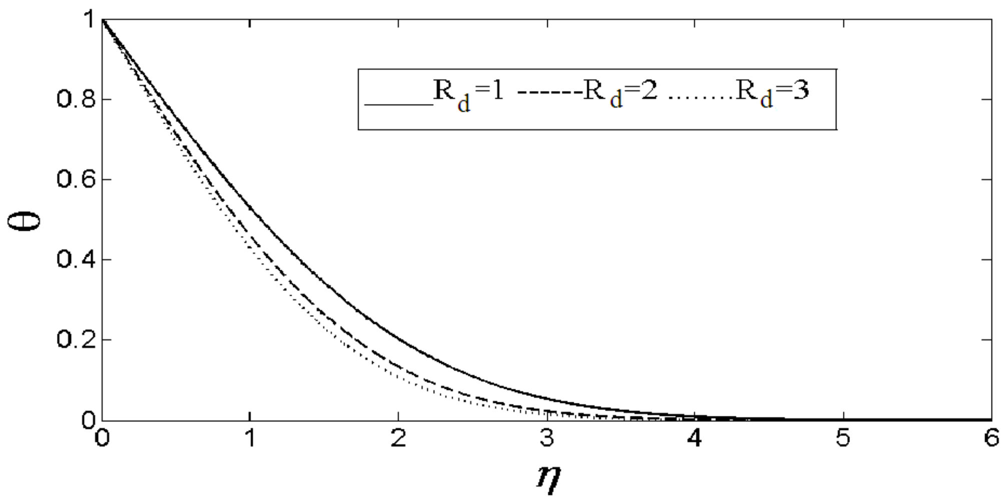

Fig. 8. Effect of R_d on the profile of θ at K=0.5, H_a=0.5, C=2, P_r=0.7, E_c=2 and B=2.

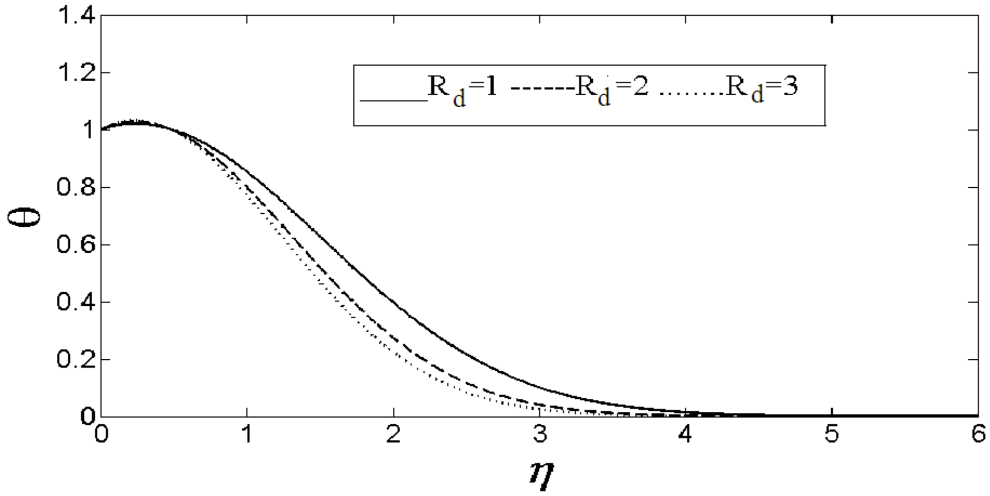

Fig. 9. Effect of R_d on the profile of θ at K=1, H_a=1, C=1.5, P_r=0.7, E_c=1 and B=2.

