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Large deflection of a circular plate under non-uniform load pertaining to Aleph-Functions

Research Article

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Abstract: The main object of the present paper is to obtain the large deflection and bending stresses for a clamped circular plate under non-uniform load by using Berger's approximate method. The load shape considered here is an arbitrary function $p(x)$ involving Jacobi polynomial, Fox-Wright function and Aleph-functions. The small deflection case is also considered as a particular case of large deflection. The obtained results of this paper provide an extension of the results given by the literature.

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Keywords: ℵ-function • I-function • Jacobi polynomial • Fox-Wright Function

1. Introduction

A lot of research work has been recently come up on the study and development of a function that is more general than I-function and Fox's H-function, known as the Aleph (ℵ)-function. The Aleph (ℵ)-function introduced by *Südland* et al. [1], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals (see also [2]):

$$\aleph[z]=\aleph^{m,n}_{p_i,q_i,\tau_i;r}[Z]=\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[z\Big|^{(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}}\right]=\frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)z^{-\xi}d\xi \tag{1}$$

for all $z\neq 0$, where $i=\sqrt{-1}$ and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j+B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^{r}\tau_i\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}\xi)} \tag{2}$$

the integration path $L=L_{i\gamma\infty},\gamma\in R$,extended from $\gamma-i\infty\ to\ \gamma+i\infty$, and is such that the poles, assumed to be simple of $\Gamma(1-a_j-A_j\xi),\ j=1,...,n$ do not coincide with the poles of $\Gamma(b_j+B_j\xi)\ j=1,...,m$ the parameter $p_i,\ q_i$ are non-negative integers satisfying $0\le n\le p_i,\ 1\le m\le q_i,\ \tau>0$ for $i=1,...,r$. The parameter $A_j,B_j,A_{ji},B_{ji}>0$ and $a_j,b_j,a_{ji},b_{ji}\in C$. The empty product in (2) is interpreted as unity. The existence conditions for the defining integral (1) are given below:

$$\phi_i>0,|arg(z)|<\frac{\pi}{2}\phi_i,\quad i=1,...,r \tag{3}$$

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$$\phi_i \geq 0, |arg(z)| < \frac{\pi}{2}\phi_i, \ \ and \ R(\xi_i) + 1 < 0 \tag{4}$$

where

$$\phi_i = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_i \left(\sum_{j=n+1}^{p_i} A_{ji} + \sum_{j=m+1}^{q_i} B_{ji} \right) \tag{5}$$

and

$$\xi_i = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_i \left(\sum_{j=m+1}^{q_i} b_{ji} - \sum_{j=n+1}^{p_i} a_{ji} \right) + \frac{1}{2}(p_i - q_i), \ i = 1, ..., r \tag{6}$$

for detailed account of the Aleph (ℵ)-function see $S\ddot{u}dland$ et al. [1, 2]. For $\tau_i = 1, \forall \ i = 1, ..., r$ in (1), we get the I-function defined as follows (see Saxena [3]):

$$I[z] = \aleph^{m,n}_{p_i,q_i,1;r}[Z] = \aleph^{m,n}_{p_i,q_i,1;r}\left[z \Big|^{(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i;r}}_{(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i;r}} \right] = \frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,1;r}(\xi) z^{-\xi} d\xi \tag{7}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;r}(\xi)$ is given in (2). The existence conditions for the integral in (7) are the same as given in (3) through (6) with $\tau_i = 1, i = 1, ..., r$. If we set r = 1, then (7) reduces to the familiar H-function defined as follows (see [4]):

$$H^{m,n}_{p,q}[z] = \aleph^{m,n}_{p_i,q_i,1;1}[Z] = \aleph^{m,n}_{p_i,q_i,1;1}\left[z \Big|^{(a_p,A_p)}_{(b_q,B_q)} \right] = \frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,1;1}(\xi) z^{-\xi} d\xi \tag{8}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;1}(\xi)$ can be obtained from (2).

