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Frictionless contact of a rigid punch indenting a transversely isotropic elastic layer

Research Article

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Abstract: This article is concerned with the study of frictionless contact between a rigid punch and a transversely isotropic elastic layer. The rigid punch is assumed to be axially symmetric and is being pressed towards the layer by an applied concentrated load. The layer is resting on a rigid base and is assumed to be sufficiently thick in comparison with the amount of indentation by the rigid punch. The relationship between the applied load P and the contact area is obtained by solving the mathematically formulated problem through use of Hankel transform of different order. Effect of indentation on the distribution of normal stress at the surface as well as the relationship between the applied load and the area of contact have been shown graphically.

MSC: 74E10 • 74R20

Keywords: Transversely isotropic medium • Integral transform • Contact problem • Alamnsi's theorem • Fredhlom integral equation

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1. Introduction

When two deformable solids are in contact and external load is applied to press one onto the other deformation occurs in the solids which may cause some changes in the contact area.There are obviously some relationships between the applied load and the contact area. The distribution of stresses in either body in and outside the contact area are subject to change significantly depending upon the nature of applied load.The determination of the stresses in and outside the contact area as well as the relationship between the applied load and contact area is the subject of study in solid mechanics for long which started through the initial investigation of Hertz[1]. With the application of load, contact area may or may not vary; accordingly contact problems have been classified as advancing (increase of contact area), receding (decrease of contact area) and stationary (contact area remaining the same). Another aspect of consideration in the study of contact problems is the frictional force at the surface of contact.Since contact is the principal method of applying loads to a deformable body, study of contact problems in various kinds of deformable media is important as well as necessary. Owing to their applications in a great variety of structural systems, such as foundations, pavements in roads and runways, automotive disk brake systems and many other technological applications, considerable progress has been made with the analysis of contact problems in solid mechanics. Among several works done, we may mention a few: Shvets et al. [2], Chaudhuri and Ray [3, 4], Comez, et al. [5], Barik et al. [6, 7], Gecit [8], Selvanduari [9], Fabrikant [10]. Various types of contact problems are discussed in books and journals, e.g. Johnson [11], Gladwell [12], Hills et al. [13], Raous et al. [14] and Rogowski[15], Chen [16] etc.

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The present investigation aims to find the elastostatic solution of an axially symmetric frictionless contact between a transversely isotropic layer and a rigid cylindrical, spherical and conical indenters which are loaded by a concentrated force P. Using the operator theory, we derive a general solution that is expressed in terms of the three potentials. These functions satisfy differential equations of the second order and are quasi-harmonic functions. Making use of these fundamental solutions, the punch problem in the aforesaid three cases,is investigated. The solution of the problem has been reduced to the solution of one Fredhlom type integral equation of second kind which requires numerical treatment. The numerical results are discussed and presented graphically to show the influence of indentation in the layer on various states of interest.

2. Formulation of the problem

We consider an elastic layer of transversely isotropic material and of thickness H lying on a rigid base.On the free surface of the layer, a rigid punch of axisymmetric character is placed with its axis of symmetry normal to the free surface of the layer. We also assume that the punch is pressed towards the layer by an applied concentrated force of magnitude P. Cylindrical coordinate system (r,θ,z) with $z-$ axis along the inward drawn normal to the free surface of the layer, will be used to specify the position of a point in the layer. We shall make the following assumptions in our discussion:

(a) the axis of symmetry of the transversely isotropic material is along the z-axis

(b) there is no force of gravity

(c) linear theory of elasticity holds

(d) the thickness of the layer is sufficiently large in comparison to the indented depth of the punch.

Because of axisymmetric structure of the indentor, the displacement vector $\mathbf{u}$ will have its cross radial component $u_\theta = 0$ i.e. $\mathbf{u} = (u_r, 0, u_z)$ and all the physical quantities are independent of θ. The solution of frictionless contact problem demands the relationship between the applied load and the contact area. The geometry of the problem is shown in Fig. 1.

The mathematical formulation of the problem consists of

Equilibrium equations:

$$\left.\begin{aligned} C_{11}\mathscr{L}_1 u_r + C_{44}D^2 u_r + (C_{13}+C_{44})D\frac{\partial u_z}{\partial r} = 0 \\ C_{44}\mathscr{L}_0 u_z + C_{33}D^2 u_z + (C_{13}+C_{44})D\frac{\partial [r u_r]}{r\partial r} = 0 \end{aligned}\right\} \tag{1}$$

where $\mathscr{L}_k = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} - \dfrac{k}{r^2} \qquad k = 0,1 \qquad D = \dfrac{\partial}{\partial z}$

The boundary conditions:

$$u_z(r,0) = f(r), \qquad 0 \le r \le a \tag{2}$$

$$u_z(r,H) = 0, \qquad r \ge 0 \tag{3}$$

$$\sigma_{rz}(r,0) = 0, \qquad r \ge 0 \tag{4}$$

$$\sigma_{zz}(r,0) = 0, \qquad r > a \tag{5}$$

$$\sigma_{rz}(r,H) = 0, \qquad r \ge 0 \tag{6}$$

The parameters C_{ij} appearing in (1) are the elastic coefficients. In addition to the boundary conditions, the displacement components should satisfy the regularity condition $u_r, u_z \to 0$ as $\sqrt{r^2+z^2} \to \infty$. At the surface of contact of the material with the indenter $0 \le r \le a$, the boundary condition will depend upon the shape of the indenter. If h is the indented depth of the solid into the material then
(a) for a cylindrical indenter the condition will be

