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Three-phase-lag elasto-thermodiffusive response in an elastic solid under hydrostatic pressure

Research Article

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Abstract: The present work deals with the investigation of elasto-thermodiffusive interactions in a homogeneous and isotropic half-space under initial hydrostatic stress in which the heat conduction equation is considered the context of three-phase-lag model, Green-Naghdi model II (i.e. the model which predicts thermoelasticity without energy dissipation) and Green-Naghdi model III (i.e. the model which predicts thermoelasticity with energy dissipation) of generalized thermoelasticity. The resulting non-dimensional coupled equations are applied to a specific problem of a half-space whose surface is situated under hydrostatic pressure, is traction-free and subjected to time-dependent chemical loadings. The analytical expressions for the displacements, stresses, temperature, mass concentration and the chemical potential are obtained in the physical domain by employing normal mode analysis. These expressions are also calculated for a copper-like material and have been depicted graphically. A comparative study of diffusive medium and thermoelastic medium is carried out, and it was seen that the effect of diffusion is significant on all the studied fields. The comparison between the models are also analyzed and the effect of hydrostatic pressure is discussed. In absence of thermodiffusion, all the results agree with the results of existing literature.

MSC: 74F05

Keywords: Generalized thermoelastic diffusion • Three-phase-lag thermoelastic model • Finite wave speed • Normal mode analysis • Hydrostatic pressure

1. Introduction

The topic thermoelastic diffusion deals with the coupling effects of the fields of temperature, mass diffusion and strain, in addition to heat and mass exchange with the environment [1]. Diffusion can be defined as movement of particles from regions of high concentration to regions of lower concentration until equilibrium is reached. Now a days, it has extensive industrial applications: for example, oil companies are interested in the process of thermodiffusion, as it is efficient in extraction of oil from oil deposits. Nowacki [2–5] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. It is well established that in classical coupled thermoelasticity theory (CCTE), the heat conduction equation represents the infinite speed of propagation of the thermal wave, which contradicts the physical observations. To overcome the paradox of infinite speed of thermal wave inherent in the CCTE theory, the subject of generalized thermoelasticity theory is developed. Lord and Shulman [6] formulated the generalized thermoelasticity theory introducing one relaxation time and thus transforming the heat conduction equation into a hyperbolic type. Green and Lindsay [7] introduced one more theory, called GL theory, which involves two relaxation times. Later Green and Naghdi [8–10] developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labeled as models I, II and III.

The next generalization to the thermoelasticity theory is known as the dual phase lag model developed by Tzou [11] and Chandrasekharaiah [12]. Tzou [11] considered micro-structural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed due to

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phonon-electron interactions on the macroscopic level. Tzou [11] introduced two-phase lags to both the heat flux vector and the temperature gradient and considered a constitutive equation to describe the lagging behavior in the heat conduction in solids. Roychoudhuri [13] has studied one dimensional thermo-elastic wave propagation in an elastic half-space in the context of dual-phase-lag model. Recently, the exponential stability [14, 15] and condition of the delay parameters in the dual-phase-lag theory under this model has been studied by Quintanilla. Kumar, Prasad and Mukhopadhyay [16] have studied the propagation of finite thermal wave in the context of dual-phase-lag model. The problem of finite thermal wave propagation in a half-space under variable thermal loading have been studied by Sur and Kanoria [17].

Recently, Roychoudhuri [18] has established a generalized mathematical model of a coupled thermoelasticity theory that includes three-phase lags in the heat flux vector, the temperature gradient and in the thermal displacement gradient. The more general model established reduces to the previous models as special cases. According to this model, the heat flux has been modified as $\vec{q}(P,t+\tau_q) = -\left[K\vec{\nabla}T(P,t+\tau_T) + K^\star\vec{\nabla}\nu(P,t+\tau_\nu)\right]$ where $\vec{\nabla}\nu$ $(\dot{\nu} = T)$ is the thermal displacement gradient and $K^\star$ is the additional material constant. To study some practical relevant problems, particularly in heat transfer problems involving very short time intervals and in the problems of very high heat fluxes, the hyperbolic equation gives significantly different results than the parabolic equation. According to this phenomenon, the lagging behavior in the heat conduction in solids should not be ignored particularly when the elapsed times during a transient process are very small, say about 10^{-7}s or the heat flux is very much high. Three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering etc., where the delay time τ_q captures the thermal wave behavior (a small scale response in time), the phase-lag τ_T captures the effect of phonon-electron interactions (a microscopic response in space), the other delay time τ_ν is effective, since, in the three-phase-lag model, the thermal displacement gradient is considered as a constitutive variable whereas in the conventional thermoelasticity theory temperature gradient is considered as a constitutive variable.

Kumar and Mukhopadhyay [19] studied the effects of three-phase-lags on an infinite medium with a cylindrical cavity. Kanoria and Mallik [20] investigated the generalized thermoviscoelastic interaction due to periodically varying heat source with three-phase-lag effect. Kar and Kanoria [21] studied generalized thermoelastic functionally graded orthotropic hollow sphere under thermal shock with three-phase- lag effect. Ezzat et al. [22] investigated fractional order theory in thermoelastic solid with three-phase-lag heat transfer. Employing this theory, Karamany and Ezzat [23] studied one problem on micropolar thermoelasticity theory. Recently, some problems on three-phase lag model have been discussed by Islam et al. [24], Sur and Kanoria [25–29], Pal et al. [30] or in the following literatures [31–33].

Recently, Sherief et al. [34] developed the generalized thermoelastic diffusion theory with one relaxation time, which allowed waves to propagate at finite speeds. Sherief and Saleh [35] investigated the problem of a thermoelastic half-space in the context of the generalized thermoelastic diffusion theory with one relaxation time. Singh discussed the reflection wave phenomena from the free surface of an elastic solid with generalized thermodiffusion with one relaxation time in [36] and with two relaxation times in [37]. Aouadi sudied diffusion in an infinitely long solid cylinder [38] and in an infinite elastic body with spherical cavity [39]. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Aouadi [40] on the basis of Laplace transform method. Aouadi [41] discussed generalized theory of thermoelastic diffusion for anisotropic media. recently, Othman et al. [42] analyzed the effects of diffusion on a two dimensional problem of generalized thermoelasticity in the context of the Green-Naghdi theory. Kumar and Kansal [43] discussed propagation of waves on free surface of a transversely isotropic body under generalized thermoelastic diffusion. Kothari and Mukhopadhyay [44] have presented thermoelastic diffusion inside a spherical shell under three different theories. Wang et al. [45] have studied for the thermoelastic dynamic solution of a multilayered spherically isotropic hollow sphere for spherically symmetric problems. Such a body is said to possess transverse isotropic about any radius vector drawn from the center of the sphere to a given point of material. Recently, employing the elasto-thermodiffusive response, several problems have been solved by A. M. El-Sayed [46], Karmakar and Kanoria [47]; Bhattacharya and Kanoria [48, 49] etc.

The objective of the present contribution is to study the generalized thermoelastic diffusion in a isotropic, thermoelastic half-space in which the bounding plane is subjected to hydrostatic pressure and a prescribed chemical potential and is free of traction. The heat conduction equation has been formulated introducing three-phase-lag model of heat conduction from which Green Naghdi models II and III can be obtained as particular cases. Introducing normal mode analysis, the governing equations have been expressed in cartesian coordinates and the numerical estimates for the thermal stress, temperature, mass concentration and chemical potential are computed for a copper-like material and have been depicted graphically and most significant points arising from our analysis are highlighted. The comparison among the models have been reported. A comparative study of diffusive medium and thermoelastic medium is carried out and the effect of hydrostatic pressure is analyzed.

2. Basic equations

The basic governing equations for homogeneous, isotropic generalized thermodiffusive elastic solid in absence of body forces and heat sources are, as follows

$$\sigma_{ij} = -\mathscr{P}(\delta_{ij} + \omega_{ij}) + 2\mu e_{ij} + \left[\lambda e_{kk} - \beta_1\theta - \beta_2 C\right]\delta_{ij}, \quad i,j = 1,2,3 \tag{1}$$

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\dot{q}_i=-K_{ij}\left(1+\tau_T\frac{\partial}{\partial t}\right)\dot{\theta}_{,j}-K^\star_{ij}\left(1+\tau_v\frac{\partial}{\partial t}\right)\theta_{,j}, \tag{2}$$