The series representation of Aleph (ℵ)-function is given by [5]:

$$\aleph^{m',n'}_{p'_i,q'_i,\tau'_i;r'}[Z] = \aleph^{m',n'}_{p'_i,q'_i,\tau'_i;r'}\left[z \Big|^{(a'_j,A'_j)_{1,n'},[\tau'_i(a'_{ji},A'_{ji})]_{n'+1,p'_i;r'}}_{(b'_j,B'_j)_{1,m'},[\tau'_i(b'_{ji},B'_{ji})]_{m'+1,q'_i;r'}} \right] = \sum_{h=1}^{m'}\sum_{k'=0}^{\infty} \frac{(-1)^{k'}\phi'(s)}{B'_h k'!} z^{-s} \tag{9}$$

where

$$\phi'(s) = \frac{\prod_{j=1}^{m'} \Gamma(b'_j + B'_j s) \prod_{j=1}^{n'} \Gamma(1 - a'_j - A'_j s)}{\sum_{i=1}^{r'} \tau'_i \prod_{j=m'+1}^{q'_i} \Gamma(1 - b'_{ji} - B'_{ji} s) \prod_{j=n'+1}^{p'_i} \Gamma(a'_{ji} + A'_{ji} s)} \tag{10}$$

and

$$s = \eta_{h,k'} = \frac{b'_h + k'}{B'_h}, p'_i < q'_i, |z| < 1.$$

Also, the Fox-Wright's function [6] is defined as

$${}_{p'}\Psi_{q'}(z) = {}_{p'}\Psi_{q'}\left[{}^{(e_j,E_j)_{1,p'};}_{(f_j,F_j)_{1,q'};} z \right] = \sum_{l=0}^{\infty} \frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j l)}{\prod_{j=1}^{q'} \Gamma(f_j + F_j l)} \frac{z^l}{l!} \tag{11}$$

where $E_j (j = 1, ..., p')$ and $F_j (j = 1, ..., q')$ are real and positive and

$$1 + \sum_{j=1}^{q'} F_j - \sum_{j=1}^{p'} E_j > 0,$$

if we set, some suitable value to the parameters involved in (11) then its reduce into some useful functions as given in [7].

Where, $P_{\beta}^{(a,b)}(z)$ is the Jacobi polynomials ([8], p.68) and is given by

$$P_{\beta}^{(a,b)}(z)=\frac{\Gamma(a+\beta+1)}{\beta!\Gamma(a+b+\beta+1)}\sum_{n'=0}^{[\beta]}\binom{\beta}{n'}\frac{\Gamma(a+b+\beta+n'+1)}{\Gamma(a+n'+1)}\left(\frac{z-1}{2}\right)^{\beta} \tag{12}$$

for more hypergeometric series , see [9].

Plates are the flat structures whose thickness t is small compared to the other in- plane dimension is the radius ρ. Plate theories are classified in many ways. One of them is based on the thickness, that is, thin and thick-plate theories. Geometrically, a plate is said to be thin if its thickness ratio t/ρ is less than $1/20$, otherwise the plate is known to be thick. The bending properties of a plate depend mainly on its thickness as compared with its other dimensions. There are several theories for plates under large deflection; the most commonly used of them is the Von-Karman plate theory which is sometimes referred to as the Kirchoff-Foppel plate theory.

In the classical theory of plates, small deflection and elastic behavior of the material are assumed. When the lateral deflection exceeds one half the plate thickness [10], the classical theory generally is not adequate and the second order effects of the vertical displacements on the membrane stresses need to be considered. Two-coupled non-linear partial differential equations considering these effects were given by [11]. Solutions based on these differential equations have been known as large deflection solutions. Berger [12] in 1955 proposed an approximate method for investigating the large deflection of initially flat isotropic plates. Here the large deflection of a clamped circular plate under non-uniform load has been calculated by using Berger's approximate method. We consider the applied external pressure p(x) in the following form:

$$P(x)=K_0\left(1-\frac{x^2}{\rho^2}\right)^{\alpha}P_{\beta}^{a,b}\left(1-\frac{2x^2}{\rho^2}\right){}_{p'}\Psi_{q'}\left\{K_1\left(1-\frac{x^2}{\rho^2}\right)\right\}\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[K_2\left(1-\frac{x^2}{\rho^2}\right)\right]\aleph_{p'_i,q'_i,\tau'_i;r'}^{m',n'}\left[K_3\left(1-\frac{x^2}{\rho^2}\right)\right] \tag{13}$$

where K_0, K_1, K_2 and K_3 are constants. Recently, some research work based on modelling had done in [13].

2. Statement of the problem

Let us assume a clamped circular plate of thickness t, radius ρ and flexural rigidity R. Then by using Berger's method, the approximate equations for a circular plate undergoing large deflections due to an externally applied load p(x) may be given as

$$\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}\right)\left(\frac{d^2w}{dx^2}+\frac{1}{x}\frac{dw}{dx}-k^2w\right)=\frac{P}{R}=\phi(x) \tag{14}$$

where k is a normalized constant of integration given by the equation

$$\frac{dy}{dx}+\frac{y}{x}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2=\frac{k^2t^2}{12} \tag{15}$$

where w is the plate deflection, normal to the middle plane of the plate and y is the radial displacement.