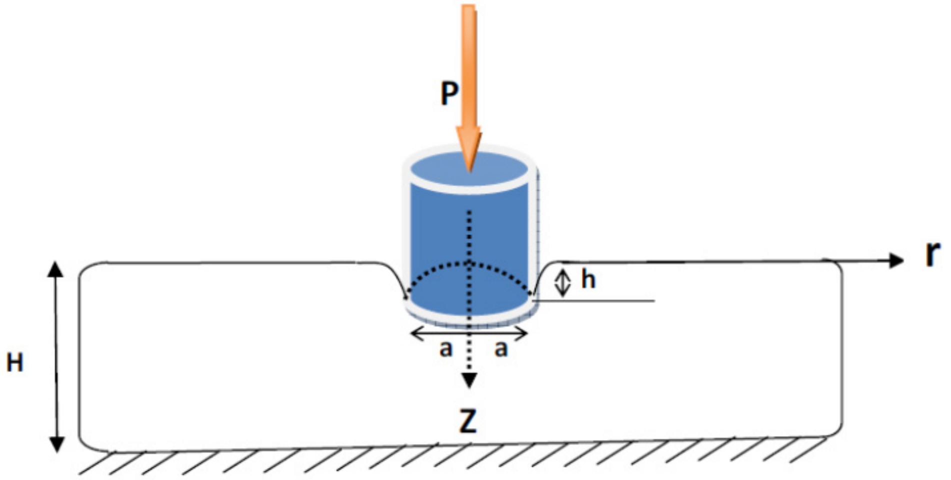
$$u_z(r,0) = f(r) = h \tag{7}$$

(b) for a conical indenter having α as the semi-vertical angle, the condition is

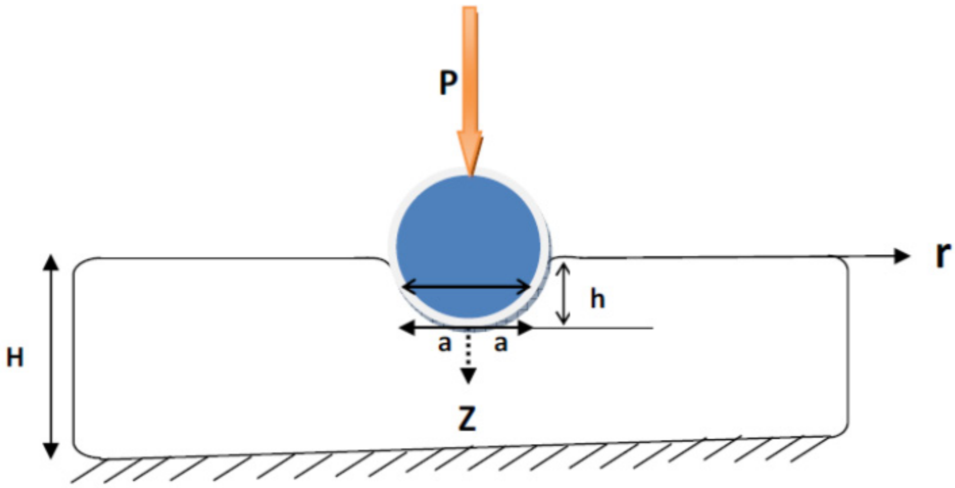
$$u_z(r,0) = f(r) = h - (a-r)\cot\alpha \tag{8}$$

(c) for a spherical indenter having radius R, the condition is

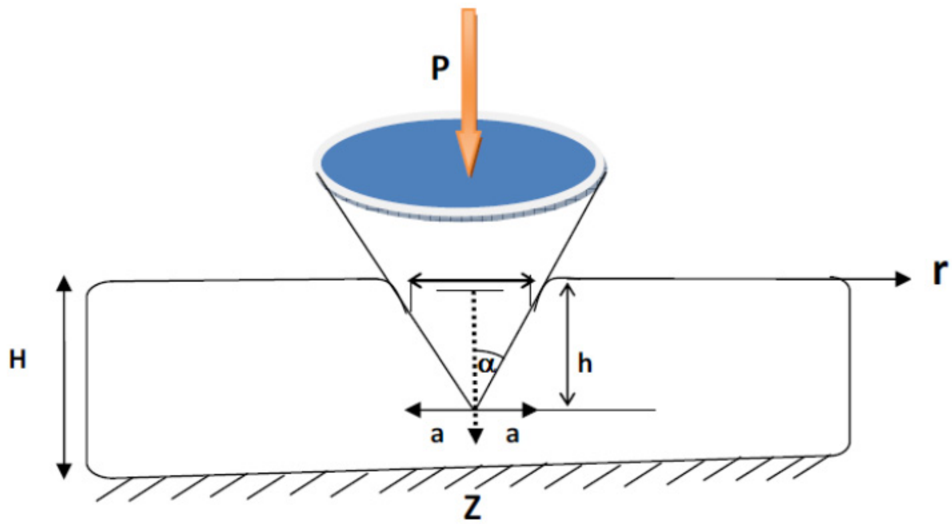
$$u_z(r,0) = f(r) = h - \frac{r^2}{2R} \tag{9}$$



(a) Flat ended cylindrical punch



(b) Sperical punch



(c) Conical punch

Fig. 1. Geometry of the problem

3. Method of solution

Solution of partial differential Eq. (1) requires use of Hankel transform technique of different order. We shall outline the method adopted by Dyka and Rogowski [17] for the first part of discussion and thereafter apply those results in our considered problem.