The entropy equation and the equation of conservation of mass are given by

$$q_{i,i}+\rho\theta_0\dot{S}+P\eta_{i,i}=0, \tag{3}$$

$$\eta_{i,i}=-\dot{C}, \tag{4}$$

The flow of the diffusion mass vector is given by

$$\left(1+\widetilde{\tau}_q\frac{\partial}{\partial t}+\frac{\widetilde{\tau_q}^2}{2}\frac{\partial^2}{\partial t^2}\right)\dot{\eta}_j=-\left[D^\star\left(1+\widetilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\widetilde{\tau}_T\frac{\partial}{\partial t}\right)\right]P_{,i}, \tag{5}$$

and, the chemical potential is given by

$$P=-\beta_2 e_{kk}+dC-c\theta, \tag{6}$$

where D and $D^\star$ are are diffusive constants, the strain tensor and ω_{ij} is the rotation tensor, $\beta_1=(3\lambda+2\mu)\alpha_t$, $\beta_2=(3\lambda+2\mu)\alpha_c$, α_t is the coefficient of linear thermal expansion, α_c is the coefficient of linear diffusion expansion, θ is the increase of temperature over the reference temperature θ_0, q_i are the components of the heat flux vector $\vec{q}$, C is the mass concentration, P is the chemical potential, S is the entropy per unit mass respectively, η_i denotes the flow of the diffusion mass vector; τ_T and τ_q are the relaxation times for Dual-phase-lag model; $\widetilde{\tau}_T$ and $\widetilde{\tau_q}$ are the diffusion relaxation times; c and d are the measures of thermo-diffusion effect and diffusive effect, respectively, where

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right). \tag{7}$$

$$\omega_{ij}=\frac{1}{2}\left(u_{j,i}-u_{i,j}\right). \tag{8}$$

Stress equation of motion in absence of body force is

$$\left(\mu-\frac{\mathscr{P}}{2}\right)u_{i,kk}+\left(\lambda+\mu+\frac{\mathscr{P}}{2}\right)u_{k,ik}-\beta_1\theta_{,i}-\beta_2 C_{,i}=\rho\ddot{u}_i,\quad i,j=1,2,3 \tag{9}$$

The heat equation for the dynamic coupled generalized thermoelasticity based on the three-phase-lag model is given by

$$K^\star\nabla^2\theta+K\tau_T\nabla^2\ddot{\theta}+\tau_v^\star\nabla^2\dot{\theta}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_v\ddot{\theta}+\beta_1\theta_0\ddot{e}+a\theta_0\ddot{C}\right). \tag{10}$$

The diffusion equation has the form

$$\left[D^\star\left(1+\widetilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\widetilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\left(-\beta_2 e_{kk,ii}+dC_{,ii}-c\theta_{,ii}\right)=\left(1+\widetilde{\tau}_q\frac{\partial}{\partial t}+\frac{\widetilde{\tau}_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\ddot{C}. \tag{11}$$

where u_i $(i=1,2,3)$ are the displacement component and ρ is the density, c_v is the specific heat at constant strain and C is the concentration of the diffusion material in the elastic body and $\tau_v^\star=K+K^\star\tau_v$. For $\tau_q=\tau_T=\tau_v=0$, Eq. (10) reduces to GN theory of type III and for $\tau_q=\tau_T=\tau_v=0$ and $K\ll K^\star$, Eq. (10) reduces to GN theory of type II.

3. Formulation of the problem

We now consider an isotropic, homogeneous and thermoelastic half-space subjected to a chemcial loading on the bounding plane to the surface $x=0$. The body is initially at rest and the surface $x=0$ is assumed to be free of traction. Introducing the rectangular cartesian system (x,y,z), the displacement vector $\vec{u}$ is taken as $(u,v,0)$. Therefore, the equation of motion, heat conduction equation and the constitutive relations can be expressed as follows

$$\sigma_{xx}=-P+2\mu e_{xx}+\lambda e-\beta_1\theta-\beta_2 C, \tag{12}$$

$$\sigma_{yy}=-P+2\mu e_{yy}+\lambda e-\beta_1\theta-\beta_2 C, \tag{13}$$

$$\sigma_{xy}=-P\omega_{xy}+2\mu e_{xy}, \tag{14}$$

where e is the cubical dilatation given by

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}. \tag{15}$$

The equations of motion along x and y directions are given by

$$V_T^2 \frac{\partial^2 u}{\partial x^2} + (V_S^2 - V_P^2)\frac{\partial^2 u}{\partial y^2} + (V_T^2 - V_S^2 + V_P^2)\frac{\partial^2 v}{\partial x \partial y} - \frac{\beta_1}{\rho}\frac{\partial \theta}{\partial x} - \frac{\beta_2}{\rho}\frac{\partial C}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{16}$$

$$V_T^2 \frac{\partial^2 v}{\partial y^2} + (V_S^2 - V_P^2)\frac{\partial^2 v}{\partial x^2} + (V_T^2 - V_S^2 + V_P^2)\frac{\partial^2 u}{\partial x \partial y} - \frac{\beta_1}{\rho}\frac{\partial \theta}{\partial y} - \frac{\beta_2}{\rho}\frac{\partial C}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{17}$$

where

$$V_T^2 = \frac{\lambda + 2\mu}{\rho}, \quad V_S^2 = \frac{\mu}{\rho}, \quad V_P^2 = \frac{\mathscr{P}}{2\rho} \quad \text{and} \quad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

The heat condition equation corresponding to three-phase-lag model is given by

$$K\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2 \dot{\theta} + K^\star\left(1 + \tau_v \frac{\partial}{\partial t}\right)\nabla^2 \theta = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_v \ddot{\theta} + \beta_1 \theta_0 \ddot{e} + a\theta_0 \ddot{C}\right), \tag{18}$$

and the diffusion equation is given by

$$\left[D^\star\left(1 + \widetilde{\tau}_v \frac{\partial}{\partial t}\right) + D\frac{\partial}{\partial t}\left(1 + \widetilde{\tau}_T \frac{\partial}{\partial t}\right)\right]\left(-\beta_2 \nabla^2 e + d\nabla^2 C - c\nabla^2 \theta\right) = \frac{\partial^2}{\partial t^2}\left(1 + \widetilde{\tau}_q \frac{\partial}{\partial t} + \frac{\widetilde{\tau}_q^2}{2}\frac{\partial^2}{\partial t^2}\right)C. \tag{19}$$

The chemical potential is given by

$$P = -\beta_2 e + dC - c\theta. \tag{20}$$

Introducing the following nondimensional variables

$$x' = \frac{x}{V_T \omega_1}, \quad y' = \frac{y}{V_T \omega_1}, \quad u' = \frac{u}{V_T \omega_1}, \quad v' = \frac{v}{V_T \omega_1}, \quad \theta' = \frac{\beta_1 \theta}{\lambda + 2\mu}, \quad C' = \frac{\beta_2 C}{\lambda + 2\mu},$$
$$\varphi' = \frac{\varphi}{(V_T \omega_1)^2}, \quad \psi' = \frac{\psi}{(V_T \omega_1)^2}, \quad t' = \frac{t}{\omega_1}, \quad \tau_{q'} = \frac{\tau_q}{\omega_1}, \quad \tau_{T'} = \frac{\tau_T}{\omega_1}, \quad \tau_{v'} = \frac{\tau_v}{\omega_1},$$
$$\tau_{q'} = \frac{\tau_q}{\omega_1}, \quad \tau_{T'} = \frac{\tau_T}{\omega_1}, \quad \tau_{v'} = \frac{\tau_v}{\omega_1}, \quad \sigma_{ij'} = \frac{\sigma_{ij}}{\lambda + 2\mu}, \quad \omega_1 = \frac{K^\star}{\rho c_v V_T^2}, \quad P' = \frac{P}{\beta_2}, \quad D' = \frac{D}{\omega_1},$$

and after removing primes, the above equations can be written in non-dimensional form as follows

$$\sigma_{xx} = -2R_P + \frac{\partial u}{\partial x} + (1 - 2R_H)\frac{\partial v}{\partial y} - \theta - C, \tag{21}$$

$$\sigma_{yy} = -2R_P + \frac{\partial v}{\partial y} + (1 - 2R_H)\frac{\partial u}{\partial x} - \theta - C, \tag{22}$$

$$\sigma_{yx} = (\lambda + 2\mu)\left[(R_H + R_P)\frac{\partial u}{\partial y} + (R_H - R_P)\frac{\partial v}{\partial x}\right], \tag{23}$$

where

$$R_H = \frac{V_S^2}{V_T^2} \quad \text{and} \quad R_P = \frac{V_P^2}{V_T^2}.$$

The equations of motions in non-dimensional form are given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + (R_H - R_P)\left[\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x \partial y}\right] - \frac{\partial \theta}{\partial x} - \frac{\partial C}{\partial x} = \ddot{u}, \tag{24}$$

$$\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 u}{\partial x\partial y}+(R_H-R_P)\left[\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right]-\frac{\partial\theta}{\partial y}-\frac{\partial C}{\partial y}=\ddot{v}, \tag{25}$$

The heat conduction equation and the diffusion equation take the form

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\ddot{\theta}+\varepsilon_1\ddot{e}+\varepsilon_1\alpha_1\ddot{C}]=\varepsilon_2\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla^2\theta+\varepsilon_3\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla^2\dot{\theta}, \tag{26}$$

$$\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^2 e+\alpha_1\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^2\theta$$

$$+\alpha_2\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\ddot{C}=\alpha_3\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^2 C, \tag{27}$$

and, the chemical potential is given by

$$P=-e+\alpha_3 C-\alpha_1\theta, \tag{28}$$

where,

$$\varepsilon_1=\frac{\beta_1^2\theta_0}{\rho^2 c_v V_T^2},\quad \varepsilon_2=\frac{K^\star}{\rho c_v V_T^2},\quad \varepsilon_3=\frac{K}{\rho c_v V_T^2\omega_1},$$

$$\alpha_1=\frac{c(\lambda+2\mu)}{\beta_1\beta_2},\quad \alpha_2=\frac{V_T^2(\lambda+2\mu)}{\beta_2^2},\quad \alpha_3=\frac{d(\lambda+2\mu)}{\beta_2^2}.$$