The boundary condition of the problem are:

(i) $w=0=\frac{dw}{dx},\ at\ x=\rho$

(ii) $y=0,\ at\ x=\rho$

Solution of the problem

Let us consider

$$w=\sum_i G_i\left[J_0(xt_i)-J_0(\rho t_i)\right] \tag{16}$$

where t_i is the ith root of $J_1(\rho t_i)=0$.

It is clear that boundary conditions are satisfied by the above equation. Now using (16) in the Eq. (14), we find

$$\sum_i G_i t_i^2(k^2+t_i^2)J_0(xt_i)=\phi(x) \tag{17}$$

now expanding $\phi(x)$ in a series of Bessel's function, we obtain on integration

$$\int_0^{\rho} G_i t_i^2(k^2+t_i^2)J_0^2(xt_i)x\,dx=\int_0^{\rho}\phi(x)J_0(xt_i)\rho\,dx \tag{18}$$

now by left hand side of (18)

$$\int_0^{\rho} x J_0^2(x t_i)\,dx = \frac{\rho^2}{2} J_0^2(\rho t_i) \tag{19}$$

and (18) becomes

$$G_i t_i^2 (k^2 + t_i^2) \frac{\rho^2}{2} J_0^2(\rho t_i) = \int_0^{\rho} \phi(x) J_0(x t_i)\, x\,dx$$

or

$$G_i = \frac{2\int_0^{\rho} x\phi(x) J_0(x t_i)\,dx}{\rho^2 t_i^2 (k^2 + t_i^2) J_0^2(\rho t_i)}. \tag{20}$$

Now using [14], Eqs. (2), (9) and (10), the definition of Bessel function and interchanging the order of summations and integration, we find

$$\int_0^1 \theta^{2\lambda+1}(1-\theta^2)^{\alpha} P_{\beta}^{a,b}(1-2\theta^2)\,{}_{p'}\psi_{q'}[K_1(1-\theta^2)]\aleph^{m,n}_{p_i,q_i,\tau_i;r}[K_2(1-\theta^2)]\aleph^{m',n'}_{p'_i,q'_i,\tau'_i;r'}[K_3(1-\theta^2)]J_{\mu}(\theta\tau)\,d\theta$$
$$= \sum_{l=0}^{\infty}\sum_{n'=0}^{\beta}\sum_{n''=0}^{\infty}\sum_{h=1}^{m'}\sum_{k'=0}^{\infty} \frac{K_1^l K_3^{-s}(-1)^{k'+n''}(-\beta)_{n'}\phi'(s)\left(\frac{\tau}{2}\right)^{\mu+2n''}}{2 l! n'! n''! k'! B'_h}$$
$$\cdot \frac{\prod_{j=1}^{p'} \Gamma(e_j+E_j l)\Gamma(1+a+\beta)(1+a+b+\beta)_{n'}\Gamma\left(\lambda+n'+n''+\frac{\mu}{2}+1\right)}{\prod_{j=1}^{q'} \Gamma(f_j+F_j n)\Gamma(1+a+n')\Gamma(1+\mu+n'')}$$
$$.\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[K_2 \left|{}^{(-\alpha-l+s,1),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(-1-\lambda-l-n'-n''-\alpha+s-\frac{\mu}{2},1)}\right.\right] \tag{21}$$

where

$$Re(a) > -1, Re(b) > -1, Re(\lambda) > -1, Re(\alpha) > -1,$$

$$Re(\mu) > -\frac{1}{2}, Re(\alpha + \frac{b_j}{B_j}) > 0, (j = 1,...,q_i;\ here\ i = 1,...,r)$$

and

$$Re(\alpha + \frac{b'_j}{B'_j}) > 0, (j = 1,...,q'_i;\ here\ i = 1,...,r')$$

.