Table 5. The variation of θ at $P_r = 0.5, 1$ and 1.5 where others parameters are constant

η	θ					
	K=1, H_a=1, C=0.5, E_c=0, R_d=10^5 and B=0			K=1, H_a=1, C=1.5, E_c=1, R_d=10^5 and B=2		
	$P_r = 0.5$	$P_r = 1$	$P_r = 1.5$	$P_r = 0.5$	$P_r = 1$	$P_r = 1.5$
0	1	1	1	0 1	1	1
1	0.557351053	0.394519016	0.290250215	1.147736076	1.00771245	0.847490691
2	0.252196998	0.099658761	0.041871567	0.516097608	0.171154267	0.05371016
3	0.091284804	0.01591444	0.002976408	0.101957068	0.005917289	0.000323479
4	0.025892084	0.001599627	0.000104334	0.009156854	4.36706E-05	1.95421E-07
5	0.005196315	9.86865E-05	1.81585E-06	0.000378814	7.29657E-08	1.2848E-11
6	0	0	0	0	0	0

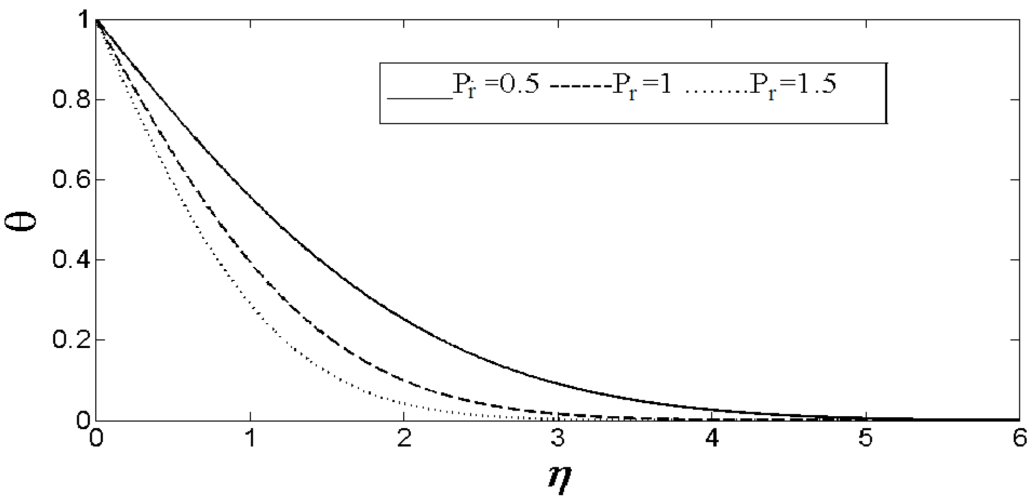

Fig. 10. Effect of P_r on the profile of θ at K=1, H_a=1, C=0.5, E_c=0, R_d=10^5 and B=0.

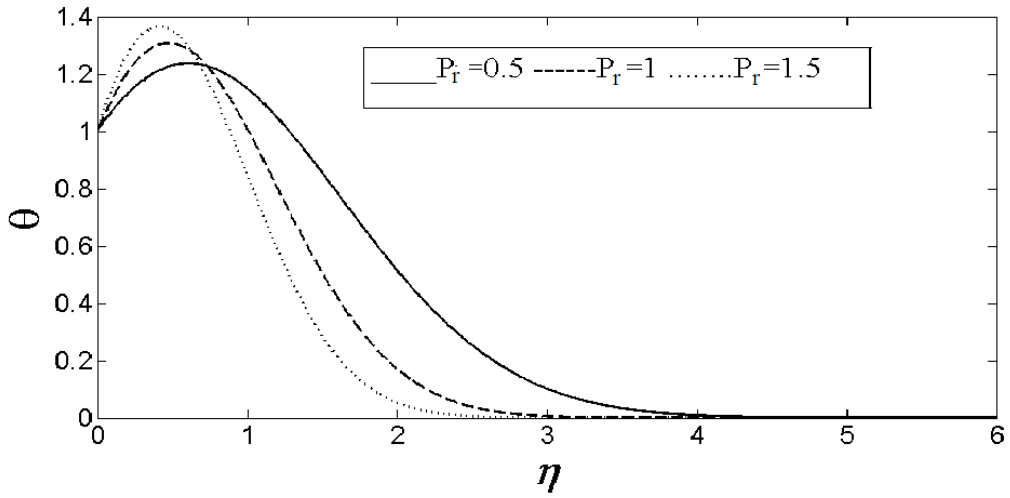

Fig. 11. Effect of P_r on the profile of θ at K=1, H_a=1, C=1.5, E_c=1, R_d=10^5 and B=2.