We introduce the displacement potential functions φ and ψ by the following relations

$$u=\varphi_{,x}+\psi_{,y},\quad v=\varphi_{,y}-\psi_{,x}, \tag{29}$$

Using Eq. (29), Eqs. (24)-(27) reduce to

$$\frac{\partial^2\varphi}{\partial t^2}=\nabla^2\varphi-\theta-C, \tag{30}$$

$$\frac{\partial^2\psi}{\partial t^2}=(R_H-R_P)\nabla^2\psi, \tag{31}$$

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\ddot{\theta}+\varepsilon_1\nabla^2\ddot{\varphi}+\alpha_1\varepsilon_1\ddot{C}]=\varepsilon_2\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla^2\theta+\varepsilon_3\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla^2\dot{\theta}, \tag{32}$$

$$\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^4\varphi+\alpha_1\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^2\theta$$

$$+\alpha_2\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\ddot{C}=\alpha_3\left[D^\star\left(1+\tilde{\tau}_v\frac{\partial}{\partial t}\right)+D\frac{\partial}{\partial t}\left(1+\tilde{\tau}_T\frac{\partial}{\partial t}\right)\right]\nabla^2 C, \tag{33}$$

where $e=\nabla^2\varphi$.

4. Normal mode Analysis

In this method, the solutions of the physical variables can be decomposed in terms of normal modes in the following form

$$[u, v, e, \theta, \varphi, \psi, \sigma_{ij}, P, C](x, y, t) = [u^\star, v^\star, e^\star, \theta^\star, \varphi^\star, \psi^\star, \sigma^\star_{ij}, P^\star, C^\star](x) \exp[\varpi t + iay], \tag{34}$$

where $u^\star(x)$, $v^\star(x)$, $e^\star(x)$, $\theta^\star(x)$ $\varphi^\star(x)$, $\psi^\star(x)$, $\sigma^\star_{ij}(x)$, $P^\star(x)$ and $C^\star(x)$ are the amplitudes of the functions $i = \sqrt{-1}$, ϖ is the angular frequency, a is the wave numbers in y direction.
Using the normal modes, the above equations can be written as

$$(D^2 - a^2 - \varpi^2)\varphi^\star(x) - \theta^\star(x) - C^\star(x) = 0, \tag{35}$$

$$(R_H - R_P)(D^2 - a^2)\psi^\star(x) = \varpi^2\psi^\star(x), \tag{36}$$

$$\varpi^2\left(1 + \tau_q\varpi + \frac{\tau_q^2\varpi^2}{2}\right)\left[\theta^\star(x) + \varepsilon_1(D^2 - a^2)\varphi^\star(x) + \varepsilon_1\alpha_1 C^\star(x)\right] =$$

$$(D^2 - a^2)\left[\varepsilon_2(1 + \tau_v\varpi) + \varepsilon_3(1 + \tau_T\varpi)\varpi\right]\theta^\star(x), \tag{37}$$

$$(D^2 - a^2)^2\varphi^\star(x) + \alpha_1(D^2 - a^2)\theta^\star(x) + \alpha_2\zeta\varpi^2 C^\star(x) = \alpha_3(D^2 - a^2)C^\star(x). \tag{38}$$

where $\zeta = \dfrac{1 + \tilde{\tau}_q\varpi + \frac{\tilde{\tau}_q^2\varpi^2}{2}}{D^\star(1 + \tilde{\tau}_v\rho) + D\varpi(1 + \tilde{\tau}_T\varpi)}$.
Eliminating $C^\star(x)$ from (35), (37) and (38), we arrive at

$$C_{41}D^4\varphi^\star(x) - C_{42}D^2\varphi^\star(x) + C_{43}\varphi^\star(x) = C_{44}D^2\theta^\star(x) - C_{45}\theta^\star(x), \tag{39}$$

$$C_{51}D^2\varphi^\star(x) - C_{52}\varphi^\star(x) = C_{53}D^2\theta^\star(x) - C_{54}\theta^\star(x), \tag{40}$$

where

$$C_{41} = \alpha_3,$$

$$C_{42} = \alpha_3 a^2 + \alpha_2\zeta\varpi^2 + \alpha_3(a^2 + \varpi^2) + 1,$$

$$C_{43} = (a^2 + \varpi^2)(\alpha_3 a^2 + \alpha_2\zeta\varpi^2) + a^2,$$

$$C_{44} = \alpha_1 + \alpha_3,$$

$$C_{45} = \alpha_3 a^2 + \alpha_2\zeta\varpi^2 + \alpha_1 a^2,$$

$$C_{51} = (\alpha_1 + 1)\varepsilon_1\varpi^2\left(1 + \tau_q\varpi + \frac{\tau_q^2\varpi^2}{2}\right),$$

$$C_{52} = \varepsilon_1\varpi^2\left(1 + \tau_q\varpi + \frac{\tau_q^2\varpi^2}{2}\right)\{(\alpha_1 + 1)a^2 + \alpha_1\varpi^2\},$$

$$C_{53} = \varepsilon_2(1 + \tau_v\varpi) + \varepsilon_3\varpi(1 + \tau_T\varpi),$$

$$C_{54} = a^2\varepsilon_2(1 + \tau_v\varpi) + a^2\varepsilon_3\varpi(1 + \tau_T\varpi) + \varpi^2(1 + \alpha_1\varepsilon_1)\left(1 + \tau_q\varpi + \frac{\tau_q^2\varpi^2}{2}\right).$$

Eliminating $\theta^\star(x)$ between (39) and (40), we arrive at

$$D^6\varphi^\star(x) - LD^4\varphi^\star(x) + MD^2\varphi^\star(x) - N\varphi^\star(x) = 0, \tag{41}$$

where,

$$L = \frac{C_{42}C_{53} + C_{54}C_{41} + C_{44}C_{51}}{C_{53}C_{41}},$$

$$M = \frac{C_{54}C_{42} + C_{43}C_{54} + C_{44}C_{52} + C_{45}C_{51}}{C_{53}C_{41}},$$

$$N = \frac{C_{54}C_{43} + C_{45}C_{52}}{C_{53}C_{41}}.$$

Since the intent is that the solutions vanish at infinity so as to satisfy the regularity condition at infinity, we now cancel the exponential part that has positive power. Therefore, we have

$$\varphi^\star(x) = \sum_{j=1}^{3} R_j(a,\varpi)e^{-k_j x}, \tag{42}$$

where k_j^2 are the roots of the equation

$$k^3 - Lk^2 + Mk - N = 0, \tag{43}$$

Similarly, it can be shown that $\theta^\star(x)$ and $C^\star(x)$ satisfy the equation

$$\left[D^6 - LD^4 + MD^2 - N\right]\{\theta^\star(x), C^\star(x)\} = 0. \tag{44}$$

Whose solutions are given by

$$\theta^\star(x) = \sum_{j=1}^{3} R'_j(a,\varpi)e^{-k_j x}, \tag{45}$$

$$C^\star(x) = \sum_{j=1}^{3} R''_j(a,\varpi)e^{-k_j x}. \tag{46}$$

Substituting from (42), (45) and (46) in eqs. (40) and (35) respectively, we have

$$R'_j(a,\varpi) = \frac{C_{51}k_j^2 - C_{52}}{C_{53}k_j^2 - C_{54}} R_j(a,\varpi) = p_j R_j(a,\varpi) \quad \text{(say)},$$

$$R''_j(a,\varpi) = (k_j^2 - a^2 - \varpi^2 - p_j)R_j(a,\varpi) = q_j R_j(a,\varpi) \quad \text{(say)}.$$

The solution of Eq. (36), which is bounded as $x \to \infty$, is given by

$$\psi^\star(x) = \mathscr{C}(a,\varpi)e^{-mx}, \tag{47}$$

where $\mathscr{C}(a,\varpi)$ is some parameter depending on a and ϖ, and

$$m = \sqrt{a^2 + \frac{\varpi^2}{R_H - R_P}}. \tag{48}$$

Substituting (42) and (47) into (29), we have

$$u^\star(x) = -\sum_{j=1}^{3} k_j R_j(a,\varpi)e^{-k_j x} + ia\mathscr{C}(a,\varpi)e^{-mx}, \tag{49}$$

$$v^\star(x) = \sum_{j=1}^{3} iaR_j(a,\varpi)e^{-k_j x} + m\mathscr{C}(a,\varpi)e^{-mx}. \tag{50}$$