Using (21) in view of (13) and (14), we get

$$G_i = \frac{K_0\Gamma(1+a+\beta)}{R t_i^2 \beta!(k^2+t_i^2)J_0^2(\rho t_i)} \sum_{l=0}^{\infty}\sum_{n'=0}^{\beta}\sum_{n''=0}^{\infty}\sum_{h=1}^{m'}\sum_{k'=0}^{\infty} \frac{K_1^l K_3^{-s}(-1)^{k'+n''}(-\beta)_{n'}}{l! n'! n''! k'! B'_h}\phi'(s)$$
$$\cdot \frac{\prod_{j=1}^{p'} \Gamma(e_j+E_j l)\Gamma(1+n'+n'')(1+a+b+\beta)_{n'}}{\prod_{j=1}^{q'} \Gamma(f_j+F_j l)\Gamma(1+a+n')\Gamma(1+n'')}$$
$$.\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[K_2 \left|{}^{(-\alpha-l+s,1),(a_j,A_j)_{l,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{l,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(-1-l-n'-n''-\alpha+s,1)}\right.\right]. \tag{22}$$

Now combining the Eqs. (16) and (22), we get

$$w = L_1 \sum_i \frac{L_2}{k^2 + t_i^2} [J_0(x t_i) - J_0(\rho t_i)] \tag{23}$$

where

$$L_1 = \frac{K_0 \Gamma(1 + a + \beta)}{R \beta!}$$

and

$$L_2 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{l=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{n''=0}^{\infty} \sum_{h=1}^{m'} \sum_{k'=0}^{\infty} \frac{K_1^l K_3^{-s} (-1)^{k'+n''} (-\beta)_{n'}}{l! n'! n''! k'! B'_h} \phi'(s)$$
$$\cdot \frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j l) \Gamma(1 + n' + n'')(1 + a + b + \beta)_{n'}}{\prod_{j=1}^{q'} \Gamma(f_j + F_j l) \Gamma(1 + a + n') \Gamma(1 + n'')}$$
$$\cdot \aleph^{m,n+1}_{p_i+1, q_i+1, \tau_i; r} [K_2 |^{(-\alpha-l+s,1),(a_j,A_j)_{l,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{l,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(-1-l-n'-n''-\alpha+s,1)}].$$

Now the radial displacement y can be obtained by using Eqs. (15) and (16) as

$$y = \frac{k^2 t^2 x}{24} - \frac{1}{2} \sum_{i=1}^{\infty} G_i^2 t_i^2 \left[\frac{x}{2} \left\{ J_1'^2(x t_i) + \left(1 - \frac{1}{x^2 t_i^2} \right) J_1^2(x t_i) \right\} \right]$$
$$- \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} G_i G_j t_i t_j \left[\frac{t_i J_2(x t_i) J_1(x t_j) - t_j J_2(x t_j) J_1(x t_i)}{t_i^2 - t_j^2} \right] + C_1, i \neq j \tag{24}$$

where C_1 is the constant of integration.
Applying the boundary condition y =0 at $x = \rho$ and $J_1(\rho t_i) = 0$, we get

$$C_1 = -\frac{k^2 t^2 \rho}{24} + \frac{1}{4} \sum_{i=1}^{\infty} G_i^2 t_i^2 \rho J_1'^2(\rho t_i) \tag{25}$$

hence the radial displacement y is established as

$$y = \frac{k^2 t^2 (x - \rho)}{24} - \frac{1}{2} \sum_{i=1}^{\infty} G_i^2 t_i^2 \left[\frac{x}{2} \left\{ J_1'^2(x t_i) + \left(1 - \frac{1}{x^2 t_i^2} \right) J_1^2(x t_i) \right\} \right]$$
$$- \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} G_i G_j t_i t_j \left[\frac{t_i J_2(x t_i) J_1(x t_j) - t_j J_2(x t_j) J_1(x t_i)}{t_i^2 - t_j^2} \right] + \frac{1}{4} \sum_{i=1}^{\infty} G_i^2 t_i^2 \rho J_0^2(\rho t_i).$$

3. Applications

(3.A) The deflection given by Eq. (23) can be used to evaluate the boundary stresses at the surface of the plate which for the circular plate, are given by [12] as

$$\sigma_x = \frac{-6R}{t^2} \left(\frac{d^2 w}{dx^2} + \frac{v}{x} \frac{dw}{dx} \right) \tag{26}$$

and

$$\sigma_\theta = \frac{-6R}{t^2} \left(v \frac{d^2 w}{dx^2} + \frac{1}{x} \frac{dw}{dx} \right) \tag{27}$$

where v is the Poisson's ratio.

