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Reliability forecast of a parallel redundant power supply plant by using B. F. Technique

Research Article

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Abstract: In this paper, the author has considered a power supply plant for reliability forecast by Boolean function technique. The authors have used algebra of logics and Boolean function technique for the purpose of formulation of mathematical model and its solution. Reliability and M.T.T.F. for the system have evaluated. Some particular cases have also given to improve practical utility of the model. Graphical illustration followed by a numerical example has appended in last to highlight the important results of study. It is Observed that reliability of the system decreases rapidly in case, when failures follow Weibull time distribution, while it decreases smoothly and in better way when failures follow exponential time distribution.

MSC: 90B25 • 68M15

Keywords: Boolean function technique • Reliability • Mean time to failure • Special cases

1. Introduction

In this paper, the author has considered a power supply plant for reliability forecast .In this power plant, power is generated by the generators namely G_1, G_2 and G_3. These generators are working in parallel redundancy. These power generators are connected by switch Boards SD_1, SD_2; sub power board SPB; output board OPB. All these components are connected with hundred percent reliable cables $C_1, C_2, ...C_{10}$. The aim of plant is to supply the power generated by generators to critical consumer through output board OPB. Only one generator is sufficient to fulfill the requirement of consumer. On failure of one generator other has to work to fulfill the same requirement. The supply of power can fail if atleast one component in all the routes of supply, fails. System Configuration has shown in Fig. 1. The reliability of every component of power plant is known in advance.

Mathematical logic and Boolean function technique have been used to formulate and solve the symbolic model. The reliability of power plant is calculated considering that failure times of various components of power plant follow exponential and Weibull time distribution. Moreover an important reliability parameter M.T.T.F. has also been calculated for exponential failure time distribution. A numerical example with graphical illustration has given at the end to highlight the obtained results.

2. Formulation of mathematical model

Assumptions

The following assumptions have been associated with this model:

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Nomenclature

x_K	K^{th} State
$K = 1,2,3$	Generators G_1, G_2 and G_3
$K = 4,5$	Switch Boards SD_1 and SD_2
$K = 6$	Sub power Board SPB
$K = 7$	Out put Board OPB
x'_K	Negation of K^{th} state quad $= \begin{cases} 0, & \text{in failed state} \\ 1, & \text{in good state} \end{cases}$, $K = 1,2,3,....7$
| |	Logical Matrix
$\wedge, \vee$	Conjunction, Disjunction
$C_1, C_2, ...C_{10}$	Cables to connect different components of power plant

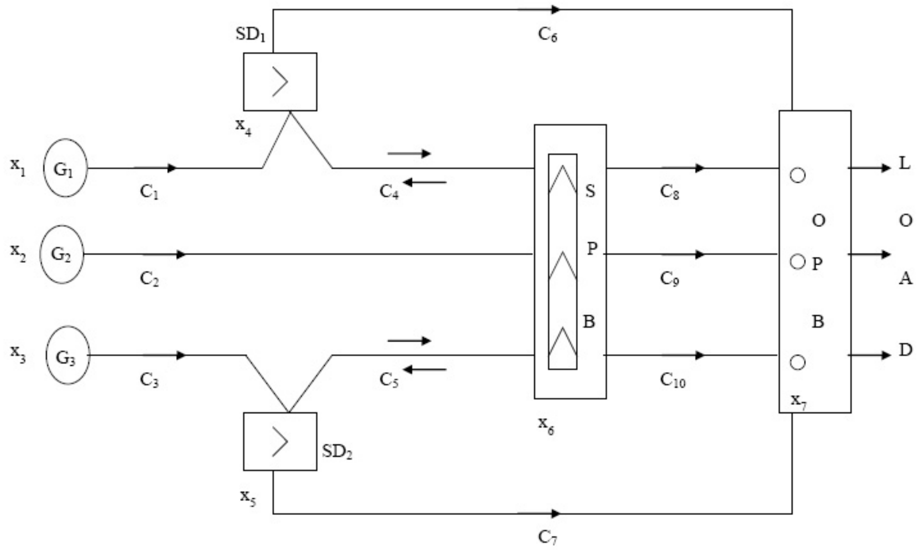

Fig. 1. System configuration

1. Initially, all the components of power plant are new and are working with full efficiency.
2. There is no repair facility to the failed component.
3. The supply of power can fail only if atleast one component in all routes of power supply fails.
4. Failures are statistically independent.
5. The state of each component and of the whole power plant can be either good or bad.
6. The cables used to connect the different components of power plants, are hundred percent reliable.
7. The reliability of each component is pre-calculated.