The stress components in terms of normal modes are given by

$$\sigma^\star_{xx}(x) = -2R_P^\star + \sum_{j=1}^{3}\left[k_j^2 - a^2(1-2R_H) - p_j - q_j\right]R_j(a,\varpi)e^{-k_j x} - 2iamR_H\mathscr{C}(a,\varpi)e^{-mx}, \tag{51}$$

$$\sigma^\star_{yy}(x) = -2R_P^\star + \sum_{j=1}^{3}\left[(1-2R_H)k_j^2 - a^2 - p_j - q_j\right]R_j(a,\varpi)e^{-k_j x} - 2iamR_H\mathscr{C}(a,\varpi)e^{-mx}, \tag{52}$$

$$\sigma^\star_{yx}(x) = -\sum_{j=1}^{3} 2iaR_H k_j R_j(a,\varpi)e^{-k_j x} - \left[(m^2+a^2)R_H - (m^2-a^2)R_P\right]\mathscr{C}(a,\varpi)e^{-mx}. \tag{53}$$

The chemical potential is given by

$$P^\star(x) = \sum_{j=1}^{3}(1 - k_j^2 + \alpha_3 q_j - \alpha_1 p_j)R_j(a,\varpi)e^{-k_j x}. \tag{54}$$

5. Boundary conditions

The problem is to solved subjected to the following boundary conditions
For the stress-free boundary $x = 0$, the boundary conditions are given by

$$\sigma_{yy} = -2P, \quad \sigma_{yx} = 0 \quad \text{on} \quad x = 0, \tag{55}$$

$$\frac{\partial\theta}{\partial y} + h\theta = 0 \quad \text{on} \quad x = 0, \tag{56}$$

and the time dependent chemical potential on $x = 0$ is represented by

$$P(x, y, t) = g(y, t) \quad \text{on} \quad x = 0, \tag{57}$$

where $h \to 0$ corresponds to the thermal insulated boundary and $h \to \infty$ to the isothermal boundary conditions. The dimensionless boundary conditions can be written as

$$\sigma_{yy} = -4R_P, \quad \sigma_{yx} = 0 \quad \text{on} \quad x = 0, \tag{58}$$

$$\frac{\partial\theta}{\partial y} + hV_T\omega_1\theta = 0 \quad \text{on} \quad x = 0, \tag{59}$$

$$P = g \quad \text{on} \quad x = 0, \tag{60}$$

In order ro determine the parameters $R_j(a,\varpi)$ $(j = 1,2,3)$ and $\mathscr{C}(a,\varpi)$, we need to consider the boundary conditions at $x = 0$, which gives

$$\sum_{j=1}^{3} \left[(1-2R_H)k_j^2 - a^2 - p_j - q_j\right] R_j(a,\varpi) + 2iamR_H\mathscr{C}(a,\varpi) = -2R_P^\star, \tag{61}$$

$$-\sum_{j=1}^{3} 2iaR_H k_j R_j(a,\varpi) - \left[(m^2 + a^2)R_H - (m^2 - a^2)R_P\right]\mathscr{C}(a,\varpi) = 0, \tag{62}$$

$$\sum_{j=1}^{3} p_j R_j(a,\varpi) = 0, \tag{63}$$

$$\sum_{j=1}^{3} (1 - k_j^2 + \alpha_3 q_j - \alpha_1 p_j) R_j(a,\varpi) = g^\star. \tag{64}$$

Therefore, solving eqs. (61)-(64), we have the parameters $R_j(a,\varpi)$ $(j = 1,2,3)$ and $\mathscr{C}(a,\varpi)$ in the following forms

$$R_1(a,\varpi) = \frac{g^\star\alpha_{11} + 2R_P^\star\alpha_{22}}{Z_1\beta_{11} + Z_2\beta_{22} + Z_3\beta_{33}},$$

$$R_2(a,\varpi) = \frac{g^\star\gamma_{11} + 2R_P^\star\gamma_{22}}{Z_1\beta_{11} + Z_2\beta_{22} + Z_3\beta_{33}},$$

$$R_3(a,\varpi) = \frac{g^\star\delta_{11} + 2R_P^\star\delta_{22}}{Z_1\beta_{11} + Z_2\beta_{22} + Z_3\beta_{33}},$$

$$\mathscr{C}(a,\varpi) = \frac{g^\star\eta_{11} + 2R_P^\star\eta_{22}}{Z_1\beta_{11} + Z_2\beta_{22} + Z_3\beta_{33}},$$

where,

$$\alpha_{11} = \left[Q_2(M_4N_3 - M_3N_4) + Q_3(M_2N_4 - M_4N_2) + Q_4(M_3N_2 - M_2N_3)\right],$$

$$\alpha_{22} = \left[Z_2(N_4Q_3 - N_3Q_4) + Z_3(N_2Q_4 - N_4Q_2)\right],$$

$$\gamma_{11} = \left[Q_1(M_3N_4 - M_4N_3) + Q_3(M_4N_1 - M_1N_4)\right],$$

$$\gamma_{22} = \left[N_4(Q_1Z_3 - Z_1Q_3)\right],$$

$$\delta_{11} = \left[Q_1(M_4N_2 - M_2N_4) + Q_2(M_1N_4 - M_4N_1)\right],$$

$$\delta_{22} = \left[N_4(Q_2Z_1 - Q_1Z_2)\right],$$

$$\eta_{11} = \left[Q_1(M_2N_3 - M_3N_2) + Q_2(M_3N_1 - M_1N_3) + Q_3(M_1N_2 - M_2N_1)\right],$$

$$\eta_{22} = \left[Z_1(N_2Q_3 - N_3Q_2) + Z_2(N_3Q_1 - N_1Q_3) + Z_3(N_1Q_2 - Q_1N_2)\right],$$

$$\beta_{11} = \left[Q_2(M_4N_3 - M_3N_4) + Q_3(M_2N_4 - M_4N_2)\right],$$

$$\beta_{22} = \left[Q_1(M_3N_4 - M_4N_3) + Q_3(M_4N_1 - M_1N_4)\right],$$

$$\beta_{33} = \left[Q_1(M_4N_2 - M_2N_4) + Q_2(M_1N_4 - M_4N_1)\right].$$

$$M_j = (1 - 2R_H)k_j^2 - a^2 - p_j - q_j, \quad j = 1, 2, 3$$

$$M_4 = 2iamR_H,$$

$$N_j = -2iaR_Hk_j, \quad j = 1, 2, 3$$

$$N_4 = -[(m^2 + a^2)R_H - (m^2 - a^2)R_P],$$

$$Q_j = p_j, \quad j = 1, 2, 3$$

$$Q_4 = 0,$$

$$Z_j = (1 - k_j^2 + \alpha_3 q_j - \alpha_1 p_j), \quad j = 1, 2, 3$$

$$Z_4 = 0.$$

6. Numerical results and discussions

With an aim to illustrate the results obtained in the preceding section, we now present the analytical results numerically. In the numerical computation, we have considered a copper-like material. Since ϖ is the complex time constant, we have $\varpi = \omega_0 + i\zeta$, then $e^{\varpi t} = e^{\omega_0 t}(\cos\zeta t + i\sin\zeta t)$. The values of the material constants are taken to be

$$\lambda = 7.76 \times 10^{10}\text{N/m}^2, \quad \mu = 3.86 \times 10^{10}\text{N/m}^2, \quad \rho = 8954\ \text{kg}\cdot\text{m}^{-3}, \quad K = 386\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1},$$

$$\nu = 0.33, \quad \theta_0 = 293\ \text{K}, \quad \alpha_t = 1.78 \times 10^{-5}\text{K}^{-1}, \quad \alpha_c = 1.98 \times 10^{-4}\text{K}^{-1},$$

$$c_v = 383.1\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}, \quad \tau_T = 0.15\ \text{s}, \quad \tau_q = 0.2\ \text{s}, \quad \widetilde{\tau}_q = 0.1\ \text{s}, \quad \widetilde{\tau}_T = 0.01\ \text{s}, \quad \alpha = 0.001\ \text{K}^{-1},$$

$$\omega_0 = 1.0, \quad \zeta = 0.2, \quad K^\star = 200,$$

which agrees with the stability condition of Quintanilla and Racke [50].

Further, the values of the other non-dimensional parameters arising in the present analysis are taken to be

$$g^\star = 1.3, \quad a = 1.2.$$

In order to study the effect of diffusion on the thermophysical quantities for three-phase-lag model, GN II and GN III models in presence of the hydrostatic pressure $R_P = 0.2$ when $y = 0.1$ and time $t = 0.1$ respectively, figures 1-7 have been plotted. In these figures, the continuous lines represent the graphs corresponding to the case with diffusion medium (WD) and the dotted lines represent the graphs corresponding to the case without diffusion (WOD).

Fig. 1 depicts the variation of the displacement u against the distance x for $y = 0.1$, $t = 0.1$ and for $R_P = 0.2$ for 3P lag model, GN II and GN III models. As seen from the figure, u attains maximum value on the plane $x = 0$ and decays sharply in $0 < x < 0.5$. As x increases, the magnitude of u decreases and finally diminishes to zero value.