Table 6. The variation of θ at $B = 0, 0.5$ and 1 where others parameters are constant

η	θ					
	K=0.5, H_a=0.5, C=1, P_r=0.5, R_d=10^5 and E_c=1.5			K=1, H_a=1, C=1.5, P_r=0.7, R_d=10^5 and E_c=1		
	$B = 0$	$B = 0.5$	$B = 1$	$B = 0$	$B = 0.5$	$B = 1$
0	1	1	1	0 1	1	1
1	0.478612287	0.586980047	0.777729041	0.343527855	0.415770598	0.5235627
2	0.157697804	0.227659418	0.36824574	0.053159456	0.074051098	0.109000104
3	0.033874066	0.055847737	0.105104072	0.003251253	0.005042827	0.008321505
4	0.004635196	0.008516369	0.018034256	7.5045E-05	0.000126679	0.000228271
5	0.000387055	0.00077283	0.001785617	6.53869E-07	1.18177E-06	2.28404E-06
6	0	0	0	0	0	0

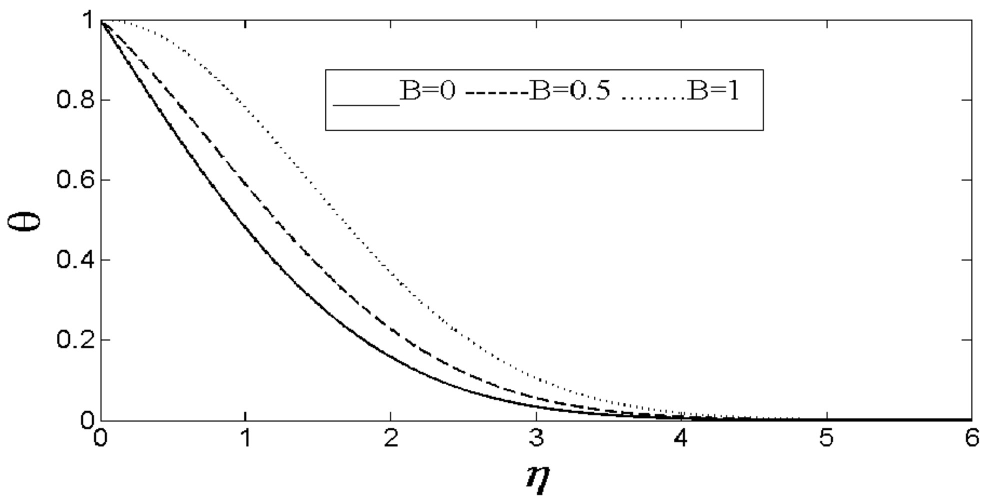

Fig. 12. Effect of B on the profile of θ at K=0.5, H_a=0.5, C=1, P_r=0.5, R_d=10^5 and E_c=1.5.