By using B.F. Technique, one may obtain the following logical matrix, which gives the conditions of capability of successful operation of the power plant under consideration:

$$F(x_1, x_2, ...x_7) = \begin{vmatrix} x_1 & x_4 & x_7 & \\ x_1 & x_6 & x_7 & \\ x_1 & x_5 & x_6 & x_7 \\ x_2 & x_6 & x_7 & \\ x_2 & x_4 & x_6 & x_7 \\ x_2 & x_5 & x_6 & x_7 \\ x_3 & x_5 & x_7 & \\ x_3 & x_6 & x_7 & \\ x_3 & x_4 & x_6 & x_7 \end{vmatrix} \tag{1}$$

3. Solution of the model

Applying algebra of logic, Eq. (1) may be rewritten as:

$$F(x_1, x_2 \ldots x_7) = |x_1 \wedge f(x_1, x_2, \ldots x_7)| \tag{2}$$

$$f(x_1, x_2, \ldots x_6) = \begin{vmatrix} x_1 & x_4 & \\ x_1 & x_6 & \\ x_1 & x_5 & x_6 \\ x_2 & x_6 & \\ x_2 & x_4 & x_6 \\ x_2 & x_5 & x_6 \\ x_3 & x_5 & \\ x_3 & x_6 & \\ x_3 & x_4 & x_6 \end{vmatrix} = \begin{vmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \end{vmatrix} \tag{3}$$

where

$$M_1 = \begin{vmatrix} x_1 & x_4 \end{vmatrix}, M_2 = \begin{vmatrix} x_1 & x_6 \end{vmatrix}, \quad \text{etc.}$$

Using orthogonalisation algorithm, Eq. (3) may be written as:

$$f(x_1, x_2, \ldots x_6) = \begin{vmatrix} M_1 &&&&&&&& \\ M_1 & M_2 &&&&&&& \\ M_1 & M_2 & M_3 &&&&&& \\ M_1 & M_2 & M_3 & M_4 &&&&& \\ M_1 & M_2 & M_3 & M_4 & M_5 &&&& \\ M_1 & M_2 & M_3 & M_4 & M_5 & M_6 &&& \\ M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 && \\ M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 & \\ M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 & M_9 \end{vmatrix} \tag{4}$$

By making use of algebra of logic, we obtained the following results:

$$M_1 = \begin{vmatrix} x_1 & x_4 \end{vmatrix}, \quad M_2 = \begin{vmatrix} x_1 & x_6 \end{vmatrix}$$

$$M_1' = \begin{vmatrix} x'_1 & \\ x'_1 & x'_4 \end{vmatrix} \tag{5}$$

$$M_1' M_2 = \begin{vmatrix} x'_1 & \\ x'_1 & x'_4 \end{vmatrix} \wedge \begin{vmatrix} x_1 & x_6 \end{vmatrix} = \begin{vmatrix} x_1 & x'_4 & x_6 \end{vmatrix} \tag{6}$$

Similarly,

$$M'_1 M'_2 M_3 = \begin{vmatrix} x_1 & x'_4 & x_5 & x_6 \end{vmatrix} \tag{7}$$

$$M'_1 M'_2 M'_3 M_4 = \begin{vmatrix} x'_1 & x_2 & x_6 \end{vmatrix} \tag{8}$$

$$M'_1 M'_2 M'_3 M'_4 M_5 = \begin{vmatrix} x'_1 & x_2 & x_4 & x_6 \end{vmatrix} \tag{9}$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M_6 = \begin{vmatrix} x'_1 & x_2 & x_5 & x_6 \end{vmatrix} \tag{10}$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 M_7 = \begin{vmatrix} x'_1 & x'_2 & x_3 & x_5 \end{vmatrix} \tag{11}$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 M'_7 M_8 = \begin{vmatrix} x'_1 & x'_2 & x_3 & x'_5 & x_6 \end{vmatrix} \tag{12}$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 M'_7 M'_8 M_9 = \begin{vmatrix} x'_1 & x'_2 & x_3 & x_4 & x'_5 & x_6 \end{vmatrix} \tag{13}$$