Fig. 2 is plotted to study the effect of diffusion on the displacement component v against the distance x for $y = 0.1$, $t = 0.1$ and for $R_P = 0.2$. It is observed that the presence of diffusion has a tendency to accelerate the magnitude of the displacement v. Also, the magnitude of v is maximum on the plane $x = 0$ and then as we move far from the boundary, the magnitude of v diminishes to zero value. Also, the profile of v is larger for GN II model than that of 3P lag model, which is again larger than GN III model.

Fig. 3 is plotted to study the thermodiffusive effect in the variation of the temperature θ for the same set of parameters. From the figure, it is seen that θ has an increasing effect in $0 < x < 0.6$ to attain the maximum value near $x = 0.6$ and then as we move far from the boundary, the temperature also diminishes to zero value. Here also, the magnitude of θ is larger for the three models due to the presence of the diffusive effect. For both diffusive medium (WD) and in the absence of diffusion (WOD), the magnitude of θ is larger or GN II model than that of 3P lag model, which is larger than GN III model.

Fig. 4 is plotted to study the variation of the shearing stress σ_{yx} against the distance x for the same set of parameters for three different models. It is observed that σ_{yx} vanishes on the plane $x = 0$, satisfying the mechanical boundary condition of our problem as laid down in Eq. (55). The stress is compressive in nature near the bounding plane and, it attains maximum magnitude near $x = 0.5$, and finally diminishes to zero value with the increase of x.