By using (23), we get

$$\sigma_x = \frac{-6R}{t^2} L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} \left[J_0''(xt_i) + \frac{\nu}{x} J_0'(xt_i) \right] \tag{28}$$

and

$$\sigma_\theta = \frac{-6R}{t^2} L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} \left[\nu J_0''(xt_i) + \frac{1}{x} J_0'(xt_i) \right] \tag{29}$$

Now, putting x = 0 in (28) and (29), we get the bending stresses at the centre of the plate as

$$(\sigma_x)_{x=0} = (\sigma_\theta)_{x=0} = \frac{3R}{t^2} L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} (\nu+1) t_i^2 \tag{30}$$

also by putting $x = \rho$, the bending stresses at the edge of the plate are obtained as

$$(\sigma_x)_{x=\rho} = \frac{6R}{t^2} L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} t_i^2 J_0(\rho t_i) \tag{31}$$

and

$$(\sigma_\theta)_{x=\rho} = \frac{6R}{t^2} L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} \nu t_i^2 J_0(\rho t_i) \tag{32}$$

(3.B) When k = 0, the differential Eq. (14) corresponds to that of small deflection equation and then Eq. (23) leads to

$$w = L_1 \sum_i \frac{L_2}{t_i^2} [J_0(xt_i) - J_0(\rho t_i)] \tag{33}$$

(3.C) By using x = 0, we obtain the deflection w_0 at the centre of the plate as

$$w_0 = L_1 \sum_i \frac{L_2}{(k^2+t_i^2)} [1 - J_0(\rho t_i)] \tag{34}$$

whereas the small deflection will be given by

$$w_0 = L_1 \sum_i \frac{L_2}{t_i^2} [1 - J_0(\rho t_i)] \tag{35}$$

4. Special cases

(i) By taking $\tau_i = 1\ \forall\ i = 1,...,r$ and $\tau_i' = 1\ \forall\ i = 1,...,r'$ for $\aleph^{m,n}_{p_i,q_i,\tau_i;r}$ and $\aleph^{m',n'}_{p_i',q_i',\tau_i';r'}$ in the load p(x), both the Aleph-functions reduces to the I-function [3]. Then we obtain the deflection as

$$w = D_1 \sum_i \frac{D_2}{(k^2+t_i^2)} [J_0(xt_i) - J_0(\rho t_i)] \tag{36}$$

where

$$D_1 = \frac{K_0 \Gamma(1+a+\beta)}{R\,\beta!}$$

and

$$D_2 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{l=0}^{\infty} \sum_{n'=0}^{\beta} \sum_{n''=0}^{\infty} \sum_{h=1}^{m'} \sum_{k'=0}^{\infty} \frac{K_1^l K_3^{-s} (-1)^{k'+n''} (-\beta)_n'}{l!n'!n''!k'!B_h'} \phi'(s') (\frac{\rho t_i}{2})^{2n''}$$
$$.\frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j l) \Gamma(1+n'+n'') (1+a+b+\beta)_{n'}}{\prod_{j=1}^{q'} \Gamma(f_j + F_j l) \Gamma(1+a+n') \Gamma(1+n'')}$$
$$.I^{m,n+1}_{p_i+1,q_i+1,1;r} [K_2 \,|^{(-\alpha-l+s',1),(a_j,A_j)_{l,n},(a_{ji},A_{ji})_{n+1,p_i;r}}_{(b_j,B_j)_{l,m},(b_{ji},B_{ji})_{m+1,q_i;r},(-1-l-n'-n''-\alpha+s',1)}]$$

whereas we get the small deflection as

$$w = D_1 \sum_i \frac{D_2}{t_i^2} [J_0(xt_i) - J_0(\rho t_i)] \tag{37}$$

in this case, the deflection at the centre of the plate is given by

$$w_0 = D_1 \sum_i \frac{D_2}{(k^2 + t_i^2)} [1 - J_0(\rho t_i)]. \tag{38}$$

(ii) By taking $\tau_i = 1 \ \forall \ i = 1,...,r,\ set\ r = 1$ and $\tau'_i = 1 \ \forall \ i = 1,...,r'\ set\ r' = 1$ for $\aleph^{m,n}_{p_i,q_i,\tau_i;r}$ and $\aleph^{m',n'}_{p'_i,q'_i,\tau'_i;r'}$ in the load p(x), both the Aleph-functions reduces to the familiar H-function [4]. Then we reduces to known result obtained by Chaurasia and Arya [15].

Inview of the generality of the $\aleph$-function, on specializing the various parameters, we can obtain from our results, several results involving a remarkably wide variety of useful functions, which are expressible in terms of the Mittag-Leffler function ([4], p.25, Eq. (1.137)), the generalized Wright hypergeometric function ([4], p.25,Eq. (1.140)), the generalized Bessel-Maitland function ([4], p.25, Eq.(1.139)) and their various special cases. Thus, the results presented in this paper would at once yield a very large number of results involving a large variety of special functions occurring in the problems of science, engineering and mathematical physics etc.

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