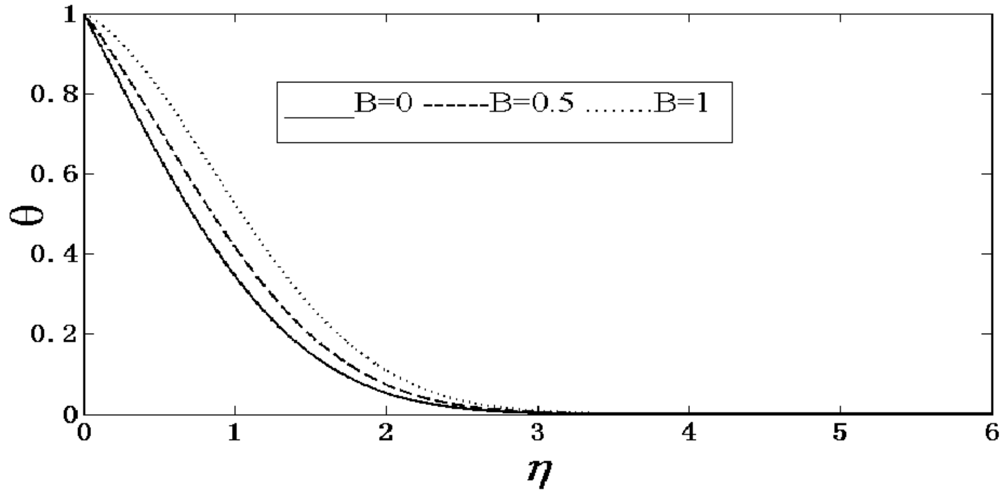

Fig. 13. Effect of B on the profile of θ at K=1, H_a=1, C=1.5, P_r=0.7, R_d=10^5 and E_c=1.

Table 7. The variation of θ at $E_c = 0, 0.5$ and 1 where others parameters are constant

η	θ					
	K=0.5, H_a=0.5, C=1, P_r=0.5, R_d=10^5 and E_c=1.5			K=1, H_a=1, C=1.5, P_r=0.7, R_d=10^5 and E_c=1		
	$E_c = 0$	$E_c = 5$	$E_c = 10$	$E_c = 0$	$E_c = 5$	$E_c = 10$
1	0.667068247	0.699194224	0.731320194	1.084871129	1.150957243	1.217043344
2	0.112465261	0.117927559	0.123389855	0.331287129	0.351723217	0.372159298
3	0.004627896	0.004852676	0.005077454	0.032627722	0.034640749	0.036653775
4	4.66913E-05	4.89592E-05	5.1227E-05	0.001079839	0.001146462	0.001213084
5	1.20113E-07	1.25947E-07	1.31781E-07	1.25026E-05	1.32739E-05	1.40453E-05
6	0	0	0	0	0	0

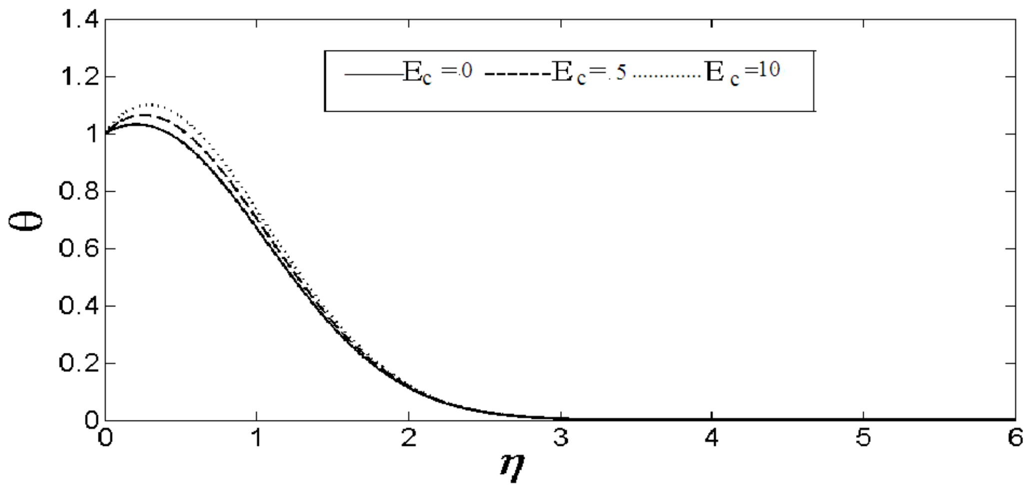

Fig. 14. Effect of E_c on the profile of θ at K=0.5, H_a=0.5, C=2, P_r=0.7, R_d=10^5 and B=2.