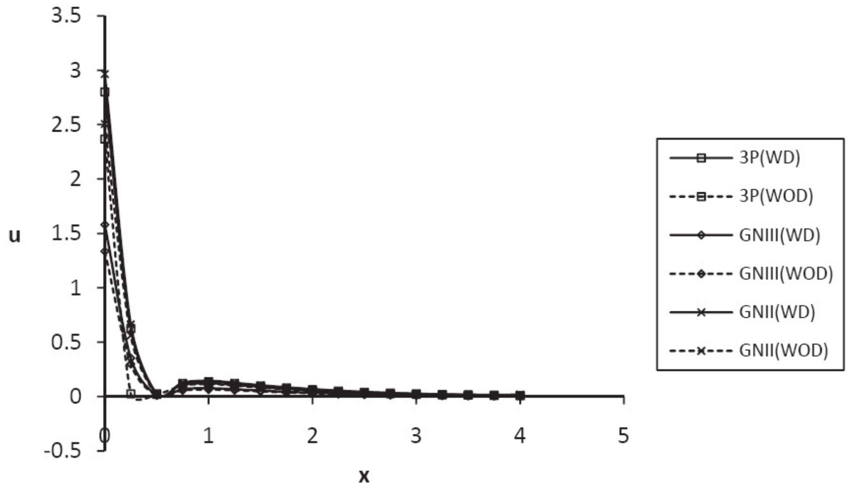

Fig. 1. variation of u versus x for $R_p = 0.2, y = 0.1, t = 0.1$

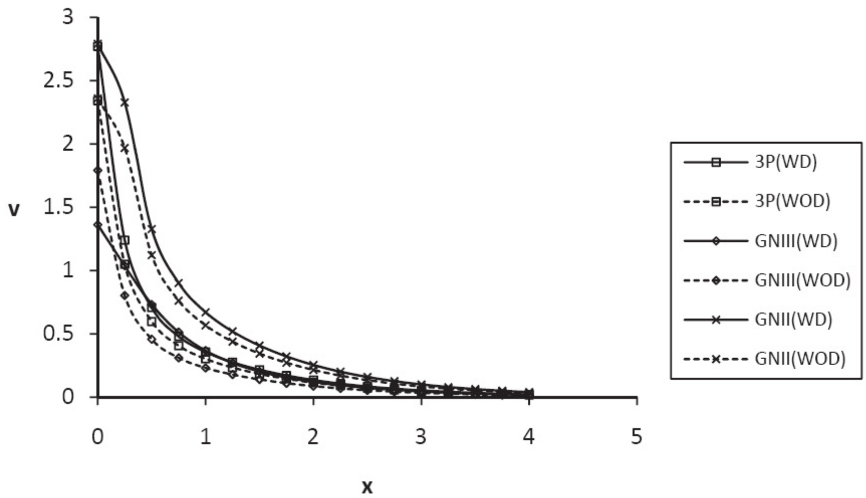

Fig. 2. variation of v versus x for $R_p = 0.2, y = 0.1, t = 0.1$

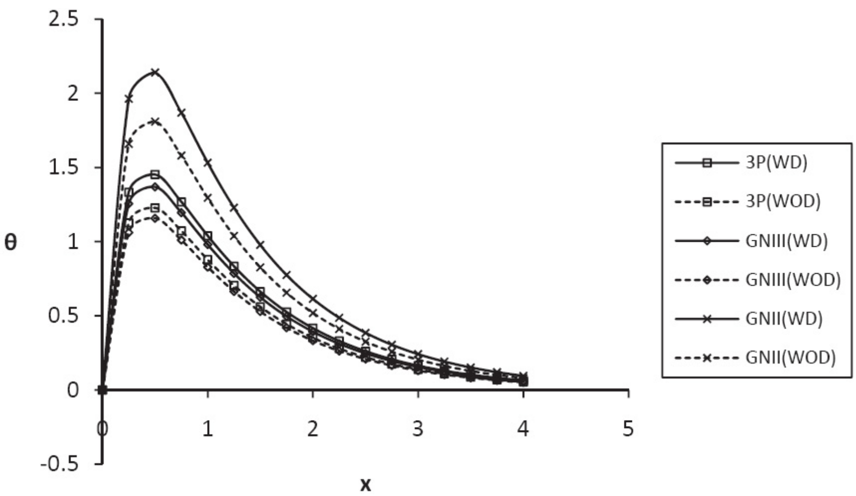

Fig. 3. variation of θ versus x for $R_p = 0.2, y = 0.1, t = 0.1$

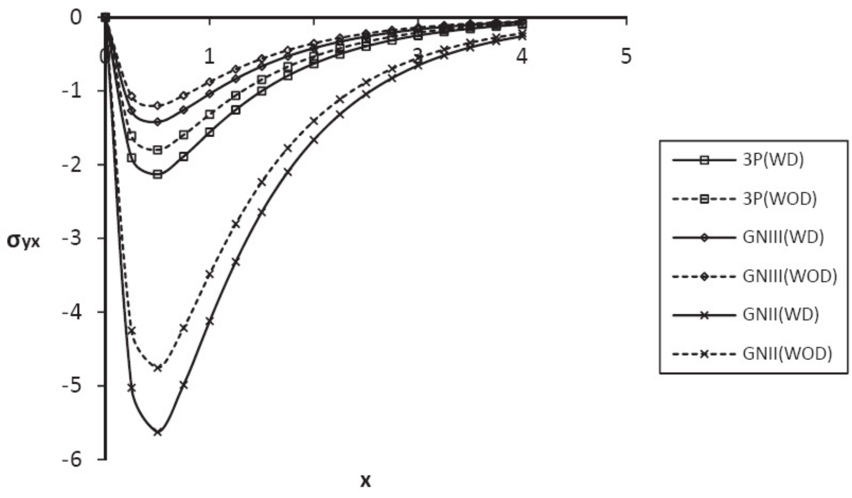

Fig. 4. variation of σ_{xy} versus x for $R_p = 0.2, y = 0.1, t = 0.1$

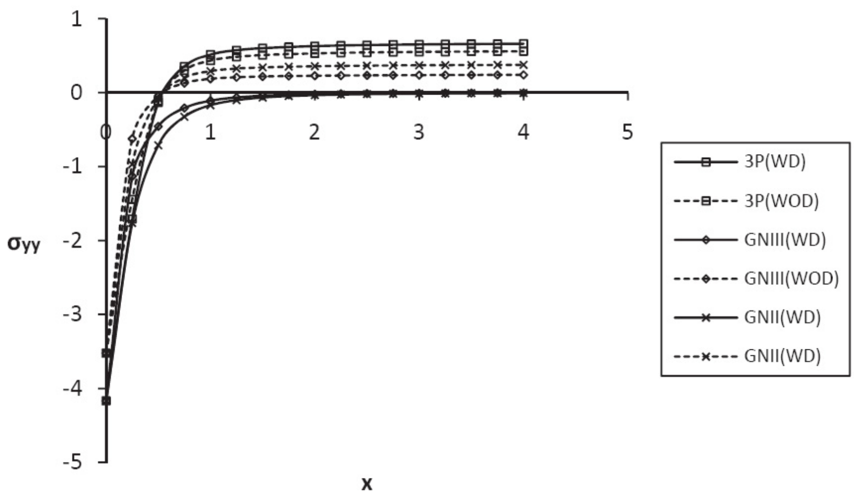

Fig. 5. variation of σ_{yy} versus x for $R_p = 0.2, y = 0.1, t = 0.1$

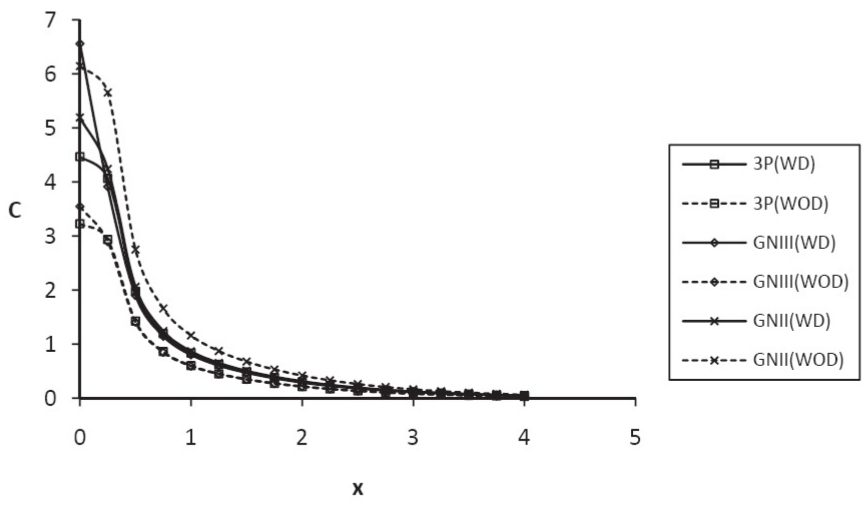

Fig. 6. variation of C versus x for $R_p = 0.2, y = 0.1, t = 0.1$

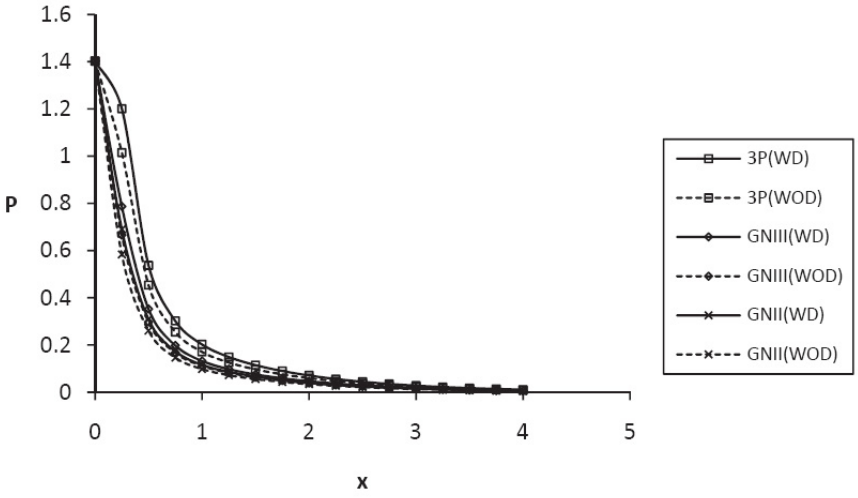

Fig. 7. variation of P versus x for $R_p = 0.2, y = 0.1, t = 0.1$

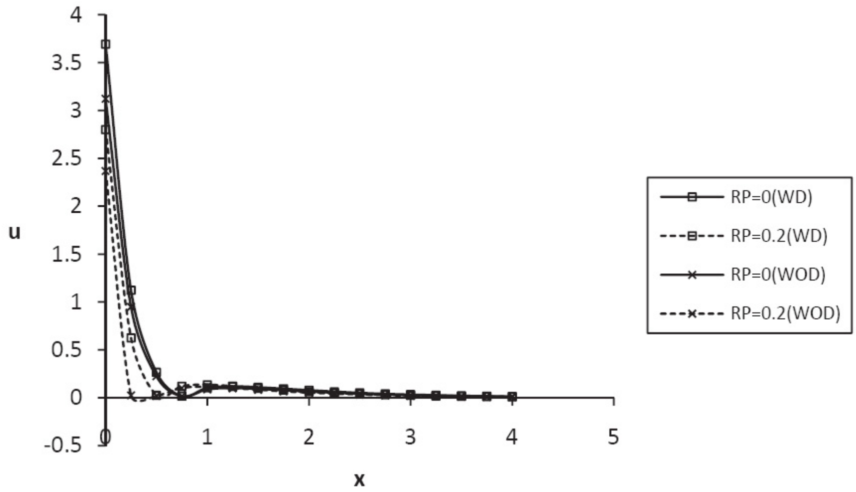

Fig. 8. variation of u versus x for $3P$ lag model for $y = 0.1, t = 0.1$

The presence of thermodiffusive effect (WD) has a tendency to accelerate the magnitude of the shearing stress for all the models.

Fig. 5 depicts the variation of the stress component σ_{yy} versus the distance x for the same set of parameters. As seen from the figure, on the plane $x = 0$, σ_{yy} satisfy the mechanical boundary condition on our problem as may seen from Eq. (55) and as x increases, the magnitude of the stress component approaches towards zero value. For GN II and GN III model, σ_{yy} disappears in $1.4 < x < 4$ in a diffusive medium (WD), whereas, in absence of diffusion (WOD), the effect of σ_{yy} is more prominent throughout $0 < x < 4$ of the body.

Fig. 6 depicts the variation of the mass concentration C when $t = 0.1$ and $y = 0.1$ for 3P lag model, GN II and GN III models for hydrostatic pressure $R_P = 0.2$. As seen from the figure, C attain the maximum value on the plane $x = 0$ for all the models. With the increase of x, the magnitude of C decay sharply and diminishes to zero value.

Fig. 7 has been plotted to study the effect of the thermodiffusion on the chemical potential P against the distance x for the same set of parameters. It is observed that on the plane $x = 0$, the chemical potential satisfies the boundary condition as laid down in Eq. (57). Here also, the magnitude of the chemical potential is larger due to the presence of diffusion (WD) than in absence of diffusion (WOD).

In order to study the effect of the non-dimensional hydrostatic pressure R_P on the thermophysical quantities for 3P lag thermoelastic model for $y = 0.1$ and $t = 0.1$ in presence of diffusion (WD) and in absence of diffusion (WOD), Figs. 8-14 have been plotted. In these figures, the continuous lines have been considered in absence of hydrostatic pressure ($R_P = 0$) and the dotted liens stand for the presence of the hydrostatic pressure ($R_P = 0.2$).

Fig. 8 depicts the variation of the displacement u against x for $y = 0.1$ and $t = 0.1$ for $R_P = 0$ and $R_P = 0.2$ respectively in WD and WOD cases. It is seen that the magnitude of u is larger on the plane $x = 0$ and then it decays to zero value as we move far from the boundary. In $0 < x < 0.8$, the magnitude of u decay sharply for $R_P = 0.2$ than that

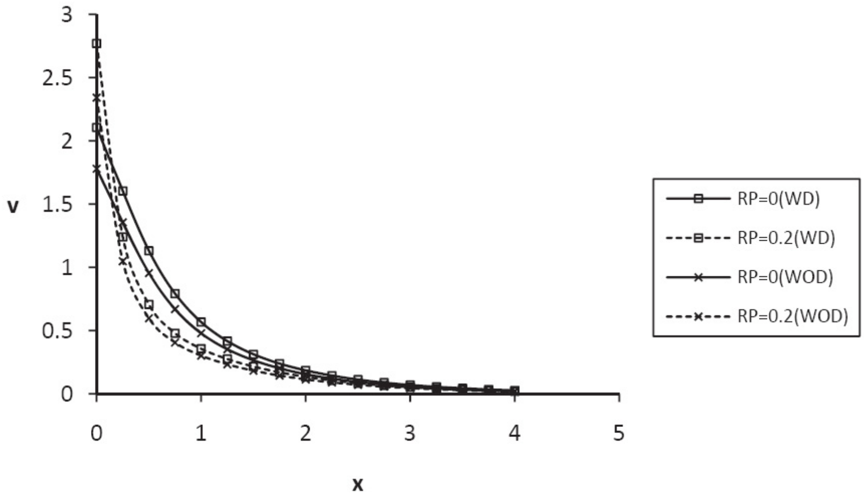

Fig. 9. variation of v versus x for $3P$ lag model for $y = 0.1$, $t = 0.1$

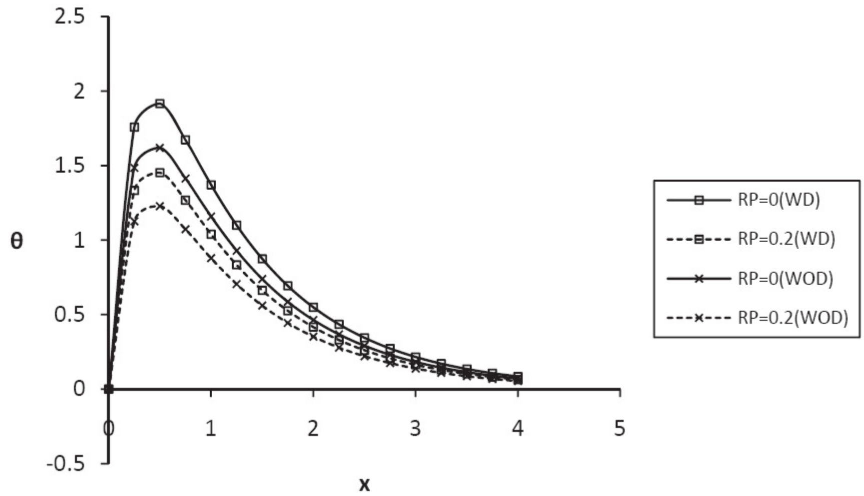

Fig. 10. variation of θ versus x for $3P$ lag model for $y = 0.1$, $t = 0.1$

of $R_P = 0$ in presence of diffusion (WD) and absence of diffusion (WOD) also.