Let us write

$$\left.\begin{aligned} \widehat{u_r}(\xi,z) &= H_1[u_r(r,z); r\to\xi] = \int_0^\infty u_r(r,z)\, r J_1(r\xi)\, dr \\ \widehat{u_z}(\xi,z) &= H_0[u_z(r,z); r\to\xi] = \int_0^\infty \{u_z(r,z)\} r J_0(r\xi)\, dr \end{aligned}\right\} \tag{10}$$

where J_0 and J_1 are the Bessel functions of the first kind and of order zero or one, respectively, and ξ is the transform parameter. We use the following properties of Hankel transforms

$$\left.\begin{aligned} & H_\nu[\mathscr{L}_\nu f(r,z); r\to\xi] = -\xi^2 \widehat{f}_\nu(\xi,z) \\ & H_1[\tfrac{\partial f(r,z)}{\partial r}; r\to\xi] = -\xi \widehat{f}_0(\xi,z) \\ & H_0[\tfrac{\partial[r f(r,z)]}{r\partial r}; r\to\xi] = \xi \widehat{f}_1(\xi,z) \end{aligned}\right\} \tag{11}$$

Applying the Hankel transformations (10) to the Eq. (1) we get a coupled ordinary differential equations, which may be written in the form

$$\mathbf{A}\begin{bmatrix} \widehat{u}_r \\ \widehat{u}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{12}$$

where $\mathbf{A}$ is the following operator matrix

$$\mathbf{A} = \begin{bmatrix} -C_{11}\xi^2 + C_{44}D^2 & -\xi(C_{13}+C_{44})D \\ \xi(C_{13}+C_{44})D & -C_{44}\xi^2 + C_{33}D^2 \end{bmatrix} \tag{13}$$

We have

$$|\mathbf{A}| = -a_0(D^2 - \lambda_1^2\xi^2)(D^2 - \lambda_2^2\xi^2) \tag{14}$$

where $\lambda_i^2 (i = 1,2)$ are the roots of the following cubic algebraic equation

$$a_0\lambda^4 + b_0\lambda^2 + c_0 = 0 \tag{15}$$

with the coefficients defined by

$$\left.\begin{aligned} a_0 &= -C_{33}C_{44} \\ b_0 &= C_{11}C_{33} - 2C_{13}C_{44} - C_{13}^2 \\ c_0 &= -C_{11}C_{44} \end{aligned}\right\} \tag{16}$$

Using the operator theory, we obtain the general solution to the Eq. (12), as

$$\left.\begin{aligned} \widehat{u}_r(\xi,z) &= A_{i1}\widehat{F}(\xi,z) \\ \widehat{u}_z(\xi,z) &= A_{i2}\widehat{F}(\xi,z) \end{aligned}\right\} \tag{17}$$

where A_{ij} are the algebraic cominors of the matrix operator and $\widehat{F}(\xi,z)$ is the zero order Hankel transform of the general solution $F(r,z)$, satisfying the equations

$$\left.\begin{aligned} & |\mathbf{A}|\widehat{F}(\xi,z) = 0 \\ & (D^2 + \lambda_1^2\Delta)(D^2 + \lambda_2^2\Delta)F(r,z) = 0 \end{aligned}\right\} \tag{18}$$

Here $\Delta = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r}$ and $D^2 = \dfrac{\partial^2}{\partial z^2}$.

Taking $i = 3$ and writing down the expression for A_{3j}, we obtain

$$\left.\begin{aligned} \widehat{u}_r(\xi,z) &= (C_{13}+C_{44})\xi D\widehat{F}(\xi,z) \\ \widehat{u}_z(\xi,z) &= (-C_{11}\xi^2 + C_{44}D^2)\widehat{F}(\xi,z) \end{aligned}\right\} \tag{19}$$

Using the inverse Hankel transforms to Eq. (19), the original solution for the displacement components are obtained as:

$$\left.\begin{aligned} u_r(r,z) &= -(C_{13}+C_{44})\tfrac{\partial^2}{\partial r\partial z}F(r,z) \\ u_z(r,z) &= (C_{11}\Delta + C_{44}D^2)F(r,z) \end{aligned}\right\} \tag{20}$$