So Eq. (4) becomes by making use of Eqs. (5) through (13)

$$f(x_1, x_2, ..., x_6) = \left| \begin{array}{llllll} x_1 & x_4 & & & & \\ x_1 & x'_4 & x_6 & & & \\ x_1 & x'_4 & x_5 & x_6 & & \\ x'_1 & x_2 & x_6 & & & \\ x'_1 & x_2 & x_4 & x_6 & & \\ x'_1 & x_2 & x_5 & x_6 & & \\ x'_1 & x'_2 & x_3 & x_5 & & \\ x'_1 & x'_2 & x_3 & x'_5 & x_6 & \\ x'_1 & x'_2 & x_3 & x_4 & x_5 & x_6 \end{array} \right| \tag{14}$$

By using Eq. (14), (2) becomes

$$F(x_1, x_2, ..., x_7) = \left| \begin{array}{lllllll} x_1 & x_4 & x_7 & & & & \\ x_1 & x'_4 & x_6 & x_7 & & & \\ x_1 & x'_4 & x_5 & x_6 & x_7 & & \\ x'_1 & x_2 & x_6 & x_7 & & & \\ x'_1 & x_2 & x_4 & x_6 & x_7 & & \\ x'_1 & x_2 & x_5 & x_6 & x_7 & & \\ x'_1 & x'_2 & x_3 & x_5 & x_7 & & \\ x'_1 & x'_2 & x_3 & x'_5 & x_6 & x_7 & \\ x'_1 & x'_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \right| \tag{15}$$

Since, R.H.S. of (15) is disjunction of pair wise disjoint conjunctions, so that the reliability of the considered power plant is given by:

$$\begin{aligned} R_s &= Pr¿F(x_1, x_2, ...x_7) = 1À \\ &= R_7[R_1R_4 + R_1(1 -- R_4)R_6 + R_1(1 - R_4)R_5R_6 + (1 -- R_1)R_2R_6 \\ &\quad + (1 - R_1)R_2R_4R_6 + (1 - R_1)R_2R_5R_6 + (1 - R_1)(1 - R_2)R_3R_5 + (1 - R_1)(1 - R_2)R_3(1 - R_5)R_6 \\ &\quad + (1 - R_1)(1 - R_2)R_3R_4(1 - R_5)R_6] \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad R_s = R_7[&R_1R_4 + R_1R_6 - R_1R_4R_6 + R_1R_5R_6 - R_1R_4R_5R_6 + R_2R_6 + R_2R_5R_6 \\ &- R_1R_2R_6 - R_1R_2R_4R_6 - R_1R_2R_5R_6 + R_3R_5 + R_3R_6 - R_3R_4R_6 - R_3R_5R_6 + R_2R_4R_6 \\ &- R_3R_4R_5R_6 - R_1R_3R_5 - R_1R_3R_6 + R_1R_3R_5R_6 - R_1R_3R_4R_6 + R_1R_3R_4R_5R_6 \\ &- R_2R_3R_5 - R_2R_3R_6 + R_2R_3R_5R_6 - R_2R_3R_4R_6 + R_2R_3R_4R_5R_6 + R_1R_2R_3R_5 + R_1R_2R_3R_6 \\ &- R_1R_2R_3R_5R_6 + R_1R_2R_3R_4R_6 - R_1R_2R_3R_4R_5R_6] \end{aligned} \tag{16}$$

where, $R_1, R_2, ...R_7$ are the reliabilities of the components of the considered power plant corresponding to the states $x_1, x_2, ...x_7$ respectively.