Fig. 9 depicts the variation of the displacement component v against x for $R_P = 0$, 0.2 respectively for the same set of parameters. As seen from the figure, v is maximum on the bounding plane $x = 0$ and the magnitude is larger for $R_P = 0.2$ than that of $R_P = 0$ for WD and WOD cases. Also, the decay of v is faster for $R_P = 0.2$ compared to that of $R_P = 0$.

In Fig. 10 the variation of the temperature θ is seen for the same set of parameters. A similar qualitative behavior in the propagation of θ is found as that in Fig. 9

Fig. 11 has been plotted to study the variation of the shearing stress σ_{yx} versus x for the same set of parameters. It is found that σ_{yx} vanishes on $x = 0$ for $R_p = 0$, 0.2, which validates the numerical codes prepared in our problem. As seen from the figure, the magnitude of σ_{yx} is larger for $R_P = 0$ than $R_P = 0.2$. Also, the presence of the mechanical force plays an important role in maintaining the smoothness of the profile of σ_{yx}.

In Fig. 12, the variation of σ_{yy} is observed. It is seen that in absence of the hydrostatic pressure, σ_{yy} almost disappears for $1.6 < x < 4$.

Figs. 13 and 14 show the variation of the mass concentration C and the chemical potential P for the same set of parameters. From these figures, it is seen that the magnitude of C and P are larger for $R_P = 0$ than that of $R_P = 0.2$ and the decay of the magnitudes are faster for $R_P = 0$ than $R_P = 0.2$ and the magnitude of the chemical potential and the concentration are larger in the case of diffusive medium (WD) than in absence of diffusion (WOD).

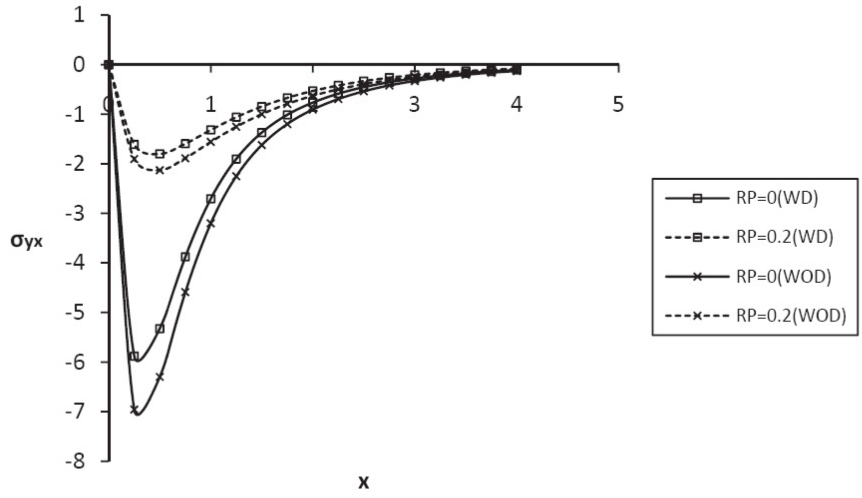

Fig. 11. variation of σ_{xy} versus x for $3P$ lag model for $y = 0.1$, $t = 0.1$

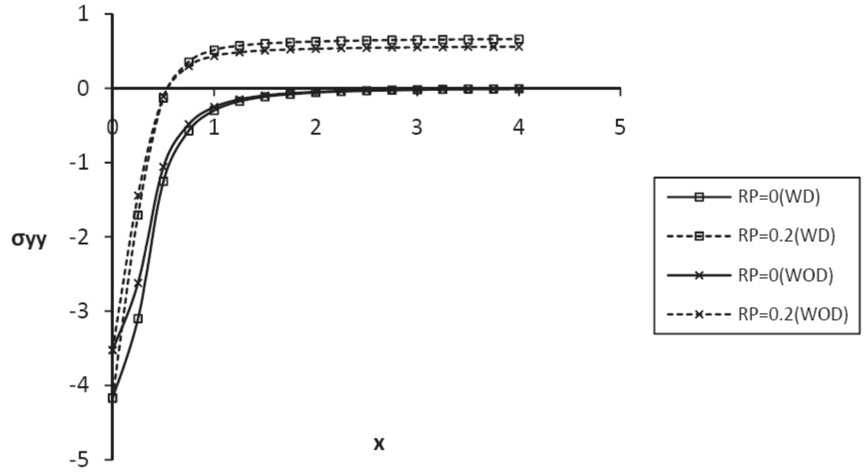

Fig. 12. variation of σ_{yy} versus x for $3P$ lag model for $y = 0.1$, $t = 0.1$

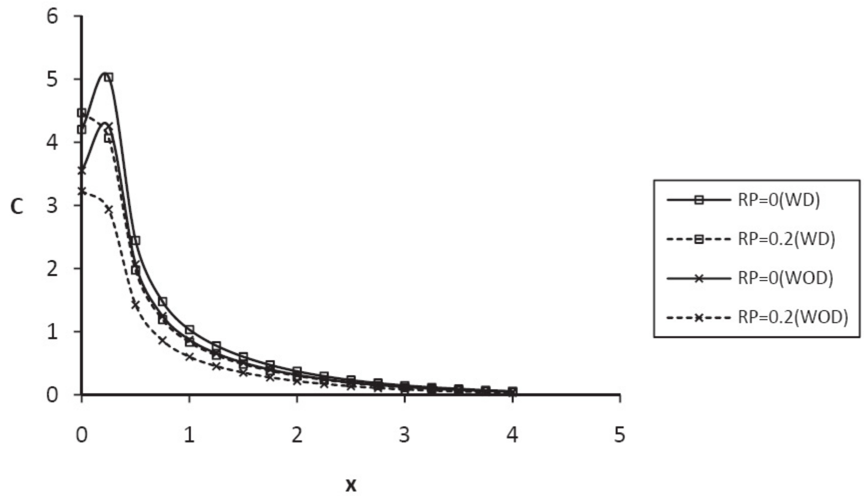

Fig. 13. variation of C versus x for $3P$ lag model for $y = 0.1$, $t = 0.1$

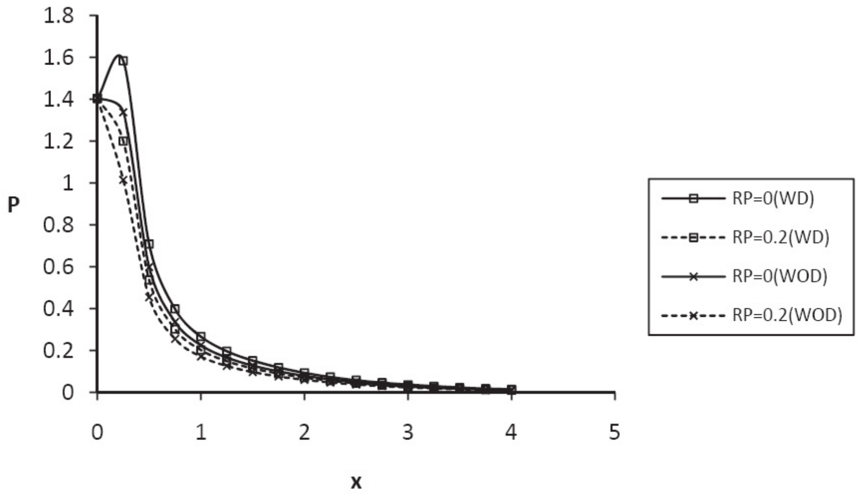

Fig. 14. variation of P versus x for $3P$ lag model for $y = 0.1, t = 0.1$

7. Conclusions

In this present analysis, a mathematical treatment has been presented to explore the effect of elasto-thermodiffusion on wave propagation in a half-space under hydrostatic pressure for three-phase-lag model from which Green Naghdi model II and III are obtained as particular cases. The problem has been solved theoretically and exemplified through specific models (3P, GN II, GN III). All the figures plotted are self-explanatory in exhibiting the different peculiarities which occur in the propagation of waves, yet the following remarks may be added.

1. The presence of the non-dimensional hydrostatic pressure has significant effect on the thermophysical quantities. The presence of the hydrostatic pressure has a tendency to decrease the magnitude of the thermophysical quantities.

2. Due to the presence of thermodiffusion, the rise in magnitude of the thermophysical quantities are observed, which supports the physical fact.

3. Here, all the results for Green Naghdi model III in absence of the thermodiffusion agree with the existing literature [51].