Using the generalized Almansi's theorem [18], the function $F(r,z)$, which satisfies Eq. (18)$_2$, can be expressed in terms of three quasi-harmonic functions

$$F = \begin{cases} F_1 + F_2 & \text{for distinct } \lambda_i \\ F_1 + zF_2 & \text{for } \lambda_1 = \lambda_2 \end{cases} \tag{21}$$

where $F_i(r,z)$ satisfies, respectively

$$(\Delta + \frac{1}{\lambda_i^2} D^2) F_i(r,z) = 0 \qquad i = 1,2 \tag{22}$$

As we shall see later that the roots of Eq. (15) are all distinct in our considered problem, so we shall consider only first solution in Eq. (21).
Using

$$\Delta F_i = -\frac{1}{\lambda_i^2} D^2 F_i \tag{23}$$

and summing in Eq. (20), we obtain

$$\left.\begin{aligned} u_r(r,z) &= -\textstyle\sum_{i=1}^2 (C_{13} + C_{44}) \frac{\partial^2 F_i}{\partial r \partial z} \\ u_z(r,z) &= -\textstyle\sum_{i=1}^2 (C_{44} - \frac{C_{11}}{\lambda_i^2}) \frac{\partial^2 F_i}{\partial z^2} \end{aligned}\right\} \tag{24}$$

Introducing functions $F_i(r,z)$ such that

$$\frac{\partial}{\partial z} F_i(r,z) = -\frac{1}{\lambda_i} \varphi_i(r,z) \tag{25}$$

Eq. (24) can be expressed as

$$\left.\begin{aligned} u_r(r,z) &= \textstyle\sum_{i=1}^2 \frac{a_{i1}}{\lambda_i} \frac{\partial \varphi_i}{\partial r} \\ u_z(r,z) &= \textstyle\sum_{i=1}^2 \frac{a_{i2}}{\lambda_i} \frac{\partial \varphi_i}{\partial z} \end{aligned}\right\} \tag{26}$$

where

$$\left.\begin{aligned} a_{i1} &= C_{13} + C_{44} \\ a_{i2} &= C_{44} - \frac{C_{11}}{\lambda_i^2} \end{aligned}\right\} \tag{27}$$

The quasi-harmonic function $\varphi_i(r,z)$ satisfies the equation

$$\left(\Delta + \frac{1}{\lambda_i^2} \frac{\partial^2}{\partial z^2} \right) \varphi_i(r,z) = 0 \tag{28}$$

The relationships between stress, displacement for a transversely isotropic elastic medium in the case of axial symmetry, are

$$\left.\begin{aligned} \sigma_{rr}(r,z) &= C_{11} \frac{\partial u_r}{\partial r} + C_{12} \frac{u_r}{r} + C_{13} \frac{\partial u_z}{\partial z} \\ \sigma_{\theta\theta}(r,z) &= C_{12} \frac{\partial u_r}{\partial r} + C_{11} \frac{u_r}{r} + C_{13} \frac{\partial u_z}{\partial z} \\ \sigma_{zz}(r,z) &= C_{13} \frac{\partial u_r}{\partial r} + C_{13} \frac{u_r}{r} + C_{33} \frac{\partial u_z}{\partial z} \\ \sigma_{zr}(r,z) &= C_{44} (\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}) \end{aligned}\right\} \tag{29}$$

Substituting Eq. (24) into Eq. (29), we obtain

$$\left.\begin{aligned} \sigma_{rr}(r,z) &= -\textstyle\sum_{i=1}^2 \frac{a_{i3}}{\lambda_i} \frac{\partial^2 \varphi_i}{\partial z^2} - (C_{11} - C_{12}) \frac{u_r}{r} \qquad & \sigma_{zz}(r,z) &= \textstyle\sum_{i=1}^2 \frac{a_{i4}}{\lambda_i} \frac{\partial^2 \varphi_i}{\partial z^2} \\ \sigma_{\theta\theta}(r,z) &= -\textstyle\sum_{i=1}^2 \frac{a_{i3}}{\lambda_i} \frac{\partial^2 \varphi_i}{\partial z^2} - (C_{11} - C_{12}) \frac{\partial u_r}{\partial r} \qquad & \sigma_{zr}(r,z) &= \textstyle\sum_{i=1}^2 \frac{a_{i5}}{\lambda_i} \frac{\partial^2 \varphi_i}{\partial r \partial z} \end{aligned}\right\} \tag{30}$$

where

$$\left.\begin{aligned} a_{i3} &= \frac{2C_{11}C_{13}+C_{11}C_{44}-C_{13}C_{44}\lambda_i^2}{\lambda_i^2} \\ a_{i4} &= \frac{-C_{13}^2-C_{13}C_{44}-C_{11}C_{33}+C_{33}C_{44}\lambda_i^2}{\lambda_i^2} \\ a_{i5} &= \frac{(2C_{44}^2+C_{13}C_{44})\lambda_i^2-C_{11}C_{44}}{\lambda_i^2} \end{aligned}\right\} \tag{31}$$

It can be easily verified that:
the equilibrium equations for stresses (Nowacki [19])

$$\left.\begin{aligned} &\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0 \\ &\frac{\partial\sigma_{zr}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{zr}}{r}=0 \end{aligned}\right\} \tag{32}$$

are satisfied.

For axially symmetric problems, the general solution of the differential Eq. (28) may be written as

$$\varphi_i(r,z)=\int_0^{\infty}[A_i(\xi)e^{-\lambda_i\xi z}+B_i(\xi)e^{\lambda_i\xi z}]J_0(r\xi)d\xi \tag{33}$$

where $A_i(\xi), B_i(\xi)(i=1,2)$ are arbitrary functions of the transform parameter ξ, which are to be determined from the boundary conditions (2)-(6) and λ_i are the roots of Eq. (15).