4. Some particular cases

Case I:

If reliability of each component of considered power plant is being R, Eq. (16) yields

$$R_s = R^3\left[5 - 3R - 2R^2 + 2R^3 - R^4\right] \tag{17}$$

Case II:

When failure rates follow Weibull distribution:

Let failure rates of the states $x_1, x_2...x_7$ are $\lambda_1, \lambda_2, ...\lambda_7$ respectively, then from Eq. (16) reliability of power plant at instant 't' is given by

$$R_{SW}(t) = \sum_{i=1}^{i=16} \exp\{-b_i t^p\} - \sum_{j=1}^{j=15} \exp\{-a_j t^p\} \tag{18}$$

where, p is a positive parameter and $bi's$ and $aj's$ are mentioned as below:

$b_1 = \lambda_1 + \lambda_4 + \lambda_7$
$b_2 = \lambda_1 + \lambda_6 + \lambda_7$
$b_3 = \lambda_1 + \lambda_5 + \lambda_6 + \lambda_7$
$b_4 = \lambda_2 + \lambda_6 + \lambda_7$
$b_5 = \lambda_2 + \lambda_4 + \lambda_6 + \lambda_7$
$b_6 = \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7$
$b_7 = \lambda_3 + \lambda_5 + \lambda_7$
$b_8 = \lambda_3 + \lambda_6 + \lambda_7$
$b_9 = \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$
$b_{10} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$
$b_{11} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
$b_{12} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$
$b_{13} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
$b_{14} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7$
$b_{15} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7$
$b_{16} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$

$a_1 = \lambda_1 + \lambda_4 + \lambda_6 + \lambda_7$
$a_2 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
$a_3 = \lambda_1 + \lambda_2 + \lambda_6 + \lambda_7$
$a_4 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_6 + \lambda_7$
$a_5 = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7$
$a_6 = \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$
$a_7 = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
$a_8 = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_7$
$a_9 = \lambda_1 + \lambda_3 + \lambda_6 + \lambda_7$
$a_{10} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$
$a_{10} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$
$a_{11} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7$
$a_{12} = \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7$
$a_{13} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$
$a_{14} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$
$a_{15} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$

Case III:

When failure rates follow exponential time distribution:

Exponential distribution is particular case of Weibull distribution for $p = 1$.Thus , the reliability of power plant in this case at time âĂŸtâĂŹ, can be obtained by putting $p = 1$, in Eq. (18) as

$$R_{SE}(t) = \sum_{i=1}^{16} \exp\{-b_i t\} - \sum_{j=1}^{15} \exp\{-a_j t\} \tag{19}$$

where, $bi's$ and $aj's$ are stated earlier in case II.

Also, an important reliability parameter $M.T.T.F.$, in this case, will be

$$M.T.T.F. = \int_0^{\infty} R_{SE}(t)\, dt = \sum_{i=1}^{16} \frac{1}{b_i} - \sum_{j=11}^{15} \frac{1}{a_j} \tag{20}$$

Table 1.

t	$R_s w(t)$	$R_S E(t)$
0	1	1
1	0.981108	0.98111
2	0.750237	0.9661
3	0.239035	0.89115
4	0.035615	0.75024
5	0.002622	0.61485
6	0.0001	0.49442
7	2.06×10^{-6}	0.392
8	2.29×10^{-8}	0.30743
9	1.40×10^{-10}	0.23904
10	4.68×10^{-13}	0.18456

Table 2.

s	$M > T.T.F.$
0	∞
0.01	70.714
0.02	35.357
0.03	23.571
0.04	17.679
0.05	14.143
0.06	11.786
0.07	10.102
0.08	8.8393
0.09	7.8571
0.1	7.0714
0.11	6.4286

5. Numerical computation

Setting $\lambda_1 = \lambda_2 = ... = \lambda_7 = s$, in relations (18) through (20), one can obtain

$$R_{SW}(t) = 5e^{-3st^p} - 3e^{-4st^p} - 2e^{-5st^p} + 2e^{-6st^p} - e^{-7st^p} \tag{21}$$

$$R_{SW}(t) = 5e^{-3st} - 3e^{-4st} - 2e^{-5st} + 2e^{-6st} - e^{-7st} \tag{22}$$

and

$$M.T.T.F = \frac{1}{s}\left[\frac{5}{3} - \frac{3}{4} - \frac{2}{5} + \frac{2}{6} - \frac{1}{7}\right] = \frac{0.7071429}{s} \tag{23}$$