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References

[1] R.S Dhaliwal, A. Singh, Dynamic Coupled Thermoelasticity, Hindustan Publication, Delhi, 1980.
[2] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids I, Bull. Acad. Pol. Sci. 22 (1974) 55-64.
[3] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids II, Bull. Acad. Pol. Sci. 22 (1974) 129-135.
[4] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids II, Bull. Acad. Pol. Sci. 22 (1974) 257-266.
[5] W. Nowacki, Dynamical problems of thermoelastic diffusion in elastic solids, Proc. Vib. Prob. 15 (1974) 105-128.
[6] H.W Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, Journal of Mechanics and Physics of Solids 15 (1967) 299-309.
[7] A.E Green, K.A Lindsay, Thermoelasticity, Journal of Elasticity 2 (1972) 1-7.
[8] A.E Green, P.M Naghdi, A re-examination of the basic postulates of thermomechanics, Proceedings of Royal Society London Series A 432 (1991) 171-194.
[9] A.E Green, P.M Naghdi, On undamped heat waves in an elastic solid, Journal of Thermal Stresses 15 (1992) 252-264.
[10] A.E Green, P.M Naghdi, Thermoelasticity without energy dissipation, Journal of Elasticity, 31 (1993) 189-208.
[11] D.Y Tzou, A unified field approach for heat conduction from macro to micro scales, ASME Journal of Heat Transfer, 117 (1995) 8-16.

[12] D.S Chandrasekharaiah, Hyperbolic thermoelasticity: A review of recent literature, Applied Mechanics Review 51 (1998) 705-729.
[13] S.K Roychoudhury, One-dimensional thermoelastic waves in elastic half-space with dual-phase-lag effect, Journal of Mechanics of Materials and Structures 2 (2007) 489-502.
[14] R. Quintanilla, Exponential stability in the dual-phase-lag heat conduction theory, Journal of Non-Equilibrium Thermodynamics 27 217-227.
[15] R. Quintanilla, A condition on the delay parameters in the one-dimensional dual-phase-lag thermoelastic theory, Journal of Thermal Stresses 26 (2003) 713-721.
[16] R. Prasad, R. Kumar, S. Mukhopadhyay, Propagation of harmonic plane waves under thermoelasticity with dual-phase-lags, International Journal of Engineering Science 48 (2010), 2028-2043.
[17] A. Sur, M. Kanoria, Finite thermal wave propagation in a half-space due to variable thermal loading, Applications and Applied Mathematics, An International Journal 9(1) (2014), 94-120.
[18] S. K. Roychoudhuri, On a thermoelastic three-phase-lag model, Journal of Thermal Stresses 30 (2007) 231-238.
[19] R. Kumar, S. Mukhopadhyay, Effects of three-phase-lag on generalized thermoelasticity for an infinite medium with a cylindrical cavity, Journal of Thermal Stresses 32(11) (2009) 1149-1165.
[20] M. Kanoria, S.H Mallik, Generalized thermoviscoelastic interaction due to periodically varying heat source with three-phase-lag effect, European Journal of Mechanics A/Solids 29 (2010) 695-703.
[21] A. Kar, M. Kanoria, Generalized thermoelastic functionally graded orthotropic hollow sphere under thermal shock with three-phase-lag effect, European Journal of Mechanics A/Solids 28 (2009) 757-767.
[22] M.A Ezzat, A.S El Karamany, M.A Fayik, Fractional order theory in thermoelastic solid with three-phase lag heat transfer, Archive of Applied Mechanics http://dx.doi.org/10.1007/s00419-011-0572-6 (2011).
[23] A.S El-Karamany, M.A Ezzat, On the three-phase-lag linear micropolar thermoelasticity theory, European Journal of Mechanics A/ Solids 40 (2013) 198-208.
[24] M. Islam, A. Kar, M. Kanoria, Thermoelastic response in a symmetric spherical shell, Acta Mechanica 225 (2014) 2841-2864.
[25] A. Sur, M. Kanoria, Thermoelastic interaction in a viscoelastic functionally graded half-space under three phase lag model, European Journal of Computational Mechanics 23(5-6) (2014) 179-198.
[26] A. Sur, M. Kanoria, Thermoelastic gold Nano-beam under ramp-type laser pulse with three-phase-lag effect, International Journal of Applied Mathematics and Mechanics 10(5) (2014) 86-104.
[27] A. Sur, M. Kanoria, Fractional order generalized thermoelastic functionally graded solid with variable material properties, Journal of Solid Mechanics 6(1) (2014) 54-69.
[28] A. Sur, M. Kanoria, Propagation of thermal waves in a functionally graded thick plate, Mathematics and Mechanics of Solids (2015) DOI: 10.1177/1081286515609652.
[29] A. Sur, M. Kanoria, Fibre reinforced magneto-thermoelastic rotating medium with fractional heat conduction, Procedia Engineering (2015) DOI: 10.1016/j.proeng.2015.11.351.
[30] P. Pal, A. Sur, M. Kanoria, Thermoelastic interaction in a viscoelastic unbounded medium subjected to fractional heat equation, Journal of Solid Mechanics (2015) In Press.
[31] R. Patra, S.P Barik, P.K Chaudhuri, An internal crack problem in an infinite transversely isotropic elastic layer, International Journal of Advances in Applied Mathematics and Mechanics 3(1) (2015) 62-70.
[32] F. Barati, H. Farahani, A. Nezamabadi, M. Veiskarami, Buckling analysis of functionally graded beams with imperfectly integrated surface piezoelectric layers under low velocity, International Journal of Advances in Applied Mathematics and Mechanics 2(1) (2014) 64-73.
[33] K.J Chauhan, D.M Patel, The similarity solutions of concentration dependent diffusion equation, International Journal of Advances in Applied Mathematics and Mechanics, 1(2) (2013) 80ï£¡85.
[34] H.H Sherief, F.A Hamza, H.A Saleh, The theory of generalized thermoelastic diffusion, International Journal of Engineering Science 42 (2004) 591-608.
[35] H.H. Sherief, H.A. Saleh, A Half-space problem in the theory of generalized thermoelastic diffusion, International Journal of Solids and Structures 42 (2005) 4484-4493.
[36] B. Singh, Reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion, Journal of Earth System Science 114 (2005) 159-168.
[37] B. Singh, Reflection of SV waves from the free surface of an elastic solid in generalized thermoelastic diffusion, Journal of Sound and Vibration 291 (2006) 764-778.
[38] M. Aouadi, A generalized thermoelastic diffusion problem for an infinitely long solid cylinder, International Journal of Mathematics and Mathematical Sciences (2006) 1-15.
[39] M. Aouadi, A problem for an infinite elastic body with a spherical cavity in the theory of generalized thermoelastic diffusion, International Journal of Solids and Structures 44 (2007) 5711-5722.
[40] M. Aouadi, Uniqueness and reciprocity theorems in the theory of generalized thermoelastic diffusion, Journal of Thermal Stresses 30 (2007) 665-678.
[41] M. Aouadi, Generalized theory of thermoelastic diffusion for anisotropic media, Journal of Thermal Stresses 31 (2008) 270-285.
[42] M.I.A Othman, S.Y Atwa, R.M Farouk, The Effect of diffusion on two dimensional problem of generalized thermoelasticity with Green Naghdi theory, International Communications in Heat Mass transfer 36 (2009) 857-864.

[43] R. Kumar, T. Kansal, Propagation of Rayleigh waves on free surface of transversely isotropic generalized thermoelastic diffusion, Applied Mathematics and Mechanics England Edition 29(11) (2008) 1451-1462.

[44] S. Kothari, S. Mukhopadhyay, A study of influence of diffusion inside a spherical shell under thermoelastic diffusion with relaxation times, Mathematics and Mechanics of Solids DOI:10.1177/1081286512446829 (2012).

[45] H.M Wang, H.J Ding, Y.M Chen, Thermoelastic dynamic solution of a multilayered spherically isotropic hollow sphere for spherically symetric problems, Acta Mechanica 173 (2004) 131-145.

[46] A.M El-Sayed, A two-dimensional generalized thermoelastic diffusion problem for a half-space, Mathematics and Mechanics of Solids DOI: 10.1177/1081286514549877 (2014).

[47] R. Karmakar, M. Kanoria, Elasto-thermodiffusive response in a spherically isotropic hollow sphere, Journal of Thermal Stresses 38 (2015) 427-446.

[48] D. Bhattacharya, M. Kanoria, The influence of two temperature generalized thermoelastic diffusion inside a spherical shell, International Journal of Engineering and Technical Research 2 (2014) 151-159.

[49] D. Bhattacharya, M. Kanoria, The infulence of two-temperature fractional order generalized thermoelastic diffusion inside a spherical shell, International Journal of Application or Innovation in Engineering and Mangement 3(8) (2014) 96-108.

[50] R. Quintanilla, R. Racke, A note on stability in three-phase-lag heat conduction, International Journal of Heat and Mass Transfer, 51 (2008) 24-29.

[51] M.I.A Othman, S.Y Atwa, Thermoelastic plane waves for a elastic solid half-space under hydrostatic initial stress of type III, Meccanica 47 (2012) 1337-1347.