Using Eqs. (33), (26) and (30) into the boundary conditions (2)-(6) we obtain

$$\sum_{i=1}^{2}a_{i5}[A_i(\xi)-B_i(\xi)]=0, \qquad r\geq 0 \tag{34}$$

$$\sum_{i=1}^{2}a_{i5}[A_i(\xi)e^{-\lambda_i\xi H}-B_i(\xi)e^{\lambda_i\xi H}]=0, \qquad r\geq 0 \tag{35}$$

$$\sum_{i=1}^{2}[A_i(\xi)e^{-\lambda_i\xi H}-B_i(\xi)e^{\lambda_i\xi H}]=0, \qquad r\geq 0 \tag{36}$$

$$\sum_{i=1}^{2}\int_0^{\infty}[-A_i(\xi)+B_i(\xi)]\xi J_0(r\xi)d\xi=f(r), \qquad 0\leq r\leq a \tag{37}$$

$$\sum_{i=1}^{2}\int_0^{\infty}\frac{a_{i4}}{\lambda_i}[A_i(\xi)+B_i(\xi)]\xi^2 J_0(r\xi)d\xi=0 \qquad r>a \tag{38}$$

Eqs. (35)-(36), yield

$$B_i(\xi)=A_i(\xi)e^{-2\lambda_i\xi H} \qquad (i=1,2) \tag{39}$$

and

$$A_1(\xi)=\chi_1(\xi)A_2(\xi) \tag{40}$$

where

$$\chi_1(\xi)=-\frac{a_{25}}{a_{15}}\frac{(1-e^{-2\lambda_2\xi H})}{(1-e^{-2\lambda_1\xi H})} \tag{41}$$

Using Eq. (39) in Eq. (37) and Eq. (38) we get, respectively,

$$\int_0^{\infty}[-(1-e^{-2\lambda_1\xi H})\chi_1(\xi)-(1-e^{-2\lambda_2\xi H})]A_2(\xi)\xi J_0(r\xi)d\xi=f(r), \qquad 0\leq r\leq a \tag{42}$$

$$\int_0^{\infty} \chi_2(\xi) A_2(\xi) \xi^2 J_0(r\xi) d\xi = 0, \qquad r > a \tag{43}$$

where

$$\chi_2(\xi) = \frac{a_{14}}{\lambda_1}(1 + e^{-2\lambda_1 \xi H})\chi_1(\xi) + \frac{a_{24}}{\lambda_2}(1 + e^{-2\lambda_2 \xi H}) \tag{44}$$

Now we assume that

$$\chi_2(\xi) A_2(\xi)\xi = \sqrt{\frac{2}{\pi}} \int_0^a \varphi_1(x) cos(\xi x) dx \tag{45}$$

Then Eq. (43) is automatically satisfied. Solving Eqs. (40) and (45) we get

$$A_1(\xi) = \frac{\chi_1(\xi)}{\chi_2(\xi)\xi} \sqrt{\frac{2}{\pi}} \int_0^a \varphi_1(x) cos(\xi x) dx \tag{46}$$

and

$$A_2(\xi) = \frac{1}{\chi_2(\xi)\xi} \sqrt{\frac{2}{\pi}} \int_0^a \varphi_1(x) cos(\xi x) dx \tag{47}$$

From Eqs. (42) and (47) we get

$$\int_0^a \varphi_1(x) k_{11}(r, x) dx = f(r), \qquad 0 \le r \le a \tag{48}$$

where

$$k_{11}(r, x) = \int_0^{\infty} G(\xi) J_0(\xi r) cos(\xi x) d\xi, \tag{49}$$

$$G(\xi) = \sqrt{\frac{2}{\pi}} \frac{1}{\chi_2(\xi)} [-(1 - e^{-2\lambda_1 \xi H})\chi_1(\xi) - (1 - e^{-2\lambda_2 \xi H})] \tag{50}$$

Now Eq. (48) can be written as

$$\int_0^r \frac{dx}{\sqrt{x^2 - r^2}} [\varphi_1(x) + \int_0^a \varphi_1(u) L(u, x) du] = f(r) \tag{51}$$

which is a Abel type integral equation. After some working we get the integral equation in φ_1 as

$$\varphi_1(x) + \int_0^a \varphi_1(u) L(u, x) du = \frac{2}{\pi} g(x), \qquad 0 \le x \le a \tag{52}$$

where

$$\begin{aligned} g(x) = \frac{d}{dx}\int_0^x \frac{r f(r)}{\sqrt{x^2 - r^2}} dr \;&=\; h, && \text{for cylindrical indentor} \\ &=\; h + (\tfrac{\pi}{2} x - a)\cot\alpha, && \text{for conical indentor} \\ &=\; h - \tfrac{x^2}{R} && \text{for spherical indentor} \end{aligned} \tag{53}$$

$$L(u, x) = \frac{2}{\pi} \int_0^{\infty} \Omega(\xi) cos(\xi u) cos(\xi x) d\xi \tag{54}$$

$$\Omega(\xi) = G(\xi) - 1 \tag{55}$$

Before further proceeding it will be convenient to introduce non-dimensional variables u', x' and r' by rescaling by length scale a:

$$u' = \frac{u}{a}, \; x' = \frac{x}{a}, \; r' = \frac{r}{a} \tag{56}$$

For notational convenience, we shall use only dimensionless variables and shall ignore the dashes on the transformed variables and the integral Eq. (52) becomes

$$\varphi_1(x) + \int_0^1 \varphi_1(u)L(u,x)du = \frac{2}{\pi}g(x), \qquad 0 \le x \le 1 \tag{57}$$

This equation determines the function φ_1.