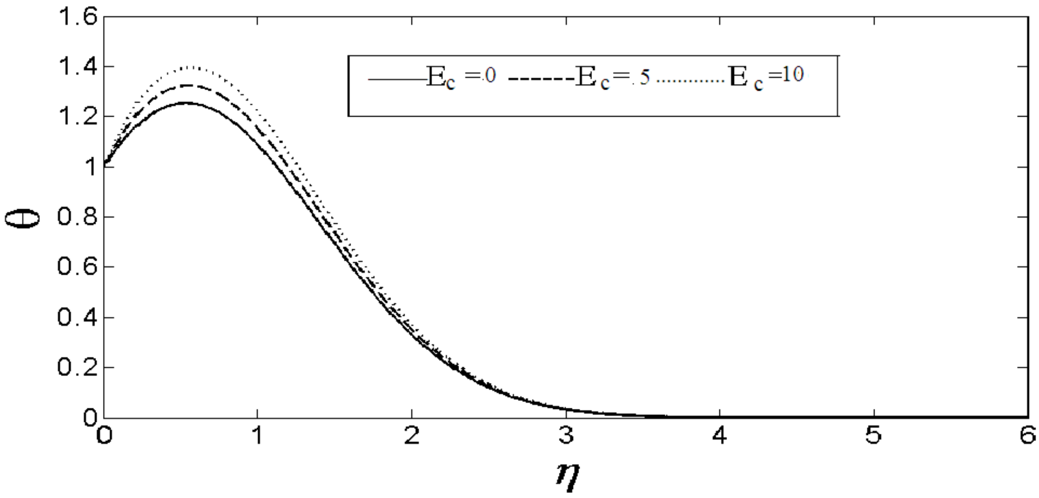

Fig. 15. Effect of E_c on the profile of θ at K=1, H_a=1, C=1.5, P_r=0.7, R_d=10^5 and B=2.

In order to verify the accuracy of the numerical solution of present work by using finite difference method we have compared these results for variation of skin friction $C_f Re_x^{\frac{1}{2}}$ with Hayat et al. [35] and Ishak and Nazar [27]. Table 8 showed that this approximate numerical solution is in good agreement with them.

Table 8. Comparison the present results with Hayat et al [35] and Ishak and Nazar [27] for skin friction $C_f Re_x^{\frac{1}{2}}$ when the material parameter K=0, $n=\frac{1}{2}$ and varies values of a stretching parameter C

		$C_f Re_x^{\frac{1}{2}}$	
C	Hayat et al. [35]	Ishak and Nazar [27]	present
0.1	-0.96938	-0.969387	-0.969424803244958
0.2	-0.91810	-0.918108	-0.918102413293237
0.5	-0.66732	-0.667265	-0.667255958315867
1	0.00000	0.000000	0.00000000
2	2.01750	2.017531	2.017481708640449
3	4.72928	4.729283	4.729232944214061

6. Conclusion

The problem of the stagnation point mhd flow and heat transfer of micropolar fluid over a stretching sheet in the presence of radiation, heat generation and Joule dissipation has been investigated. The effects of stretching parameter, material parameter, magnetic parameter, prandtl number, radiation parameter brinkman number and heat generation parameter on velocity and temperature profiles. A similarity transformation was used to convert the governing partial differential equations to ordinary ones. The transformed equations with associated boundary conditions were then solved numerically by finite difference method. The results for different parameters are summarized as follows:

- $f(\eta)$ and $f'(\eta)$ increase with increasing the stretching parameter C but temperature profile θ decreases with increasing of C.
- The material parameter K varies inversely with the three profiles of $f(\eta)$, $f'(\eta)$ and θ.
- The temperature profiles θ decrease with an increase in the radiation parameter R_d while it increases and then decreases with an increase of the Prandtl number P_r depending on the value of η.
- The temperature profiles θ increases with increasing both the heat generation/absorption parameter B and the Eckert number E_C.

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