In Eq. (21), put $p = 2, s = 0.1$ and $t = 0, 1, 2,$ In Eq. (22), put $s = 0.1$ and $t = 0, 1, 2,$ In Eq. (23), put $s = 0.01, 0.02.$ One can compute the Table 1 and Table 2, and sketch the graphs as shown in Fig. 1 and Fig. 2, respectively.

6. Interpretation of result

Fig. 2 has drawn by using Table 1 and it shows the reliability of the power plant decreases in which manner as we make increases in the time 't' for Weibull and exponential time distributions. A critical examination of graph reveals that $R_{sw}(t)$ decreases rapidly with increase in time while RSE(t) decreases in a uniform way.

Fig. 3 has drawn by using Table 2 and it show how the $M.T.T.F.$ of power plant decreases with increase in failure rate 's'. A critical examination of graph reveals that for $s = 0.01 to 0.04$, $M.T.T.F.$ decreases very rapidly but after $s = 0.05$it decreases approximately in a uniform manner.

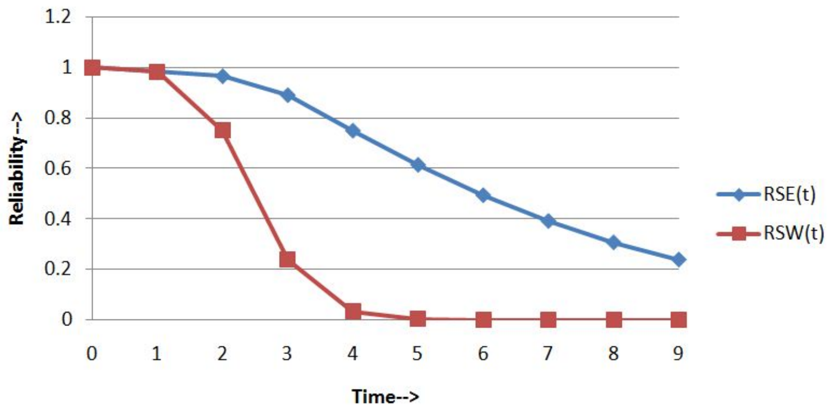

Fig. 2. Reliability Vs Time

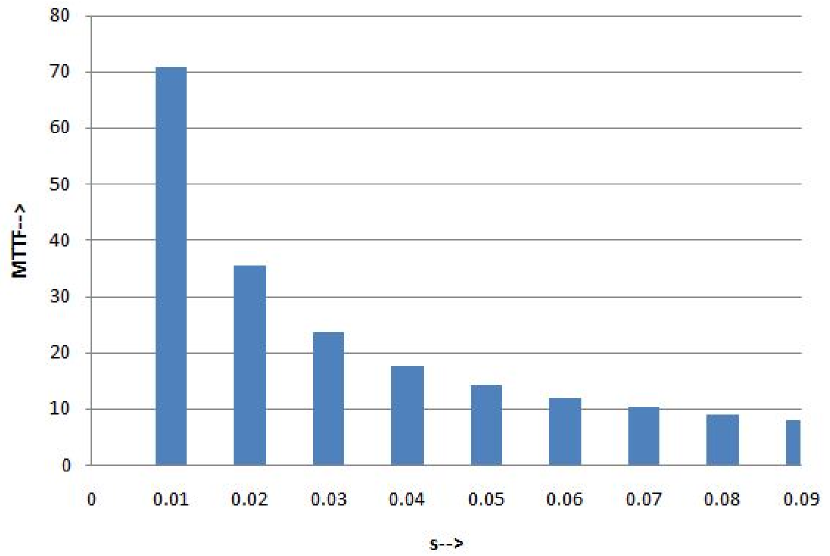

Fig. 3. M.T.T.F. Vs s

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