Now equilibrium condition demands

$$P + \int_0^{a^*} 2\pi r dr \sigma_{zz}(r,0) = 0$$

$$\Rightarrow P + 2\pi \int_0^\infty M(\omega)[\int_0^{a^*} J_0(\omega r) r dr] d\omega = 0 \tag{58}$$

where

$$M(\omega) = \sqrt{\frac{2}{\pi}}\omega[\{\beta_1(\omega) + \beta_2(\omega)\} \int_0^1 \varphi_1(x) cos(\omega x) dx] \tag{59}$$

$$\beta_1(\omega) = \frac{a_{14}\lambda_1\chi_1(\omega)}{\chi_2(\omega)}(1 + e^{-2\lambda_1\omega}),\ \ \beta_2(\omega) = \frac{a_{24}\lambda_2}{\chi_2(\omega)}(1 + e^{-2\lambda_2\omega}) \text{ and } a^* = \frac{a}{h},\ \omega = \xi h.$$

The Eq. (58) is the relationship between the applied load P and the radius of the contact area.

4. Numerical results and discussions:

The present study aims at investigating a frictionless contact problem in a finite transversely isotropic elastic layer. The main objective of the present discussion is to study the effects of indentation on the load contact area relationship as well as normal stress distribution. Numerical evaluation requires numerical solution of the integral equation Eq. (52).

In our present discussion we have considered the transversely isotropic material as Cobalt, Magnesium and Titanium to illustrate theoretical results. The numerical values of the elastic coefficients for the materials are listed in Table 1 [20].

Table 1. Basic data for three transversely isotropic materials.

Quantity	Unit	Cobalt	Titanium	Magnesium
C_{11}	GPa	307.0	162.4	59.7
C_{33}	GPa	358.1	180.7	61.7
C_{44}	GPa	78.3	46.7	16.4
C_{12}	GPa	165.0	92.0	26.2
C_{13}	GPa	103.0	69.0	21.7

Our numerical study will cover three different types of rigid indenter, namely, cylindrical shaped, spherical shaped and conical shaped.

Fig. 2 relate results for cylindrical punch, Fig. 3 relate results for spherical punch, while Fig. 4 relate results for conical punch. Fig. 2(a) shows effect of indentation h on $\sigma_{zz}(r,0)$ for flat-ended cylindrical punch for various values of h in a transversely isotropic medium like titanium. As expected, more indentation will require more normal stress. Fig. 2(b) shows variation of $\sigma_{zz}(r,0)$ for three different materials with fixed indentation h of the flat-ended cylindrical punch.It is evident from Fig. 2(b) that increase in rigidity will generate less normal stress. There are not much significant changes in the behaviour of the stresses in the case of spherical indenter from the corresponding results of cylindrical punch as shown in Fig. 3. But here we see that the values of $\sigma_{zz}(r,0) \to 0$ as $r \to 1$. In the case of conical punch some kind of dissimilarities from the above two punches are observed. Firstly, in this case the stresses act oppositely and have decreasing numerical values with increasing h. This is shown in Fig. 4(a). Fig. 4(b) shows a comparison between results of three materials for fixed h. The results are similar to those in Fig. 2(b). Variations of normal stress with r have been also studied with different shapes of the conical punch by varying the semi-vertical angle. Results are shown in Fig. 4(c).

Fig. 5 show the variation of the applied load P with contact radius $\frac{a}{h}$ for all kinds of considered punches. In case of cylindrical punch, as expected, Fig. 5(a) shows that greater magnitude of applied load is required to have indentation of cylinder having greater radius.

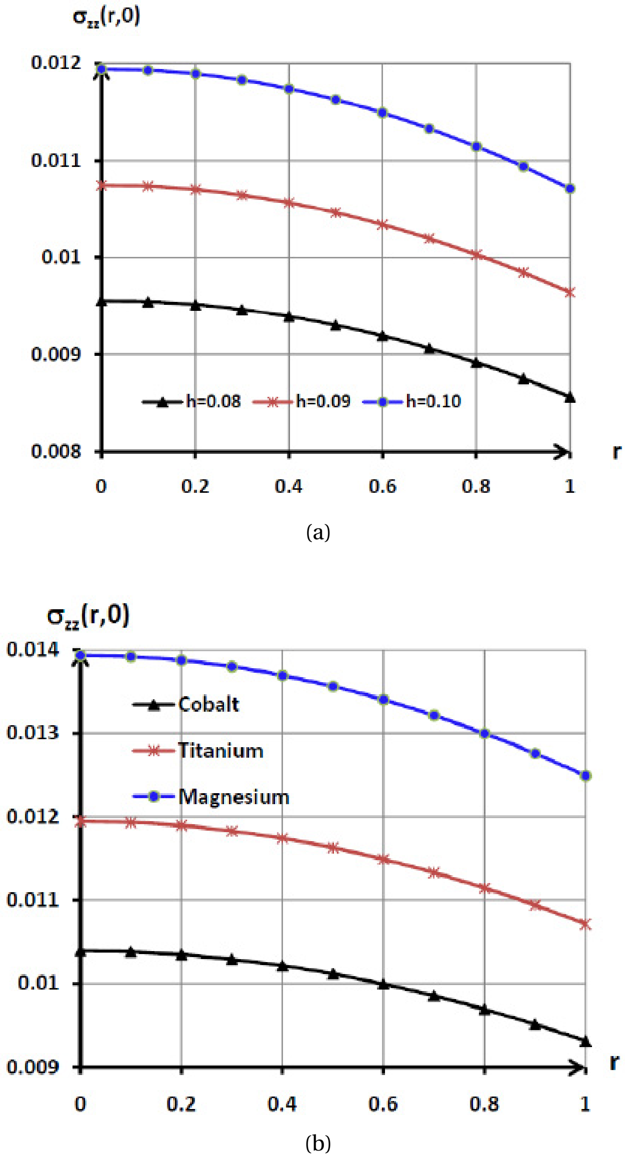


Fig. 2. (a) Effect of indentation h on $\sigma_{zz}(r,0)$ for flat ended cylindrical punch (Titanium) (b) Variation of $\sigma_{zz}(r,0)$ for various material with fixed indentation of the flat ended cylindrical punch ($h = 0.1$)

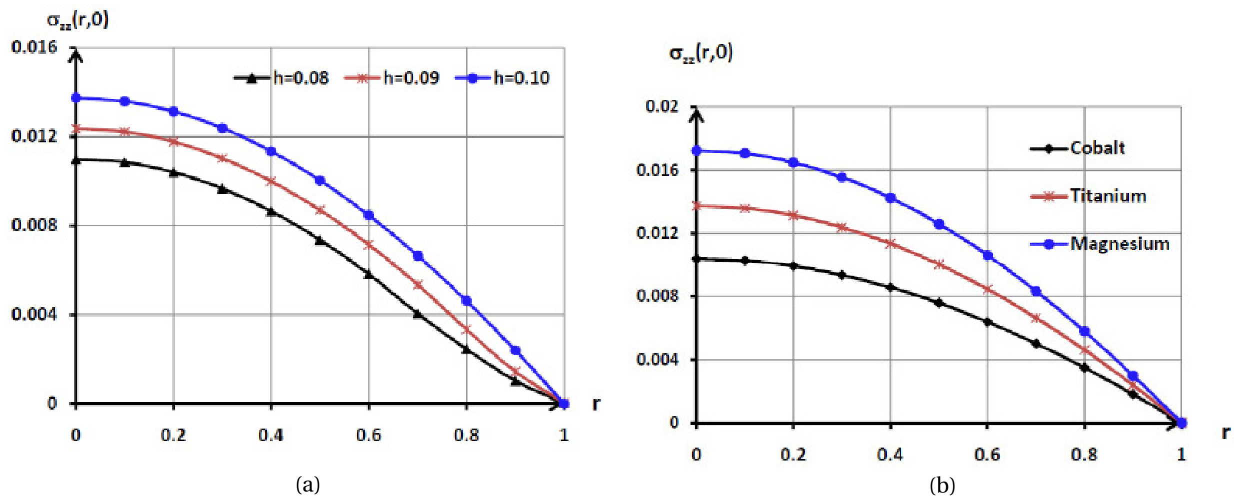
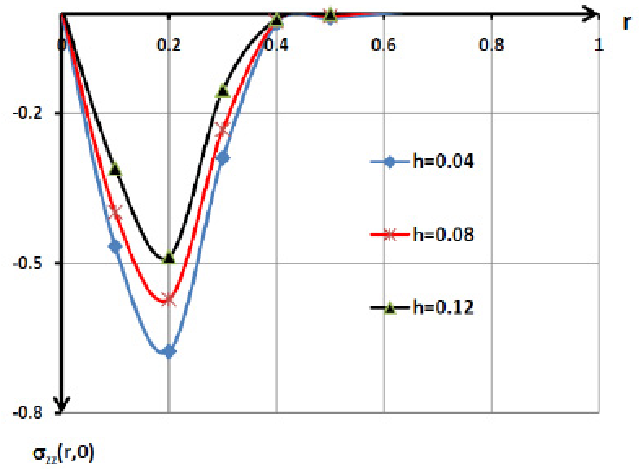
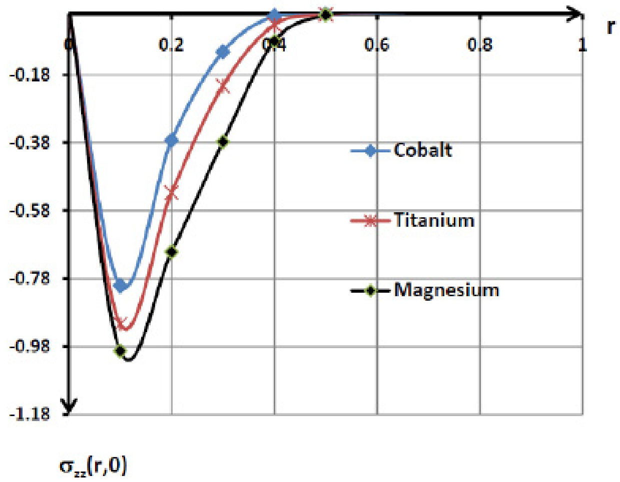


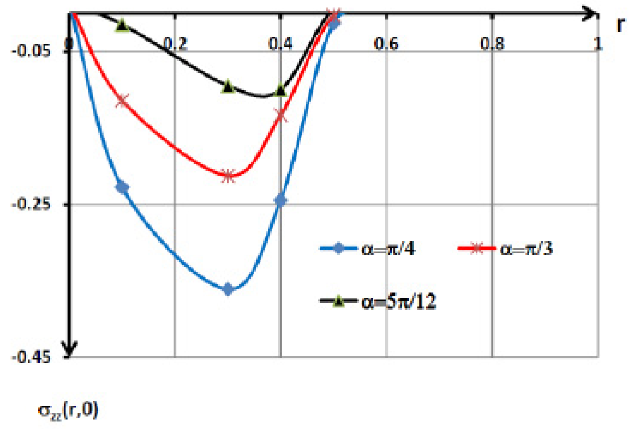
Fig. 3. (a) Effect of indentation h on $\sigma_{zz}(r,0)$ for spherical punch ($R = 10$) (b) Variation of $\sigma_{zz}(r,0)$ for various material with fixed indentation of the spherical punch ($h = 0.1, R = 10$)



(a)

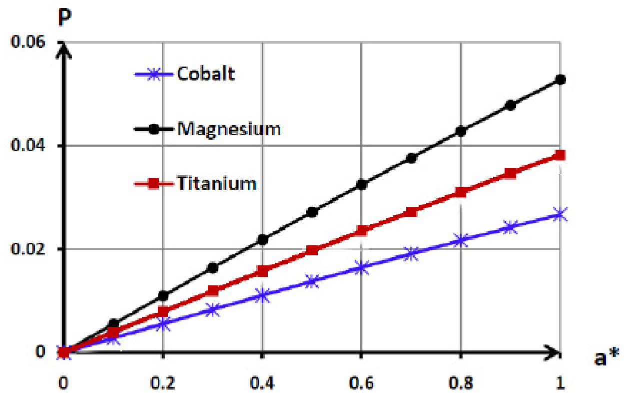


(b)

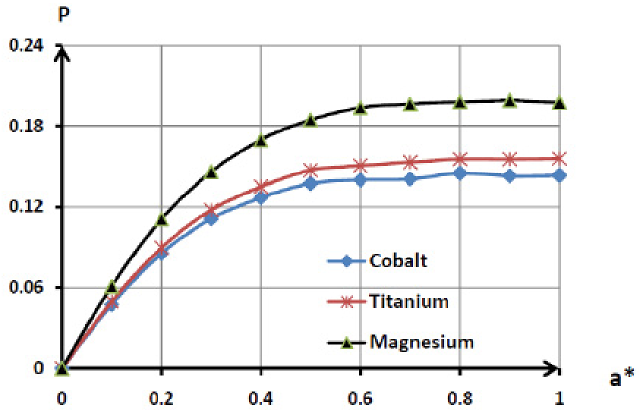


(c)

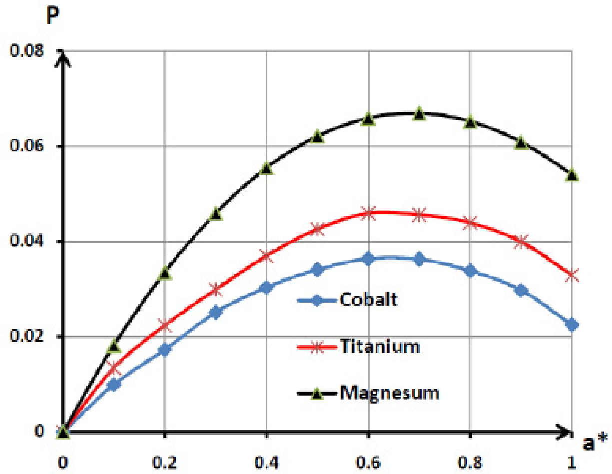
Fig. 4. (a) Effect of indentation h on $\sigma_{zz}(r,0)$ for conical punch ($\alpha = \pi/3$) (b) Variation of $\sigma_{zz}(r,0)$ for various material with fixed indentation of the conical punch ($h = 0.1, \alpha = \pi/3$) (c) Variation of $\sigma_{zz}(r,0)$ for different α of conical punch ($h = 0.1$)



(a)



(b)



(c)

Fig. 5. (a) Variation of total force P with contact radius with fixed indentation ($h = 0.1$) of the flat ended cylindrical punch (b) Variation of total force P with contact radius with fixed indentation ($h = 0.1$) of the spherical punch ($R = 10$) (c) Variation of total force P with contact radius with fixed indentation ($h = 0.1$) of the conical punch ($\alpha = \pi/3$)

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